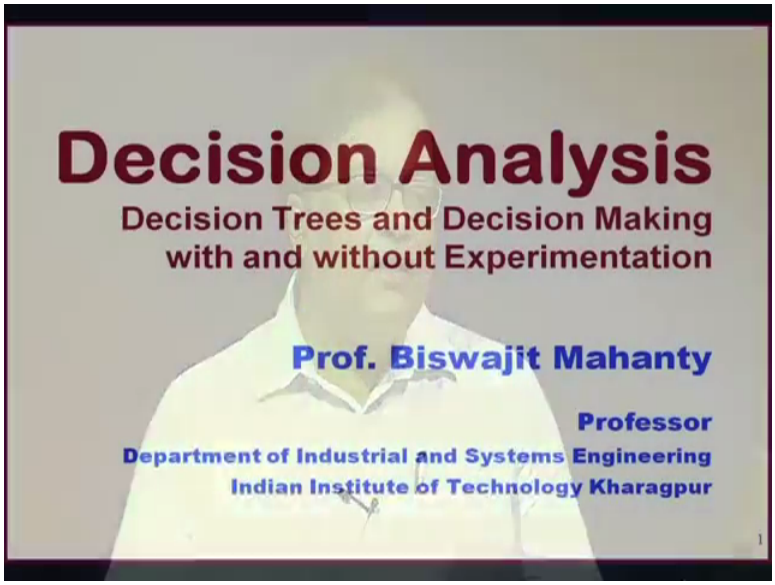
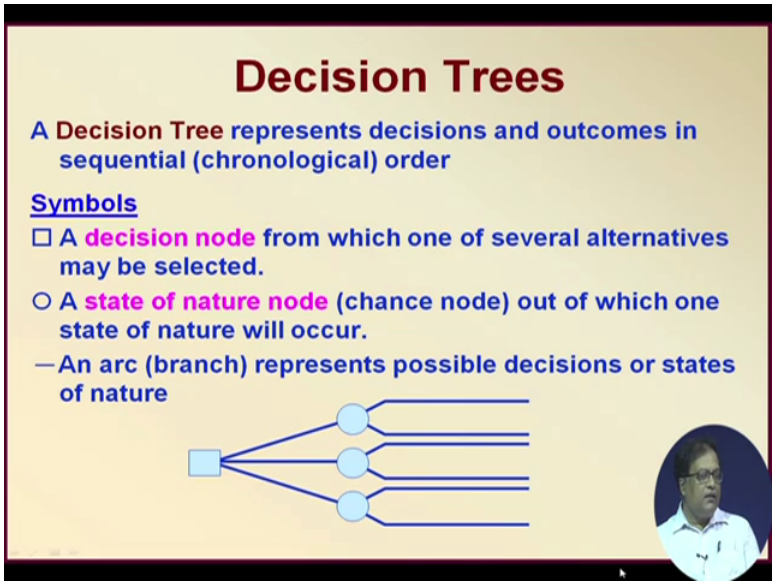
**Decision Modelling.**
**Professor Biswajit Mahanty.**
**Department of Industrial and Systems Engineering.**
**Indian Institute of Technology, Kharagpur.**
**Lecture-07.**
**Decision Tree.**

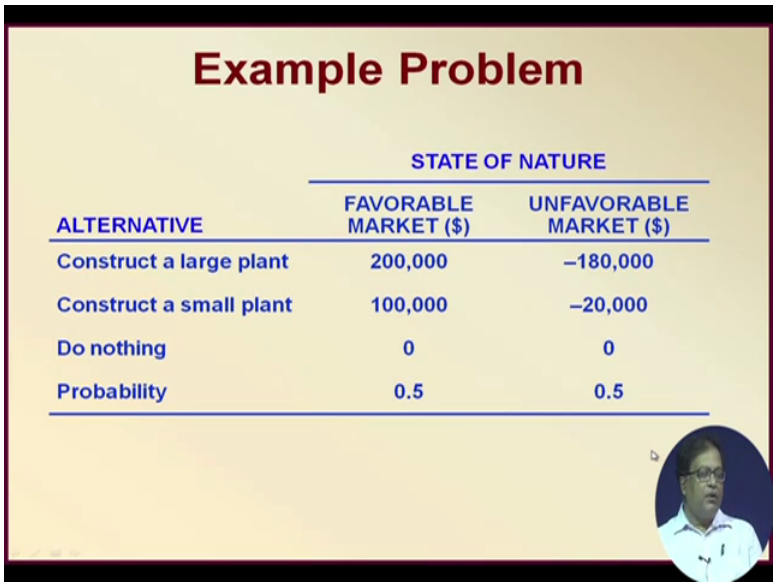
(Refer Slide Time: 0:22)




