Design and Analysis of Experiments
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Lecture - 27
Randomized Complete Block Design (RCBD): Balanced Incomplete of Block Design (BIBD)

Welcome today we will discussed balanced incomplete block design BIBD it is a special case for Randomized Complete Block Design.

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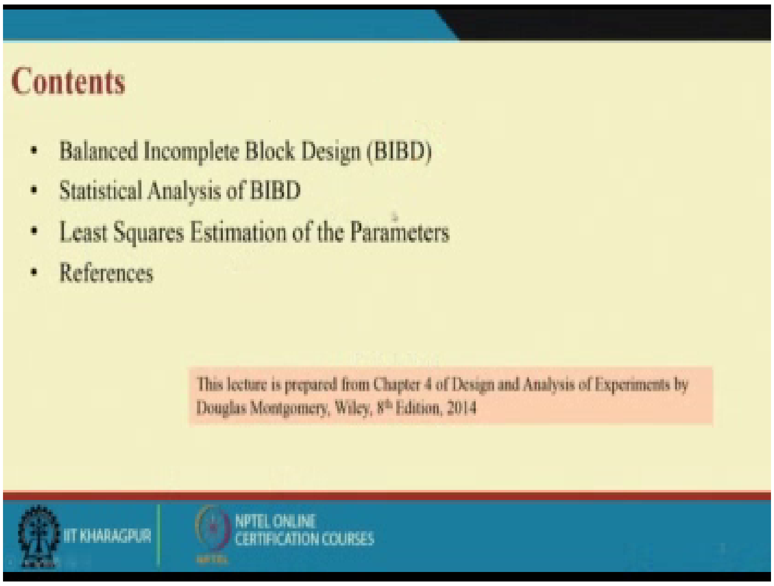

Now first what we see that what is BIBD balanced incomplete block design, then how the statistical analysis of BIBD will be done, then we will see the least square estimation of the parameters and some references primarily the materials taken from this book design and analysis of experiments by Douglas Montgomery.

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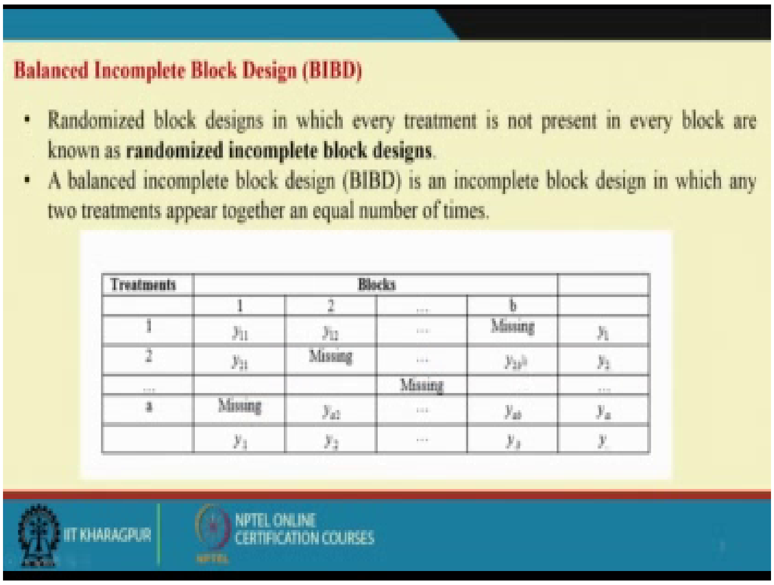

So, what is incomplete block design? Randomized block designs in which every treatment is not present in every block are known as randomized incomplete block design. And a balanced incomplete block design BIBD is an incomplete block design in which any 2 treatments appeared together an equal number of times.

So, you see this table in this table there are a treatments b blocks, but what it is seen that in every blocks some of the treatments are missing in the sense experiment for that treatment was not done.

So, in column 1 some missing column 2 some missing similarly in column b some missing. So, it is primarily because of the may be unavailability of raw materials if raw material is blocked that the that batches of raw material will not be able to accommodate all the treatments or maybe the if the operator is blocked then operator may not be available for all the treatments.

So, under such situation incomplete block design is made and it will be it will be balanced if that equal number of the total is any 2 pair of treatments will occur equal number of times in each block.

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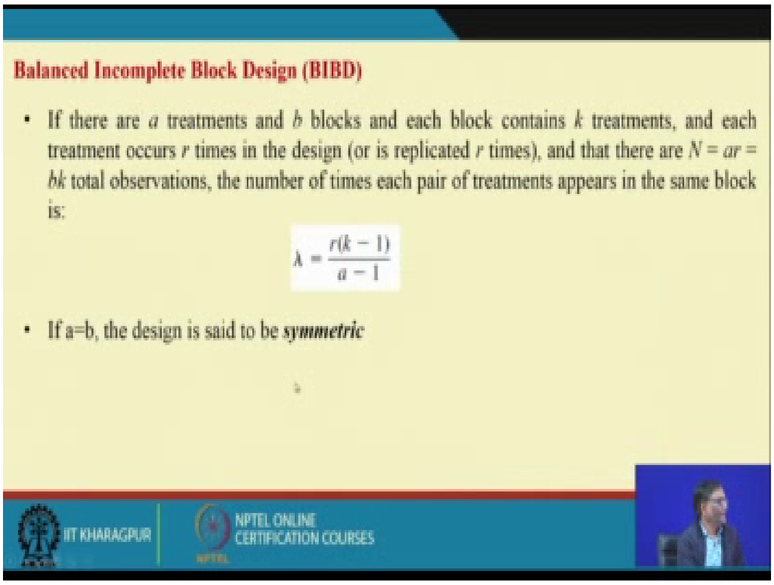

