Course on Decision Modeling
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Mod08 Lecture 40
Shortest Path Problems (Contd.)

So in our last class, we were solving shortest path problems. Now we will continue from there. Very quickly in our last lecture, we have seen the Bellmans optimality principle that if we have reached a particular point, no matter how we have reached, the remaining distances should be optimally found, right. So that is the essential idea and based on which we have an algorithm call Dijkstras algorithm.

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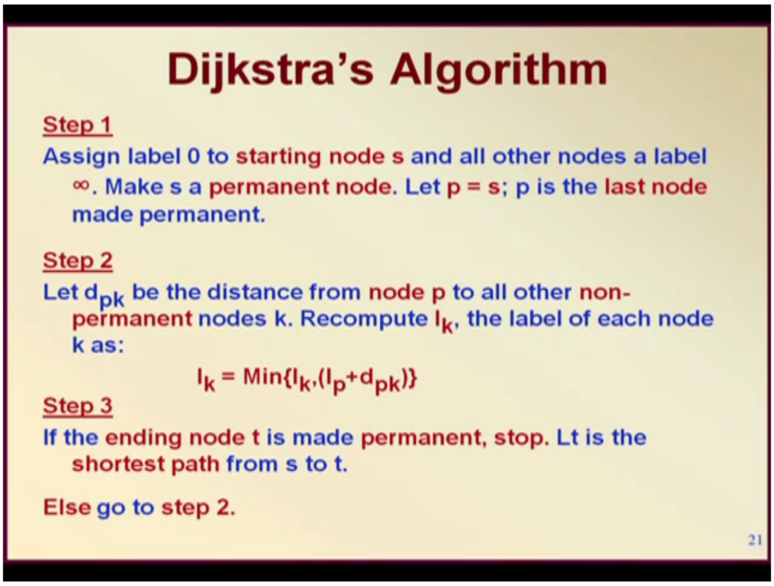

Now this is how the official statement of the Dijkstras algorithm, let us note it down. First of all assign a level 0 to the starting node s and all other nodes a level infinity. Make s a permanent node, let p equal to s where p is the last node made permanent that is the step 1. At the step 2, Dpk be the distance from node p to all other non-permanent nodes k. Recomputed lk means the level of all the remaining nodes as minimum of lk the current level and lp plus Dpk, right. So that is step 2. Step 3, if the ending node t is made permanent then stops. Lt is the shortest path from s to t, is all right, else go to step 2 otherwise keep computing. So idea is that all the nodes should be made permanent when all the nodes are permanent then the Dijkstras algorithm really stops, right.

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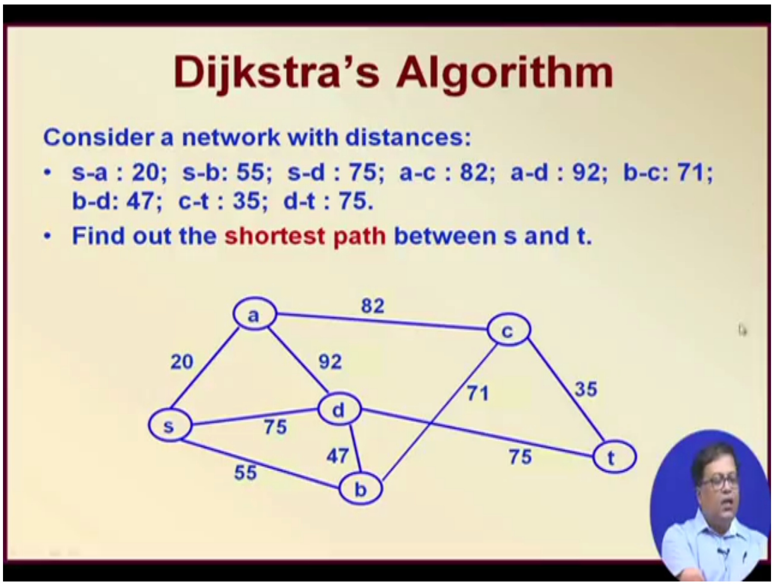

Now let us look at one more problem. So this is a network then we have these distances s to a and a to c etcetera and you know we have to find let us say, the shortest path between s and t. really you see, like here you are having the network shown, a many a time it is not always the network will be drawn for you, right. Basically all you have is a matrix something like a distance matrix that s, a, b all the nodes on one side and the other nodes on the other side and the distances between them that is the matrix that will be available to you, right. So if we have to follow the Dijkstras algorithm how do we go about?

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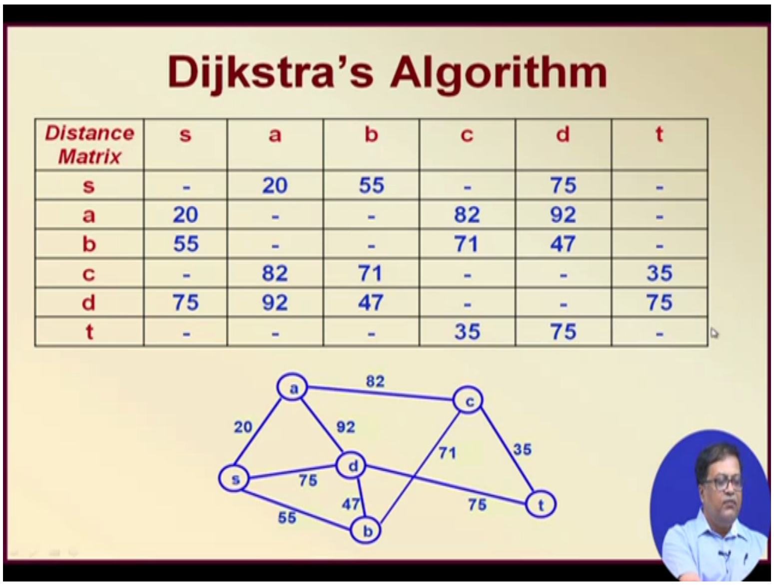

So if you look at it that we create the distance matrix. So here is the distance matrix, this side is the nodes that side is also the nodes and s to a, s to b, s to d and if you see, you know the node s to node a is 20 and node a to node s is also 20, then this particular all the rows, the first row and first column, second row and second column they will be identical, but if the distance s to a or a to s are different then we have different rows and column values. So anyhow this is the distance matrix, we obtain first.

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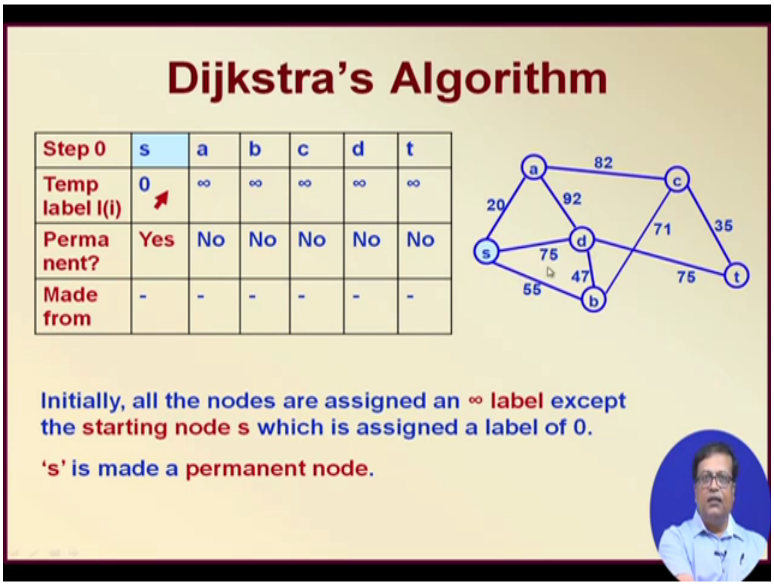

Now after the distance matrix is obtained you see, first we have made s permanent right. s is made permanent and temporary level is assigned to s as 0, then a,b,c,d,t all the remaining nodes they are assigned as infinity as the your temporary level. Now is anything is permanent. Now s is made permanent, so this is s and this arrow means that this is the last node that is to have been made permanent and made from I mean nothing, because after all s has to be made from s what we need or write it, right. So that is the first step.

Now what we should do? Please think, I have already shown one example, in our last lecture. So how what should you do from s? What are the points you can reach? You can reach a, d and b, there are direct paths where direct path is not there, we consider the distance to be infinity and since, infinity as a level is already there, we need not compute them. So what will be the new level of a? You see, it should be the lower of current level of a, which is infinity then 0 plus, because level of s is 0 last node to have made permanent plus the distance. So 0 plus 20 is 20, so 20 is lower of 20 and infinity. So level of a should be now 20 instead of infinity. How much should be b, b should be then 55 and d should be 75.

