Decision Modelling.
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Lecture-06.
Bayes Theorem.

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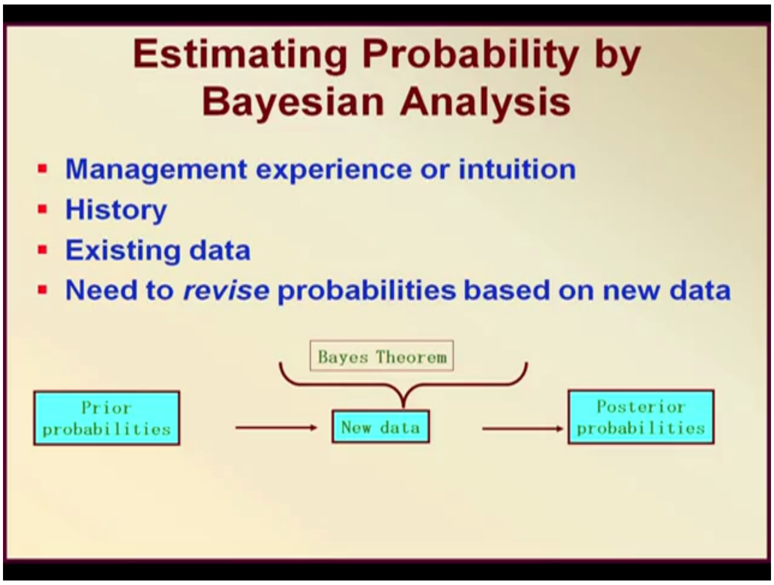

Today we shall start a topic, that is estimating probability by Bayesian analysis. The Bayesian analysis is a very very important component in today's world and it is used extensively in many different situations, particularly on the decision situations. So what is it, what is so important about this Bayesian analysis? If you look at this particular slide, basically that is something called a management experience or intuition, there is a thing about history, then is existing data and there is a need to revise probabilities based on the new data.

So it is like the kind of probability which exist can be called as prior probabilities and based on the new data on the Bayesian theorem, what we get is the posterior probability. So one hand we have the prior probability, one hand we have the data and on that data we get what is known as the posterior probability. In fact let us straightaway jump to an example and you know let us let us look at a problem on Bayesian theorem, Bayes theorem and try to understand what it is.

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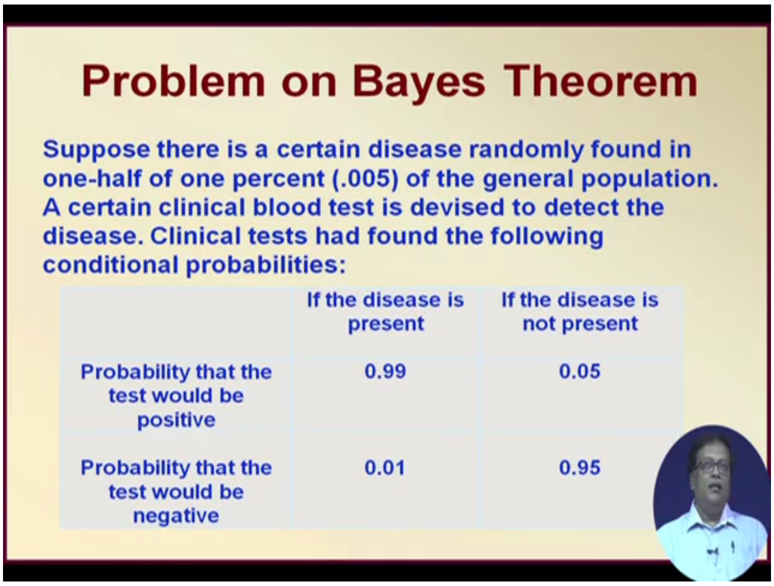

The case is something like this, suppose, there is a certain disease randomly found in one half of one percent people, that is 0.005 of the general population. A certain clinical blood test is devised to detect the disease, right. The clinical tests had found the following conditional probability. What are they, if the disease is present, probability that the test would be positive is 0.99, right and probability the test would be negative is 0.01. If the disease is not present, then 0.05, you see that the probability that the test would be positive and 0.95, probably that test would be negative. Now carefully look at these probabilities.

The test has been devised, whenever we devise a test, the test has to do 2 purpose, number-one, it catches the people who have the disease but it should not catch the people who do not have the disease. Most often the 1st task is done pretty okay, in this case 99 percent probability that person is having the disease and test is finding him positive. But even if the person is not having busy, then also there is a probability that the person catches the disease. So what is the a priori probability?

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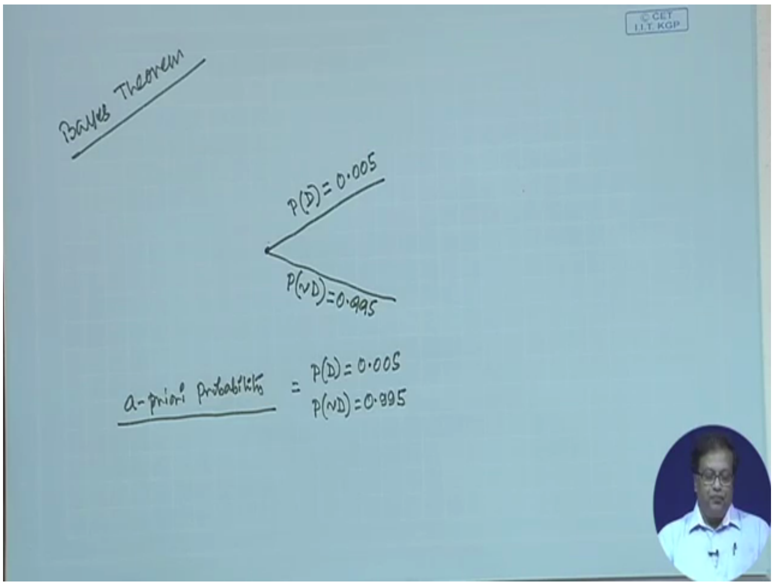