Today we are going to cover what is known as decision trees and decision making with and without experimentation. In the previous week we have seen the decision analysis, the various aspects of decision analysis, particularly what is decision analysis , the specific idea of payoff matrices, the value of information and thereafter we have also seen the probability concepts and particularly the Bayesian analysis. So how all of these combine together and we can actually analyse a decision-making situation particularly with and without experimentation that these are going to be covered, will be covered in the subsequent classes.

(Refer Slide Time: 1:20)




Now the 1st topic that we are going to have is what is known as the decision tree. A decision tree represents decisions and outcomes in a sequential or chronological manner. Now as you can see in that picture there are certain nodes that will be available on the decision tree. The square node, that we can call what is known as the decision node. The decision node is from which one of the several alternatives may be selected. So as you know any decision problem, the 2 things, one, the set of alternatives amongst which we actually make a decision and the 2nd is the states of nature and these states of nature sometimes also called as chance nodes out of which one state of nature will occur.

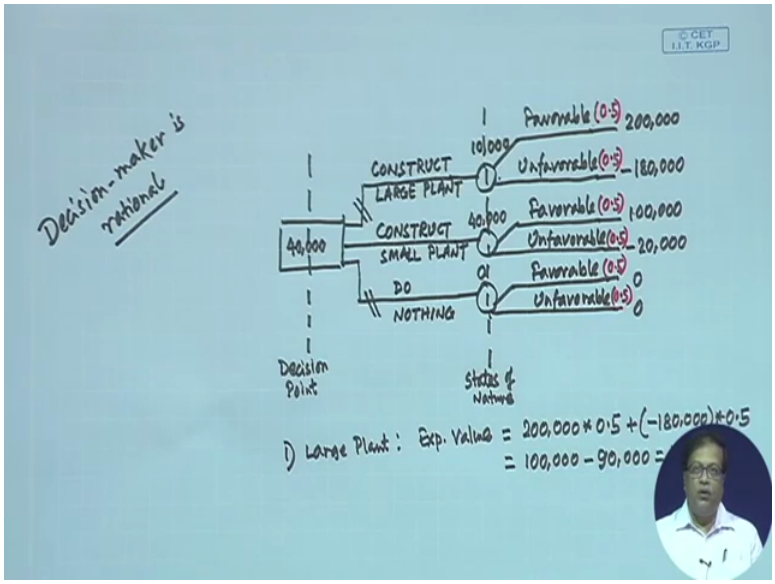
But why they are states of nature, because on this we do not have any control. The decision alternatives are within the decision maker's choice but the states of nature are not within the

choice of the decision maker. Then the arc or the branch represents possible decisions of states of nature. So that is how the structure will be and let us see a particular problem, this is the problem that we have dealt with previously. Suppose there are 2 states of nature, one is the favourable market, the other is the unfavourable market and the payoffs are given, there are basically 3 alternatives, we can construct a large plant or we can construct a small plant.

If we construct a large plant, the payoffs will be 200,000 or - 180,000 depending on whether the market is favourable or unfavourable. On the other hand if we construct a small plant, we can get what is known as either 100,000 or - 20,000 as payoffs depending on whether it is favourable or unfavourable market. And there is a 3rd alternative also, if you do nothing, then we are not going to get any payoffs whether it is favourable or unfavourable market. Now with these and there are some probabilities, there are 0.5 each of favourable market and unfavourable market.

Please remember probabilities are a priori or prior probabilities. What do they mean, that if we do not conduct any experiment, then before that experiment these are the probabilities of the favourable market or the unfavourable market. Now suppose we have to construct it to this payoff matrix to a decision problem, then how would we go about it?

