Design and Analysis of Experiments
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Lecture - 29
Randomized Complete Block Design (RCBD): Graeco – Latin Square Design

Welcome we will discuss now Graeco Latin square.

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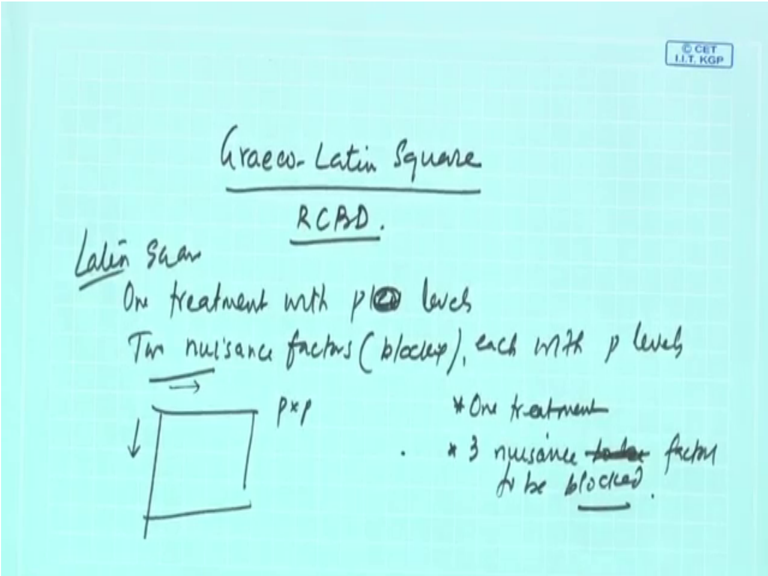

Latin square this is a special type of design in randomized complete block design.

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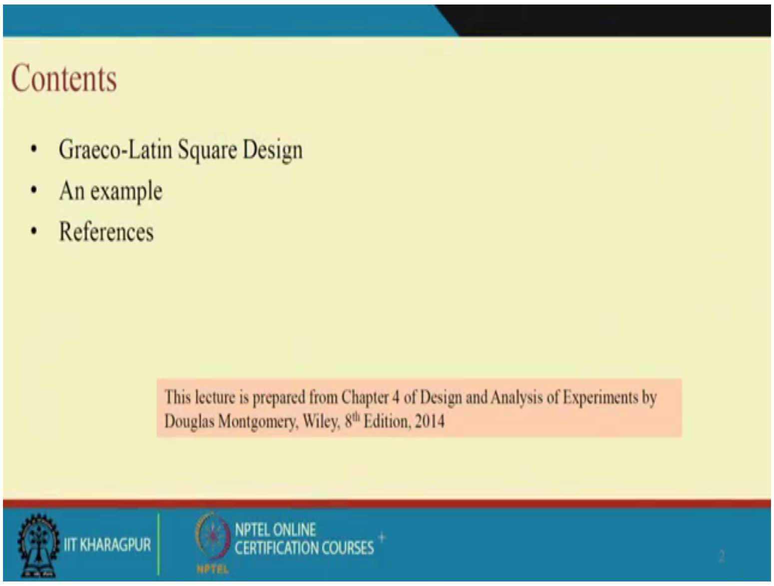

Let us see that what are the topics we will cover today first we will discuss what is Graeco Latin square design and then it is statistical analysis and I will give an example some kind of tutorial here and references your text book where from I have taken the material is Montgomery is design and analysis of experiments.

Then 2 nuisance factor factors or we are saying blocking factors each with P levels that is that is what you have seen in Latin square and then you made a square P cross P square where row represent 1 blocking factor columns represent another blocking factors factor with levels and the each of the individual cell will be assigned to individual that the treatments.

Now, suppose you want to block another factor you have one treatment and 3 nuisance to be 3 nuisance factors to be blocked how can you do it that will be done through Graeco Latin square design and that this resultant design is Graeco Latin square design.