So you see if I if I draw a diagram in the which is actually is a form of you know decision tree, then for the Bayes theorem case, let us say that the probability of the disease, what is the probability of disease? If you look at this, then we say that one half of one percent, then 0.005, that is the probability of disease. So what is the probability of not having disease, right, that is 0.995. What are these, these are called a priori probabilities, a priori means before, the probability which are known before our Bayesian experiment or before our tests.

So these are, a priori probabilities are probability of disease 0.005, probability of not having disease is 0.995, because this is somewhat of a rare disease, right, not many people get it, these are the a priori probabilities. Now the question is that a person goes to a doctor and the Doctor says that you may be having the disease, why do not you do the test. So the person goes, test is made and the test comes out to be positive, right. So the person is therefore very fearful, you know the test has come out to be positive, now what is my probability, that do I really have the disease or I do not have the disease, what is the probability that I have the disease, right, so in view of the fact that the test is positive.

So here a question of conditional probability comes in, right. The person is apprehensive that what is the probability, is it very high, rather high or low, right, depending on that the person has to really think about the future course of action. So what has happened, that we had some a priori from diabetes, probability of disease 0.005, probability of not having the disease 0.995. New fact, a fact has been you know taken into account that is a clinical blood test is done and that clinical blood test has come out to be positive. So let us look at all the options here in the form of a decision tree.

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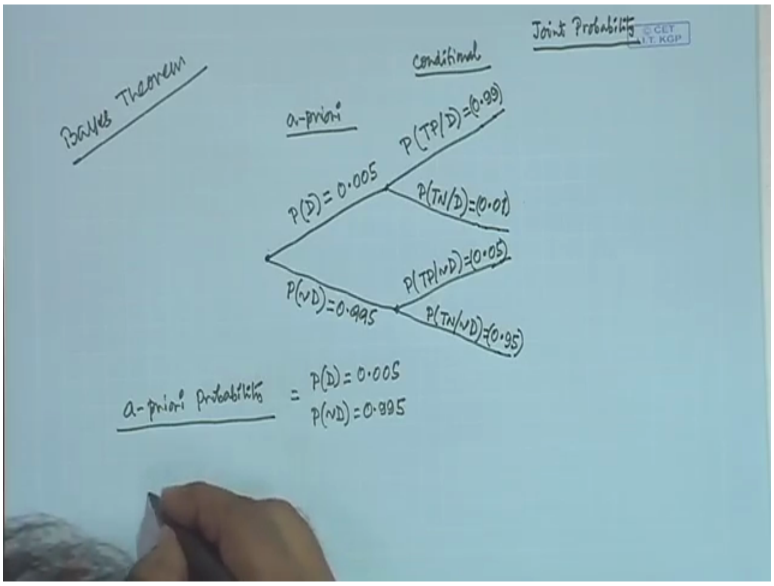

So you see 4 possible things are possible, what are they and you now see that these are our a priori probabilities, right, so a priori probabilities I have written here. And here, after that here we shall write what is known as conditional probabilities, right. What are some conditional probabilities, there are 4 possible conditional probabilities. One is that the test is positive, right so that we are writing as probability of T, test positive, let us say Tp, probability of test positive, given a person is having disease, this is one conditional probability. Another conditional probability is probability of test negative given the person is having disease because this side is disease.

This side is probability of test positive, given he does not have the disease and this side is probability of test negative given that he does not have the disease. What are these probabilities? Now look at the slide once again. The slide says that probability of the test would be positive if the disease is present, so if the disease is present, that is the given D side, that is 0.99, is it all right. The probability that test would be negative is 0.01. So that means that if the test is positive, so now here probability of Tp given D is 0.99. Right. That means if the disease is present, test would be positive for 99 percent of the people.

And if the test is negative that would be 0.01, right. Again look at the slide, you see if the disease is not present, that means not D, that probably that the test would be positive is 5 percent and probability that test would be negative is 95 percent. So let us look at those figures also, so probability of test is positive given that there is no disease is 0.05, test is negative given that there is no disease is 0.95. So these are some figures that we have, right. So 0.99, 0.01, 0.05 and 0.95, this is all right. Now once we have these kinds of probabilities, what we can do by using these a priori and the conditional probabilities, what we can do, we can find out the joint probabilities, right.