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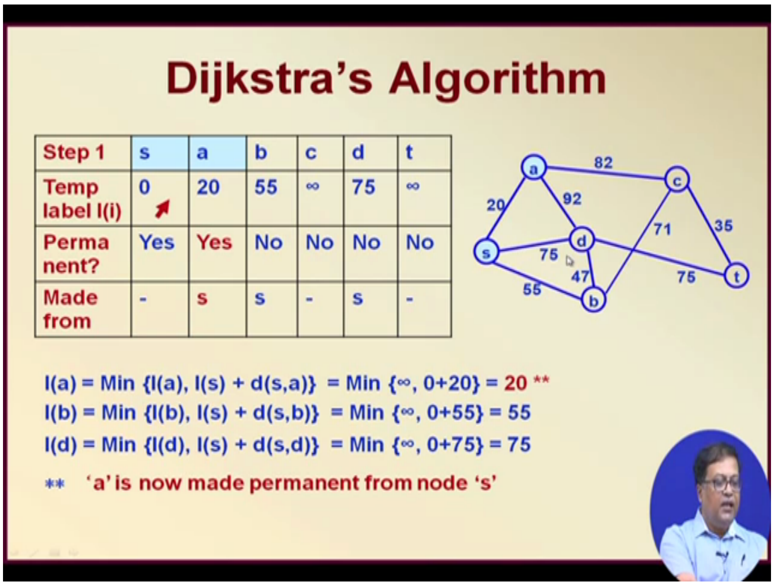

So exactly that is what is done here that 0, 20,55 and 75. Now which one should be made permanent you have to make out of all the node see, 0 is already taken out, because it is already permanent out of the remaining nodes which one is the minimum? A is the minimum. So then it is you know optimal distance from s to a will be 20 and then we make a as also permanent. So a is made permanent and a is made permanent from whom from s, so that made from s note it also. So s and a are now permanent a is the last node to have made permanent right, a is the now made permanent from node s. So a is the last node to have made permanent.

So now the next distances are to be computed from a. So a you see you can reach 2 nodes c and d. So if you go to c then it should be 20 plus 82, 102, but current value is infinity. So which one is lower 102, so we can have a value of 102 now, but what about d. The d value was 75 and if you go from a to d, a value is 20, so it will become 112. So no point exploring that path 75 should remain.

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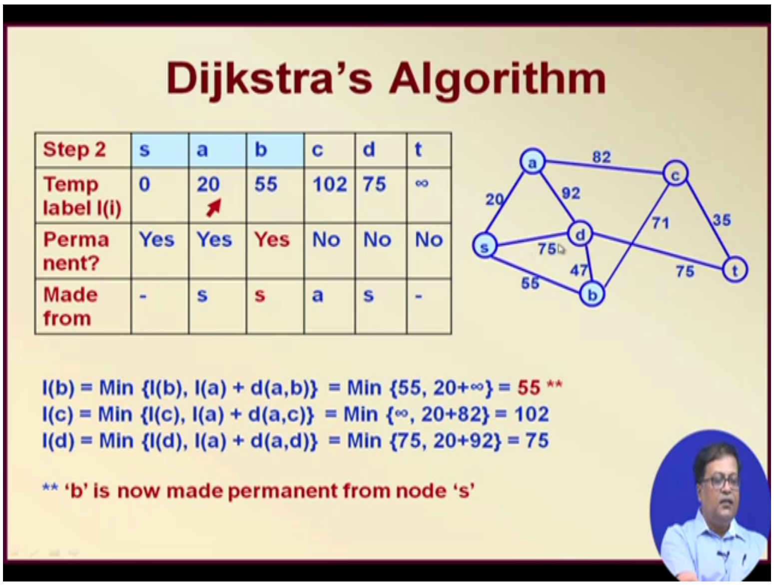

So that is what is shown here that here as remained and c becomes 102. So look we have recomputed c as 102, t still remains infinity last node to have made permanent was a and through which we have computed all these values and out of the remaining values, because these 2 were already permanent. Now 55 is the lowest, so we make it permanent. So now b is made permanent, so now s and b they are made permanent and this is also made from how the b, look here, we computed currently. So this mistake people make sometimes, so at this time we have made d and c permanent.

So you may be thinking that one of them should be now sorry d and c we have to recomputed. So one of them should be made permanent, no it should be the global minimum out of all the remaining nodes. So that is where the dynamic programming comes up, right. so it is note that their recent calculated values only should be made permanent out of all the remaining nodes, see s and a where already permanent out of all the remaining nodes that node should be made permanent, which one is the lowest possible level. So since, the level of b was 55, so b is made permanent. What the point so at the next round, the b is made permanent. So again we recomputed all the values, so you see from b you can reach c you can reach d and you can reach t, you cannot reach t. So if you reach a c, the value is 55 plus 71, which is you know higher than 102. If you reach d again that value is higher than 75.

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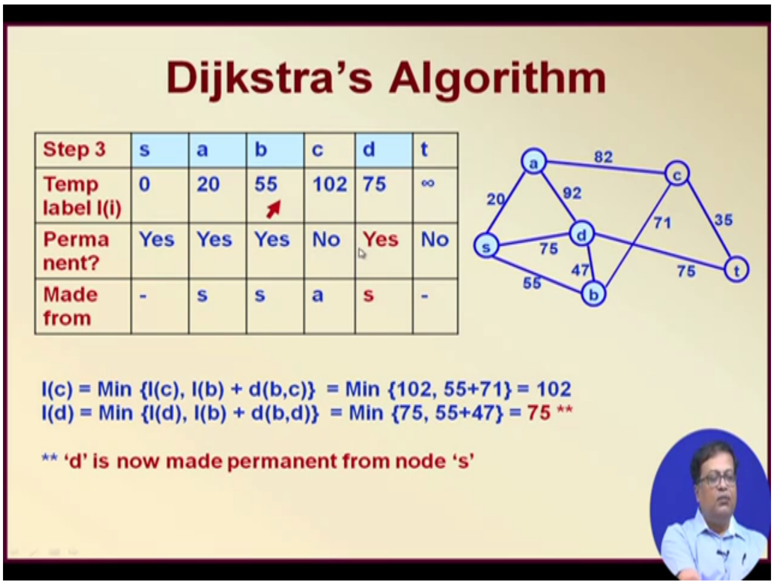

So at this point when b is made permanent and last node to have made permanent is b we have node you computation, right. The values that were available this t, but out of the remaining 3, now d becomes permanent, because d is the value which is 75. So now d is also made from see s not c s, so therefore now which one is lowest, so d. so d should be made permanent now.

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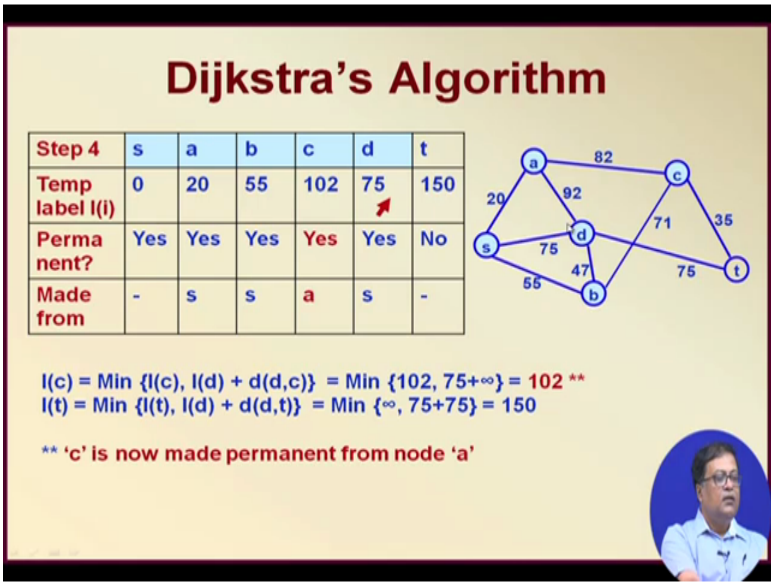

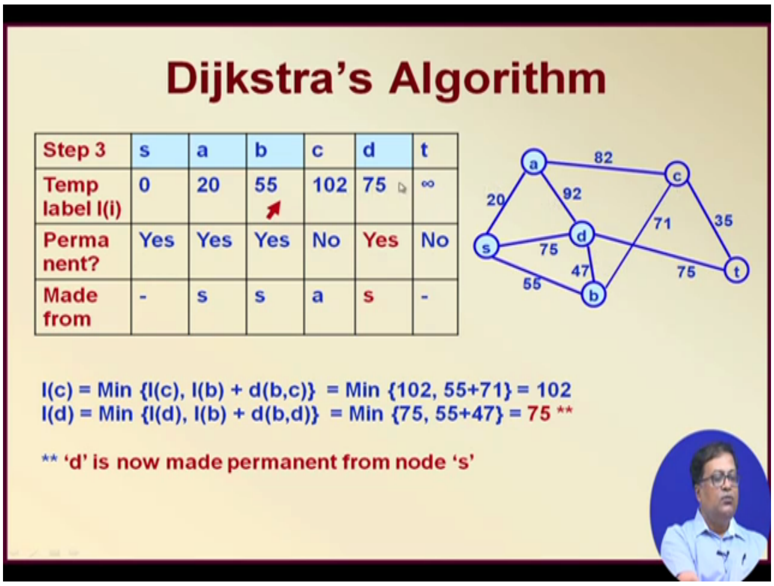

So at this round we have made d also as the permanent node, right.

