Design and Analysis of Experiments
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Lecture – 11
Hypothesis Testing

Welcome today we will discuss another very important topic for design and analysis of experiment. This is hypothesis testing.

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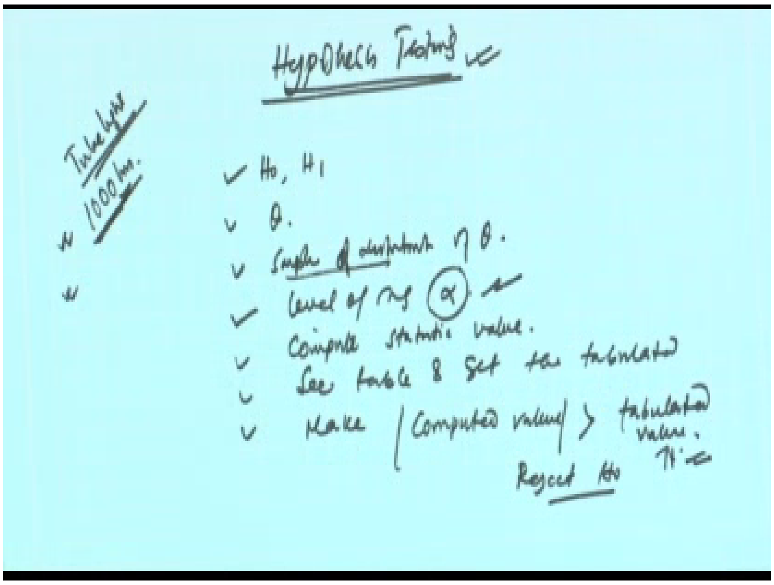

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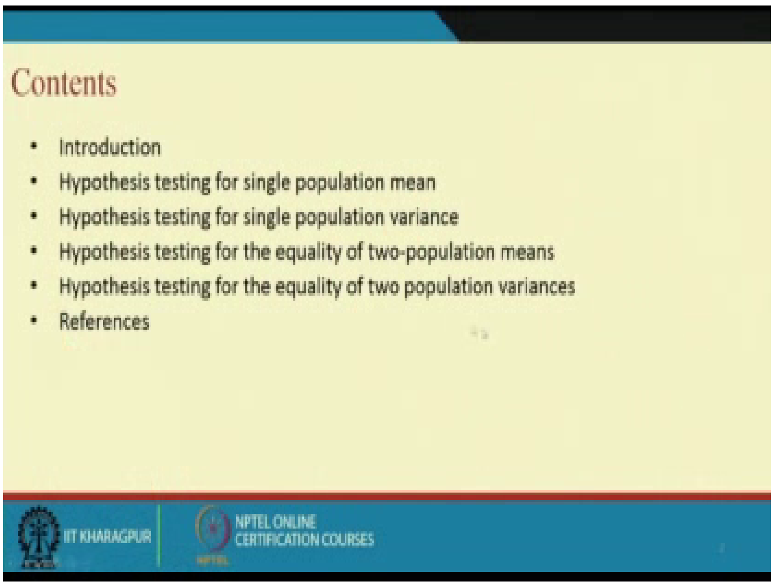

So, let us see the contents. So, we will, I have introduced what is hypothesis and what do you mean by hypothesis testing. Different kinds of hypothesis testing; such as hypothesis testing for single population mean, single population variance, equality of two population means and E equality of two population variances. So, it has much similarity with confidence interval, although the approach is different, but from confidence interval estimation also we will be able to take some decision and hypothesis test to be tested.

So, the variation point of view as there are lots of similarities, I will skip most of the variations, but whether how hypothesis testing is done and how to take decisions that is important. So, we will discuss more of those kinds of things.

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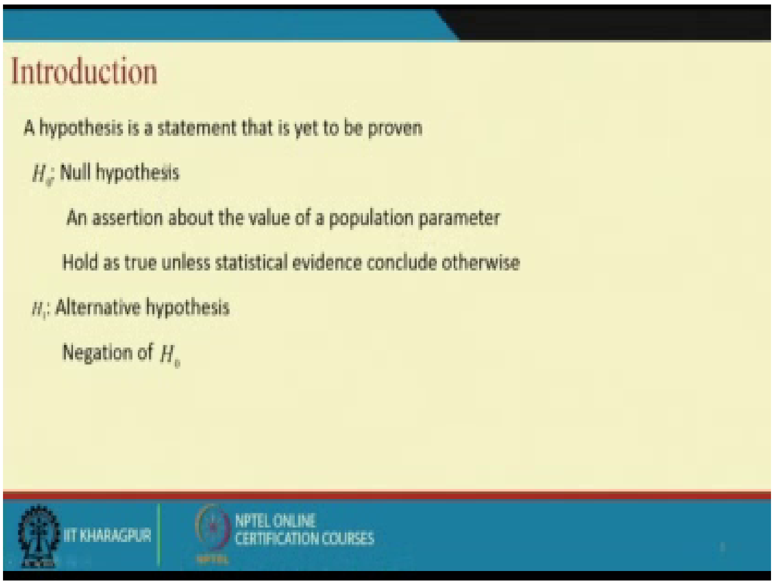

So, what is hypothesis? A hypothesis is a statement that is yet to be proven. So, for example, a manufacture says that, suppose a manufacture manufacturer producing, let it be a tube light.

Now you want to know that what is the, how long it will burn or blow continuously. So, manufacturer may say that, yeah 1000 hours it will continuously burn. So, that mean the manufacturer is making a statement

That my, the tube light my company produces it has on an average 1000 hours of continuous burning. Suppose another one maybe if just think of a medical test gone started medical test, and then there is some measures may be cholesterol, may be blood pressure, may be something other some kind of diseases related test. So, it is hypothesis that by value 100 or 150 or 120, it is basically it leads to certain kind of diseases.

So, whether if the blood pressure is for a certain age group, blood pressure is more than 140 then for it is high blood pressure, and it is from the proven from the medical science. And if this continuous with the continue with the high blood pressure for may be 20 hours together you will have some accident, or some kind of Ha ill affects, let us assume a kind of statement. So, suppose a victim is taken to the court in front of the judge. The judge may think that he is an innocent guy, he unless it is proven through facts I cannot tell he is guilty. So, he is innocent that is also a statement.

