Design and Analysis of Experiments
Prof. Jhareswar Maiti
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 16
ANOVA – Pair-Wise Comparisons; Tukey's Test and Fisher's LSD Test

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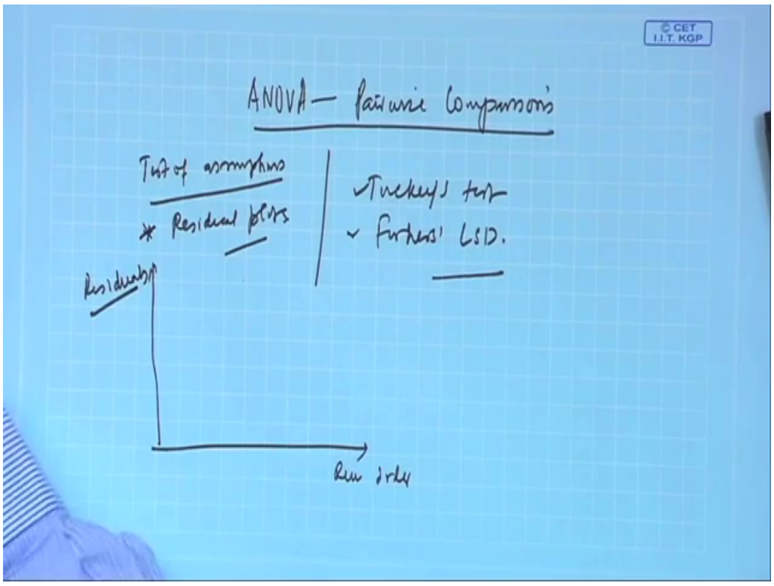

Welcome to the DOE lectures. This lecture we will primarily concentrate on pair wise comparisons, pair wise comparisons. So, last class we ended with test of assumptions. Test of assumption mean normality test, homoskedercity test, independence and identically distributed test. So, I will quickly go through now on the residual plots, residual plots, and then we will come for the, we will concentrate on pair wise comparisons under this, this Tukey's test and fishers, Fishers LSD. This two test will be discussed.

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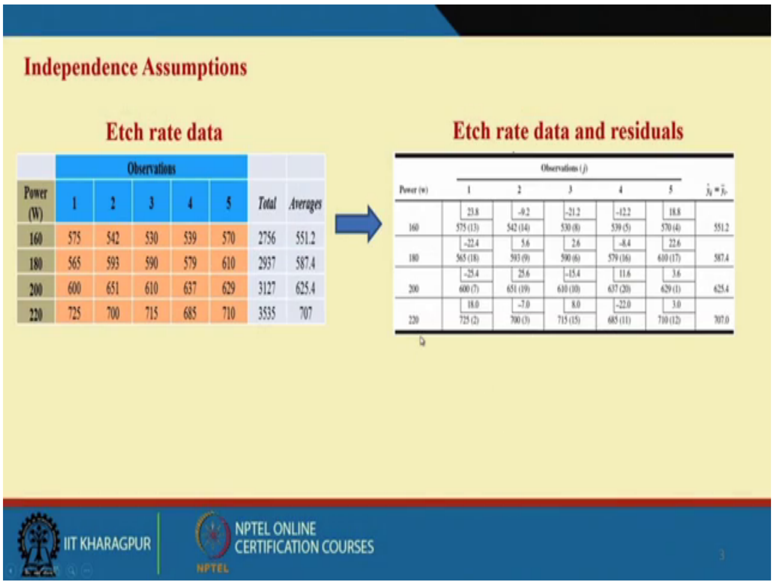

So, let us see the first that is etch rat data, and we have developed the ANOVA model, and finally, we computed the residuals; like in the first cell 23.8 is the residual second cell minus 9.2 like this. In this table you see that there are original of y i variable y values. There are residuals or error, estimated error value and there are run orders within first bracket in which order the experiment was conducted.

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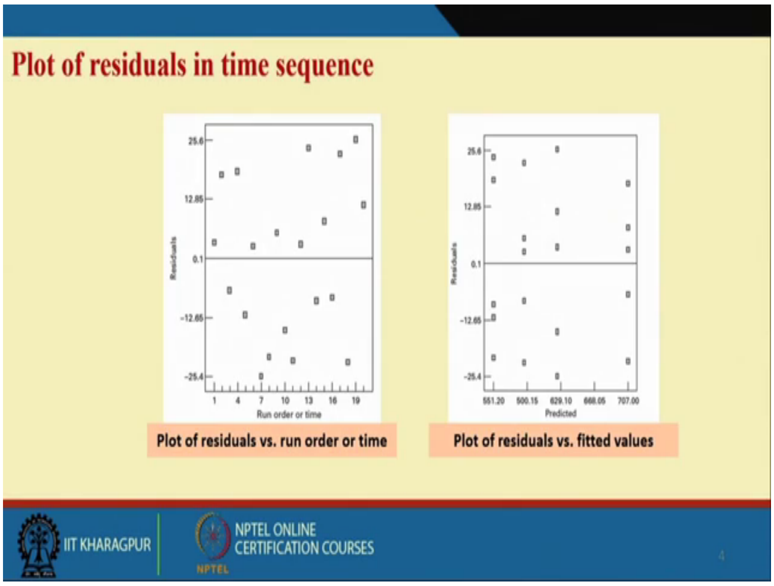

So, first we will see the plot of run order versus residuals, run order versus residual. This plot gives you an idea about independence, whether the observations collected through the experiment are independent or not. See the plot. So, in the left hand side the plot is run order of time, or the observation order and y axis is residuals, you see the plot shows random pattern.