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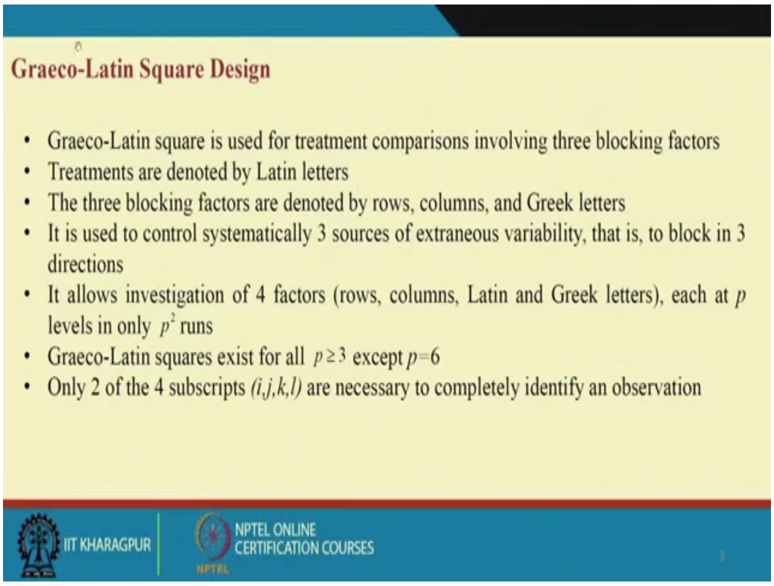

Let us read some of the sentences here which will give you more realisation about Graeco Latin square Graeco Latin square is used for treatment comparisons involving 3 blocking factors treatments are denoted by Latin letters ABCD etcetera 3 blocking factors are denoted by row block rows columns and Greek letters. We will use Greek letters to represent 1 blocking factor.

It is used to control systematically 3 sources of extraneous variability that is to block in 3 directions it allows investigation of 4 factors rows column Latin and Greek letters each at P levels in only P square runs Graeco Latin square exist for all P greater than 3 levels except P equal to 6 only 2 of the 4 subscript y i j k l will be the general observation are necessary to completely identify an observations.

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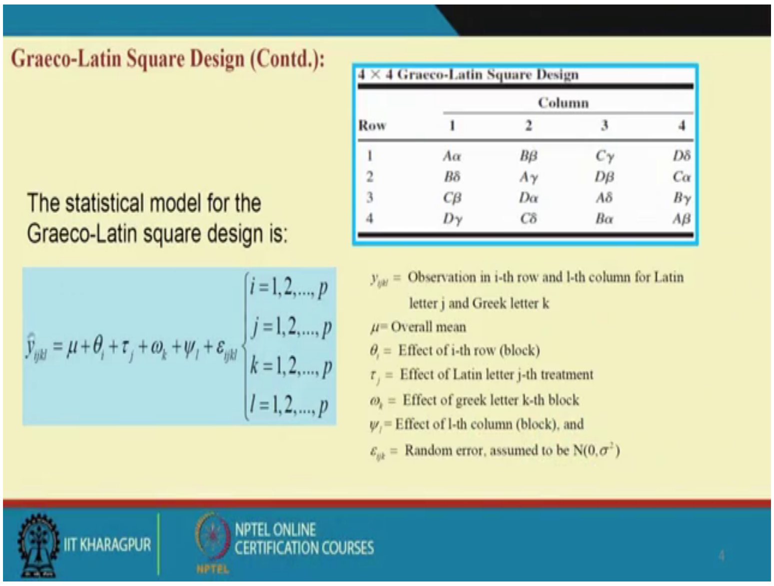

Let us see how do what is there then that means.