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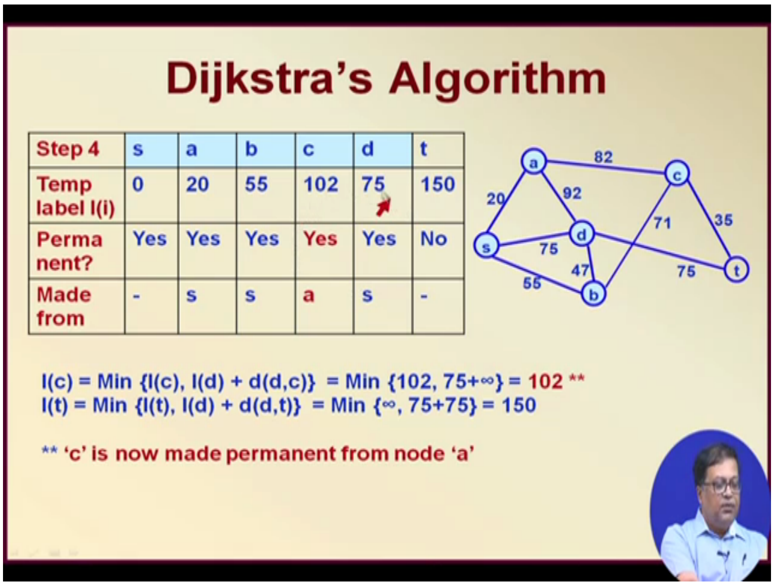

So actually now the at this round d was made permanent, after d is made permanent we again calculate at d you see we can compute t as 150 and c we do not change. So out of 102 and 150, since 75 is also there not permanent. So at this point 75 sorry, 75 was a last node to have made permanent. So we make 102 that is c as permanent, right.

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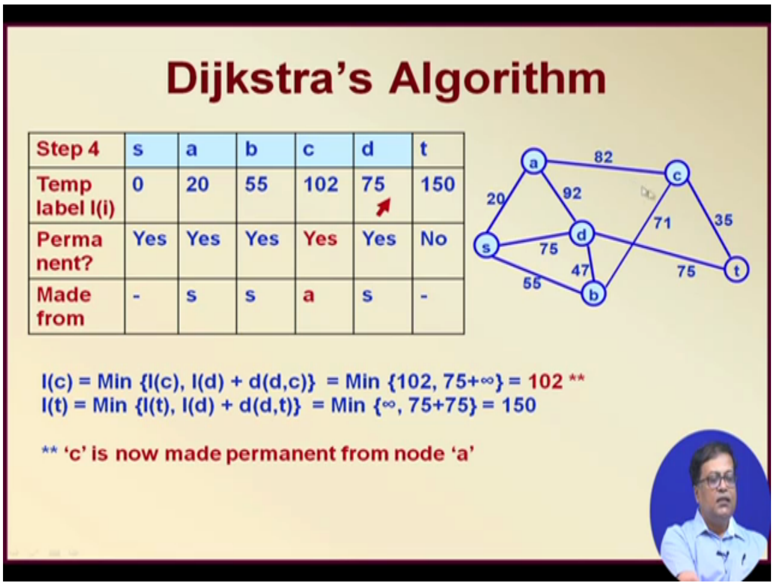

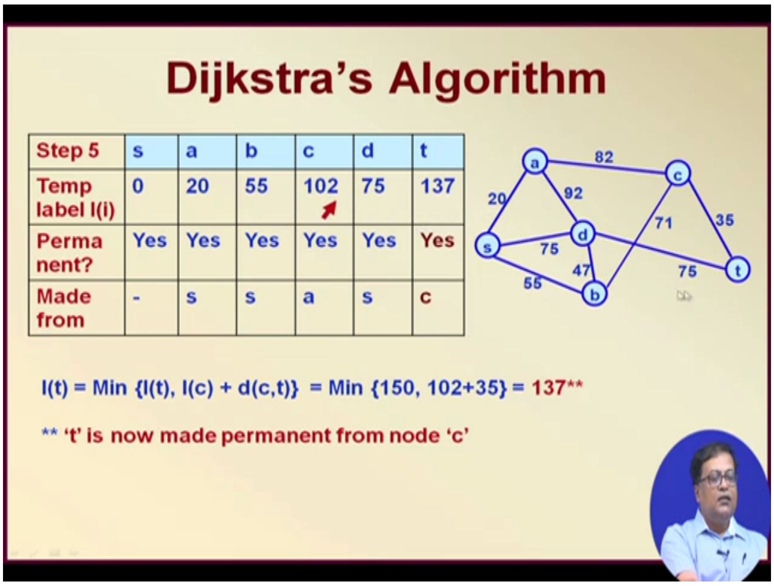

Proceeding like this at the last step, here we see that (())(10:26) value was 150 from c the distance is lower so we make at this point 137, because the value of c was 102 and now t will become 137. So t is now 137 because the value of c was 102 and now t will become 137. So t is now made permanent and we have completed all the nodes are now having permanent level. Is it all right? Sometimes if you really want only s to t distance if other nodes are not permanent, but t is permanent s is permanent we can get the distance from s to t if that is all that we want, right.

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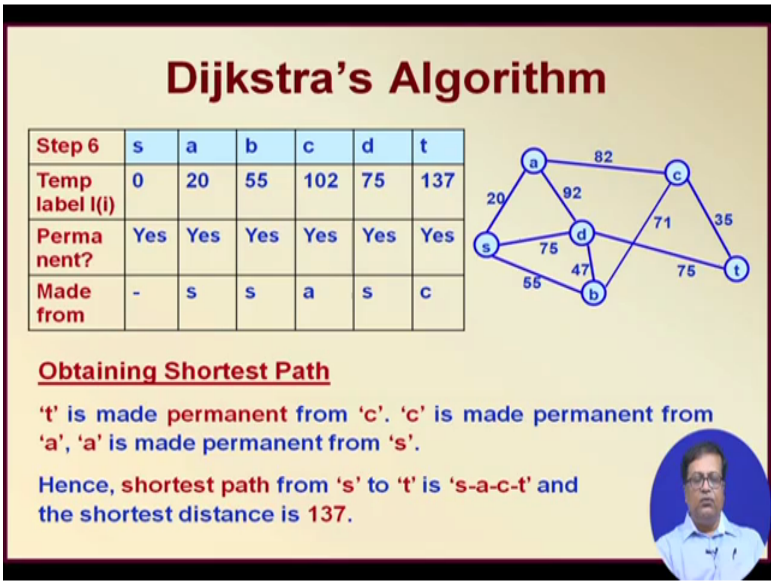

So now this is our complete table. These are the labels no more temporary, they are all permanent. If all the levels are permanent and made from least is also available. So what we do t is made permanent from c, c is made permanent from a, a is made permanent from s. So s to a to c to t that is the shortest path and that distance is 137, right. So that is Dijkstras algorithm explained to you once more time.

What I said that when there are negative distances then we have the bellman Bellmans Ford. This algorithm is nothing but an extension of force sorry, that Dijkstras algorithm, but only difference is that we cannot make any of the nodes permanent. Is it all right? So it is like an exploratory process. all the nodes has to be used for recompilations of the levels. So at every round keep computing the you know, the what you call the values again and again until you get a situation that no further levels are changing. So we will not discuss that in detail, but just remember when negative distances are involved, one requirement is there should be a loop, which has a total negative distance and second thing, we cannot make any of the nodes permanent.

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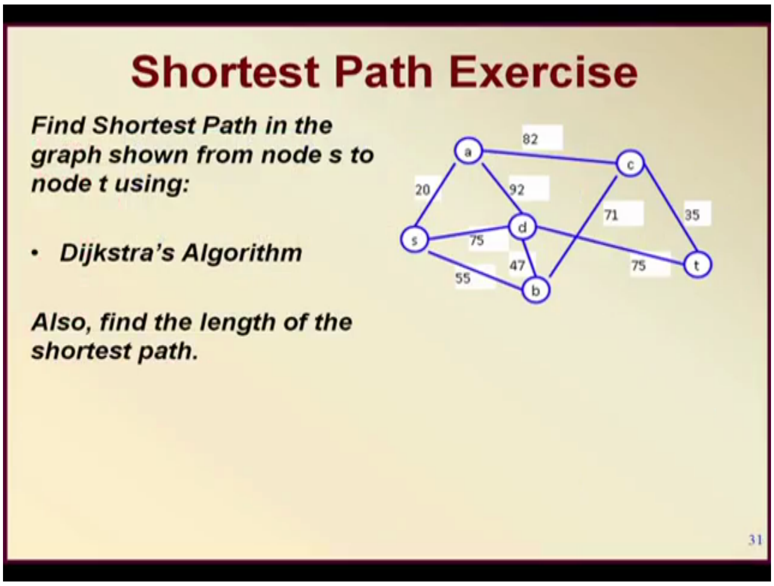

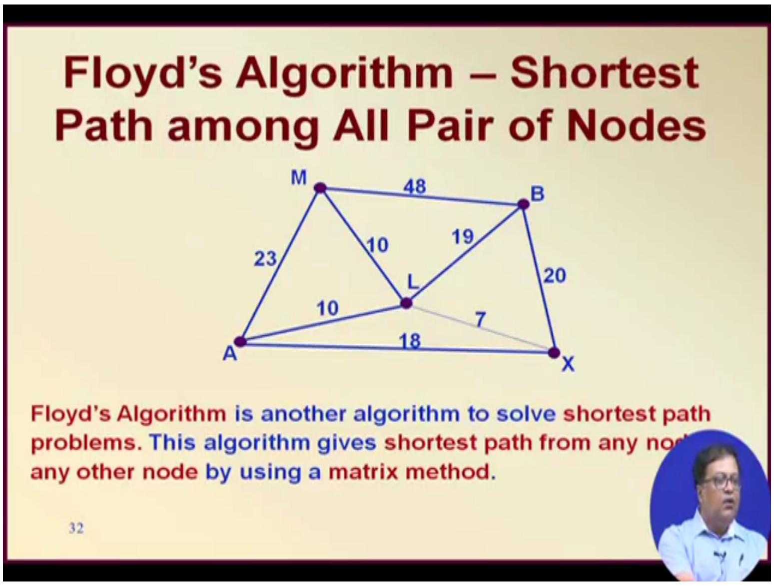

So all the nodes are candidates for calculation at all the rounds. So that is the difference, but we discuss now what is known as the Floyds algorithm. The Floyds algorithm essentially the idea is to by this method using a matrix method we can find out the shortest path from any anode to any other node, right. So that is algorithm known as the Floyds algorithm. How do we do that? So let us take this particular shortest path problem where there are 1, 2,3,4,5 nodes and these are the distances. So we need to find out the shortest path from any node to any other node. How do we proceed?