(Refer Slide Time: 4:27)




What we need to do, we know to 1st of all make a decision point. So this is our decision point, so at this decision point basically we have 3 arrows will be emanating from here, maybe we can put it like this, these are the 3 arrows. What are they, these are our 3 alternatives, what are they, the 1st one is construct large plant. The 2nd one, construct small plant, 3rd one, do

nothing. So these are my decision alternatives. Now for each of the decision alternative, there are 2 states of nature. As I said that the states of nature will be shown by circles, for each of them there are 2 states of nature.

And these states of nature are one is favourable market, unfavourable market. Now, after this we have to write the payoffs. So what are the payoffs, as we can see that the payoff for the large plant and if we have a favourable market, then the payoff is 200,000 and if it is unfavourable, then the payoff is - 180,000. And for a small plant, these are 100,000 and -20,000 but if it is do nothing, then the payoffs are 0. Now each of these also have some, the states of nature has probability, we write those probabilities here. So all these probabilities we know are 0.5 and 0.5.

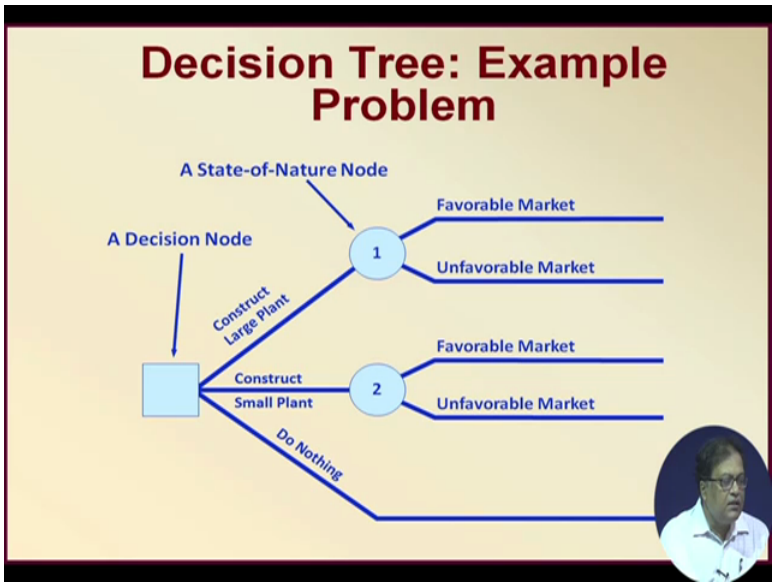
Now if you recall in our 1st or 2nd class, we had dealt with the types of decision makers. So you know again here it definitely depends on what is the type of the decision maker. Now if the decision maker is let us say rational, then we can expect that he will be an expected value operator. Obviously he is pessimist, he would take the, he will ignore the probabilities and would like to believe that worst state of nature is going to happen. But anyhow we are assuming that the decision maker is what is known as rational. So decision maker is rational.

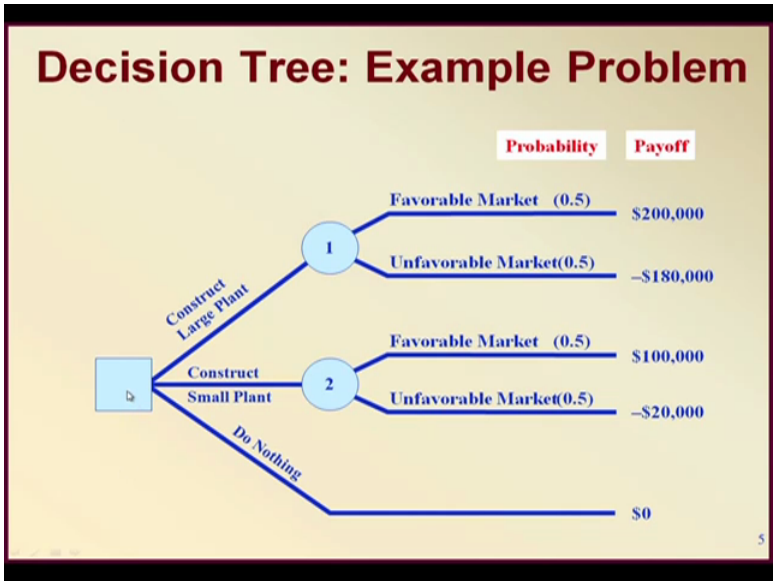
So when decision maker is rational, then we can calculate what is known as the expected values for each of the situations. So what will be the expected value if we construct a large plant. 1st of all you look the properties are 0.5 and 0.5 for favourable and unfavourable market and these are the payoffs. So what would be the expected value? Expected value for the 1st case is let say case 1, that is for the large plant, the large plant expected value would be 200,000 multiplied by 0.5 + - 1 80,000 multiplied by 0.5. So this will be 100,000 - 90,000 equal to 10,000, right. So this 10,000 we actually write here.