So, more or less Ha in our subject, we will be looking for the quantitative statement point of view, not the subjective or qualitative statement by like he is good or bad, but here most of the times, we will be looking after certain mean value, certain standard deviation value, or certain mean difference values like this. For example, I say that mean burning time is 1000 hours or the difference between two between the diameter or bulb bearing produced by two different machines 0.00 at 2 centimeters, such kind of statement we will make, and that is what is, but that is what is the hypothesis, and what do you require.

You require from the data may be through experiments or other way, but here it is the through experimental data, you say that. Yes, it is correct or wrong, it will be accepted or it will be rejected or it will be rejected and you will fail to reject the hypothesis. So, the hypothesis what you want to accept or fail to reject, or other way I can reject. This is known as null hypothesis.

So, there are two kind of hypothesis; null hypothesis and alternative hypothesis. Null hypothesis an assertion about the value of a population parameter hold as true, unless statistical evidence conclude; otherwise for example, when a person a is being to the court, the judge thing that he is an innocent. The reason is the maximum people, when if you consider may be the, any civilizations they will find out the most of the people are civilized in 99.19 percent people. They are basically innocent people, civil people, then not have been, they are not doing any criminal kind of things.

So, that is why so long it is not proven that is, he is guilty, it is true that he is innocent, because that is what is the mass says. So, that is ends, what is alternative hypothesis negation of the null hypothesis; that means, the null hypothesis is fall.

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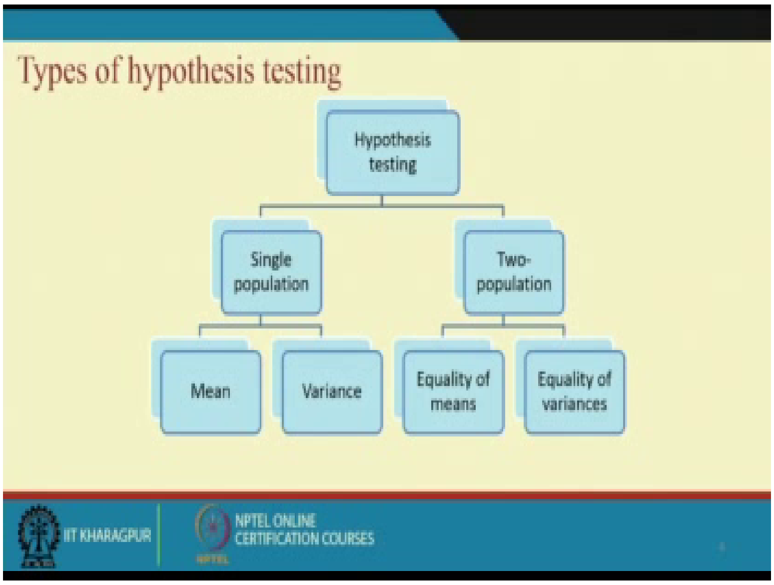

So, here in this half an hour 40 minutes of lecture. Now what we will do. We will see that the single and both double population case, and both mean and variance case, and how do we do the hypothesis testing.

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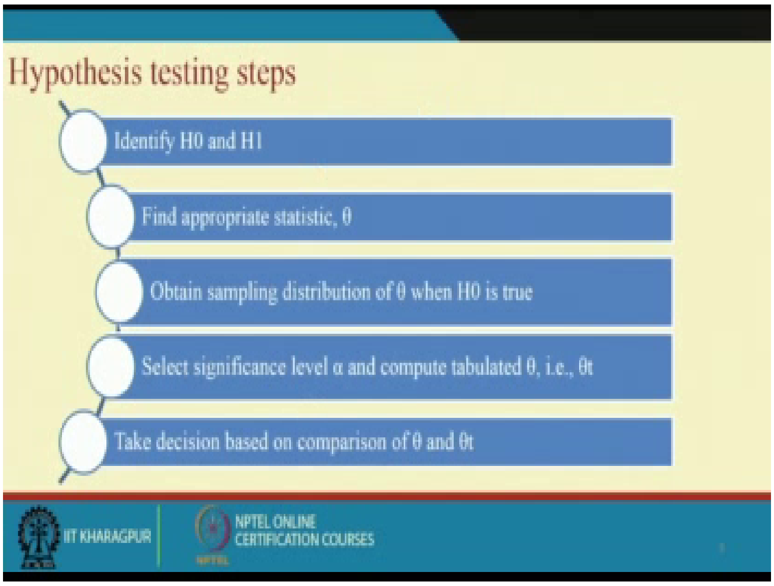

So, these steps are very important. First you find out the null hypothesis, you find out alternative hypothesis, then find the appropriate statistics which will be used to test hypothesis, then obtain sampling distribution of theta with theta is the appropriate statistics when H 0 is true, keep in mind, because everything you are testing through under the null hypothesis in the sense that you are assuming the null hypothesis is true, and you are testing. And after test you will find out the null hypothesis you cannot be accepted, you reject it.

Select significance level alpha and compute tabulated theta; that is theta tabulated and then take decisions based on this. So, what I mean to say. First you select H 0 and H 1, then find out appropriate statistics theta, then find out the sampling distribution of theta sampling distribution of theta, and then you choose the level of significance; that is alpha and then compute the statistics value.

See table and get the tabulated value and then make decision that mean if tabulated value is a, as a, if the computed value is suppose computed value mode of this is greater than the tabulated value; obviously, you know the distribution, you will be using different kind of tabulation, then reject H 0 null hypothesis. So, where from you go tabulated value, you will use different kinds of tables. So, you have to know what is the appropriate sampling distribution and according you find the tabulated value for appropriate significance level.