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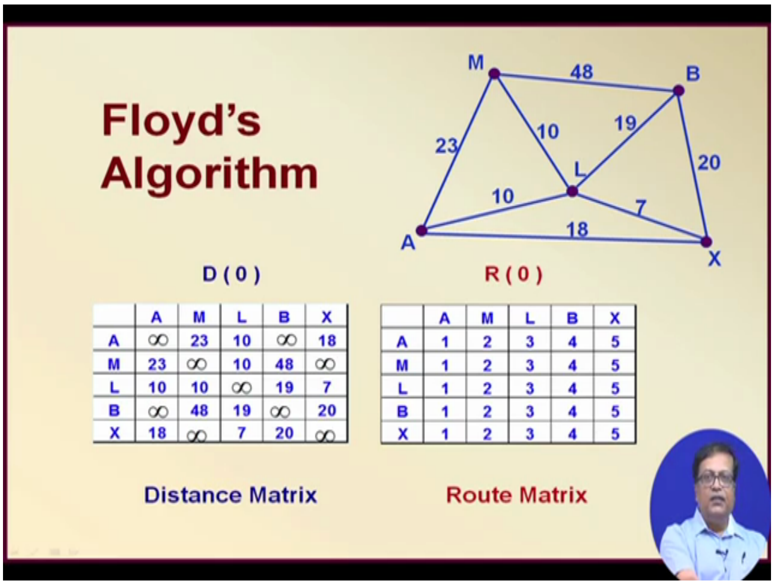

So we make 2 matrixes, the first matrix is call the distance matrix and the second matrix is called the Route matrix. The distance matrix is you know the distances like infinity A to A and you know they are all infinity, X to M there is no such path. So it is also infinity, then like

M to X look no path from M to X, so infinity. So this is call the distance matrix and in this distance matrix, you know this is symmetrical in the sense that, because up and down both distances are same, but it need not be you know you can also take distances if A to M and M to A are different, right.

Now route matrix is a matrix which is very simple, simply take (1,1,1,1,1),(2,2,2,2,2) (3,3,3,3,3) and 4 and all 4 and all 5, right. All ones, all 2s, all 3s, all 4s and all 5. So simply take it. Now for convention although the node names are A, M,L,B and X we call them as 1,2,3,4,5 also at some time. Just remember A is 1, M is 2, L is 3, B is 4, X is 5. Now why they are not named as 1,2,3,4, 5 in the beginning because it will confuse you otherwise, because here also 1 and here also 1. So we will not be clear, but anyhow just remember that. So this is the first round.

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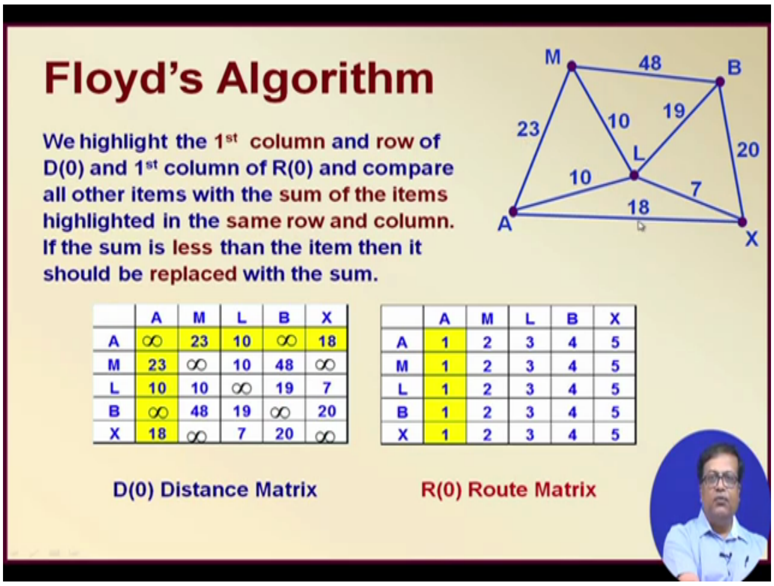

At the second round what we do? We highlight the first column and the first row, right we have highlighted the first column and the first row and here this column. So what is we do now we try to find out the distances through A, right. All the distances we are now finding through A, so M to X what is the distance through A? B to L what is the distance through A? See B to L current distance is 19. What is the distance from B to L through A you see A to L is 10 and B to A is infinity. So B to L through A is infinity out of infinity and 19 which one is lower, 19. So we keep 19, but B to A what is the current distance from A to B, infinity. What is the distance of B to A through A? There is no path, so we do not put any change to remain (())(16:35) it, let keep it infinity. What is the current distance from M to X or X to M is infinity. What is a distance from M to X through A through A the distance is 23 and 18 you see, through A, the distance because A is now highlighted. So M to X through A will be 23 plus 18 that is 41, look here also, M to A is 23, A to X is 18, so 23 plus 18 is 41.

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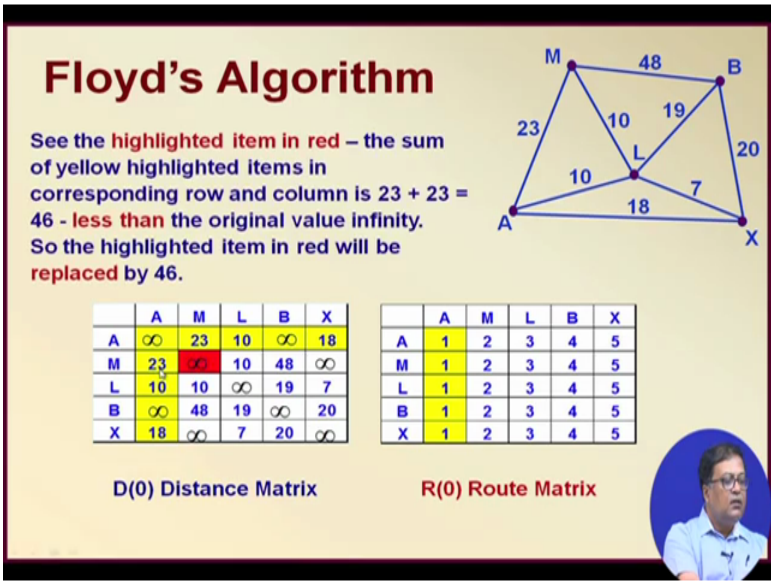

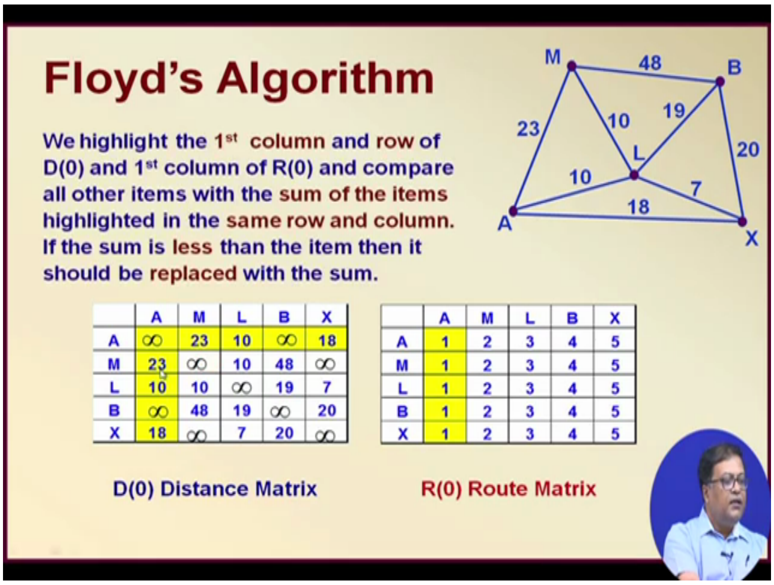

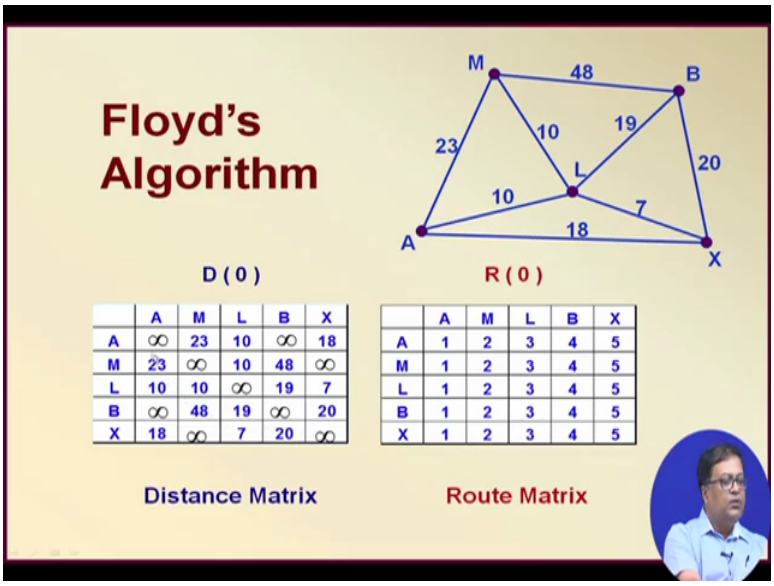

So very simply you simply look at the corresponding row and corresponding sorry, column and row and M to X is 23, sorry 18 here.