So, general case is that suppose there are a treatments and b blocks and in each block contain k treatments and each treatment occurs r time in the design then there are N equal to a r equal to b k total observations and it is obvious, if you see the this slide you see the table what happen although there are a number of rows because a number of treatment are there, but some treatments are missing

Hence there are k number of treatments in each block and if you see that the number of total number of blocks consider under each statement that it also not b it is something less than b depending on the number missing suppose that k number of a r number of blocks are used for treatment 1 lambda, which basically the number of times each pair of treatment appears in the same block.

Suppose block number one. So, number of times each pair may be 1 2 1 3 2 2 2 3 2 4 1 like this appears that the what is that number that is lambda and we will find out that the simple mathematics that lambda will be r k minus 1 by a minus 1.

So, then what is the quantity lambda number of times each pair of treatments appear lambda equal to number of times each pair of treatments appear in appear in the same block.

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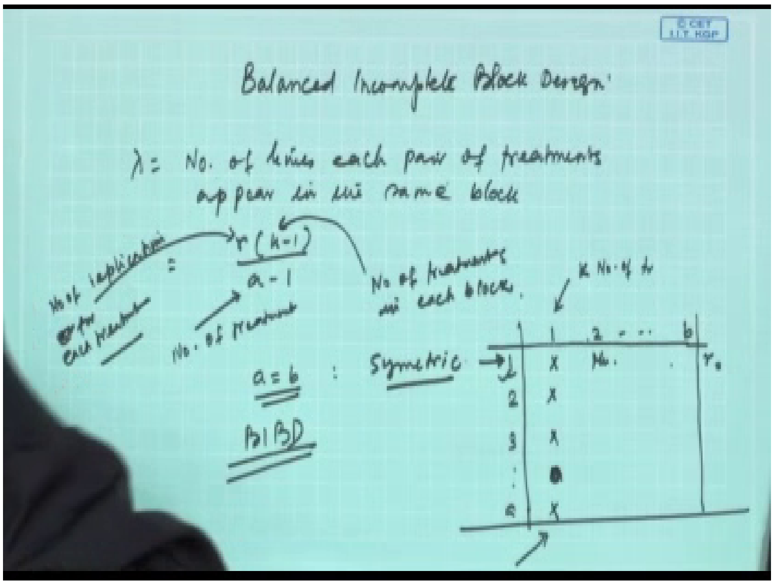

This will be r k minus 1 by a minus 1 where a is the number of treatments k is the number of treatments treatments in each block and r is the number of replications against this treatment for each treatment for each treatment.

For example when a equal to b this design is known as symmetric. So, let us understand then that what is the layout for balanced incomplete block design, it will be something like this there will be 1 2 3 like a number of treatments there will be 1 2 like b number of blocks. So, there will be data, but there may be some missing points in detail here.

So, you may what will happen in each block there will be k number of actual treatments k number of treatments against like you know if this may be missing. So, what will happen against each treatment there will be r number of blocks will be used. So, r will be the replication per treatment, in addition what will happen it will be balance because every 2 treatment that will occur in a same block is same number of times.

So, this condition is known as BIBD balanced incomplete block design repeat a balance incomplete block design is an incomplete block design in which any 2 treatments appear together an equal number of times votes.

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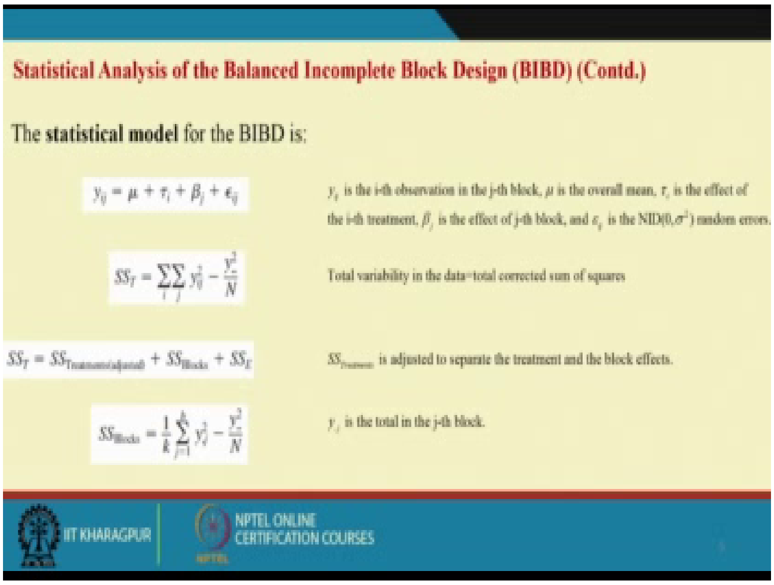

So, what will be the statistical model the statistical model remain same as RCBD in RCBD what you have seen you have seen y i j equal to mu plus tau i plus beta j plus epsilon i j when i equal to 1 2 a j equal to 1 2 b.