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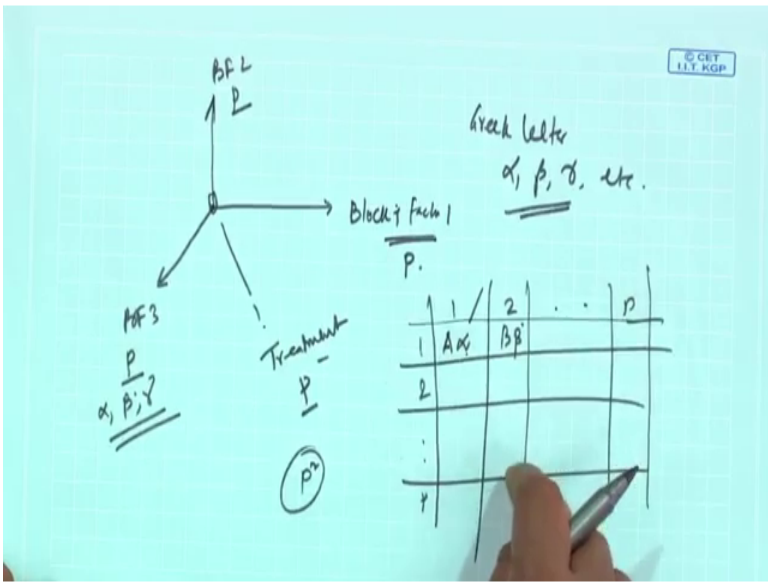

I have 3 blocking factors nuisance factor 1 or I can say blocking factor 1 blocking factors 2 blocking factor 3 and in addition you have another factor which is known as treatments.

If treatment has P levels you take for first blocking factor P P levels this also P levels this also P levels and then your layout will be here row 1 2 P column 1 2 P and each row and column here you will first assign suppose treatments a suppose here treatment B, like this you will treatment.

Now, you will introduced another letter called Greek letters Greek letters like alpha beta gamma etcetera which will be suppose if I mean use here A alpha this or here B beta something like this. Then this alpha beta they are also they are basically for the third blocking factors alpha beta gamma.

In P cross P P square observations of the lessons you are able to include the effect of 1 treatment and 3 nuisance factors that is a special class of design and Graeco Latin square design what will be the general observation then general observation will be Y I J K l these will be divided into different sources or attributed to different sources of variability like 1 is the grand mean fine then this is the row effect then this is the treatment effect then this is your column effect and then what will happen that another block.

I J K l I stands and J K l all basically vary from 1 to P, if I say what is I J K del then the reading is like this observation in ith row and jth lth column for Latin letter j and Greek letter K your Latin letter j jth Latin letter whatever may be there that will basically represent the treatment ith letter lth column and Greek letter K that represent the blocking factors.

Here MU is over mean theta is effect of ith row TAU j is the effect of Latin letter jth treatment omega K is effect of Greek letter kth block and psi l is effect of lth column block and epsilon I J K l epsilon I J K l this will be random error.

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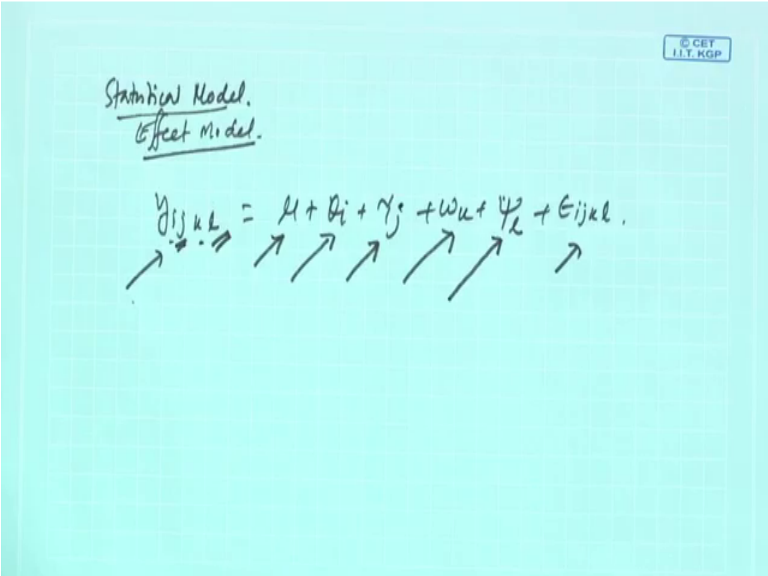

What is happening I am repeating this, what is your statistical model then for here it is basically effect model effect model for Latin square, here Y I J K l this will be MU plus your theta I TAU J plus omega K plus psi l plus epsilon I J K l.