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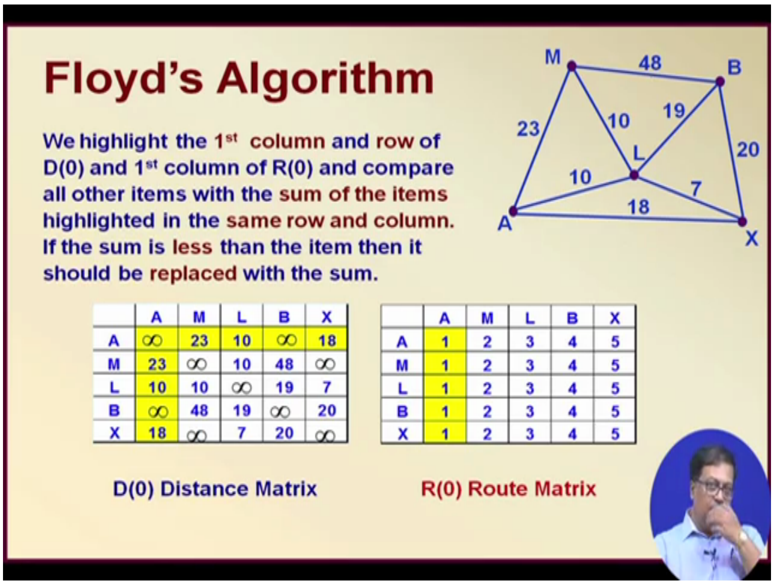

So this is 23 plus 18 that will be 41, right. So that is how we proceed in this particular situation

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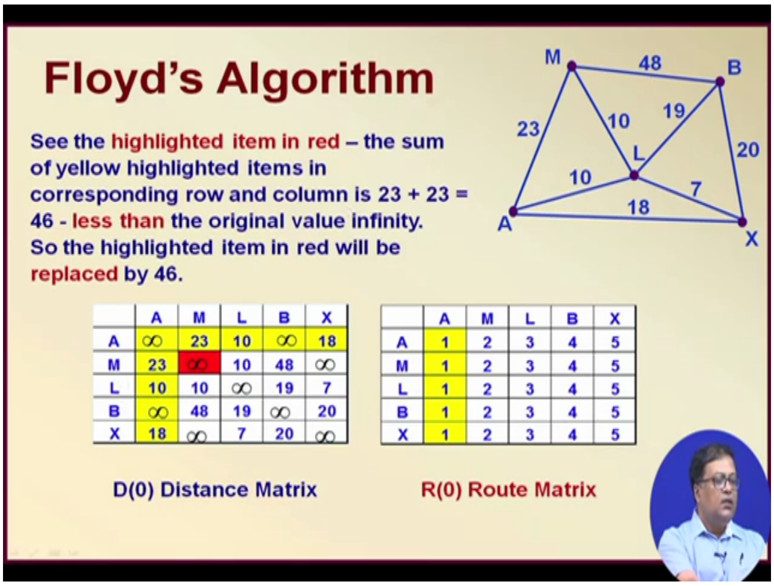

So look at this now M to M through A, you see M to M through A will be 46. Why because 23 plus 23 is 46, right.

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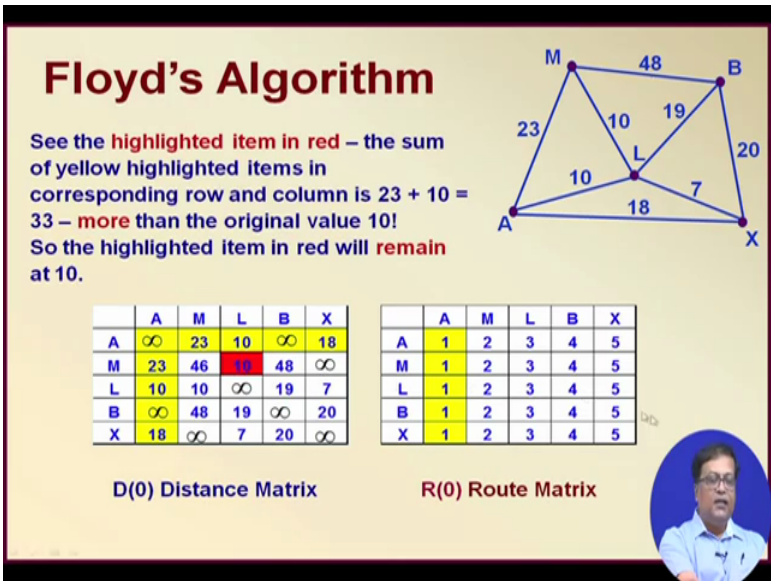

So look at this. This is currently 10 it should be how much, because 23 plus 10 is 33 more than 10, so it remains at 10.

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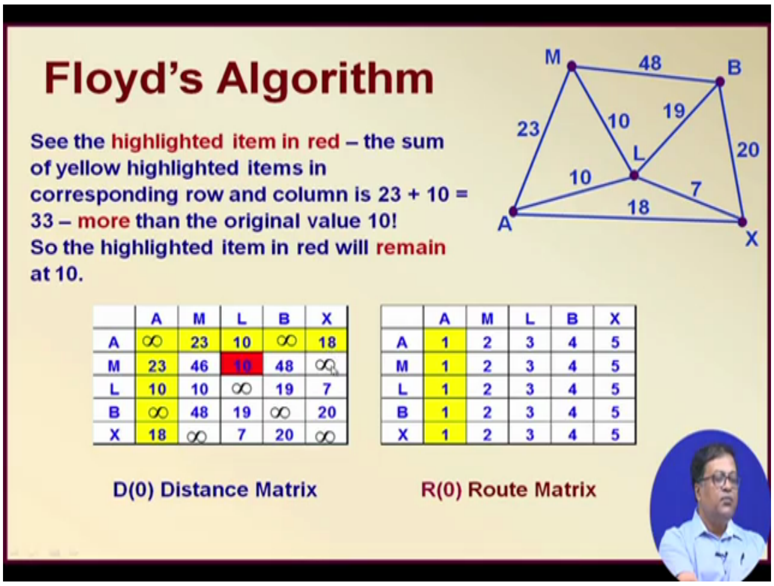

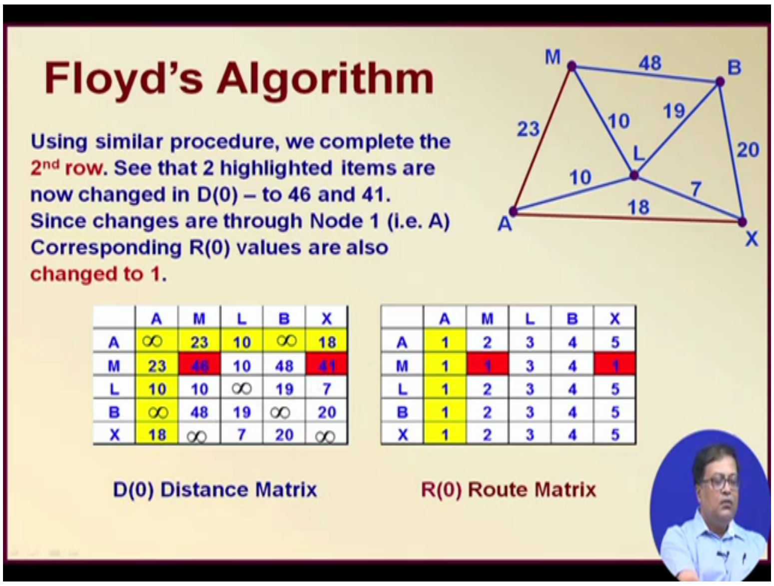

Now look at these value is currently you know it was infinity, but 23 plus 18 is 41, so it will now become 41, but every time you change you see like here we have change it to 46. So we change it to 1. Here we have change it to 41 and that is through 1 that is point A. So A so we write 1 here, but the point the so that is why is called route matrix that means M to X is 41 through 1 right that is a minimum 1.