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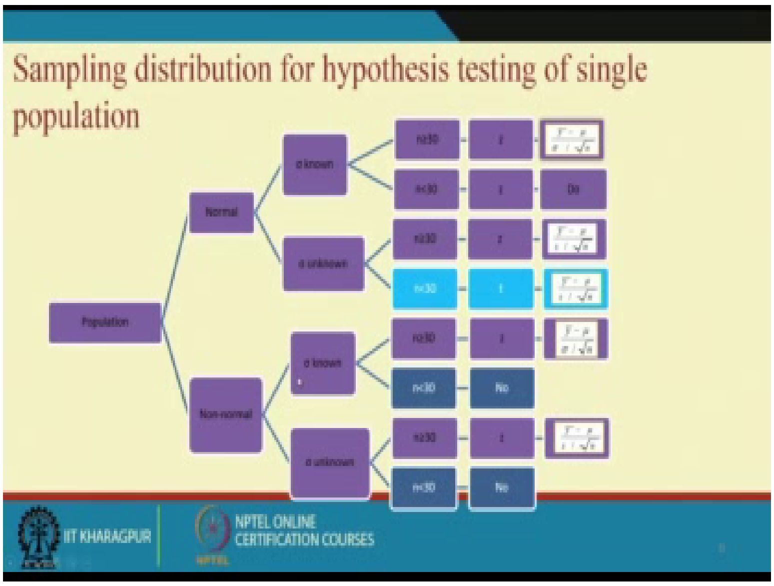

This is very simple and straight forward things to do it. So, I will just quickly say that this is nothing, but what we have already seen under estimation population can be normal and non normal, and under this through under the normal and non normal, and with different types of sample size what are the different statistics that are important. So, it is a normal population sigma known and sample size small or large, you expected to, this is the z distribution and y bar minus mu sigma by rout in follow z distribution.

If sigma unknown, then if its sample size is large then it will follow z distribution; otherwise T distribution, if non normal case sigma known n greater than n is large then z distribution, if sigma unknown n is large then will also z distribution, but if sigma in any case, whether sigma known or sigma unknown if sample size is low less then not known the distribution not low.

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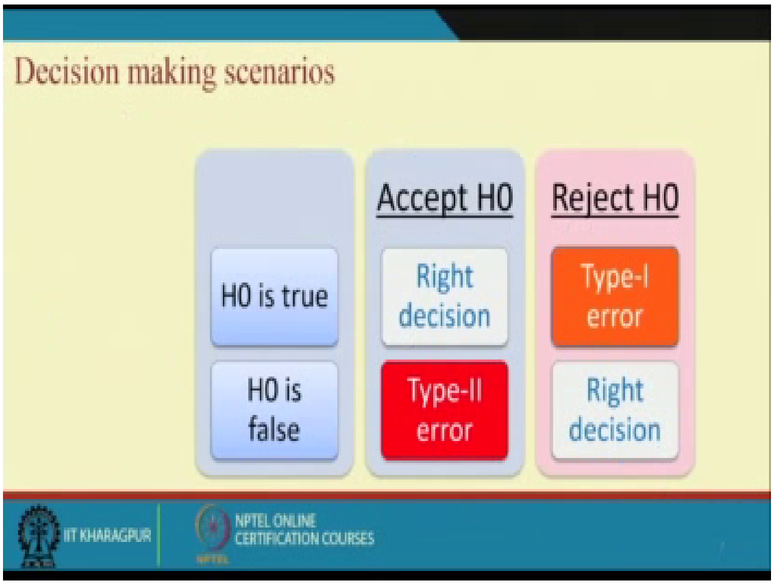

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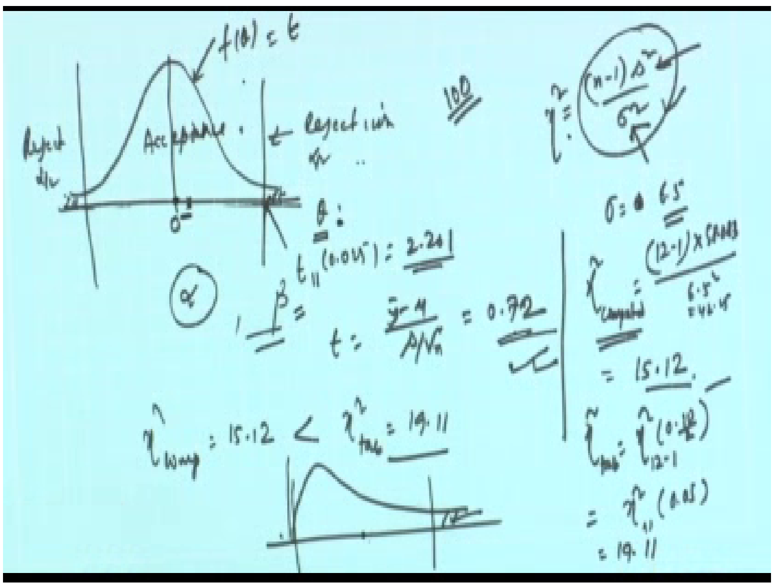

So, another very important concept here that what kind of decision making scenario it will happen. So, as I told you that when you talk about any statistic and its result and distribution, then when you take decision, you will create a range of rejection zone, rejection and this side rejection, this side acceptance, this side acceptance, this is the rejection, rejection.

So, but theoretically all possible, all values are possible for theta. So, if you reject by any error. So, that error will talk about level of significance. So, now, that because error is there into which either error is basically the statist statement is falls, you can accept it then that is one can be error, statement is correct, but you are rejecting based on (Refer Time: 11:01) test that is another kind of error, this is what is given here.

So, in reality the statement is true or statement is falls through test. You may accept the statement you may reject the statement. If in reality statement is true, and you are rejecting the statement, then this type 1 error with probability a alpha type 1 with probability alpha. And another one is that in reality the statement is false, but you are accepted it, then it is type 2 error with probability beta. So, this type 1 error and type 2 error are very important one, that suppose you are producing something and your some sampling skill and through sampling you are finding out, you are basically rejecting a lot which is basically a good quality not sending to the customer.

So, in that case, it is a type 1 error and what happened, your loss is the production loss. But it may so happen that your sampling scheme is not able to detect that bad lot. It has been sent to the customer, because you are accepted the bad lot as good lot and that enter item that has been gone to customer, and customer wears a huge loss, because not only the production lost, the logistics supply, chain supply lost. There is reputation lost and many more lost. So, it is the huge one.

In a medical test suppose someone is having a illness, and your test is not, test is saying that he is not, he is having illness irrespective of he is not having that illness, then this is a type 1 error, because even though he is not (Refer Time: 12:57) illness He will be treated in the sense that, but he if he having the disease and you are saying, he is not having the disease, and he is not treated accordingly then his lose will be much more, but; obviously, its many a time the medical error is such that the, and not having disease, but detecting disease also very costly.

But whatever may be the thing. So, the situation is here, accepting the statement provided it is falls, this is type 2 error, accepting the rejecting the statement provided it is true; that is type 1 error. Type 1 error in most of the production situations, and all those cases, it is basically less severe than type 2 error. Type 1 error is known as alpha error, type 2 known as beta error. So, when it is type 2 error is coming, it is basically the population is not having the distribution (Refer Time: 14:01) under H 0, it maybe; however, different distribution.