There is no pattern; it is a completely random one. So, if this is the case, then it is independent, the observations are independent in nature. You see the right hand side plot where is the predicted value versus residual also. These also showing there is no pattern, it is purely random one.

So, this one, not only this one gives another proof for homoskadersicity assumption. So, there is no pattern in the data. There is no funneling right to left.

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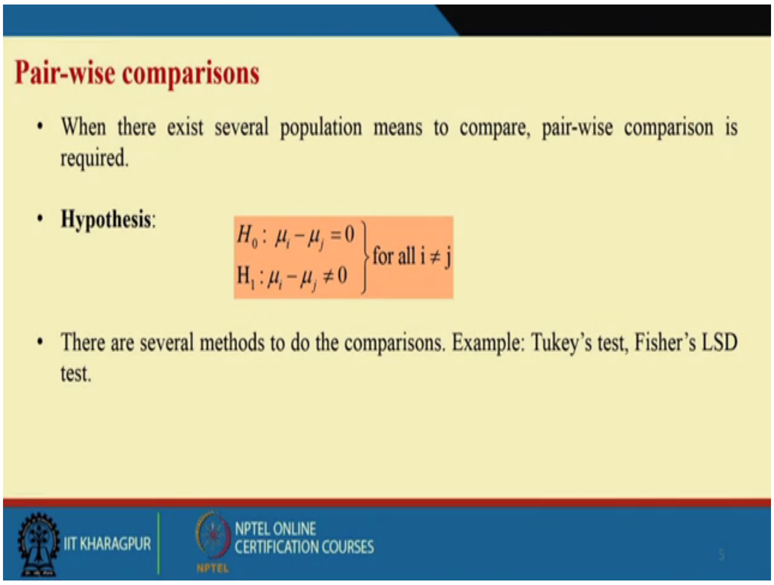

So, now as you have seen that the ANOVA hypothesis is so that, mu 1 equal to mu 2 equal to mu a.

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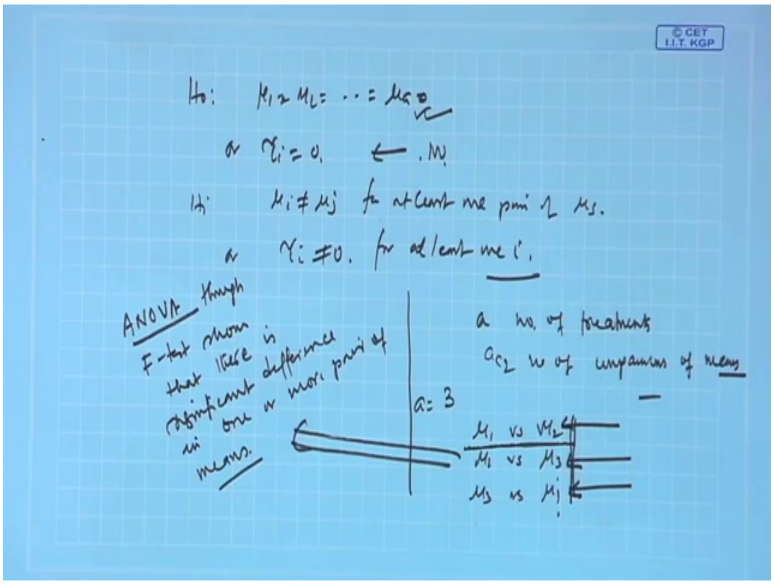

This is basically null hypothesis, the all means are equal. All or we say tau i equal to 0 that is no treatment effects. And alternative hypothesis is that at mu y not equal to mu j for at least one pair, at least one pair, pair of mu s, or alternatively tau i not equal to 0, for at least one i, at least one i ok.

Suppose ANOVA this test, this ANOVA says that, ANOVA through f test, through f test shows that there is significant difference in one or more pair of means. So, ANOVA, through ANOVA table through f test what we can observe, either there will be no difference between pair of means, or no treatment effect, or there will be at least one pair of mean that is difference. So, one or more pair of means will be different, and that mean there are one or more treatment effects.

Now, if there are a number of treatments. So, what will happen, you can have ac 2 number of comparisons of means basically. So, that mean mu 1, suppose 3 a equal to 3 then comparison will be mu 1 versus mu 2, mu 2 versus mu 3 and mu 3 versus mu 1. So, that mean three pairs of comparison of means there. Why this pair wise comparison is essential, because of this.

If there is one or more pairs which are different, then ANOVA that through f test will not tell you that which pairs are different, it will tell you there are one or more pair or pair of means they are different. So, in order to know which pair is different, you require test for this mu 1 versus mu 2, mu 1 versus mu 3 and mu 3 versus mu 1. The question is then why not then independently you test mu 1 versus mu 2, mu 1 versus mu 3 and mu 3 versus mu 1 from the data itself, without doing ANOVA before hand, then what will happen, what will happen, this there will be errors. Error, because you cannot test separately this here the pair wise comparison we will make, and that the error will be adjusted. So, that is why the special kind of test, this Tukey's test, fishers LSD test will be used.