General observation on this ith row kth column jth jth Latin letter lth Greek letter this is overall grand mean this is the row blocking factor treatment column blocking and then this one is Greek letter blocking and this is the error and the design is like this you see here 4 cross 4 cross 4 design in the slide, we will show the 4 cross 4 design and you see that row 1 to 4 and column 1 to 4 here ABCD these are the treatments row 1 2 3 4 this is the blocking factor 1 column 1 2 3 4 level for blocking factor 2 alpha beta gamma delta this is the Greek letters used for blocking factor 3, you will be getting this kind of design and please remember you may have to maintain the orthogonality.

Here what happened same thing what we have done in Latin square in Graeco Latin square also we will find out the all sources of variability they are sum square they are degrees of freedom then we will create the F statistics for the treatment and we will compare with the where the where the threshold 1 from the table and if the if the treatment a value computed from the data is more than the theoretical 1 we will say that the treatment effect is there otherwise.

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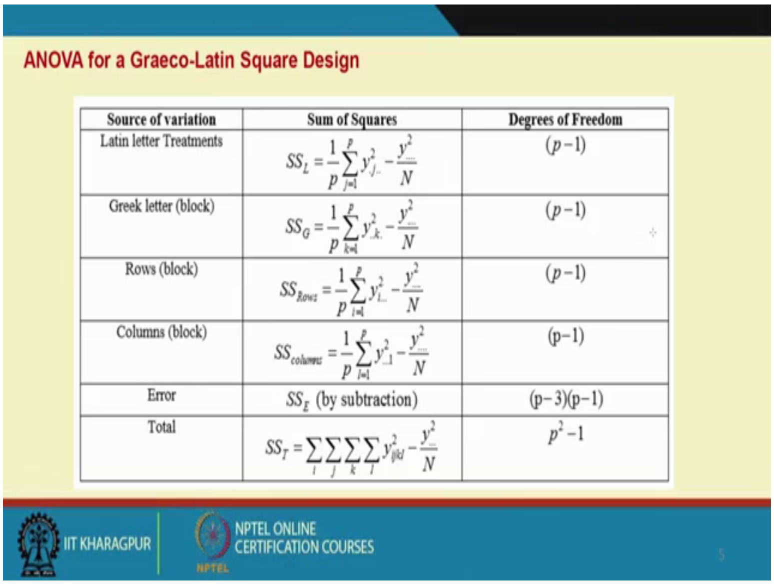

We will reject the null accept the null hypothesis that treatment effect is not there, here the calculation part. 1 is Latin letter treatment J from 1 to P other things as it is earlier 1 only you have 4 dots because your control you are basically estimating in 4 directions controlling 4 directions ABCD not from the and the treatment point of view 4 different factors point of view.

Row column treatment and Latin letters and Greek letters, Greek letter this is K changing rows I columns l. That mean what you are saying row I column l in between J and K J stand for the Latin letters for the treatment K for the Greek letters that is the blocks and then everywhere the degrees of freedom for treatment will be P minus 1 block row Greek letter block P minus 1 row block P minus 1 column block P minus 1 because your total P square observation and then by subtraction you will get P minus 3 into P minus 1 this is what is the error degrees of freedom.