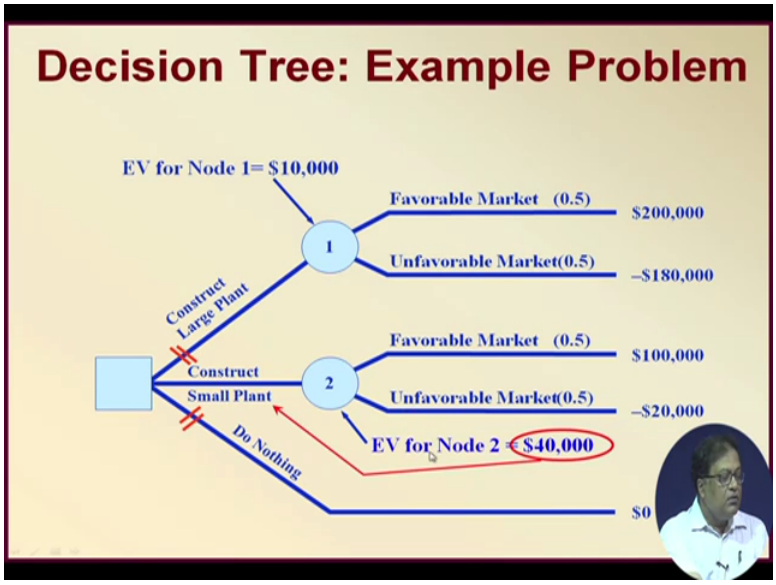
So this 10,000 is the payoff at state of nature and similarly for the 2nd decision, that is construct a small plant, we are saying that again it is 100,000 and -20,000, so it will be 50,000 - 10,000, the expected payoff would be 40,000, right. So this is the 2nd one and what would be, because both are 0, so obviously the expected payoff will be 0. So if we really now evaluate the expected payoffs, at large plant it is 10,000, small plant 40,000, do nothing 0. Therefore the highest payoffs in this case is the 40,000, so we expect that kind of payoffs to obtain, that is 40,000 which we write at the decision point.

And therefore the other payoffs we cross out, so by looking at this diagram it is very clear that it is advisable to construct a small plant because that is giving us the higher payoff. So once again you know we let us look at what is that in the decision tree, we have a decision point, right and the other we have the states of nature. Right, so here we have the states of nature, here we have decision point. Now this is a simple problem, there is a single decision point, in a more complex problem we may have a number of decision points.

(Refer Slide Time: 12:01)