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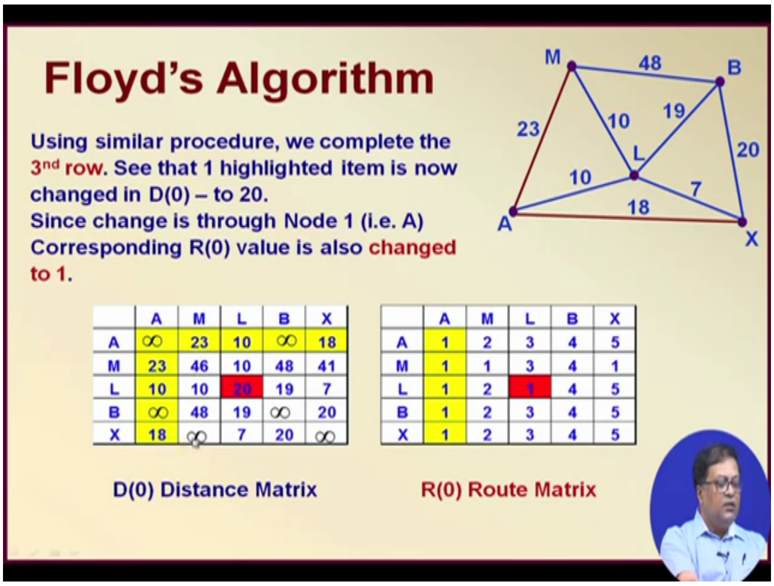
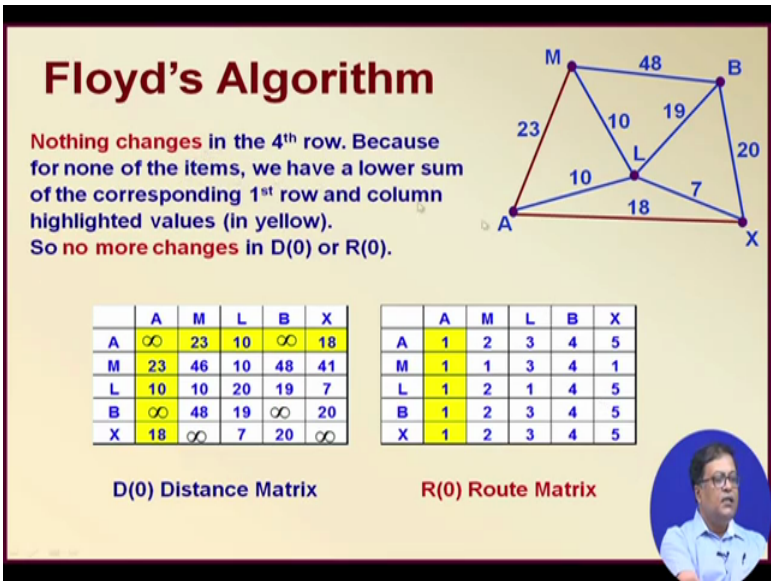


	A	M	L	B	X
A	∞	23	10	∞	18
M	23	46	10	48	41
L	10	10	20	19	7
B	∞	48	19	∞	20
X	18	∞	7	20	∞

	A	M	L	B	X
A	1	2	3	4	5
M	1	1	3	4	1
L	1	2	1	4	5
B	1	2	3	4	5
X	1	2	3	4	5



	A	M	L	B	X
A	∞	23	10	∞	18
M	23	46	10	48	41
L	10	10	20	19	7
B	∞	48	19	∞	20
X	18	∞	7	20	∞

	A	M	L	B	X
A	1	2	3	4	5
M	1	1	3	4	1
L	1	2	1	4	5
B	1	2	3	4	5
X	1	2	3	4	5

So now that we have made all computations from A to M you know we leave it here and we call it now this you see, the other side also this is 10 to 10 20 and this also changes. So through 1. So all the values we look at so these infinity will be 41 also. So nothing changes in the 4th row, because items.

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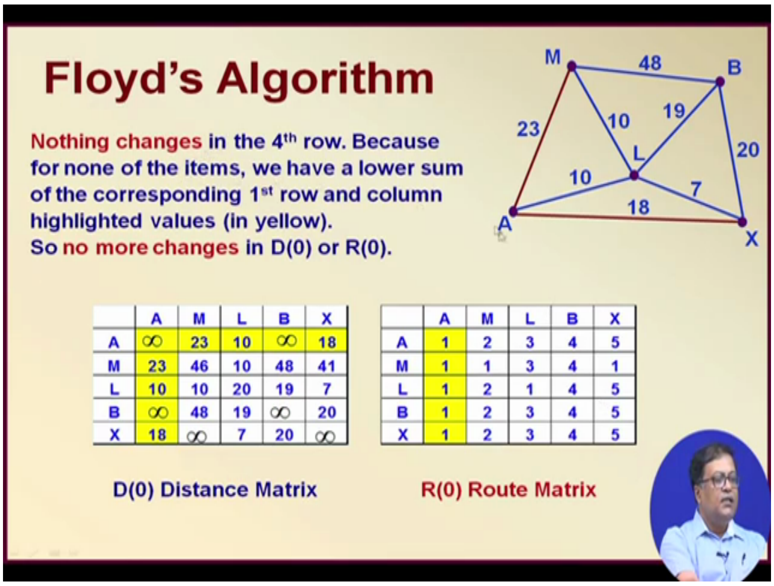

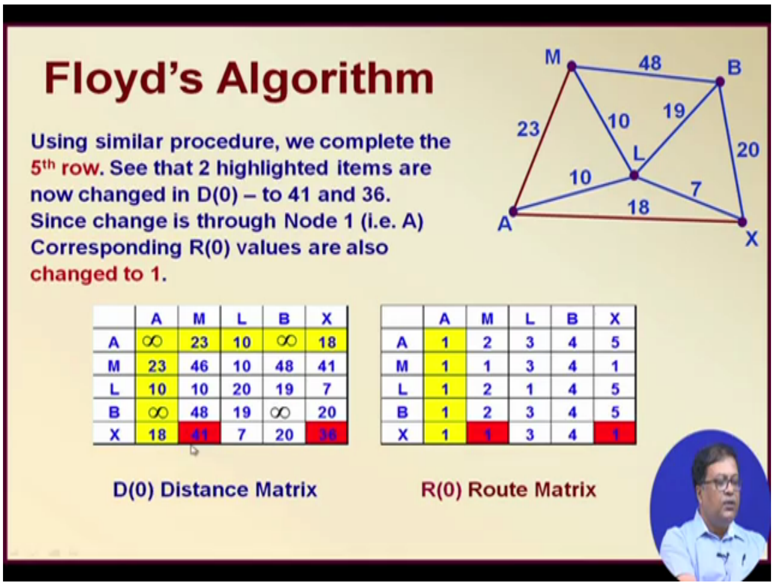

And then the 5th row you see, this 41 it was infinity infinity. Now it becomes 41 and 36 and there through 1.

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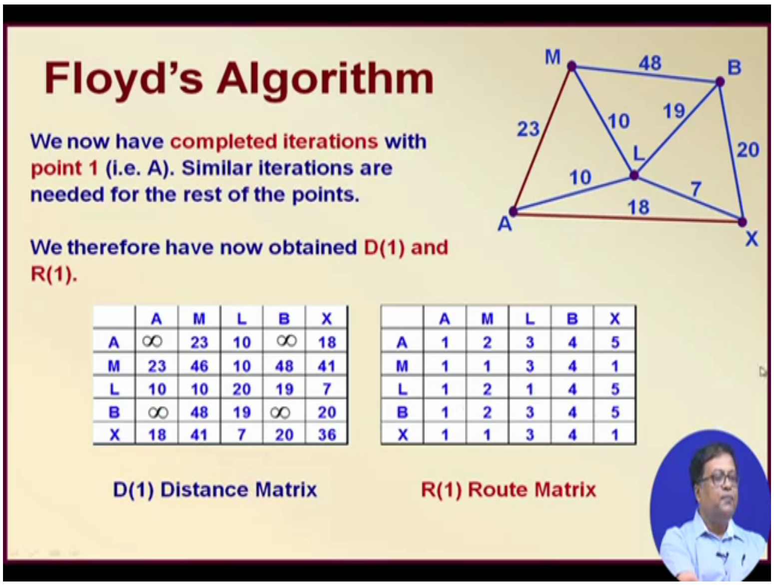

So now we have computed and we get these two matrices which we can called D(1) and R(1). D (1) is the distance matrix; R (1) is the route matrix. How we have observe this? We have now got the original distances or all distances through 1 that is through A. through the point A all distances are computed and all those values that has been changed they are now changed to 1, right. So this is my new distance matrix and we have got, but all that thing through 1. So this is the D (1), right.