So, let us see the hypothesis for the pair wise comparison that mu i minus mu j equal to 0, and mu i minus mu j not equal to 0. So, we will see Tukey's test and fishers LSD test.

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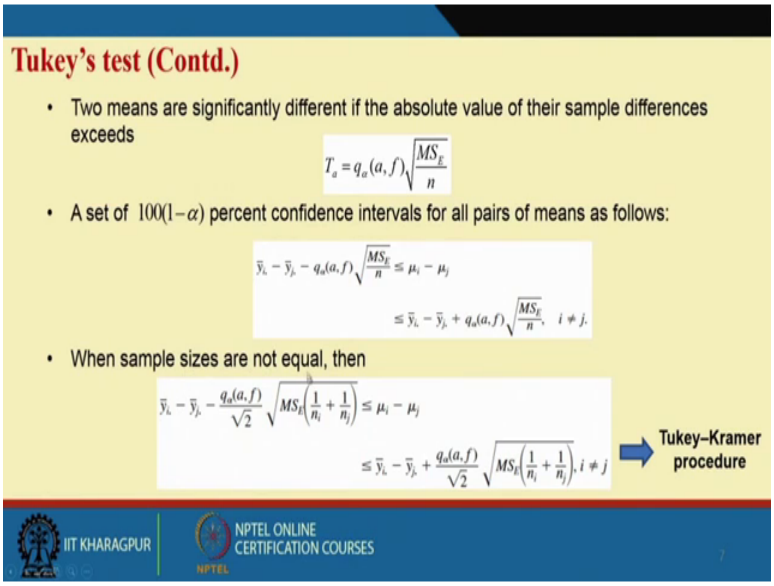

What is Tukey test? Tukey test actually, first they there will be a statistical q which is known as studentized range statistics, which is y bar max minus y bar min by root over MS E by n. You see all other quantity or right hand side is known to you. What is y bar max? You have a number of treatments. So, you have a number of means or average here from the experimental data. So, there will be a average value which is maximum.

There will be another one minimum. So, the difference between maximum of the average and the minimum of the average values for all the a treatments; that their difference divided by MS E. MS E is mean square errors which already you have computed. So, that by n, this is a statistic which basically will be, is used by tukey to develop the pair wise comparison test. So, you see what we have to consider here. We have considered out of a group of p sample means also, p number of sample means are.

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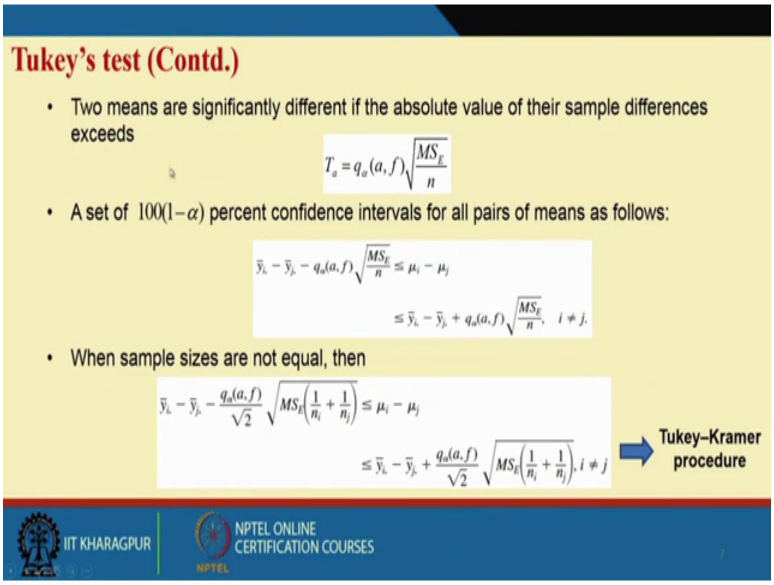

So, then two means are significantly different if the absolute value of their sample difference exceeds this. So, there is a threshold value T a which is q alpha a f root over of this, this quantity. Now a is the, a you all know that the number of treatments, and MS E also known to you, and n is also known to you. Now f is another quantity which is required to be known, and then the 100 into 1 minus alpha percent confidence interval for all pair of means will be y i dot bar minus y j dot bar minus u alpha a f root over this and this. So, if sample sizes are not equal, then the other formula will be used.

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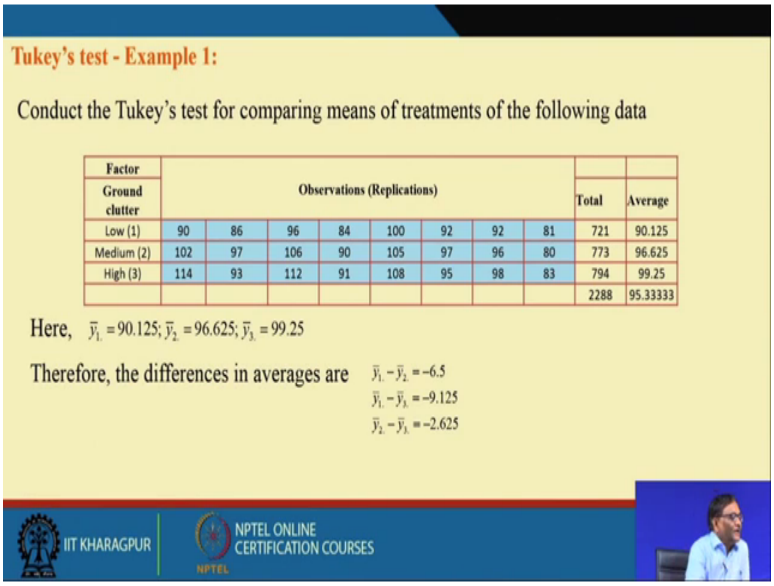

Now, see the example, the ground clutter example.