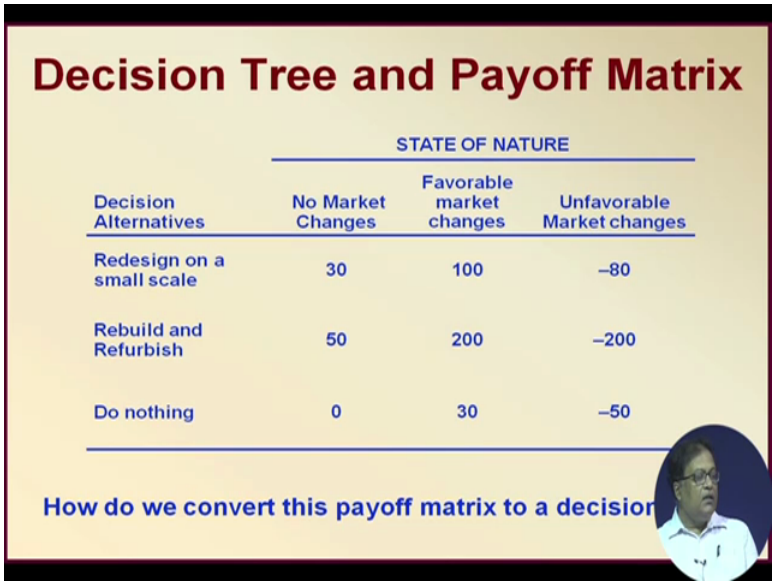






So let us look at how all of these look, you know just look at this example problem once again. This is exactly what we have done, there is a decision point, the decision node and the state of nature node and the 3 decisions are shown and those other states of nature. At the next level we have shown the probabilities, that is 0.5 and 0.5 and we have also shown the payoffs, that is 200,000, then -1 80,000, 100,000, -20,000. And these are the 2, they are the states of nature and this is the decision point. At the next stage we are computing what is known as the expected values, so already we have computed, so again it is a revision, the expected value of node 1, 10,000, expected value at node 2 40,000 and here because both are 0, then this node is a limited but we can also keep if we like, like I have shown in the example.

Now at the last stage, you see these 2 are not there, right. So finally therefore here we can write 40,000 and this is going to be our decision. So this is how decision tree problems are handled, particularly in the simple case.

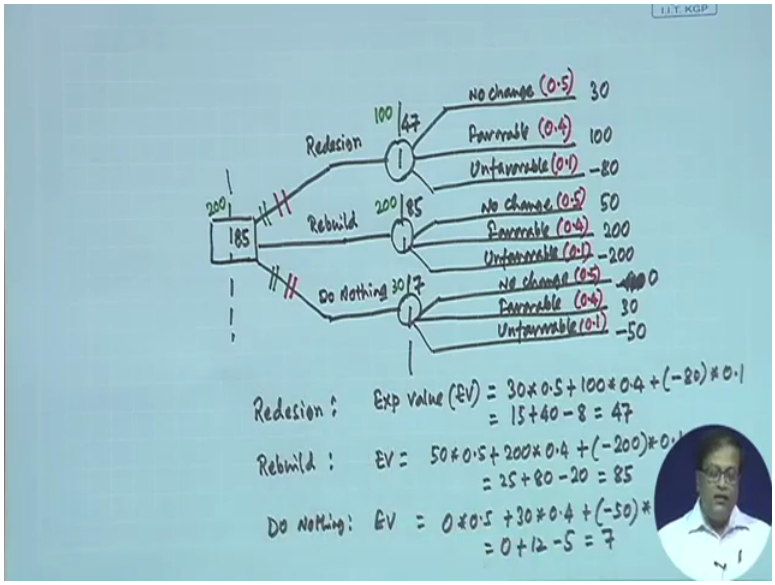
(Refer Slide Time: 13:29)




Now here, you know suppose look at this problem once again, the decision tree and the payoff matrix. So this is a payoff matrix we have considered earlier, suppose it is a question that how do we convert this payoff matrix to a decision tree. If you again recall, there are 3 decisions redesign on a small scale, rebuild and refurbish and do nothing and those are our payoffs. So suppose we have to convert this particular decision tree, you know payoff matrix to a decision tree, then how do we go about?

Let us look at this problem also. So again the 1st question that comes to our mind that what are our decision point or decision node. How many decision nodes should be there? Basically since there are 3 decision alternatives, there will be a single one because there is no secondary decision alternative. And there are 3 states of nature favourable market change, unfavourable market change and no market change.

(Refer Slide Time: 14:49)




So again we put a decision point or decision node and here we have the 3 decision alternatives, right, so 3 decision alternatives. So what are those 3 decision alternatives, let us write them down. 1st one is redesign on a small scale, right, the 2nd one, rebuild and refurbish, so let us call it rebuild and the 3rd one is do nothing. Right, so these are our 3 decisions and for each of these decisions, there are 3 decision points, 3 states of nature, not decision points, so there are 3 states of nature. So we can put them here, right.

And these are 1st of all no market changes, no change, then there are favourable market changes and unfavourable market changes, same thing for all the options, right. And the payoffs also should be put, the payoff for the 1st one 30, 100 and -80, the payoffs for the 2nd one 50, 200 and -200. And the 3rd one is -80, sorry, the 1st one, redesign on a small scale 30, 100, -80, 50, 200, -200 and this one is not -80, this one is 0, 30 and -50. Right, so let us check once again, the redesign option, the payoffs are 30, 100 and -80, the rebuild option, 50, 200 and -200 and do nothing 0, 30 and -50.