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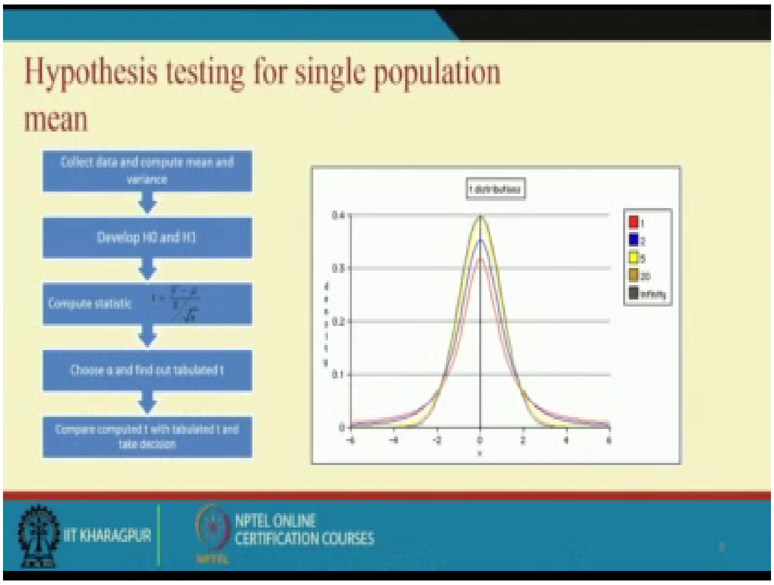

So, all of the steps are collect data compute mean and variance, develop H 0 H 1 compute statistics choose alpha level tabulate, this one. Those things we have discussed earlier, what is the T 1 here y bar minus mu by S by root n.

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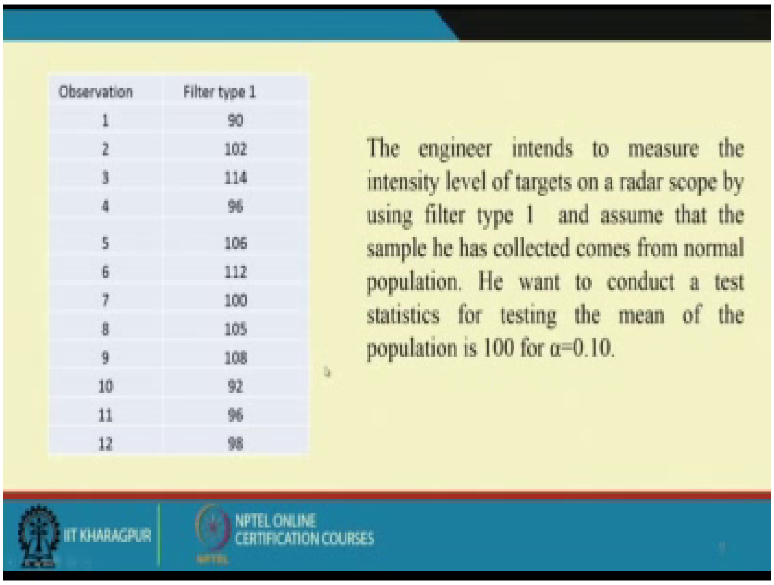

So, I will straight away go to an formula go to an example with a given formulas here suppose the when the filter 1 is used for detecting the target in the radar scope experiment suppose the mean signal that a level is 100 that mean when the signals range is at 100 level it will be detected that is what is the population mean. So, we want to test it and we are we are giving a lesser confidence that 10 per moreover 10 percent error

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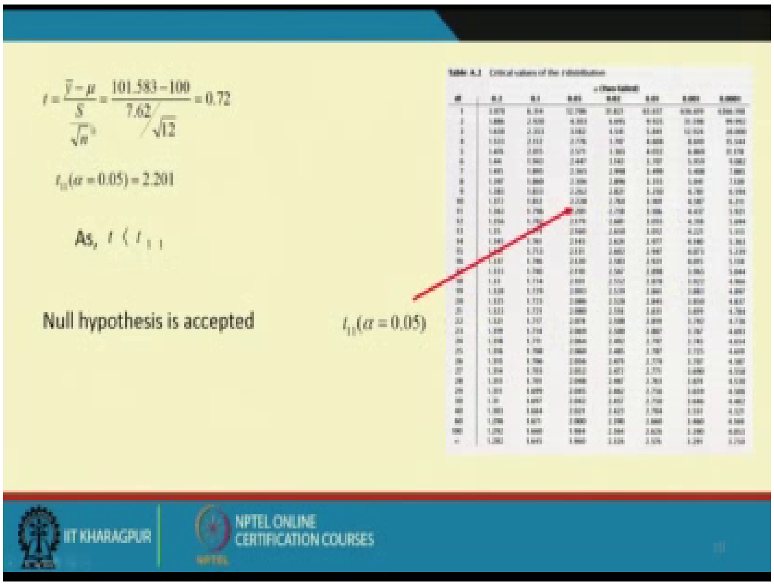

So, what you will do. You will find out the appropriate statistics t statistics. So, y bar minus mu by S by root n, because here S is not variances is not given. So, this value is

0.72. Now what is the theoretical distribution for this? This will be t n minus 1. So, t n minus 1 is t n is 12 minus 1, it is 11 t 11. Now we are considering alpha by 2 alpha by 2 is point this is alpha by 2 t alpha by 0.05. So, point t alpha by 2.05 is 2.021, because what happened you will consider alpha equal to 0.10. So, alpha by 2 will be 0.05 ok.

So, as the computed value is less than the tabulated value. So, this is my t distribution. Now this value this t 11 0.05 this value is how much 2.201, and tabulated value t. what is y bar minus mu by S by root n, this is 0.72, if this one is 2.2 and this is your 0. So, 0.7 is somewhere here, the distance from 0 it is not this distance is not much. It should not far away from this mean value. So, as a result what is happening, you are accepting the null hypothesis; that means, mean intensity level at detection is 100.