So, y 1 bar and y 2 bar and y 3 bar computed 90 96 99.25 is you just compute it. So, then the difference in average that y 1 bar minus y 2 bar is this, y 1 bar minus y 3 bar is this and y 2 bar minus y 3 bar is this, differences are computed.

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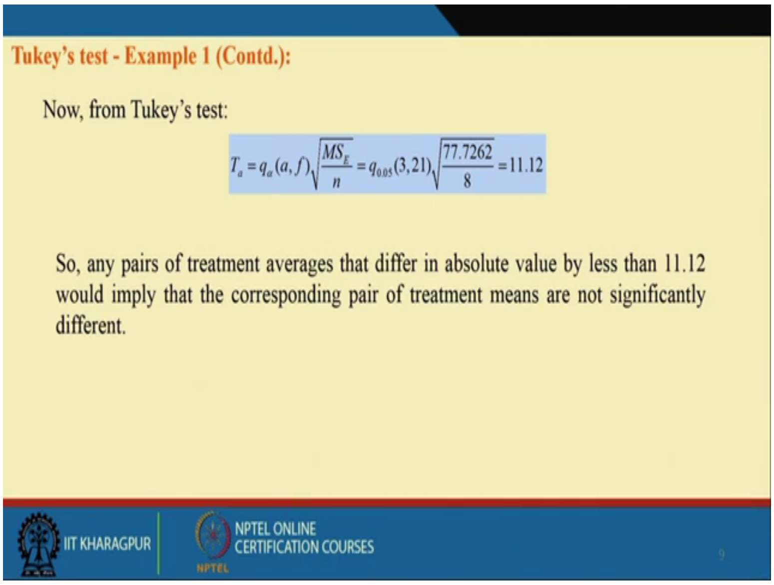

And then from Tukey test you just find out see what happened. What is the threshold value? Threshold value is q alpha a f root over MS E by n, then q 0.05 alpha you have consider 0.05. There are three levels of, there are three treatment levels and 21 is coming from the errors degree of freedom. As I told, you need to know the f value. What is f value? f value is error degrees of freedom, because for this example, there were 24 observations; 8 into 324 observations.

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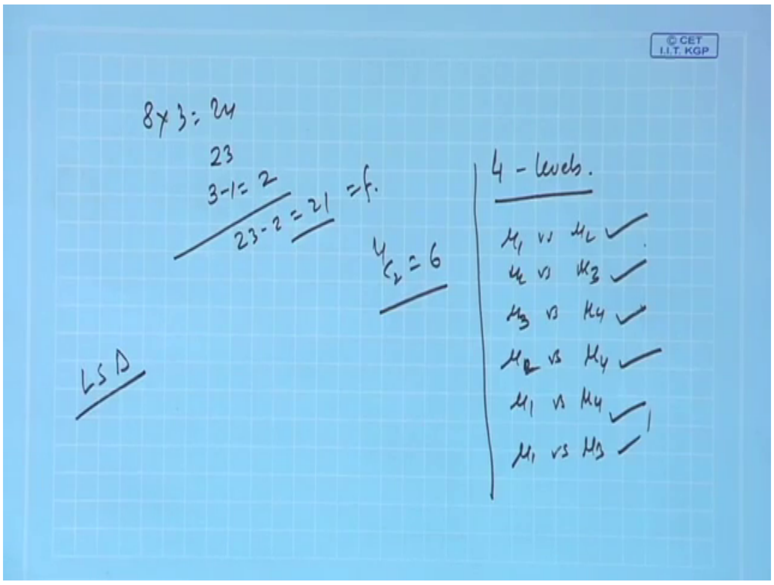

Then total degree of freedom is 23, then treatment degree of freedom is 3 minus 1 2, then error degree of freedom is 23 minus 2 equal to 21 which is also (Refer Time: 11:30) we consider here f. So, if you put this, this value this will be 11.12. Now you may be interested to know that where, how are you getting this q 0.03 21 value, this value is available from the chart developed for this Tukey test. So, there will be table or chart available to you and if you follow Montgomery, then what you will see at the.

In the appendices that mu Tukey, Tukey’s test this values are given. So, what we have done. We have basically taken q 0.053 20 alpha 20 21 that value from the Tukey, that appendix, and then this multiplied, and finally, you got 11.12. Now this is the threshold value.