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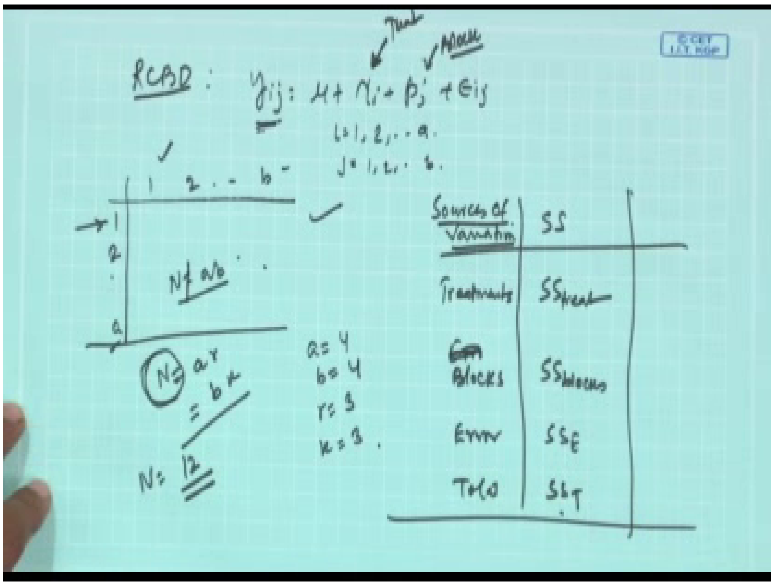

So, this the treatment effect and then this is your block, but you will not found what will happen you will see that you do not have a and this and if i say b N equal a b number of observations you do not have this number of observations this will not happen, reason it is incomplete and it is complete. So, as a result N will be either a and each treatment having r replication a r or each block having k treatments. So, b k there is a difference.

So, difference is in number of treatments total number of observations. So, difference is there will be a number of total treatments there will be b number of total blocks, but the number of total observations will be a r or b k if a equal to 4 and b b equal to suppose 4 and suppose r equal to 3 and also k equal to 3 let it be like this then what will happen N will be 12

So, accordingly what will happen your calculation for S S T, S S T, S S treatments S S error all will be different? So, now, what are the sources of variations, in this case also sources of variation this is what is the starting point in analysis sources of variation; obviously, treatment then error sorry block then error then that this will this ultimately will it to total.

And when we calculate S S what you do S S treatment S S block S S error and S S total. So, we will see that how these S S all those things are calculated. Go to slide when you see that S S t is sum total of y i j square i and j changing minus y double dot square by a that is what is the formula earlier you have used for RCBD.

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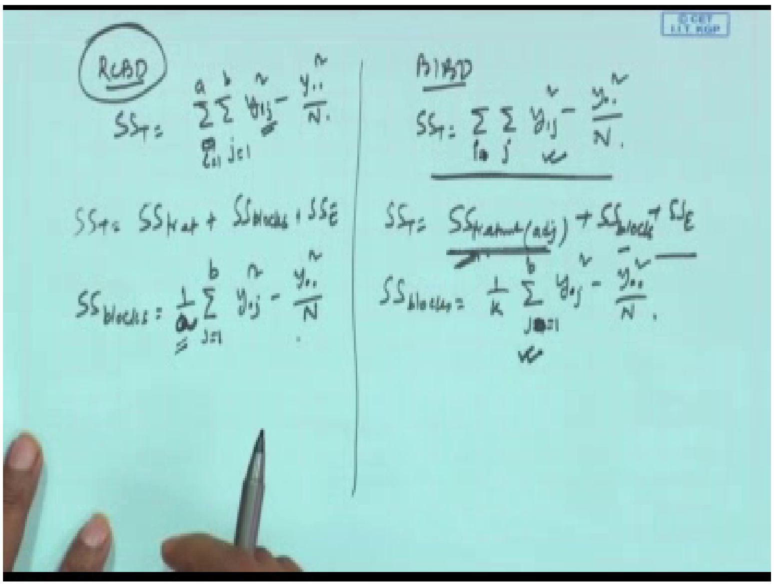

What is the formula we are used S S T equal to double sum i equal to i and j and here we have write i equal to 1 to a j equal to 1 to b y i j square minus y dot dot square by N.