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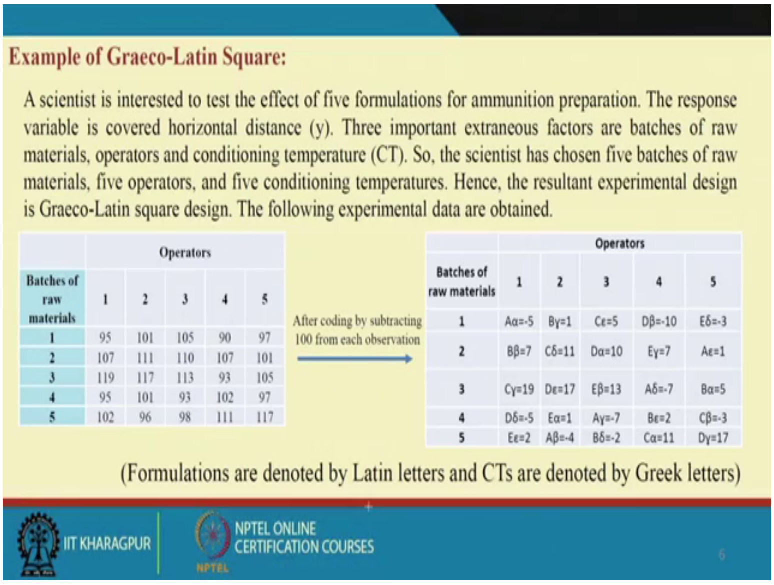

And SS T will be computed using this, with one tutorial we will see it very nicely. Let us see the data the data is a scientist interested to test the effect of 5 formulations for ammunition preparation ammunition you know that in case of rocket launching that ammunition this is this is what is used.

The response variable is covered horizontal distance. When you launch a rocket it will move certain along certain projectile, but it has a horizontal distance that it will cover. These 3 important extraneous factors are batches of raw materials operator and conditioning temperature. The scientist as chosen 5 batches of raw materials because here are 5 formulations 5 batches of raw materials 5 operators 5 conditioning temperature. That mean treatment is formulations 5 formulation blocking factor 1 is raw material 5 batches.

Second blocking that are operators 5 operators conditioning temperature is the third blocking factors with 5 levels hence the resultant experimental design is Graeco Latin square design the following experimental data suppose it is obtained please note that these data is not collected a based on certain experimentation it is it is a hypothetical one all the data set what I am showing most of the data are hypothetical in nature means we conceptualize the problem and then we put some amount of data, but in reality when you do experiment you will actually get the response variable data.

Now, we have supposed what happen 5 by 5. This is what is the data actually this is what is here what happen when I subtract by hundred then you are getting like this, but here actually we have given the dot actual design.

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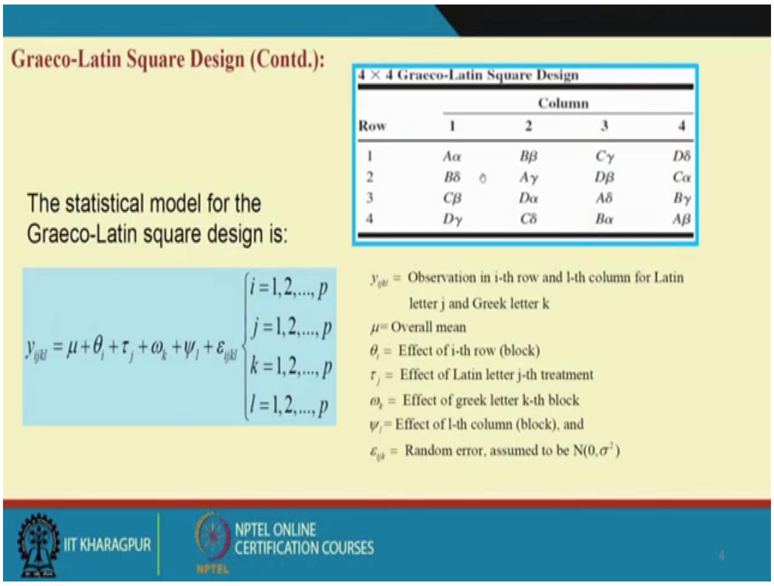

You see this design versus this design here A alpha B beta C gamma. This is what is the design.

Now, using the data that design with 5 with 5 cross 5 that matrix A alpha beta gamma gamma epsilon gamma alpha gamma epsilon beta delta. Here is alpha beta gamma delta epsilon these are the Greek letters and they are basically coming once in every rows or every column and similarly to the like the treatment also coming once in every treatment coming once in every rows and every column.