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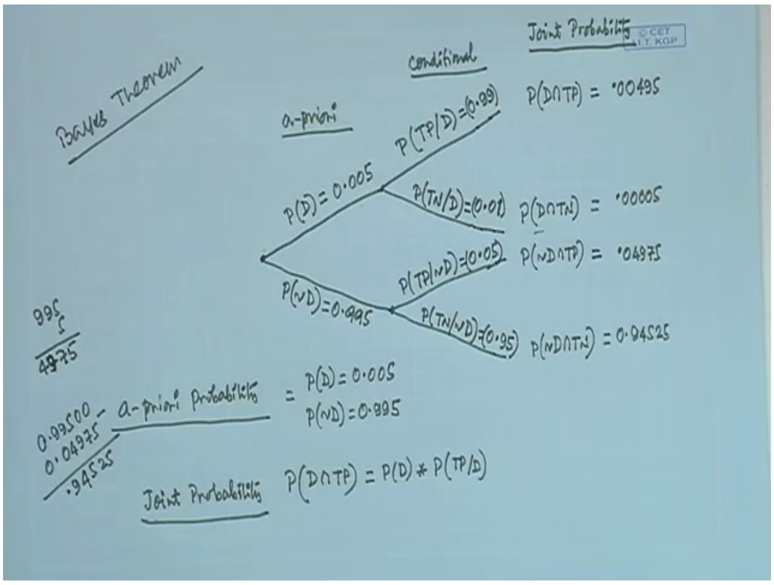

So 2 types of probability we have seen, the a priori probabilities which are before the test is done, then we have some conditional probabilities if the tests are present. Now if I multiply by the a priori probabilities and the conditional probabilities, we get another set of probabilities which will be called the joint probabilities. What are some joint probabilities? The joint probabilities are basically the multiplication of the a priori probability and the conditional probability. So let us write down the what is the joint probability of joint probability of the disease and test positive, right.

Probability of the disease and test positive, both, both are happening, that probability of disease and the test positive, this is given by probability of disease multiplied by probability of test positive given the disease is present, right, it is a multiplication. So this particular multiplication gives the joint probability of disease is available as well as the test is positive. So this, the 1st one is the ability of D and test positive, this side we have, there will be 4 such joint probabilities, probability of disease and test negative, here probability of no disease and test positive and this side probability of no disease and test negative. So these are the 4 joint probabilities which are we going to get.

So what will be this figure, so 1st of all, if you multiply 99 by 5, then how much you get, 99 by 5 will be 495, right, 495 and there are how many, 5, so equal to 0.00495, so 5, so this is how it is. And here it will be equal to 0.00005, right, all of these, so if you add these 2, you will get 0.005, right. So 0.005 into this. On the other side is 0.995 into 5, so basically this figure would be 0.995 + 0.05, much is that, so this figure will be, suppose we do small calculation, 0.995 by 5, so 5 into 2, 45, 4 and 47, right. Into 5, 2, 4, 49, so 4975, so 4975. So 0.04975, right, 0.04975.

And the remaining portion would be this, so basically 0.995 - 0.04975, so this is the, if I deduct, then I get 25, 5, 4 and 9, right, so 0.94525. So no cross check, this is 5, this is 2, this is 5, this is 4 and this is 9. So this one becomes 0.94525. So what are these probabilities? These are some joint probabilities that what are the probabilities that there will be disease and test positive, this is 0.00495. Probability of disease and test negative, 0.00005, same thing about here. So these are some joint probabilities. The next thing what we need to do is let us see what are we interested in.

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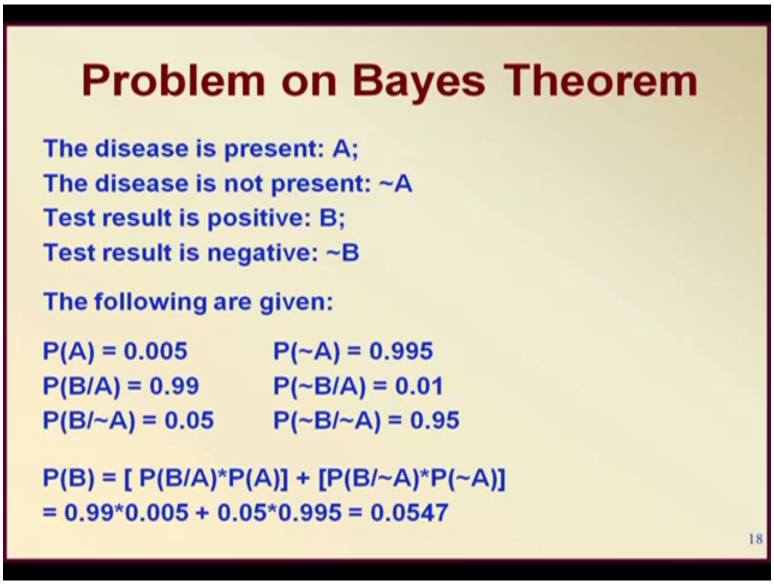