So, here what happened you will not get y i j for all a b combinations there will be some combinations were missing value will be there that will not be counted. So, that is why i for BIBD we will just to avoid this we will write S S T equal to sum i and j we are not putting 1 to b or 1 to a whether i am we are writing like this minus y dot dot square by n. So, here N equal to a r or b k and y dot dot dot dot is the total and here whatever y i j available the all y i j square this will give you your S S T.

Now, in order to calculate and here in RCBD we have written S S T equal to S S treatment plus S S blocks plus S S error here we will write S S T equal to S S treatment and that treatment will be adjusted because there are missing values. So, plus S S blocks whatever we got plus S S E S S block is not adjusted because we are interested in we want to block and we adjust S S treatment and so that we will get the actual contribution of treatment here.

So, suppose here if you calculate S S block what you do you write down j equal to 1 to b y dot j square minus y double dot j square by N this is RCBD randomized complete block design here it is incomplete block design this you write and then 1 by 1 by a 1 by a this is the formula.

So, here the formula will be S S blocks formula will be see there is not a treatments against each block against each block there will be k treatment. So, 1 by k sum total j equal to j equal to 1 to b write down, but there will be some missing value that will be excluded. So, let me write like this j then y dot j square minus y double dot square by n

So, as it is y dot j; obviously, you will be getting j equal to b the total will get. Let me go back you see even the missing value is here, but y dot 1 this is computed because rest will be totalled y 2 like this. So, that mean you will getting 1 to b. So, here 1 to b, but in this calculation you are comparing all the you are basically taking all the values all y 1 2 1 missing values you are excluding. So, we are not writing that word to word it is basically available ok.

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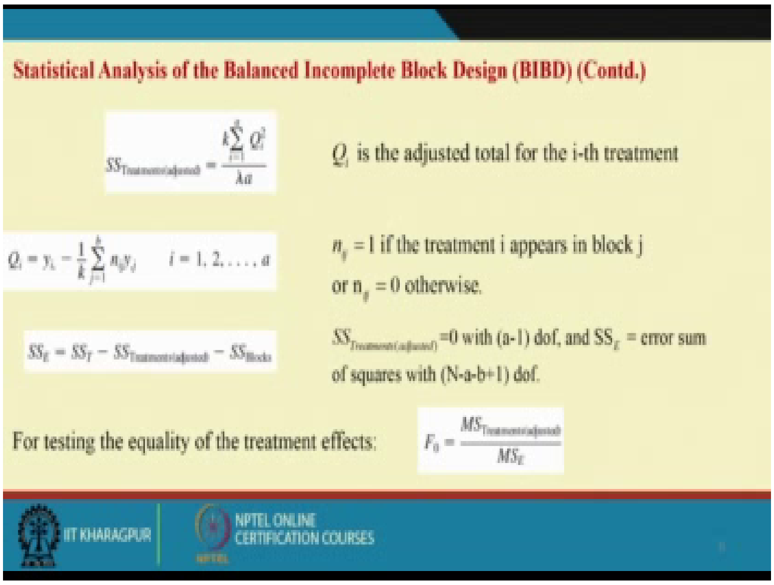

Now, how do you calculate the S S treatment adjusted? So, in order to compute S S treatment adjusted we will create a quantity called Q i.

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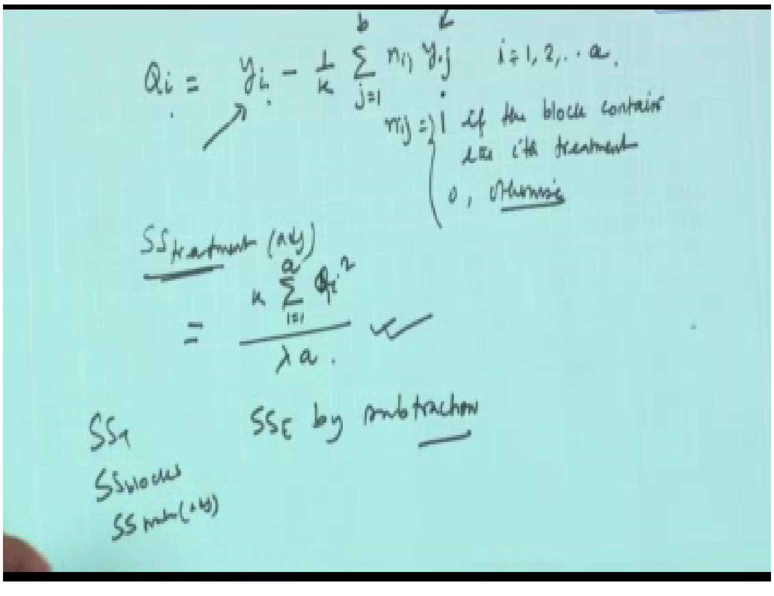

So Q i will be y i dot minus 1 by k j equal to 1 to b in i j y dot j i equal to 1 to a here n i j this will be 1 with the block contains the treatment contain the ith treatment otherwise it is 0 otherwise.