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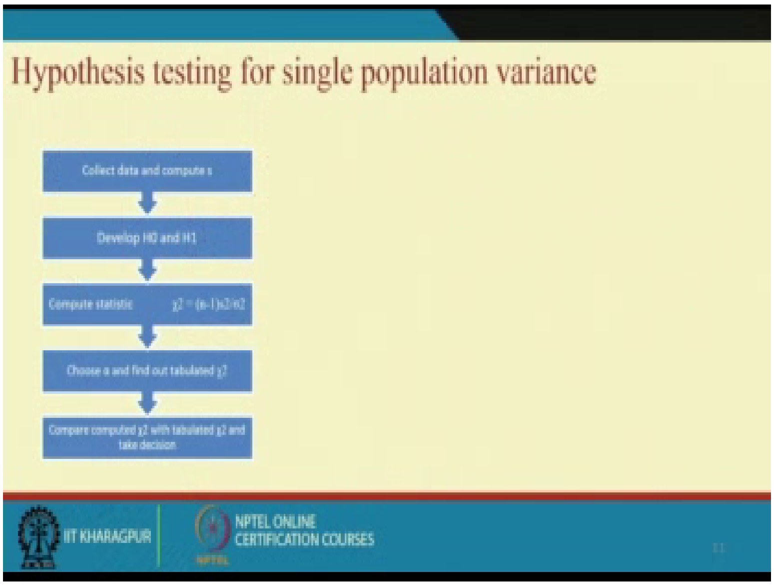

Now for variance case what happen? Variance case we know that chi square equal to n minus 1 S square by sigma square. So, you find out this quantity, if sigma square is known your sample calculate S sigma will be given find out this. So, what is the, this is or this is the problem? Problem is that the same problem (Refer Time: 17:31) you say that sigma equal to 6.5 sigma is 6.5 n is 12.

So, what is the chi square computed. Chi square computed is 12 minus 1 into a square, a square is how much. A square is 58.083 and divided by 6.5 square this is 42.0 this one is 42.25. So, this quantity is coming at 15.12. Now will see chi square tables, what is chi square tabulated value. Tabulated value mean chi square n minus 1 12 minus 1 11 degree of freedom with alpha by 2. So, 1 0 by 2.

So, what is this then chi square 11 0.05, and this value is 19.11. So, chi square compute it equal to 15.0 this is less than chi square tabulated equal to 19.1. What is it mean? That mean if I know, I know the that the distribution of this quantity, this chi square is chi square distributed, and I have created line this one. So, now, my this computed value is 19 15.12 somewhere here. So, it is not far way, it is not falling here or here, it is not far away from the so, ok.

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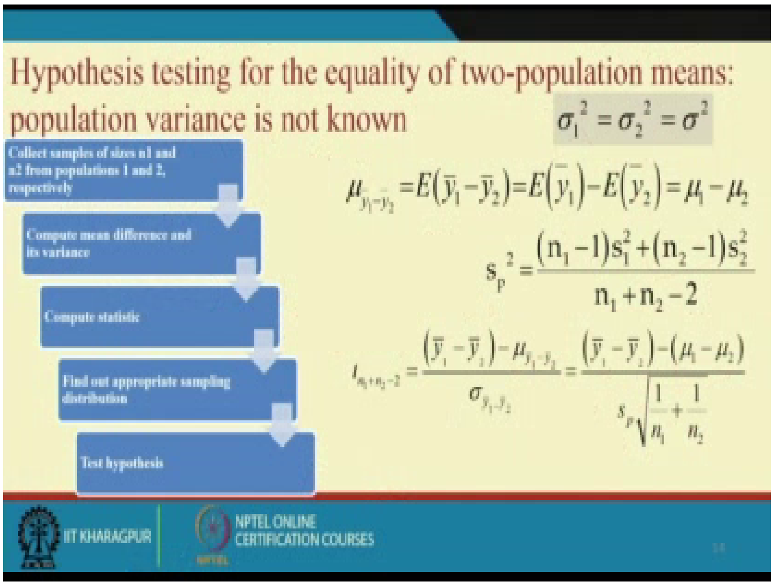

So, what we mean or what we say that it is null hypothesis is accepted. Now the same thing you think of the two population that we are talking about. Two population means equality of two population means equality of two population mean case, what is H 0. Here mu 1 equal to mu 2.

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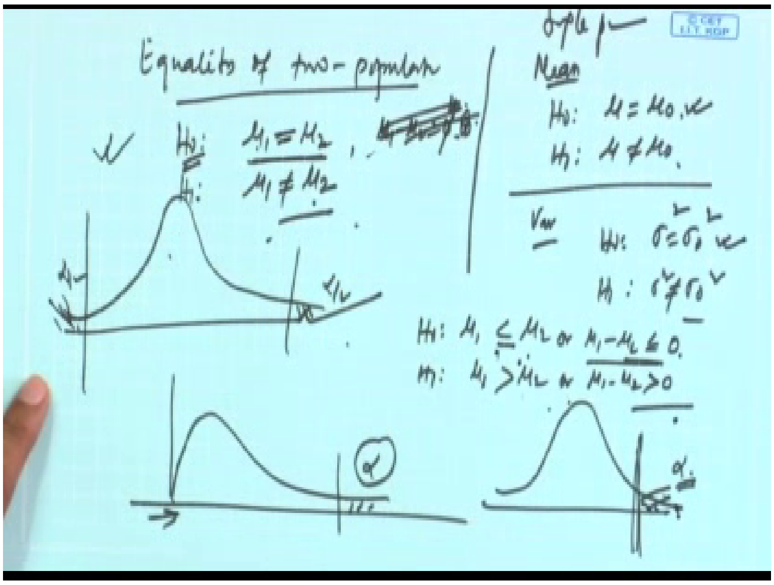

I am not explaining what is, what are those two population. All of you know. Now then what is your H 1 mu 1 greater not equal to mu 2 in the single population case, what was the H H 1 for mean case mu equal to mu 0, and H 1 value mu is not equal to mu 0 for variance case for variance case. It was H 0 sigma equal to sigma 0 or sigma square equal to sigma 0 square and H 1 sigma square not equal to sigma 0 square that was the test.

It is single population case, single population case, now two population case. So, here in single population case, this value is given, this value is given. In this case we have two population, we are considering their means are equal or not. Sometimes what happens someone you may consider like mu 1 minus mu 2 equal to 0; that is what is H 0, maybe some other value, may be given mu 1 minus mu 2 equal to theta or k it is possible so, but most likely case is this. So, we will use this one.