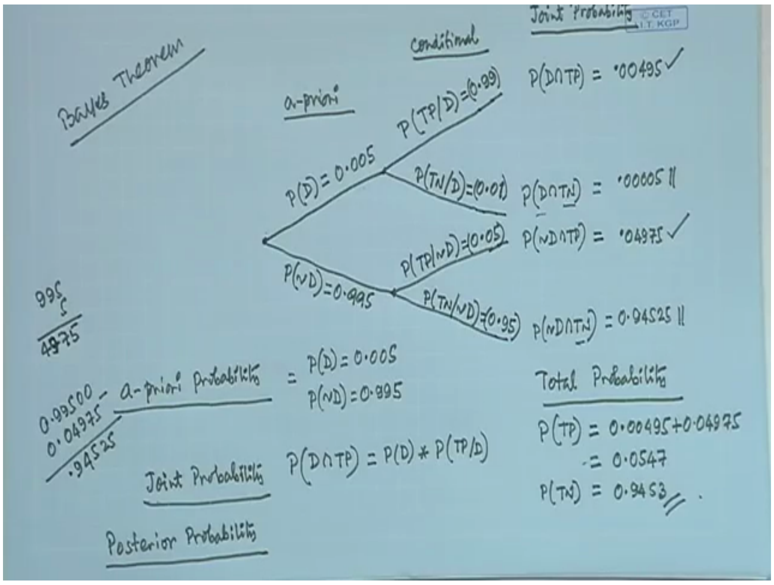

So we are interested in, so let us look at some of the calculations which are also done here, these are the here, so you can do can look at here that we have all these calculations. The probability of A, these are all that, say disease is positive, so similar things are written here. Now you see if we add, you know what is the, what is the total probability, now there is another term which is known as a total probability. The total probability is nothing but the addition of, so what is the total probability of the test result is positive, in this case probability of the test positive, so in this case TP.

What is the probability of test positive? See there are 2 terms, one is, this is, see look at, this is one time where the test is positive, this is another term where, sorry, there is a is positive. 1 is case where test is positive with disease and there is a test is positive without disease. So if you add them, that is what you will get a probability of the test positive, this is all right. So what is, what will be the test positive, that figure. This would be 0.00495 + 0.4975, right. So these 2 figures when you add, then you will get a figure of 0.0547, 0.0547.

So what is 0.0547? 0.0547 is the probability of test positive, right. So this kind of probabilities are called the total probability, these are the addition of joint probabilities in all those cases where test is positive. Similarly you can also get another term which is the total probability of test negative. Now this would be 0.00005 and 0.94525, right. So how much it would become, it would be 0.9453, right, 0.9453 because these are the addition of this number and this number, because these are cases where you see test is negative. These 2 terms will be attending these 2 terms you add, probability of test positive and probability of test negative, then what you get is 1 again.

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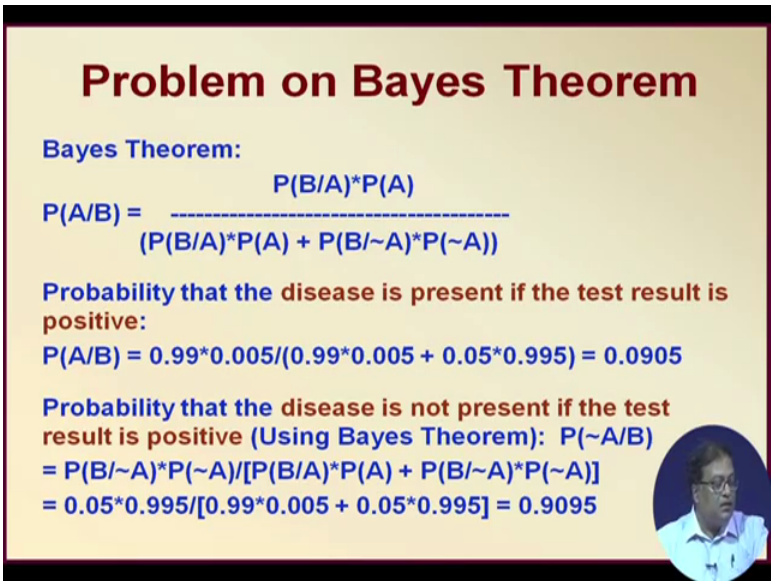

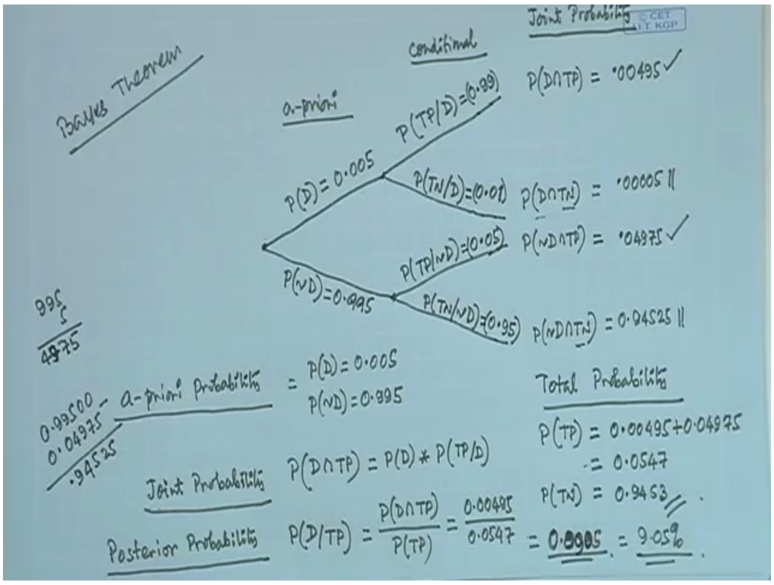

So we have not talked about 4 types of probabilities, one is the a priori probabilities, the conditional probabilities, the joint probabilities where both are present and finally the total probability which you add by taking the test positive and test negative conditions. Now comes the posterior probabilities. The posterior probability which is of concern to us because, please remember what I said right in the beginning that the basic point here is to consider, look at the person, the person was advised by the Doctor to take a test whether he or she has that particular disease or not.