So, this is your row total this is your column total and you are creating y i j Q i using this formula. And then what we will do we will write down S S treatment adjusted this equal to k times sum total of Q i square Q i square i equal to 1 to a by lambda into a. So, S S treatment adjusted will be computed using this then S S total is known S S block is known blocks known S S treatment adjusted is known. So, S S error will be by subtraction by subtraction you can do calculate by subtraction.

Now, we will see the anova table for this let us see.

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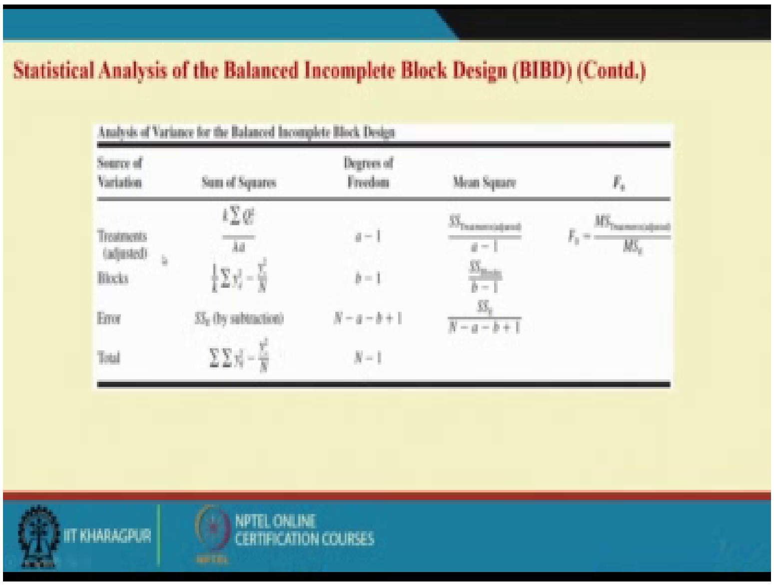

So, sources of variation treatment adjusted blocks error and total sum of square, but S S treatment is k times Q i square by lambda a degree of freedom is a minus 1 blocks 1 by k sum of dot j square minus y 1 dot dot by N degree of freedom is b minus 1, then S S E by subtraction because you know the S S T is sum of y i j square minus y dot dot y N and it is degree of freedom is N minus 1.

So, similarly N minus 1 minus a minus 1 minus b minus 1 that will give you that the error degree of freedom N minus a minus b plus 1 here N is not a b. So, if you recall if you recall incomplete block design the error degrees of freedom N minus 1 minus a minus 1 minus b minus 1.

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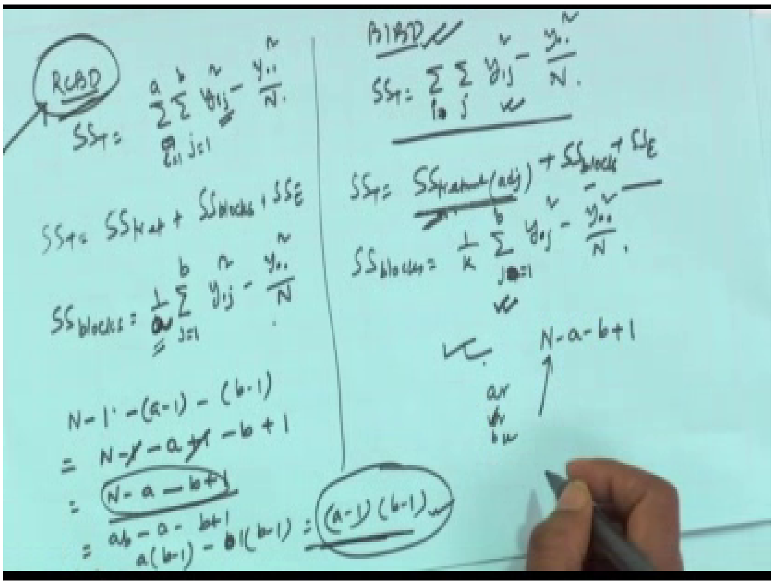

So, this is nothing, but N minus 1 minus a plus 1 minus b plus 1. So, that mean this cancel out. So, N minus a minus b plus 1 that is what we have written here in this case, but if you put N equal to a b then this minus a minus b plus 1 which is nothing, but a b minus 1 minus 1 b minus 1 which is a minus 1 in to b minus 1.

So, this is a minus 1 and b minus 1 is error degree of freedom in case of complete randomized complete block design, but when it is coming as incomplete block design then this N is a r or d k. So, you cannot write in this form.

So, as a result the incomplete case error degree of freedom N minus b plus 1 where N equal to a r or b k, the anova computation will remain same table remaining part of the table is like this that first find out the M S treatment m S block and M S error, then you see f 0 M S treatment adjusted by M S e and this will be f distributed with M S treatment will be a minus 1 and N minus a plus minus b plus 1 degree of freedom.