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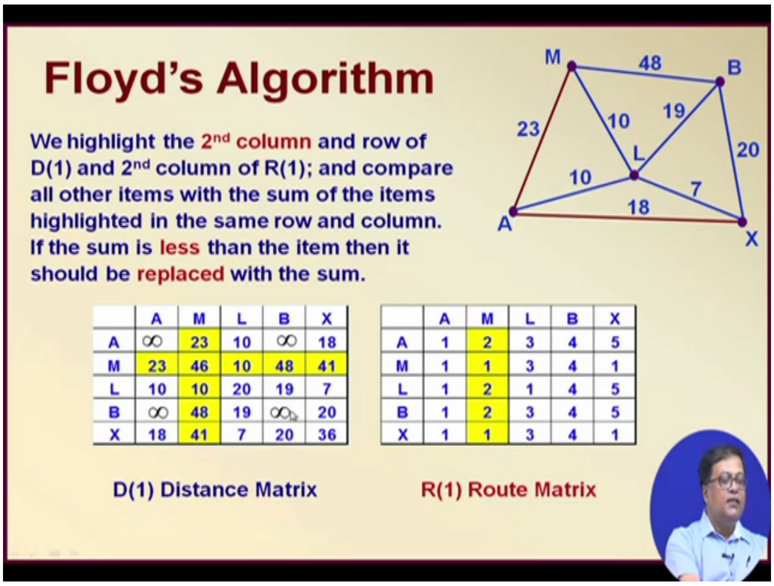

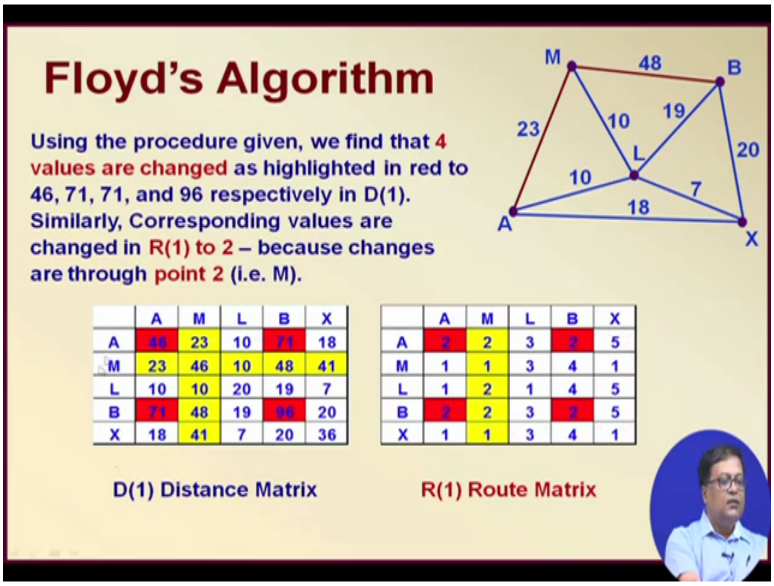

Now after we have got D(1) now we highlighted 2. What is the point 2 that is M. so we highlight the rows and the columns of M and here look at the column M, right? So now look at what will be changing. This infinity will become 46, this 10 can it be 33, no. This infinity will now become 58. How? Let us look at, what is the distance from A to B is infinity, a same thing is B to A. Now if you go through M what will be that distance? That distance will be 23 plus 48 that is 71, because go to A to M and M to B it will be 71. So this infinity will now become 71, right. So we can now compute all such distances what about A to L, do not change.

So you see keep doing it 23 plus 23, 23 plus 48, 23 plus 41, 41 plus 41, 82, but currant value is 36. 48 plus 10 current value is 19, 48 plus 48, 96 when this infinity will become 96. So like this we change. So you see these 4 values change at D(1) and R(1) through l and we get these changes in the matrix.

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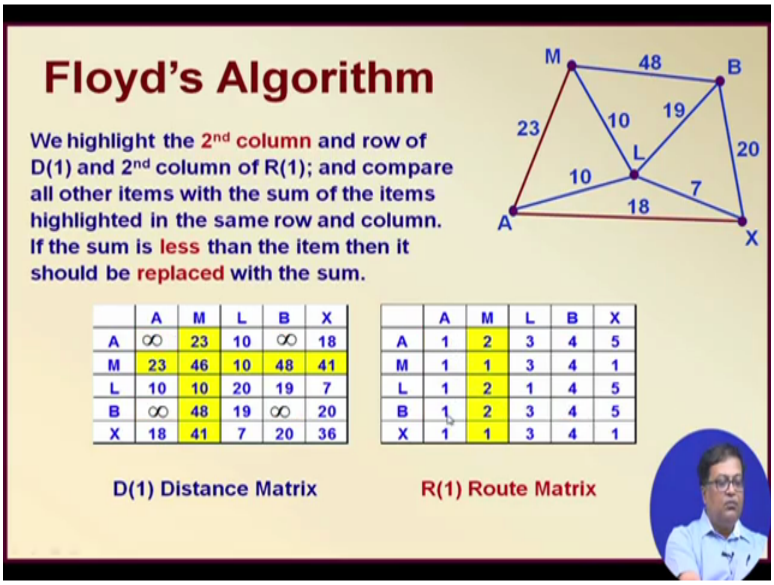

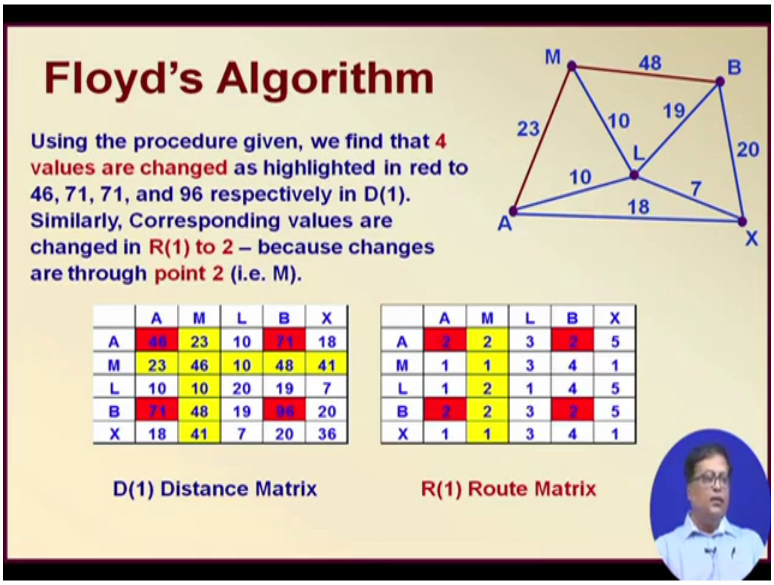

So look here these values where 1,1,4,4 we have now change them 2 to 2. So we have now got our new matrices that is D(1) and R(1) that means through 1 the values are obtained. Then what do we do logically you see and where do we stop? We started at D(0) and R(0) through all these distances we have computed now D(1) and R(1).

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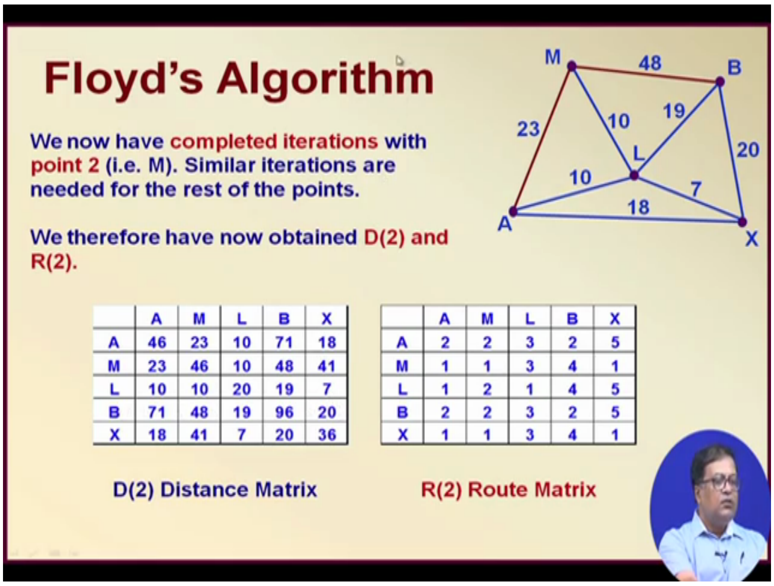
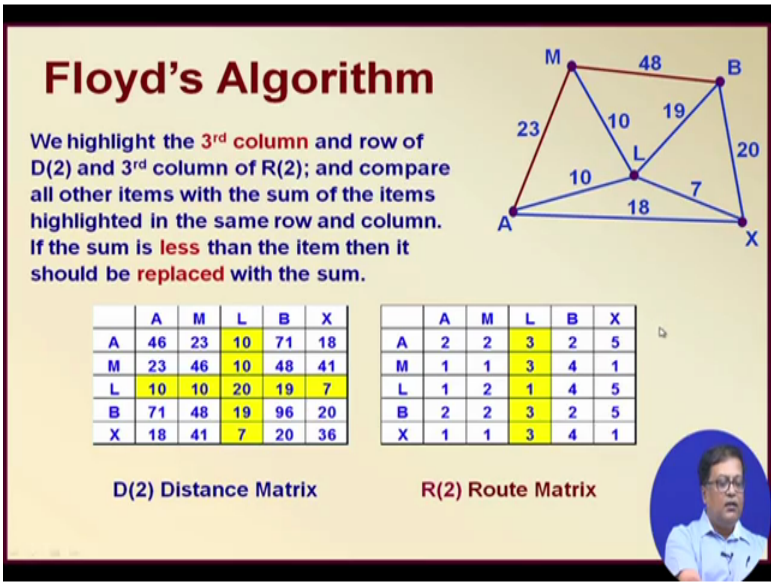


	A	M	L	B	X
A	46	23	10	71	18
M	23	46	10	48	41
L	10	10	20	19	7
B	71	48	19	96	20
X	18	41	7	20	36

	A	M	L	B	X
A	2	2	3	2	5
M	1	1	3	4	1
L	1	2	1	4	5
B	2	2	3	2	5
X	1	1	3	4	1



	A	M	L	B	X
A	46	23	10	71	18
M	23	46	10	48	41
L	10	10	20	19	7
B	71	48	19	96	20
X	18	41	7	20	36

	A	M	L	B	X
A	2	2	3	2	5
M	1	1	3	4	1
L	1	2	1	4	5
B	2	2	3	2	5
X	1	1	3	4	1

And finally we have reached D(2) and R(2) matrix right, 1 was through A and 2 is through M right. So we have now obtained both D(2) and R(2). So both these matrices are now obtained. So after obtaining both D(2) and R(2) matrix next is should be the 3rd point. What is a 3rd point that is point B So that means row and column of these 3rd point that is B sorry the 3rd point was L that should be highlighted. So L now should be highlighted. So L is highlighted.