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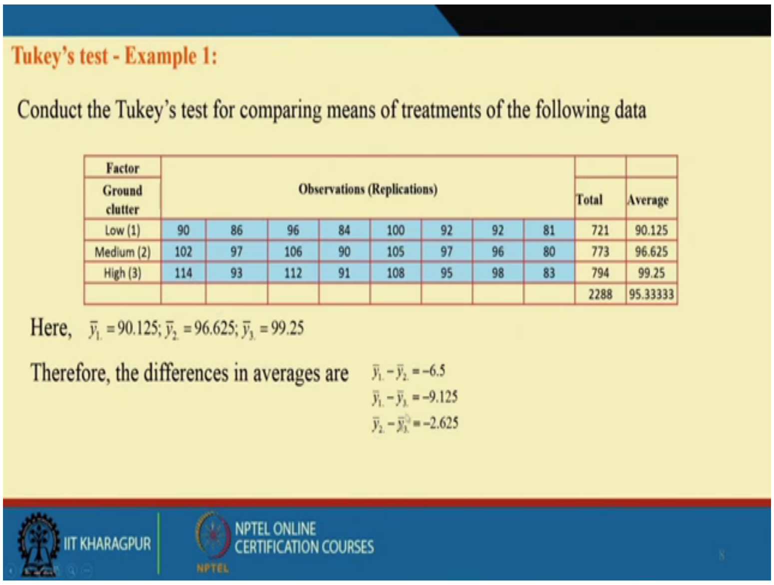

So, if any pair, any pair differences, like y 1 bar minus y 2 bar y 1 bar this differences they are in absolute value, they are more than 11.12, then we consider that pair is different, but if you recall the ANOVA, that overall test, that there we found, when we consider only ground clutter levels and there is no difference in the means. ANOVA is itself says there is no difference in means. So, the pair wise test is not required, but we have done the pair wise test in order to show you that, even the from the pair wise test also we are finding out that no two pairs are pair of means of different. So, no two pairs, in every pair that is mu 1 mu 2, mu 2, mu 3 and mu 3, mu 1, no pairs showing any significant difference ok.

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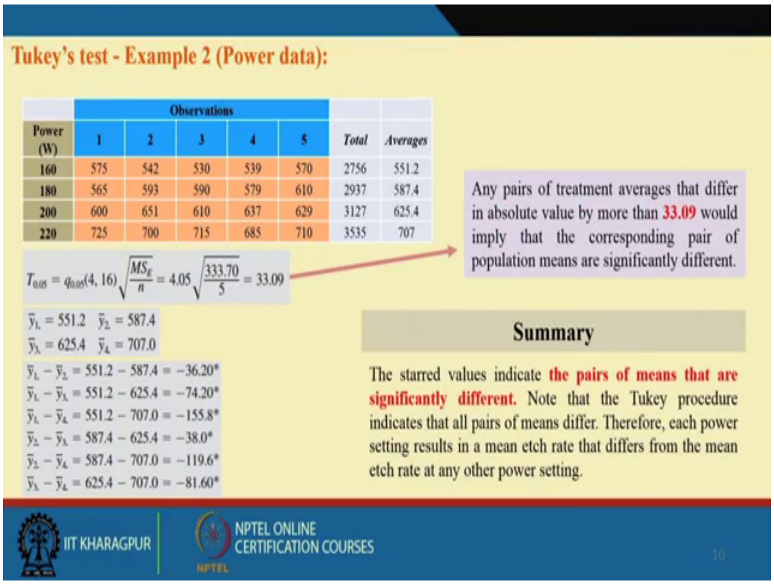

So, but if you recall the second example, where the etch rate at four level of array power factor. So, what you have found out there, that the ANOVA overall test says that, there is significant difference in means. Maybe at least one of the mean is, one of the means is different from the other means; that is what we found from the overall f test using for ANOVA. Now then we were interested to know which pair is different. So, as in the power example there are four levels, treatment levels. So, how many comparison possible mu 1 versus mu 2, mu 2 versus mu 3, mu 3 versus mu 4, then mu 2 versus mu 4, then mu 1 versus mu 4; so, mu 1 mu 2, mu 1 mu 4 and mu 1 versus mu 3, then mu 2 versus mu 3, then mu 2 versus mu 4, mu 3 versus 1 2 3 4 6.

So, that mean 4 c 2 6 pair wise comparisons, six pairs you are getting, six pair of mean differences . So, now, see here. So, we found out that T alpha value, which is q is 0.045 16 and this value is 33.09. Now when we compare all the means, you see that the all the six pairs that one is, first one is minus 36.20, second 1 is minus 74.20 like this. So, if you see the absolute value for all these, you see the none of them are less than 33.09. all are more than the threshold value.

So, it says that there is significant differences is mean means which is proven by the overall test, from pair wise test also it is further confirmed. So, that mean, here interestingly all the mean differences significant, but there may be situation when you find out that some of the mean differences will be significant, and some of the mean difference will not be significant. And in the previous example, we found that all the mean differences are not significant, and here all mean differences are significant and there will be in between. So, some other example may gives, may give you these kind of results.