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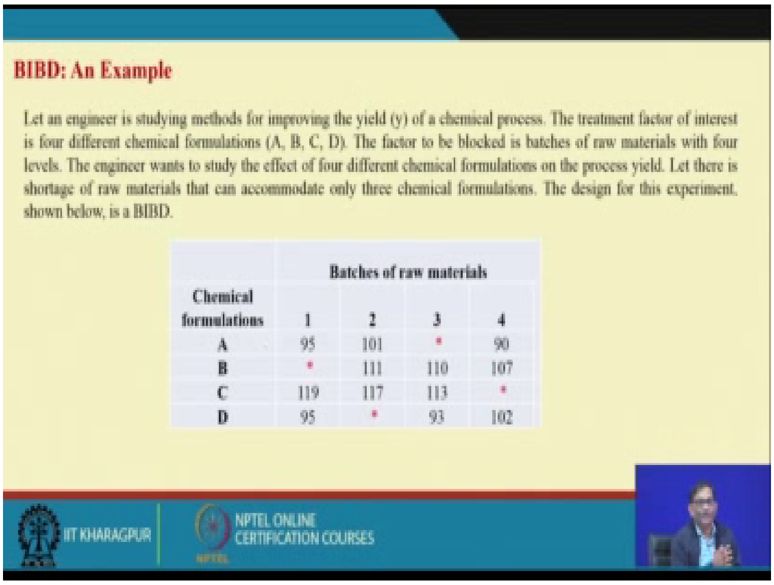

So, we will see 1 tutorial here in terms of example what is this let an engineering studying methods for improving the yield of a chemical process. The treatment factor of interested is 4 different chemical formulations A B C and D the factor to be blocked is batches of raw materials with 4 levels 1 2 3 4 levels that mean 4 different batches.

The engineer wants to study the effect of 4 different chemical formulations on the process yield let there is shortage of raw materials. So, that that the raw material. Let there is shortage of raw material that can accommodate only 3 chemical formulations although we have 4 chemical formulations, but raw material every batch is such that it cannot accommodate all 4 treatments. So, that 3 treatments can be possible and then the then this design is basically BIBD balance incomplete block design. We are so near some hypothetical data that as if you go for experiments with this chemical formulation and with these 4 batches of raw materials suppose the yield is coming like this.

So, please do not attach to any units for this yield, but what I mean to say suppose let the data is like this now yield in some unit is like this then you see that that first batch of raw material is treated with chemical formulation that is mean A C and D.

Second batch A B C third batch B C D fourth batch A B and D. So, like this here what happen it is a balanced 1 because you see you take any 2 treatments and you will find the appear equal number of times here A is 4 B is 4 k equal to 3 also r equal to 3 .

So, with these data let us calculate.

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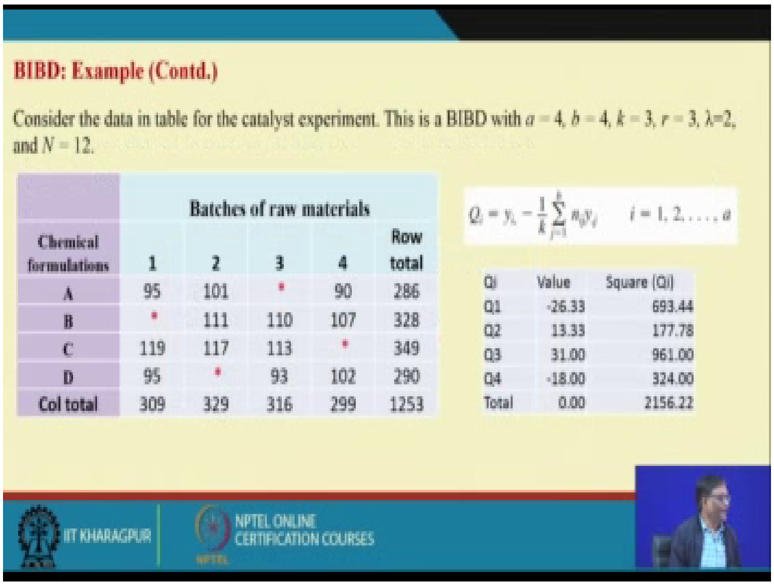

What we will do a equal to 4 b equal to 4 b equal to 4 k 3 r 3. So, lambda will be 2. So, what is what is your lambda value your lambda value we have seen earlier lambda value h is this 1 I have given you.

So, r k minus 1 by a minus 1.