Now we also need some probabilities that probability, let us assume some probabilities here. And let us say that these probabilities are 0.5, 0.4 and 0.1, right, so these are probabilities 0.5, 0.4 and 0.1, right. Now what we need to do, we need to calculate the expected values for each of the cases. The 1st case that is redesign the expected value EV would be 30 into 0.5 + 100 into 0.4 + - 80 into 0.1 equal to 15 + 40 - 8, so 55 - 8 equal to 47. So we write 47 as the expected value here, right. 15 + 40, 55 - 8, 47.

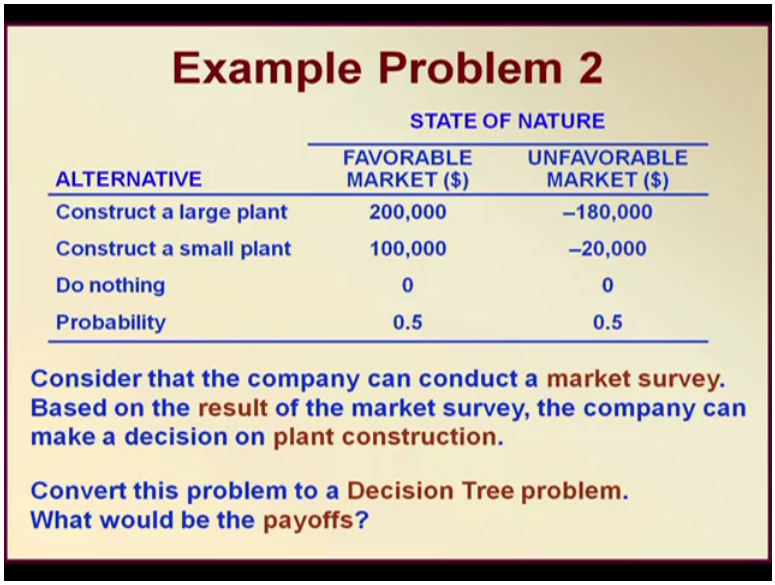
So what is the expected value for rebuild? For rebuild option, the EV would be 50 into 0.5 + 200 into 0.4 + - 200 into 0.1, again 25 + 80 - 20, this will become 85, so we put 85 here. And in case of do nothing option, EV will be 0 into 0.5 + 30 into 0.4+ - 50 into 0.1 equal to 0 + 12 - 5 equal to 7, so 7 would be the payoff for do nothing. Now let us look, what we have done, we have computed for each decision option the payoffs in terms of the expected value. So the expected values are 47, 85 and 7. So since 85 is the highest, it is expected that one would go for the rebuild option and the other 2 options, that is redesign and do nothing will not be taken.

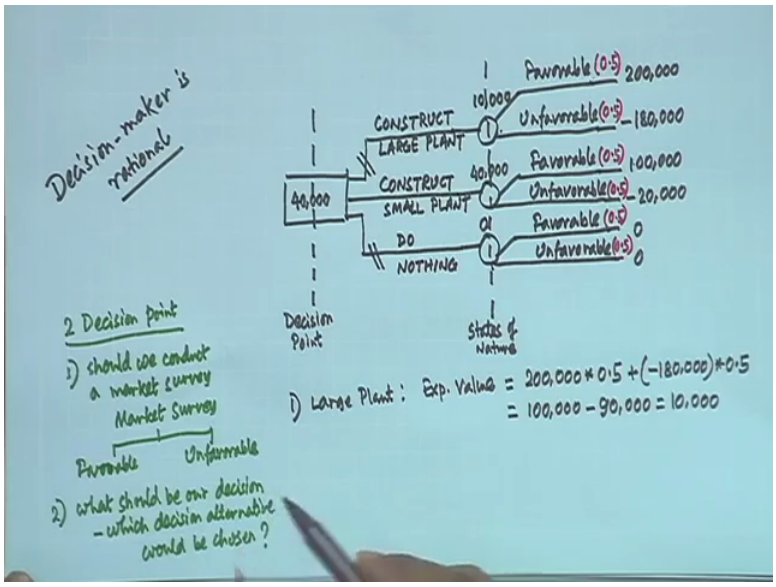
So those options we are removing by crossing and by looking at this decision tree one can understand that the people would be preferring what is known as the rebuild option because it is giving the maximum payoffs. An interesting question that comes, Supposing the person is a maximax operator, what would be his decision from this particular you know decision tree? See, very interestingly what a maximax operator would do, the maximax operator is optimist. So the optimist, he would not believe these probabilities. What are these probabilities, the 0.5, 0.4 and 0.1.