When the test was taken, it was found to be positive. Now naturally the person is apprehensive, now the test is positive, have I got the disease or not, that question comes into that person's mind. So what is that probability that person is having the disease given that, right, person is given the disease given the test is positive, right. Probability of disease are given the test is positive. So you see this would be given by the term, the probability of disease given the test positive and test positive, divided by probability of test positive, is it all right.

So this one would be nothing but 0.00495, which is this term given by the total probability, the probability of test positive which is 0.0547, right, the 0.00495 by 0.0547. So you see that term actually comes out to be, if you look at this calculation, then it comes out to be 0.9095. So this value is 0.9095. So, sorry, I am sorry, 0.0905, so they, this is wrong calculation, 0.0905, right. So this value comes out to be 0.0905, right, so because this is a lower number. So that means there is only 0.09, that is around 9 percent, right, about 9.05 percent is posterior probability that comes out that the person really have the disease given the test is positive.

So this is something very interesting that the person was very apprehensive that the test is positive, whether the person has got the disease or not, but it comes out that through the posterior probability calculation that the person is having only 9 percent chance that whether he or she is having the disease. Now why such a particular result has come, that person was advised by the Doctor, he took the test, test has come out to be positive and then we say that only 9 percent chance of that person having the disease.

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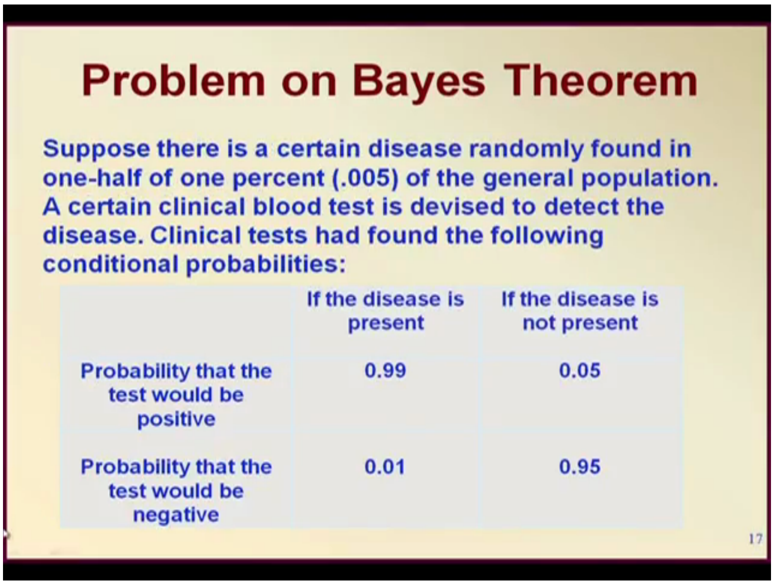

You know this result is so fantastic because please look at this, the 1st slide, it says that the probability that the test would be positive and you see only 0.005 percent, 0.005 people are actually having the disease. That means 99.5 percent people do not have the disease. And the test efficacy is not that good, the test is coming out to be positive, even when the disease is not present, right, at least for 5 percent people. It basically tells therefore if it is a rare disease, it is very important not only to catch people who have the disease but also not to catch people who are not having the disease.

That means this particular probability which was 0.05, you know, when the test would be positive, it would have been very very low, maybe 0.0005, right. So this is what exactly what we find in such situations. Now what it tells us, it tells us about a kind of inferencing system, what is that inferencing system? Suppose a person goes to a doctor, the Doctor tells, it is like, let us an example that a person is having a apprehension that he or she might be having a particular disease, say malaria, he goes to Doctor, the Doctor asks a question that are you putting your mosquito net regularly or not.

He says no, I do not put mosquito net. So you see the original probability which was there in the doctor's mind that the person may be having a malaria disease, now after having that additional fact that a person does not put mosquito net, that probability increases. So using the a priori probability or what is the probability of a given person getting malaria putting an additional fact that he or she does not put mosquito net and posterior probability is obtained, the higher chance of having malaria. Then the Doctor asks for the test, the test comes out to be positive. So again a new fact is added and inferencing is happening, the posterior probability comes up and it may be further refined.

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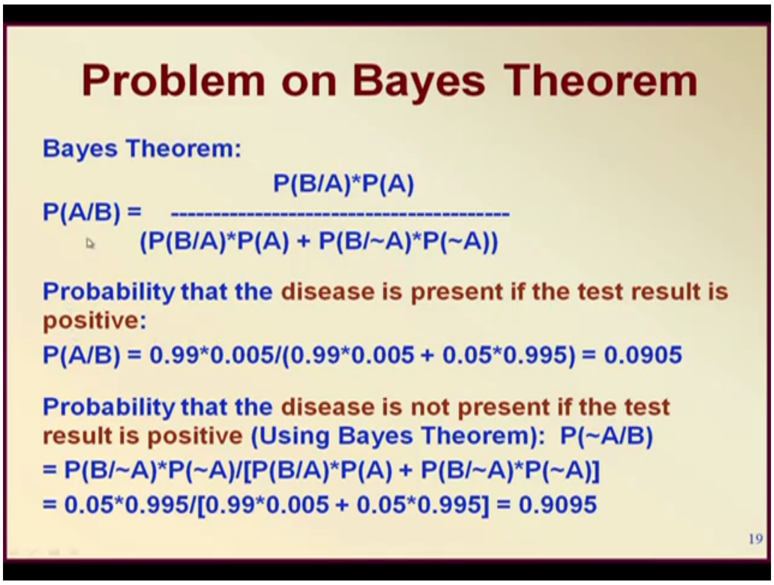

Right, so this is what the Bayesian analysis is all about. Before going further, let us look at the Bayes theorem. See the Bayes theorem, probability of A given B is probability of B given A multiplied by probability of A, that means the joint probability of that part, and multiplied by the total probability, right. So if we say this is one side of the joint probability, this is another side of the joint probability and by that part of the joint probability which we are seeking, because we are seeking A given B, so what is that, you know that A part and the not A part, so both should be added together. And when you do that, that is our Bayes theorem.