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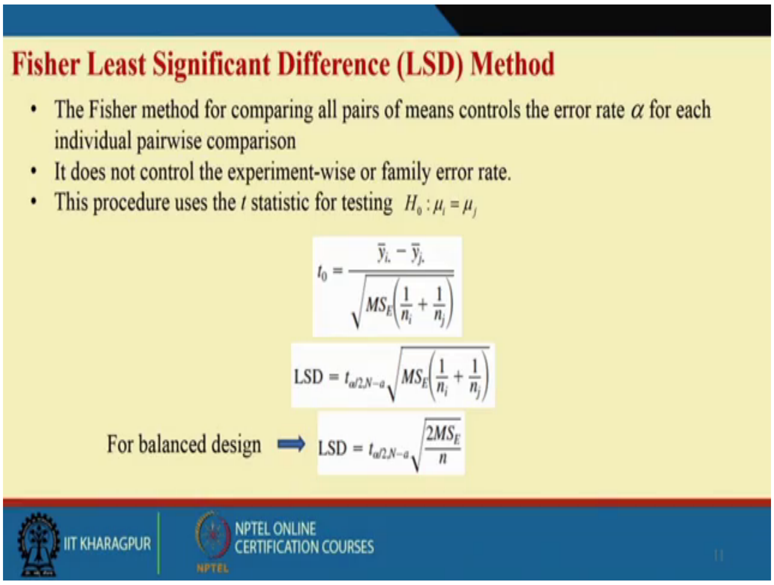

So, the same test or same thing you can do using fisher least significant difference method. LSD stands for least significant difference. Here what is done in this case we use t test which is very popular test, and it is known data, tables are available. So, fisher LSD is the easier method to know to calculate or to test, or calculate and to test whether the mean pair, means are different or not pair wise. So, you see t 0 y i bar minus y j dot bar by MS E by into this, and this is nothing, but the basically what you have seen earlier that the difference this, in confidence interval test we found out the difference between two means, two averages, and using this statistics you found the confidence interval for two difference between two means, and there you have also seen this, this kind of this variability of this ok.

now then LSD value the threshold value is t alpha by 2 n minus a, what is n minus a that is again you have taken degree into this. And if your ANOVA is balanced mean the sample sizes are equal, then what happened this quantity will become 2 by n. So, 2 MS E by n, if any pair is absolute value of any difference pair, any pair is more than the LSD value, threshold value then that mean that pair is significantly different. So, here again ground clutter data, you find this one you already computed, and LSD value 9.17, you see none of there these values in absolute term or more than 9.17, only y 1 dot bar and minus y 3 dot bar are, this is, this value is. Sorry this value is basically close to 9.17.

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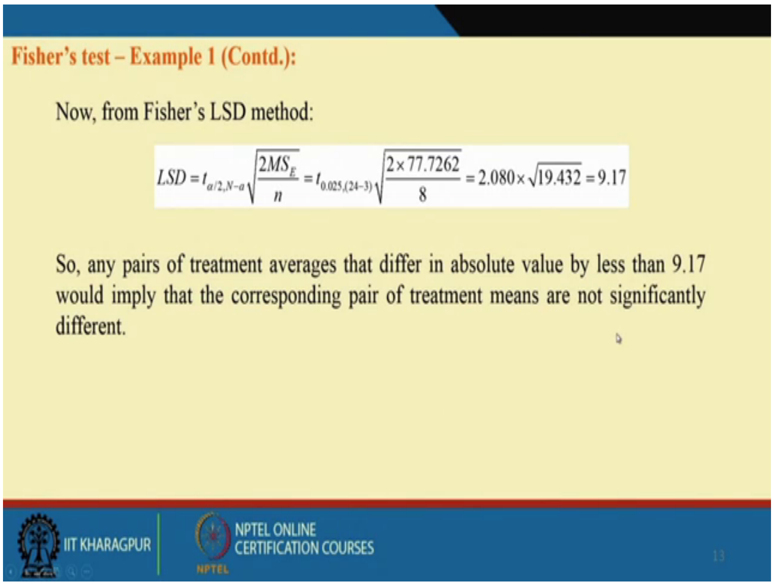

But anyhow as it is less than 9.17 we are saying that we know mean pair or means are different ok.

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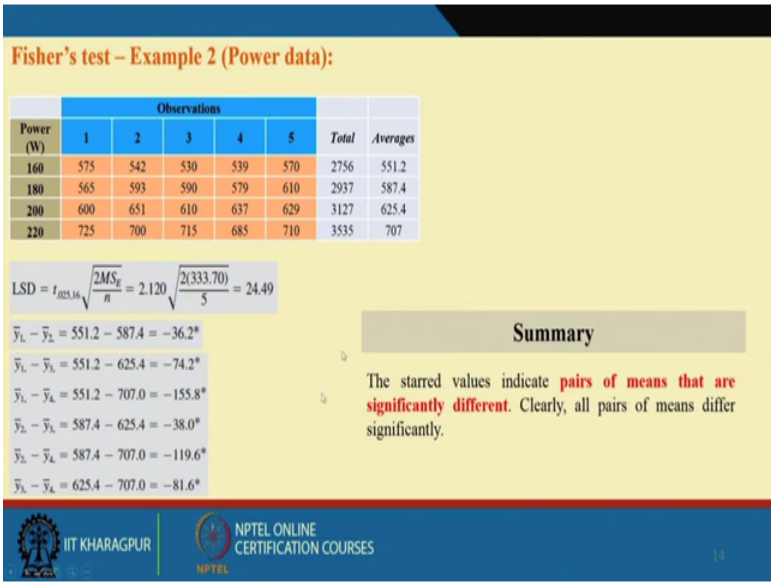

And the same will be proven and it is also done, and we have computed and found that, even using fisher LSD also, you are finding out that in the power example in the etch rate data case that all the pairs are pairs are significantly different.