So, here one three more case are there, this is when we are saying mu 1 min equal to mu 2 mu 1 less than equal to mu 2. We are talking about two tail; this tail and two tail test, this side right tail and left tail. If the computed value fall in right and left tail or other way, the reaction based on it will be rejected. There can be one tail test. Also sometimes what happen H 0 H 0 can be said H 0 mu 1 less than equal to mu 2 and H 1 mu 1 greater than mu 2, then what will happen. It is one tail like this H 1 mu 1 minus mu 2 means mu 1 minus mu 2 less than equal to 0 and mu 1 minus mu 2 greater than 0 or this is or. So, this one tail.

So, when you use two tail, then the error will be divided into two side, this side alpha by 2 and this side alpha by 2. If it is one tailed only, if it is one tailed only, then this side only alpha will be there, this side no means what we are saying. Suppose if it is that the distribution for this, for this, for this. Suppose this normal distribution fine, but my case is like this.

So, mu 1 might be less than 0 or greater than 0. We will go by one tail only. So, this side alpha alpha 1 tail alpha will be considered this right side or left side depending on what is H 0. If H 0 is greater than this one case in case of H 0 efficiency is less than, then left side or right side depending on the situation it will be chosen, but in our d o e case most of the time we will be using this two tail test, because this is most of the time suiting to our situation, keep in mind this one.

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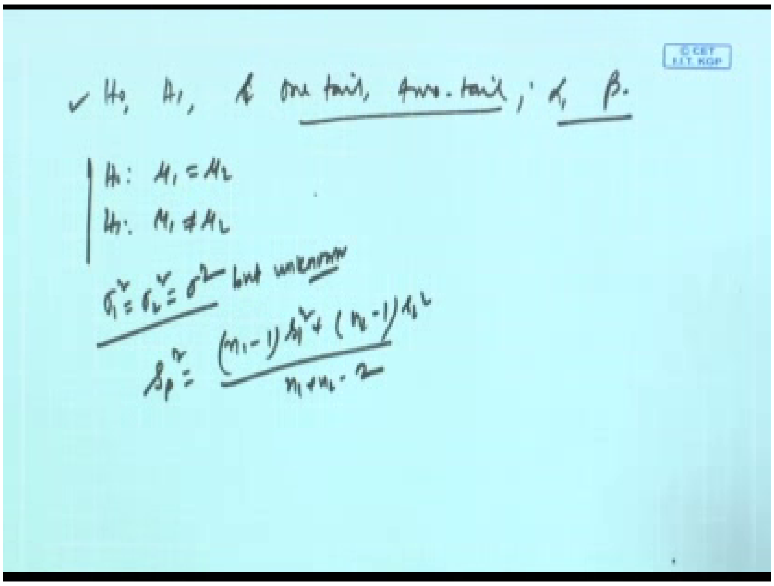

So, in statistics hypothesis testing h null hypothesis alternate hypothesis sing one tail or two tail test, very important, keep in mind and then your alpha error and beta error, I discuss keep in mind. So, with this background now two population mean case equality of two population means H 0 mu 1 equal to mu 2 H 1 mu 1 not equal to mu 2, it is the 2 2 tailed case. So, here some other assumptions that sigma 1 square equal to sigma 2 square equal to sigma square; that mean population means variances are equal and unknown, but unknown population variances are not known. Now you split your sample size, we can use to distribution here, but your sample size is much more then you can use z distribution.

Anyhow here what we have seen in confidence interval. We have created a S p square which is n 1 minus 1 S 1 square n 2 minus 1 S 2 square by n 1 plus n two minus 2, and we have created the interval

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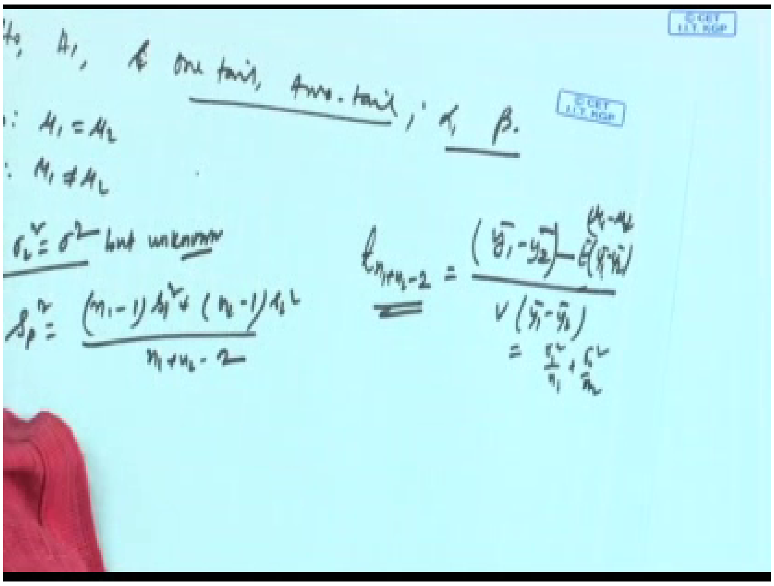

But will say that t n 1 plus n 2 minus 2 this will be this y 1 bar minus y 2 bar minus this expected value of y 1 bar minus y 2 bar divided by its variance, variance of y 1 bar minus y 2 bar this is our, this will distribution. So, accordingly what happened we found out this one is nothing, but sigma 1 square by n 1 plus sigma 2 square by n 2, and this one is mu 1 minus mu 2. Can you remember all those things and resultant, the resultant you see that resultant is that t computed value will be this t n minus that y 1 bar minus y 2 bar go to the slide 1 y 1 bar minus y 2 bar minus mu 1 minus mu 2 by S p root to bar 1 by n 1 plus 1 by n 2.

So, this the tabulated computed t values. So, under null hypothesis mu 1 equal to mu 2 this will become 0 mu 1 minus mu 2 becomes 0.