So he would like to think that if redesign option is chosen as a decision, the best state of nature would come. Which one is the best state of nature, that means where we are getting the payoff of 100. So in this case the optimist payoff, I am writing in green would be what is known as 100. What would the optimist would choose for the 2$^{nd}$ decision option, it is the rebuild, so you can see again, the best state of nature, the best payoff is obtained under favourable which is 200. So obviously here the payoff would be 200 and the 3$^{rd}$ option where do nothing, again the payoff would be 30 because that is the best state of nature.

So the payoffs for the 3 decisions would be 100, 200 and 30. So since 200 is the best option in this case the payoff would be 200 and the cross out the other 2 here and again the rebuild would be the choice. So these are interesting to note that depending on the type of decision making these calculations are going to be different and the person may choose different kind of payoff. So this is about the decision tree for simple problems but what would happen, suppose we have a slightly more complicated problem.

(Refer Slide Time: 23:57)




Let us look at this example problem 2. So if you look at this particular problem, again we find out that this is a problem that we have already done, that there are 3 alternatives, construct a large plant, a small plant and do nothing and those are the payoffs. But suppose there is an additional option, what are those additional options? Consider that the company now can conduct a market survey. Based on the result of the market survey, the company can then make a decision on plant construction. Suppose we have to convert this to a problem of a decision tree, I mean through a decision tree construction. And what would be the payoffs?

So in this particular situation, there are something more to think over. And what is that something more that we have to think of? Not only we have to look at you know the payoffs that are going to happen under the a priori situation, that is the probabilities are which are 0.5

each, but you know we have to understand that when we conduct a market survey, the market survey would give us some results and based on those results the probabilities could be modified. So 1st of all you know, even look at the decisions, what would be our decisions?

The 1st decision would be that whether to do the market survey or not, so let us let us try to think of that what are the things that are basically coming into the picture. So this is the problem that we have taken up earlier, our original decision was construct a small plant. The construct a small plant with expected value we have got is 40,000 in this particular case for a small plant because that is what we have got, 0.5 into 100 was 50,000, - 10,000 and 0, that was 40,000. So the question is that anyhow we are constructing a small plant at large plant, you know is market is coming out to be favourable, we can get as high as 200,000.

So we have definitely missed out there, we are happy with only 40,000 and which can go to a maximum of 100,000 if the market is favourable but it can lead to loss also. But the question is that suppose the market is favourable and we construct a large plant, we can get as high as 200,000. So should we take the chance? So why leave it, you know before even considering that? So those are the, that is the option before us. So therefore you see a mother decision points then, there 2 decision points to really consider. What are those decision points, 1, the 1st one is, number-one, should we conduct a market survey, that is the question, right.

So should we conduct a market survey, now if we conduct a market survey, then 2 types of answers can come. The market survey, so let us put it here that we do the market survey, the market survey can tell that whether it is a favourable mark it or whether it is an unfavourable market. Please remember that the market survey may call it favourable but we may not be able to 100 percent depend on it, that is another issue. So if the market survey prediction is favourable, then again the 2nd decision point is what should be our decision. In another language which decision alternative would be chosen, right?

So basically the problem becomes like this, 1st of all whether to conduct the market survey, now the market survey results me, to be favourable or unfavourable, if it comes favourable, then which decision alternative would be chosen and if it comes out to be unfavourable, which decision alternative would be chosen, right. So the problem is more complex and requires certain other conceptual understanding such as the Bayesian analysis which we have discussed earlier. So we stop here for this class and in our next class we shall take it up. Thank you very much.