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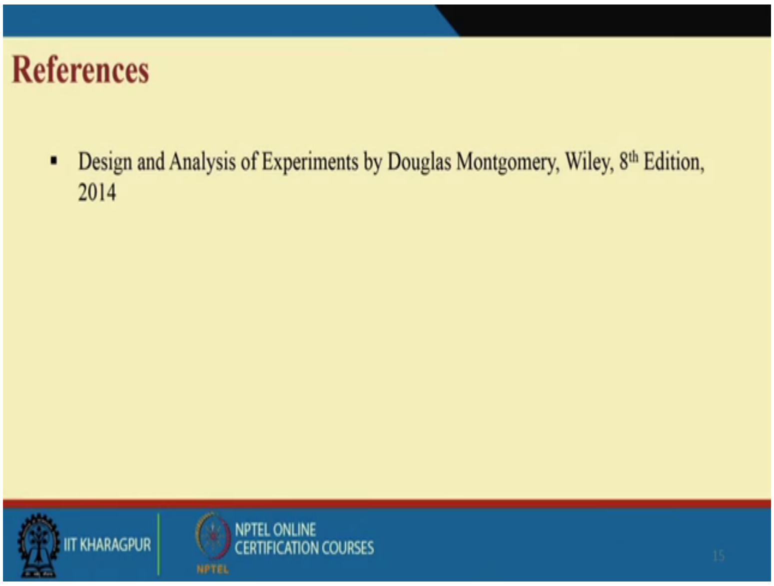

So, this is what is pair wise comparison. So, in nutshell let me tell you that conclude the one way ANOVA part.

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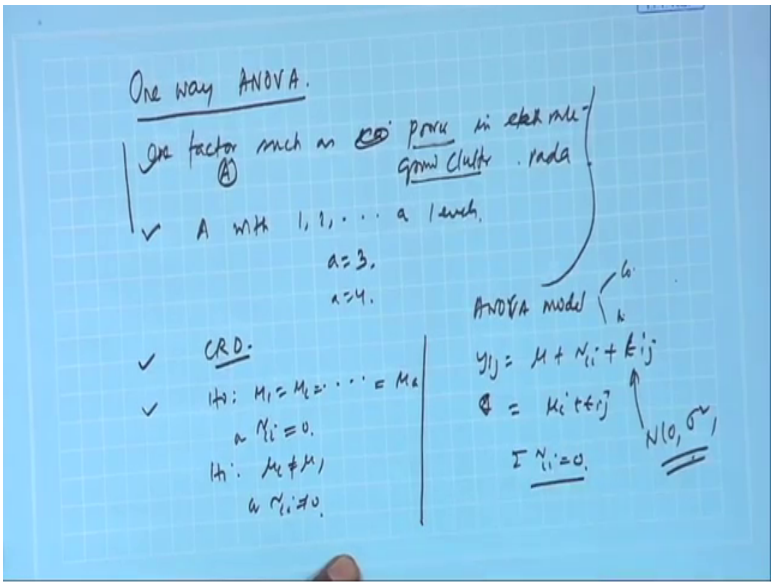

One way ANOVA so, what we have done. One way ANOVA mean there will be only one factor; such as your etch rate.

sorry such as your power in etch rate example etch etch rate example, or your ground clutter, ground clutter in case of that radar scope example may be one factor. And that factor will be having different levels. So, suppose in general if you say the factor is A. So, A with a levels 1 2 like a levels, what does it mean by level these are all treatment level. For example, in case of ground clutter a equal to 3 that is low medium high. In case of power that is rate data a equal to 4; that is 160 180 200 and 220 watt, this is the first thing. Second is that; obviously, you will you conduct random experiment.

Complete randomized design, and your ultimate hypothesis is you want to test H 0 mu 1 equal to mu 2 dot dot dot dot mu a or a tau i equal to 0 alternatively mu i not equal to mu j or tau i not equal to 0, this is the test. And you have ANOVA model with data, ANOVA model. So, there will be mean model and there will be well complete model parametric model and mean model, model will be there. So, that mean y i j will be mu plus tau i plus epsilon i j, or it can be written mu i plus epsilon i j, and this epsilon i j is normal distributed with mu sigma square (Refer Time: 21:55) squared as, and; obviously, some of tau i equal to 0, and then there are lot of assumptions, assumptions for ANOVA.