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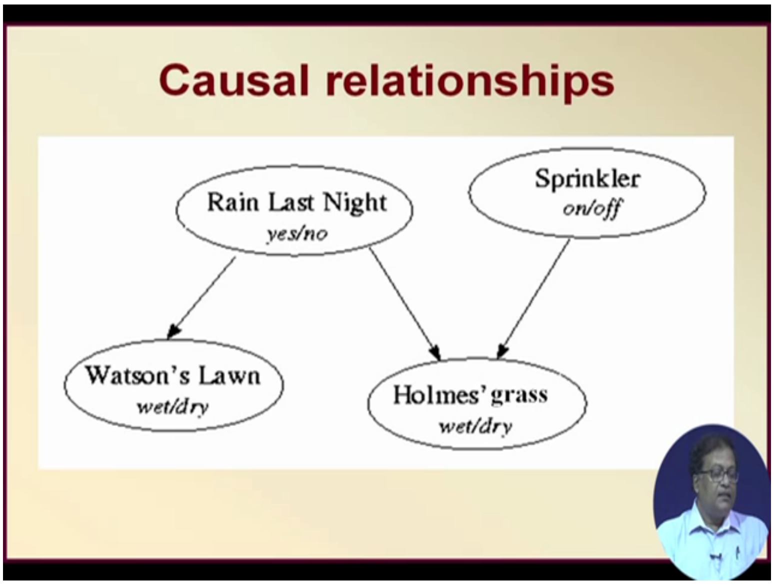

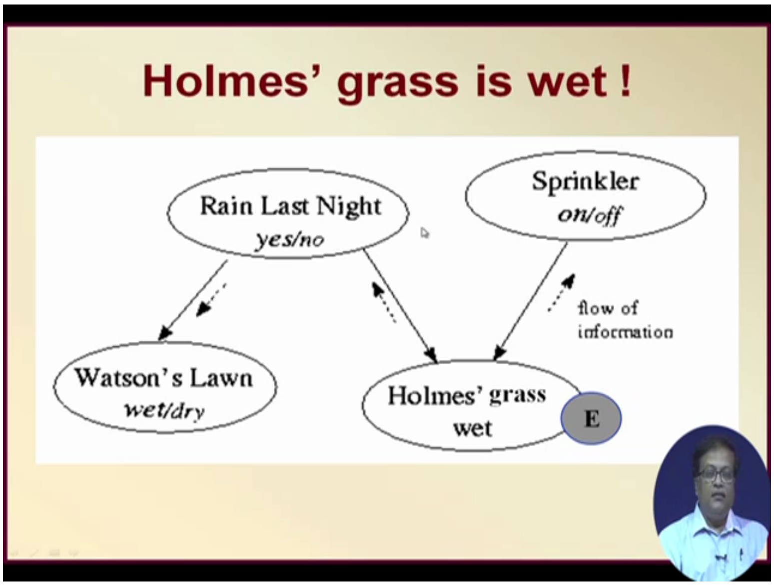

Let us take some more examples. Let us say you know the typical the Sherlock Holmes stories that, let us say Watson and Holmes, you know they are staying on side-by-side on both sides. So one fine morning vary, Sherlock Holmes gets up in the morning and Sherlock

Holmes really finds that the grass is wet. Now obviously the question that comes that you know grass is wet and maybe has he really forgot the sprinkler to be put off, I mean maybe he has not put off his sprinkler the previous night. So what Holmes does, Holmes actually looks at the next lawn which belongs to Watson and what he finds, he finds that Watson's lawn is also wet.

So what is the probability that Holmes has forgotten to put off the sprinkler and Watson is also forgotten, probability less, right, probably not so much likely. So the next thing to find out that whether it was rain last night and suppose it was found that yes there was rain last night, so the here the thing to see is the Holmes's grass is wet and Watson's lawn is also wet, probably it is not due to probability to sprinkler on or off, maybe because of rain last night. And if suppose it was found that it rained last night, therefore Holmes will find out through this experimentation that basically he has not forgotten sprinkler, the probability of that putting off comes down.