That mean this minus 5 or s 95 this is what is the distance travelled by the rocket horizontal distance and this one is result of when you are used formulation A and you are raw material batch one operator one and alpha is basically conditioning temperature 1.

Similarly here it is 2 second raw material first operator and conditioning temperature 2. In this manner you are basically doing it. This is what is the data. Now you have to analyse in order to calculate the STS what you require.

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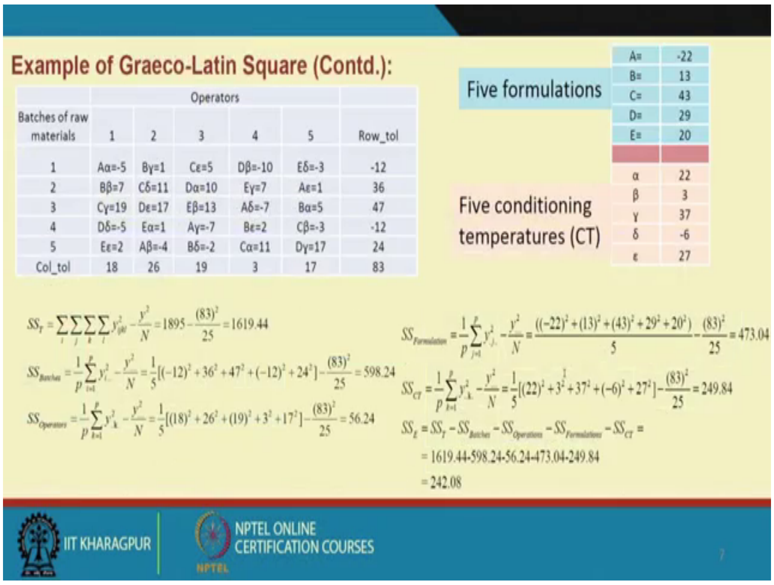

First is your total sum square total will be you square all those values 5 square 7 square like this all those values you square 1 to 5 25 values then take the sum and subtract it by the total square by total square eighty 3 divided by the number of observation these then you are getting the total variability 16 19.4 then for batches of raw material which is represented by rows you take the row total you take the row total and here row total square it and sum it take the average of this and then subtract by this, this, this 1 that 83 square by 25 you are getting s batches 0.590 this.

Similarly, for the column if you do you will get s operators. 2 more data sets are not directly visible from here the total for example, the formulation ABCD as I told you in the Latin square you find out a attached with which value in this row data in this data set.

Whenever a is there here a minus 5 and then here minus 4 and here minus 7, like this all those things you add similarly you add for B you will be getting total for a total for B total for C like this and in the same manner you find out for alpha beta gamma dell epsilon delta all those things you will be getting total of this.

Then what is the s formulation all those squares there addition divided by the number of observation 5 and then subtracted by the 83 square by 25 you are getting s formulation is 4 73.0 4 similarly S conditioning temperature is 2 49.8 4 and S error h 2 42.0 8.

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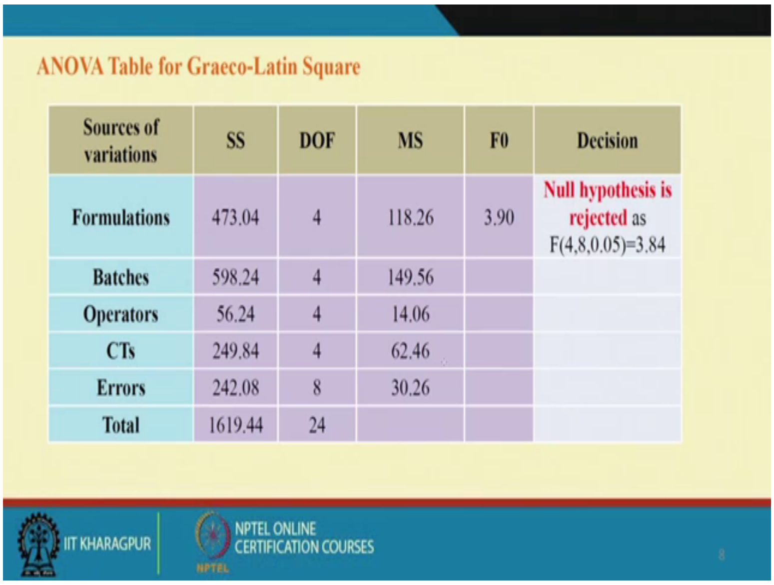