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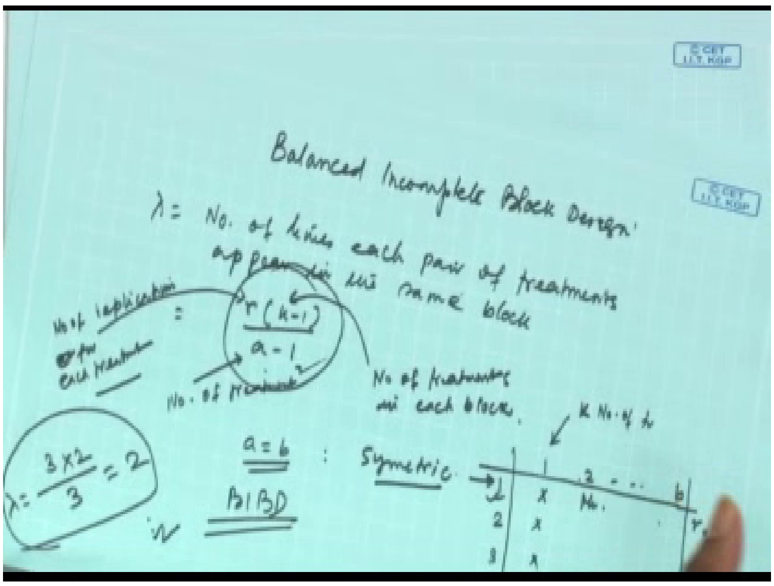

So, what is r? R is 3 k minus 1 is 2 a minus 1 is 3. So, lambda equal to 2 for this example lambda equal to 2. Now we will calculate the remaining things first of all is row total that is y 1 dot it is 286 y 2 dot y 3 dot and y 4 dot.

Similarly, y dot 1 that column total 309 329 316 and 299 and grand total is 1253; obviously, grand total is this plus this plus this plus this or this. So, now, what you will do you will calculate Q 1 for this Q Q i y i total minus 1 by k j equal to 1 to b N i j y dot j. So, i equal to 1 to a.

So, we have computed can you see the value only 1 i will show you suppose Q 1 when we are computing Q 1 that time you will write y 1 total dot minus 1 by k is 3 sum total of j equal sum total of n i j y [FL] n i means 1 suppose j is basically 1 to 4 and y dot j.

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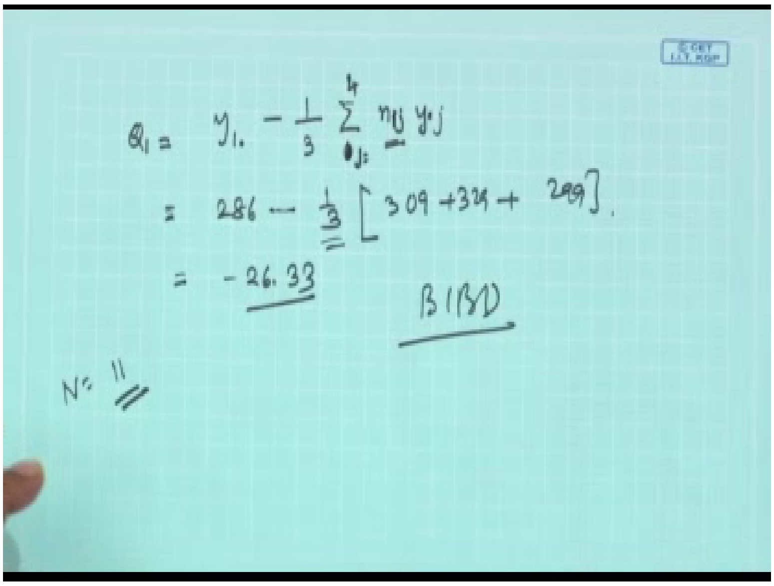

So, y 1 total is 286 minus 1 by 3 now n i j is 1 if that jth both contains that treatment otherwise it is 0. So, as a result it will be here in the first case that 3 that block 3 in case of block 3 there is no treatment. So, accordingly this this total that y dot 3 will not be counted others will be counted and it will be 309 for the first 1329 for the second 1 and 299 for the fourth 1 and divided by 3 this resulting quantity is minus 26.33.

So, Q 1 is calculated like this similarly Q 2 you calculate Q 2 is 328 minus 3 that 1 by 3 within bracket 309 then this will not this will not be counted that block, which 1 we are talking about Q 2 this will not be counted 320 309 will not be counted because for b it is not consider. So, then 329 plus 316 plus 229 by 3 and these average will be subtracted from 3 to 8 it will give you 13.3.

So, in this manner you are calculating Q i and to you square it you will be getting these values and the grand total Q a square is to 156.22 then you are using the this competition like S S treatment k i equal to 1 to a Q i square by lambda a.

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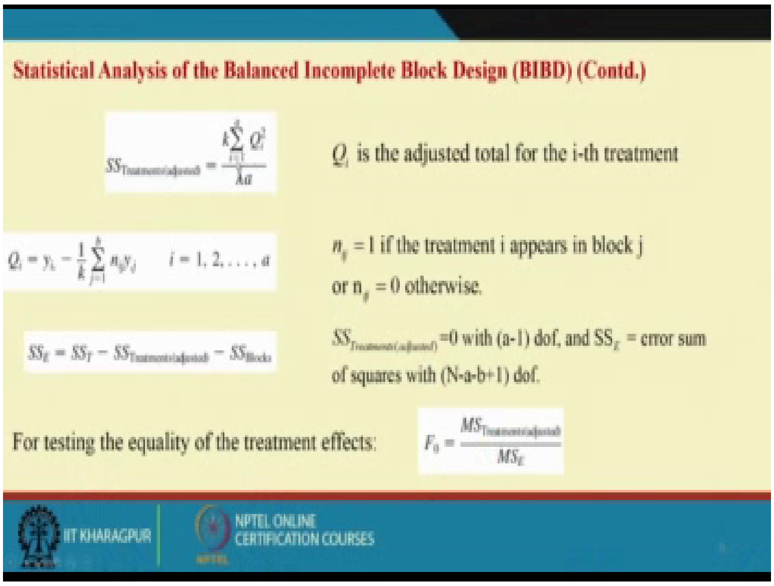

So, all Q is are known Q 1 Q 2 Q 3 Q 4 is known lambda is 2 and so you can calculate. So, k is 3 lambda is 2 a is 4 if you put you will get lambda S S treatment.