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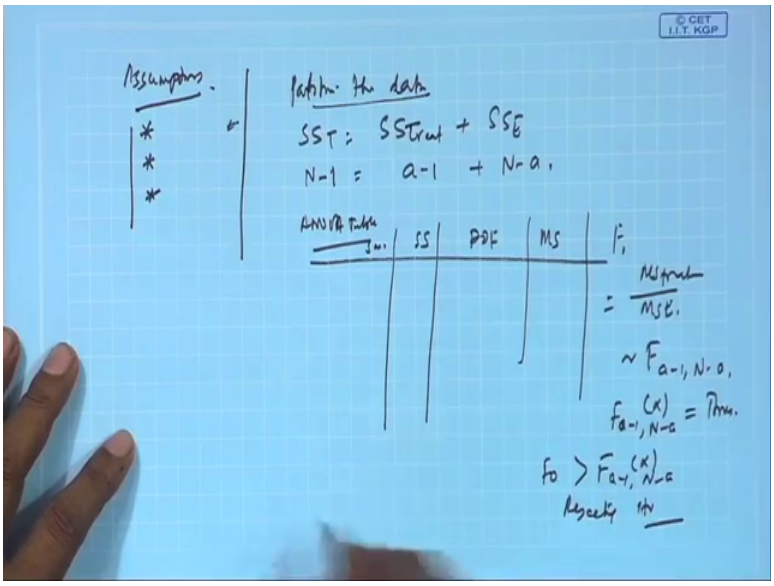

And assumptions are basically that is my normality assumptions, homoskdersity assumption, independent and identically distributed assumptions will be there. So, those assumptions are must be full filled, otherwise what will happen data may be, the results may be erroneous, but anyhow ANOVA is a robust against normality, but if there is homoskderticty problem, it is that the heteroscedacity it is difficult. So, then what happened you have what do, you partition the, you basically partition the data into different components, then sum square total is also portioned into sum square treatment plus sum square error, then here the degree of freedom is N minus 1. Here degrees of freedom is a minus 1 plus N minus a.

Then you have developed ANOVA table, ANOVA table, and from there in the table the sources of variation is important, then SS you computed, then you computed degree of freedom, then you computed MS, then you computed F F equal to MS treatment by MS error, and this quantity follows F distribution with a minus 1 and N minus a degrees of freedom. You find out a a minus 1 N minus a alpha, some threshold value, this is the threshold value. If computed like F 0, if computed F is greater than the threshold value alpha, then you are rejecting null hypothesis rejecting H 0. So, you reject H 0. Then once you reject H 0 the pair wise comparison is important.

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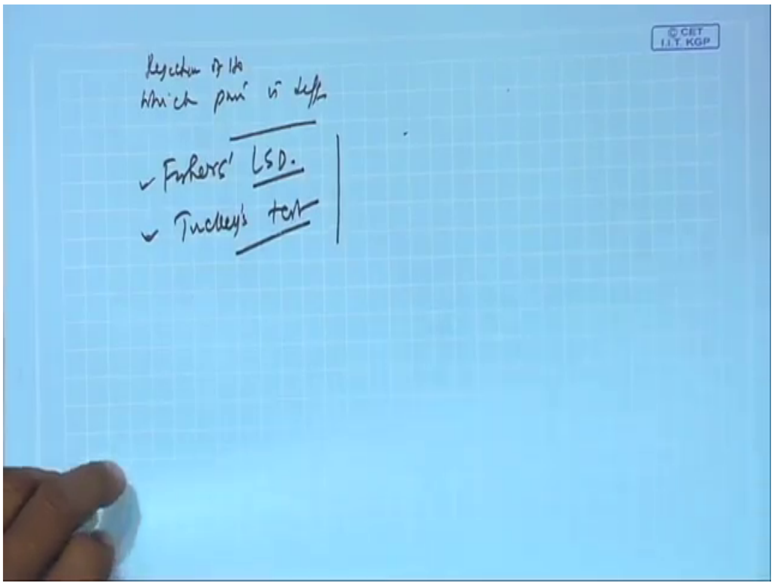

Ones you reject H 0 you are interested to know which pair is different rejection of H 0, rejection of H 0 case gives you which pair is different. So, there you have used fisher LSD least significant difference, and we have use Tukey’s test also, and in addition in the test of assumptions some amount of residuals plots are there, parameters and all those things. So, this is in nut shell the one way ANOVA.

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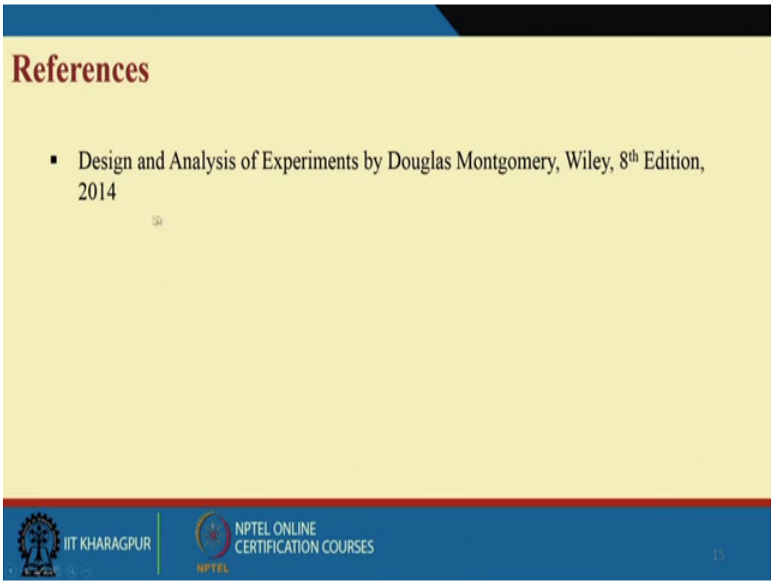

And now let me acknowledge that this lecture is prepared, based on this book Montgomery, design and analysis of experiments.

Thank you very much.