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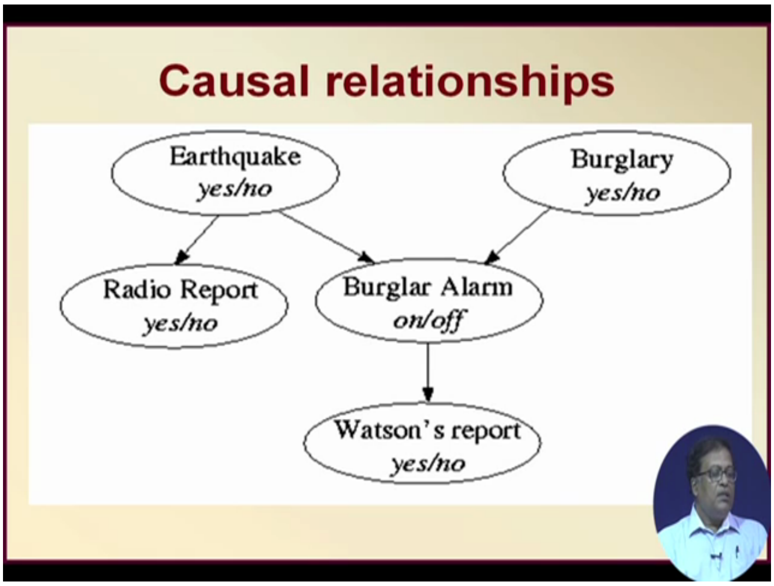

So these are the kind of things that you know you can think of. So you see maybe another example, there is a burglar alarm that has been put and there is a burglar alarm that comes out to be you know going off. So is it really a burglar or something else might have happened? Then it comes to Watson that you know earthquake can also put on the burglar's alarm. So if a radio report was found that yes there was you know an earthquake, so probably that would change Holmes mind in the sense that because of earthquake the burglar alarm might have gone off and it is not truly a burglar has come.

So it is not that it is really taken out but you know the fact that there has been an earthquake and earthquake puts burglar alarm off also is you know helps a person to inferencing.

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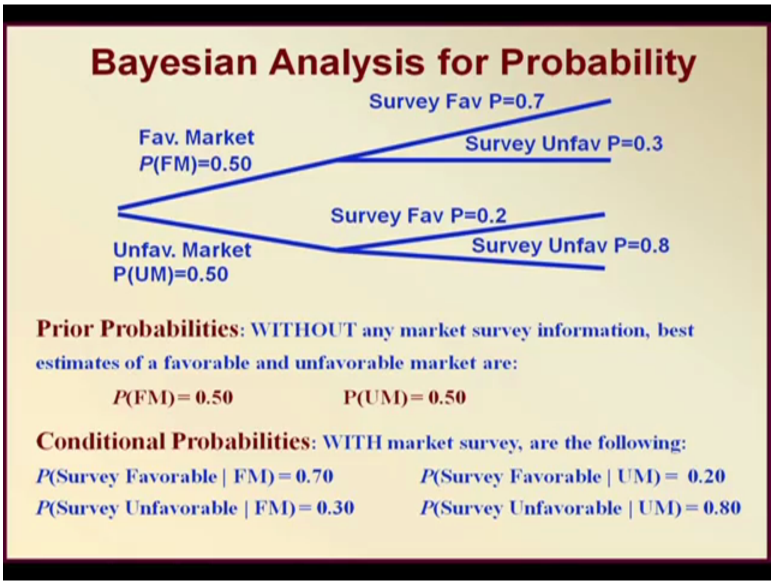