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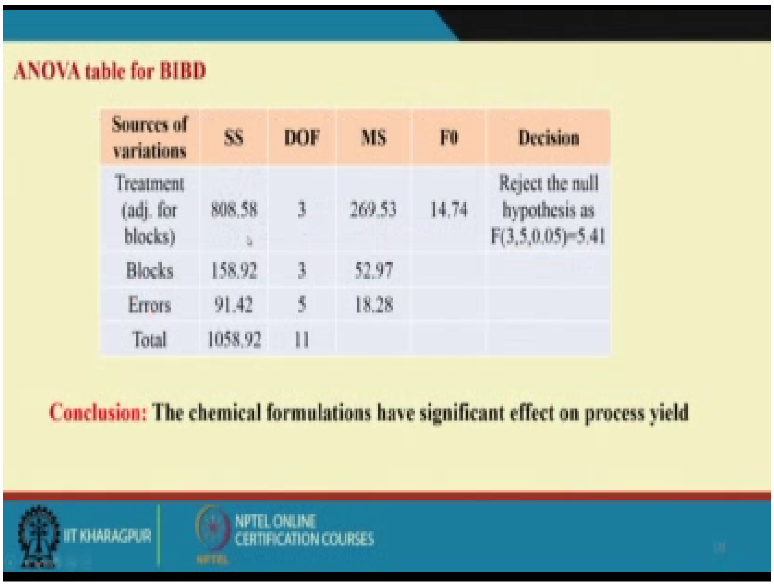

So, here is the a computation for S S treatment similarly S S block using the formula I have shown you the formula earlier I can repeat this one.

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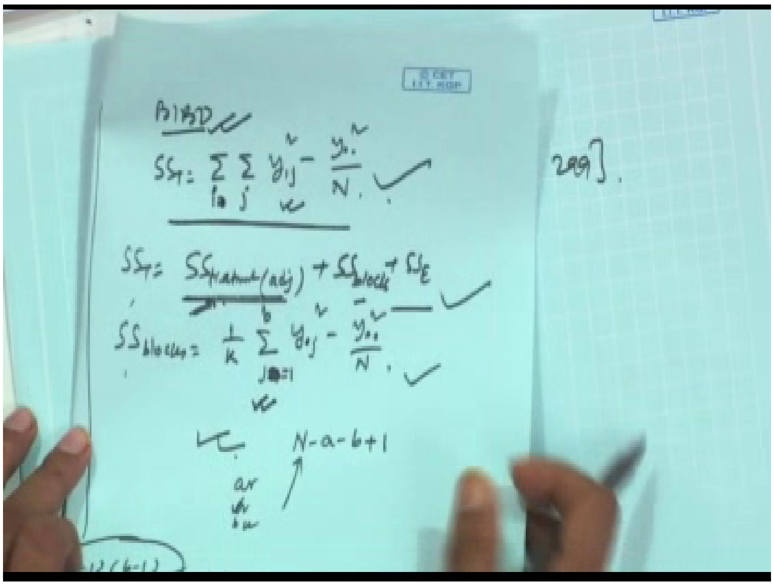

So, what you will do you will use S S T calculation this formula S S treatment adjust this formula S S block this formula and then ultimately using those formulas you will be in a position to get all those values.

So, degree of freedom a minus 1 b minus 1 and N minus a minus b plus 1 so; that means, the chemical formulation as significant effect on the process yields. So, this is what is BIBD that balance incomplete block design, now I will show you very quickly that how the regression approach is used to compute the formula it is similar to RCBD you have b i that mu N tau i and beta j this parameters and only thing you see that for when you are talking about tau i there are r times it is appearing. So, instead of instead of a r is appearing everywhere.

And similarly for beta j instead of b k times beta k times in each block the treatments are consider. So, instead of instead of your this 1 a it is k and then k times it is coming and; that means, you are getting these many equations j equal to 1 to b and i equal to a. So, a plus b plus 1 number of equations you are getting. And now solving this you will be getting this kind of equations and you have this this constant and then this will be your this will be your resultant equation and from here you will be able to find out tau i this tau i is k Q i by lambda a tau i is k Q i by lambda a. This is what is b b BIBD and I have given you the simple details as well as 1 1 numericals example hope you will be able to reproduce it.

Thank you.