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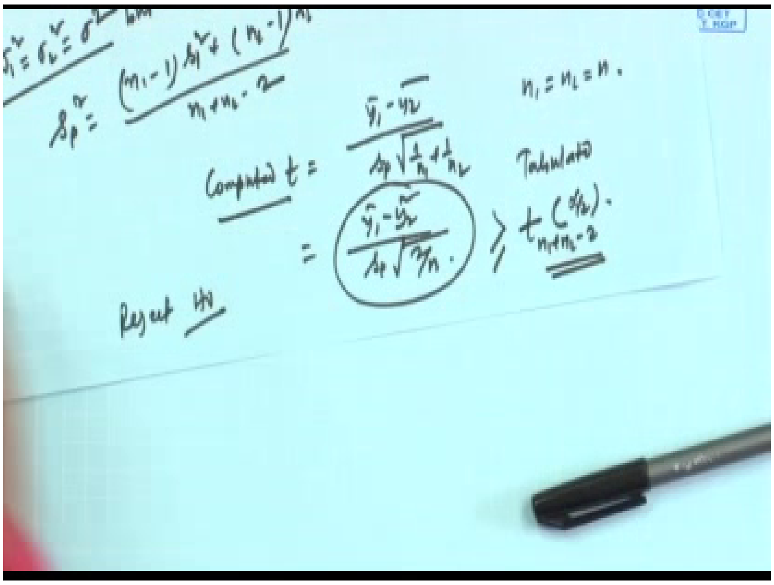

So, our computed t value will be y 1 bar minus y 2 bar by S p into root over 1 by n 1 plus 1 by n 2, and most of the time what will happen n 1 equal to n 2 equal to n. Then this will be y 1 bar minus y 2 bar by S p root over 2 by n. Then what is the tabulated t. Tabulated t is what is the degree of freedom here. Degree of freedom is t n 1 plus n 2 minus 2 and definitely its a 2 tail test. So, it will be alpha by 2.

So, what we do then, if the computed t greater than equal to tabulated t, then reject H 0 that is the decision test hypothesis.

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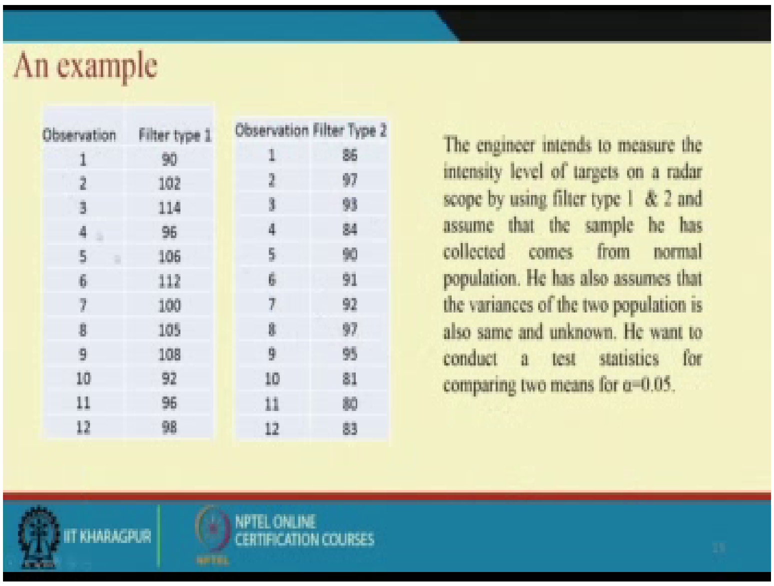

Now, come to this we are considering two different population filter 1, filter type 1 type 2 and we are interested to know see that, whether given this data experimental data we can see that whether the means are different or not, or the whether the means are equal or not.

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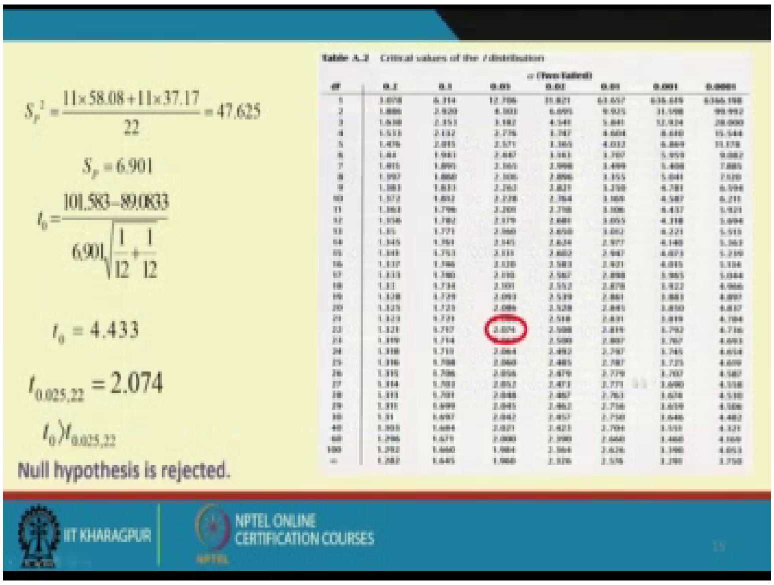

So, what we have done; first calculate the S p square which is 47.625 S p 6.9, then find out t 0, the difference is between means by S p root to bar 1 by 12 1 by 12. So, this is 4.40.

So, what is the theoretical distribution t 22 and its alpha value 0.025 that value is 2.074 from the table. Now computed value is more than the tabulated value. So, now, reject the null hypothesis. Null hypothesis rejected that mean that 2 S H two population mean differs. So, if you use filter type 1 the la signal at stain that detection intensity, at the detection will be more or less than this to op type 2.

But it is, we can see from the data the mean difference, where is this one no Ha here, here, here. So, we found that one 0 1 and 89. So, that mean in the first case it is more. Second case it is less than the first case. So, I mean this one 0 1 and 89, there Ha. This difference is large difference considering the variability waiting with the variability. So, find then another case will be that. Suppose variances are known, in that case we use this normal distribution, and accordingly the normal distribution same example.

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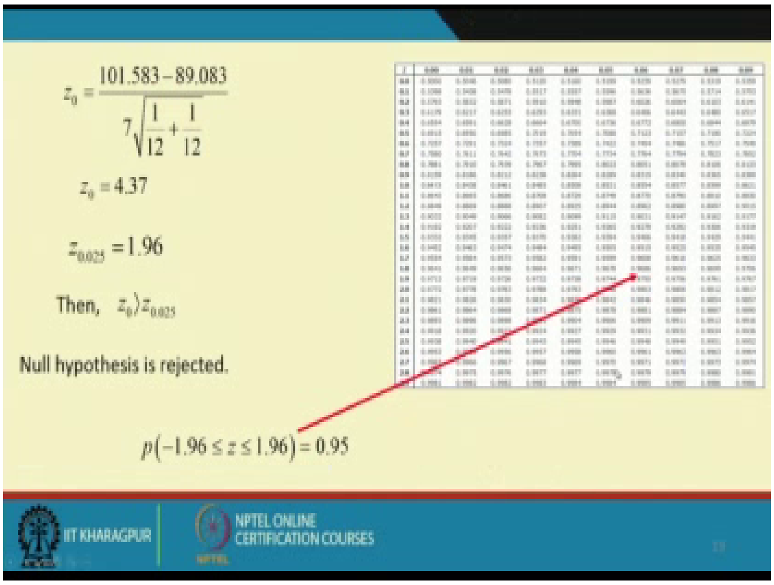