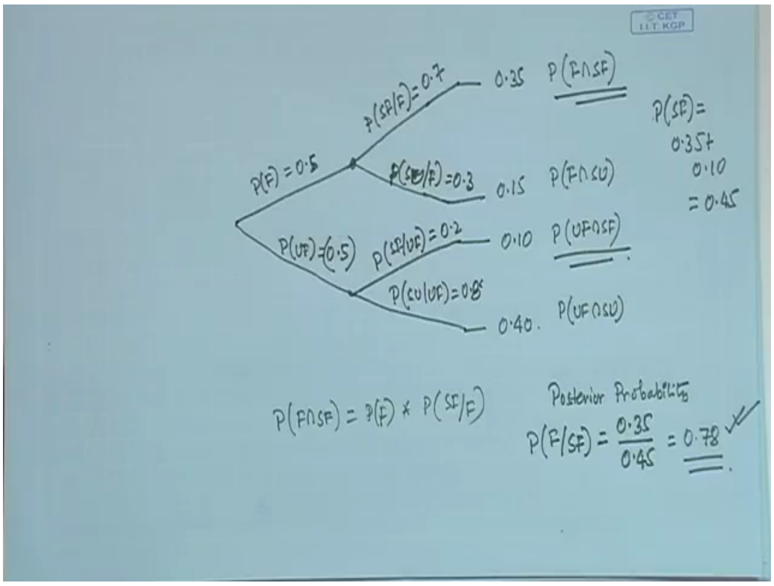

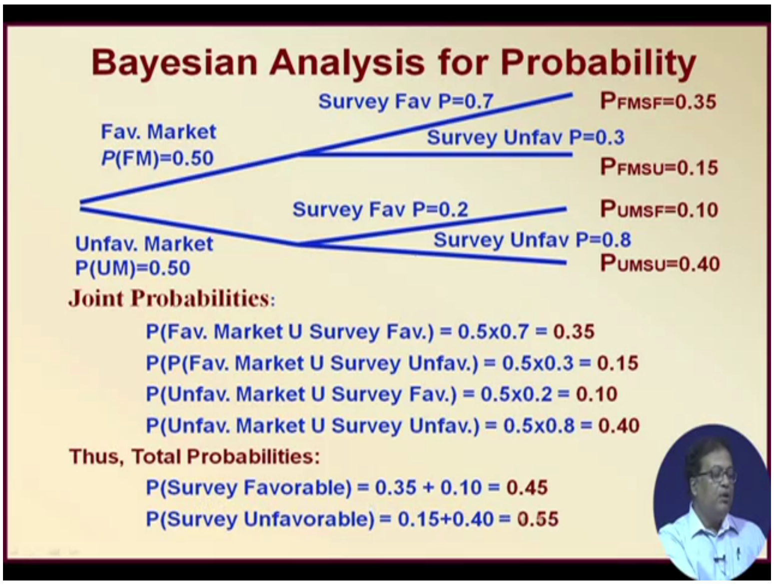

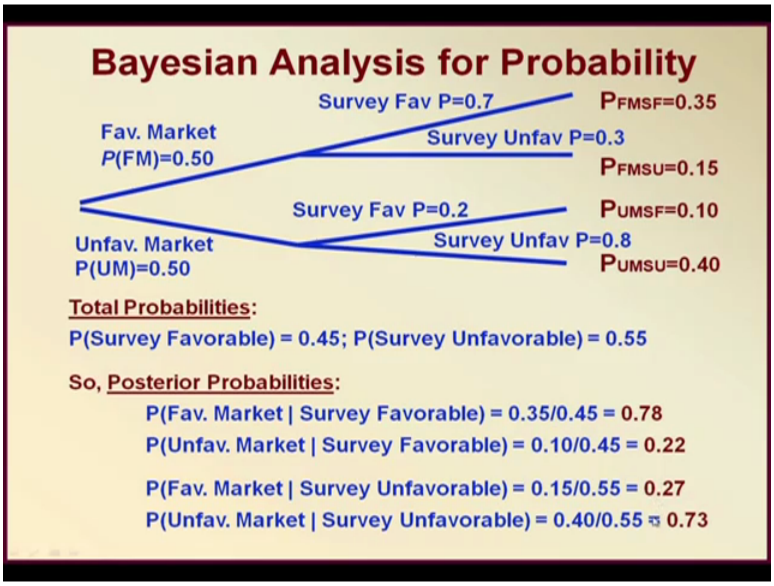

Let us take another example of the Bayesian analysis. So this example goes like this that in a particular region there is you know the people who has, really you know there is a favourable market conditions that is 50 percent and obviously automatically it means that unfavourable market conditions will be 50 percent. Now a survey has been done, what the survey says that if the market condition is favourable, the survey also will become favourable, there is a 70 percent chance. But if there is an unfavourable market condition, that the survey would be favourable, that probability is only 0.2, right.

So now if the survey has come out to be favourable, how does it change the favourable market conditions, right? So these are the prior probabilities, before the market survey, it was, the estimate was a favourable market chance is 50 percent an unfavourable market chance is also 50 percent. Now in the survey, what the survey would actually really look into. Let us quickly do that. So supposing that particular diagram that probability of favourable market is 0.5 and probability of unfavourable market is 0.5, so both are 0.5. Now there is a survey, so probability of survey favourable given the favourable market condition is 0.7.

And probability of survey favourable , survey favourable, sorry survey unfavourable given the favourable market would be 0.3, same thing here that probability of survey favourable given an unfavourable market is the 0.2 and probability of survey unfavourable given unfavourable market is 0.8. So these are the figures and what are some joint probabilities? Let us look at the joint probabilities are 0.35, 0.15, 0.1 and 0.4, right. So this is the probability of favourable market conditions and survey also favourable, this is probability of favourable and survey unfavourable, this is probability of unfavourable and survey favourable and this is the last one.

And these are, see, what is probability of F intersection SF, it is probability of F multiplied by probability of SF by F, right, so these are the probabilities. But now the question is that what is the you know if you look at the 2, these 2 terms, then these are the things, you know if you add, then you get the survey favourable total probability comes out to be 0.35 + 0.10 equal to 0.45, right. So now you can calculate the posterior probability, posterior probability of, probability of favourable given that survey is favourable, right.

So this would be 0.35 by 0.45, right, 0.35 by 0.45. So that is what you know calculations are here and this comes out to be around 0.78. So you see what is this 0.78 essentially means? 0.78 essentially means, if you look at the diagram, the probability of favourable market was 50 percent, probably of unfavourable market was also 50 percent. But after the survey we find the probability of favourable market, given it is survey, because survey, suppose survey has come out to be favourable, that is 78 percent, right. That means if the survey says that it is a favourable market, now we are that much more sure, earlier it was only 50 percent, nowadays 78 percent, right.

So look at the complete analysis very quickly. So here you see we calculate all of these, which are called the joint probability is, 0.35, 0.15, 0.10 and 0.40 by multiplying the a priori probabilities with the conditional probabilities and then when you add the, you know the corresponding items, the survey favourable 0.45 and survey unfavourable 0.55 and then we can take the ratio is to get the posterior probabilities. That means favourable market under survey favourable is 78 percent, unfavourable market is 0.22 percent. And what happens if the survey comes out not favourable?

Then those probabilities get revised as 27 percent and 73 percent, right. That means the Bayesian analysis tells that of the survey comes out to be favourable, then probably it is good to assume a favourable market may come in but if the other things happen, then it may go on the other side. So by keeping these facts in mind, a person should be able to make decisions, right. So thank you very much, in the next section we shall see how decision tree analysis can actually help decision-makers to arrive at appropriate decisions by making use of Bayesian analysis, thank you very much.