Sources of variations	SS	DOF	MS	F0	Decision
Formulations	473.04	4	118.26	3.90	Null hypothesis is rejected as F(4,8,0.05)=3.84
Batches	598.24	4	149.56		
Operators	56.24	4	14.06		
CTs	249.84	4	62.46		
Errors	242.08	8	30.26		
Total	1619.44	24			

We computed STS error now you prepare the ANOVA table ANOVA table S formulation S batches S operators S conditioning temperature S errors S total and degree of freedom are given then find out the M S and then find out F 0 for formulation it is hundred 18.26 by errors 30.26 which is 3.9 and this is this quantity the hypothetical theoretical value is a 4 8 0.05 3.84.That 3.8 4 is less than 3.90. Null hypothesis rejected it means that the formulation has effect on the response variable horizontal variable.

Far what we have discussed we have in RCBD we have discussed RCBD.

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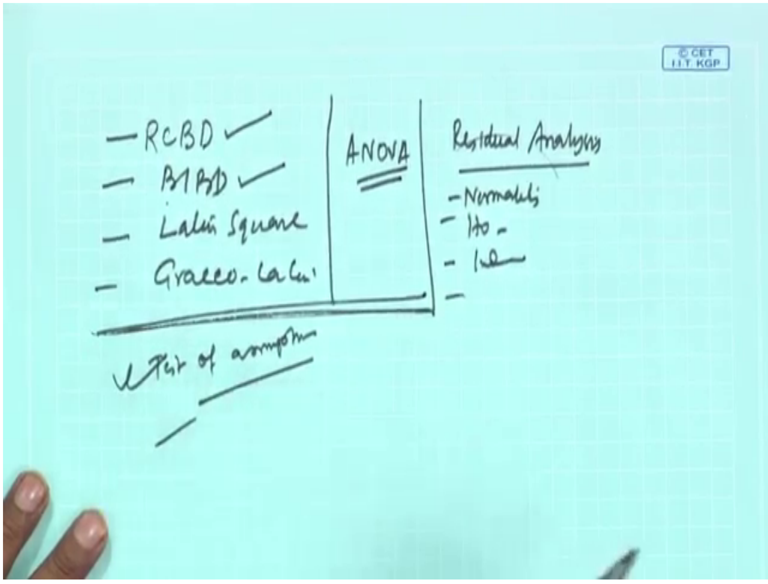

Then we have discussed B I B D this is randomized complete block design balanced incomplete block design then we have discussed Latin square design then we have discussed Graeco Latin square design Latin square design and depending and all those cases we have shown how ANOVA will be model will be used and whether the effect is there or not.

Please keep in mind that you require to do residual analysis residual analysis for test of all the assumption related to ANOVA like normality error terms that homo homoscedasticity then your independence observations all those things you have to you have to test through residual plots like and you have seen residual plots in 1 way ANOVA case completely we have described, but here also whenever if you even if you use some software please keep in mind the residual for test of assumptions test of assumptions all those things are very valid very important thing you have to do it.

Another one is that if formulations are different or if the treatments are different general case. Then what will happen which pairs are different or not (Refer Time: 19:52) all those things we have shown earlier. Similar kind of things also you required to do here in case of randomized complete block design and it is different variations.

So, I hope that you will be able to do this kind of experiment in life you have to able to analyse using ANOVA you now know that how regression approach is used to estimate the parameters that also you will be able to do a do it.

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From the from the textbook point of view that I have consulted these 2 books and some other materials from here and there ah, but primarily the first book which is our main book for this particular subject D O E hope you will do well in both in assignment and in exam related to randomized complete block design or incomplete block design or the special class of block designs.

Thank you very much.