We can use normal distribution Z distribution and we will get this 4.37. Now incase of two population variances whether they are equal or not in confidence interval type, we know that this follows this distribution. So, that way this is my computed F

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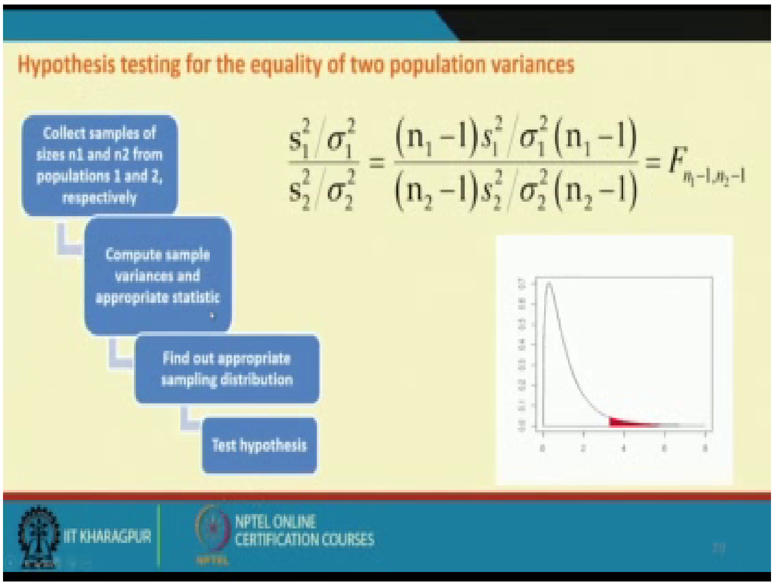

Now what will be tabulated F F n 1 minus 1 and n 2 minus 1 alpha by 2? And with these examples let us see what is happening, he wants to conduct a test statistics for testing the equality for two population variance alpha equal to 0.05.

So, F 0 S 1 square by S 2 square is this, and this quantity Ha. Here we are assuming that Ha [FL] minute Ha that two equality of two population variances under H 0 sigma 1 square equal to sigma 2 square, where sigma 1 square equal to sigma 2 square, then it will. This quantity will become S 1 square by S 2 square that is what is happening S 1 square by S 2 square. So, F 0 is 1.562, and tabulated one is 3.48. So, tabulated one is more than the computed one our computed one is less than the computed one, null hypothesis is accepted that mean the two population variances are equal.

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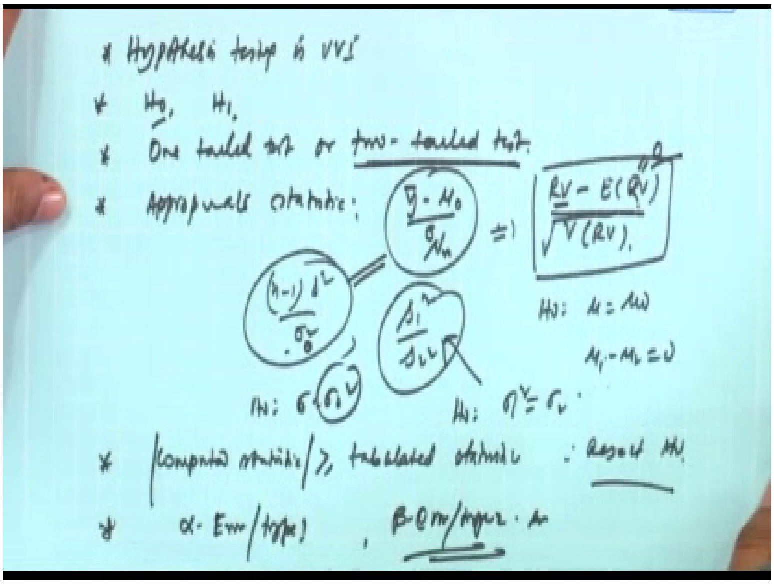

So, with this I conclude, I say that hypothesis testing is a very important concept, very important. There will be null hypothesis, there will alternative hypothesis, there will be Si 1 tail tailed test or two tailed test. We will be mostly lying on two tailed test that is in is our situation suits to this most of the time, then then what happened. It is basically you must, once you know H 0 and H 1 that you must and also data is collected, you must know the appropriate statistic what does it mean by appropriate statistic.

If it is single population mean then y bar minus mu by sigma by root n. Actually we are basically calculating random variable minus expected value of this random variable by variance of this random variable, square root of this variance of random variable. This is what is the appropriate statistics.

If it is y bar it will be y bar minus mu by sigma by routine if sigma is known. Otherwise S by routine if it is y 1 bar minus y 2 bar, then y 1 bar minus y 2 bar minus this y mu 1 minus mu 2 by this. If it is in variance case then it will be S square that is S S squares for single population, and that time what happen. We will not use this. We will use for variance case. We will use other kind of thing like n 1 n minus 1 S square by sigma square ok.

If it is single population single population, if it is two population that there it will be difference here, where here what will be. We will be interested to know S 1 square by S 2 square. The reason actually most of the time what will happen in case of single population H 0 is known mu equal to mu 0, this will be mu 0. In case of two population mu 1 Eq minus mu 2 equal to 0. So, this value will become 0.

In case of variance what will happen that H 0 mu sigma equal to sigma 0 square. So, sigma 0 this quantity will become sigma 0. And incase of equality of two population variance k sigma 1 square equal to sigma 2 square this is the quantity. So, and then what happened. You are computed statistic if it is greater than equal to, suppose tabulated stats. Then what will you reject H 0; otherwise accept it. So, another important one is that whether you accept reject H 0 when it is true, this is known as alpha error or type 1 error.

When you accept H 0 when it is falls, which is known as beta error or type 2 error , this concept is very important. Later on we will be discussing all those things, when you discuss the sample size calculation that time I will discuss about this, when on the different situation how to calculate the sample size.

Thank you very much.