Now look what are some values that will change. These 46 will be 20 right these 46 also will become 20. This 48 will now become 29; this 41 will now become 17. So can you see lot of changes through L lot of distances are changing. This 96 will become 38, this 20 26 no change, this 36 will now become 14, So lot of changes will now take place through L, because the distances are smaller, right.

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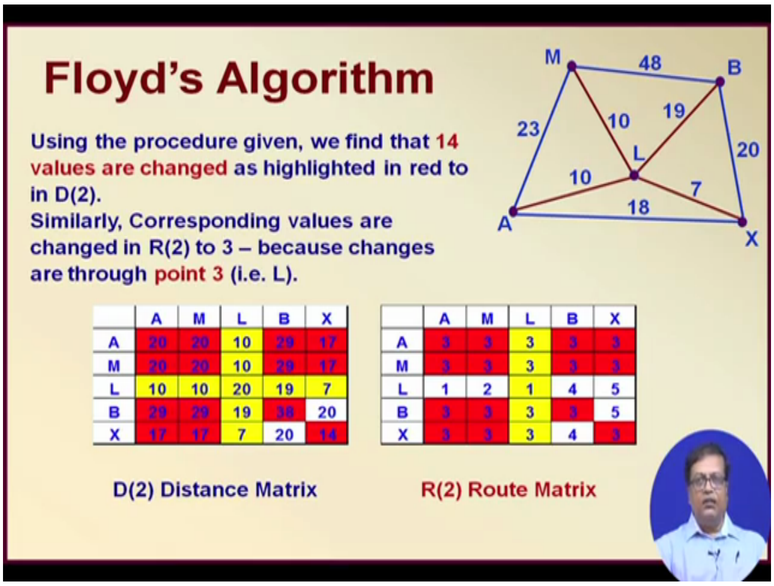

So Exactly we have made all these changes as I we said look at 7 plus 7, 14, 19 plus 1 9, 38, then 19 plus 10, 29, 19 plus 10, 29 and all these changes will be now reflected in putting in this matrix 3 at the appropriate places, right. So once again how do we get D(3) and R(3) from D(2) and R (2). Basically the L is a 3rd point, so all the distances are to be recomputed by taking this row and this column and wherever the values are lower we replace them. So what exactly is happening? What is Floyds algorithm?

The Floyds algorithm if you really understand carefully, it is nothing but the application of Dijkstras algorithm in a matrix form, right earlier we were applying Dijkstras algorithm between only 2 points in a linear one dimensional situation. Here we are using for a 2 dimensional situation and if we go from D(0) R (0) to D(5) and R(5), all possible combinations are explored. So that is the essential idea of Floyds algorithm, right.

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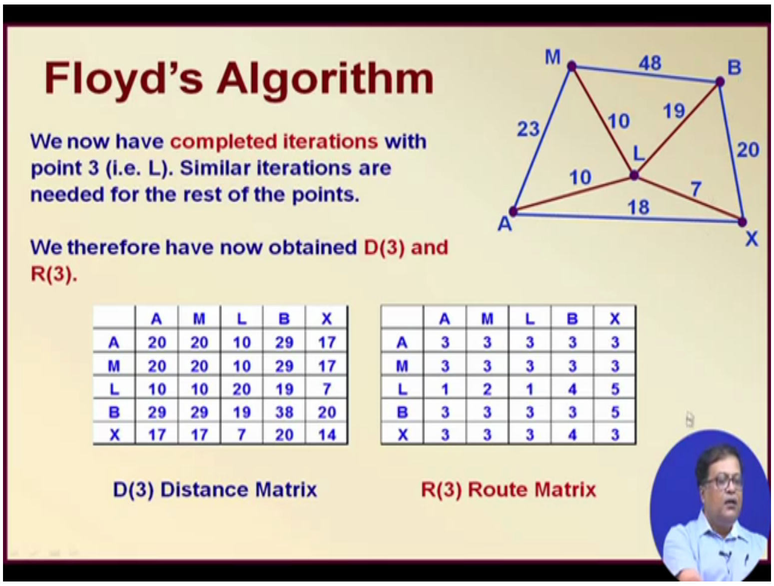

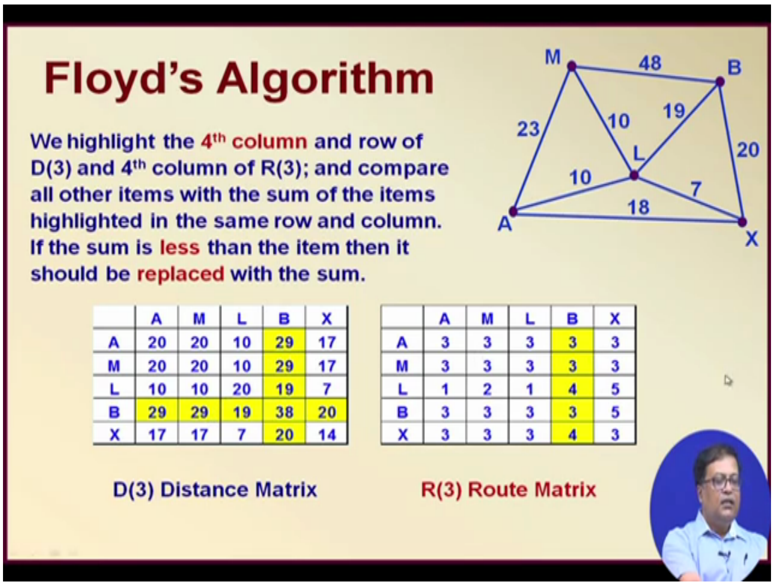

So we have obtained now D(2) and R(2) (25:01) and for from there D(3) and R(3). So the 3 points are explored. What will be our 4th point? The 4th point is B, right. So 4th point is B. Now look at the B is highlighted. Can you identify some points you know some values that will be changing at this time, just identify 29 plus 29, 29 plus 29, 19 plus 19, 19 plus 20, this think over. Are you getting anything which is changing here?

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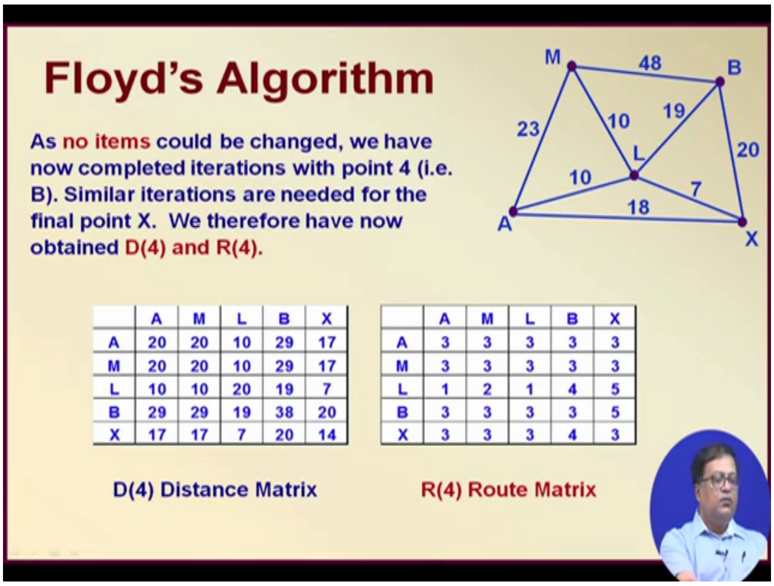

Let us see the result, we got nothing, right no item could be changed, because all the values through B, the values are higher like M to X, look at M to X, it was 17, but if you go through B, it will become 68. So we do not change anything. So we get what is known as D(4) and R(4).

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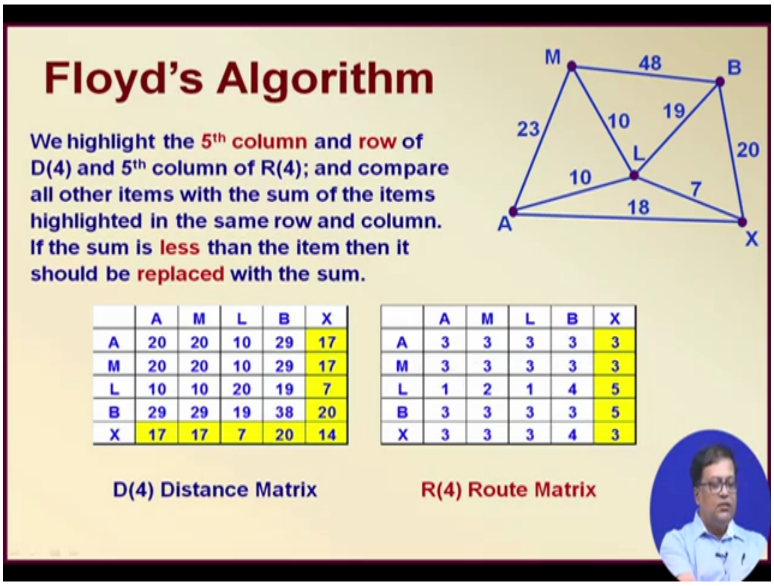

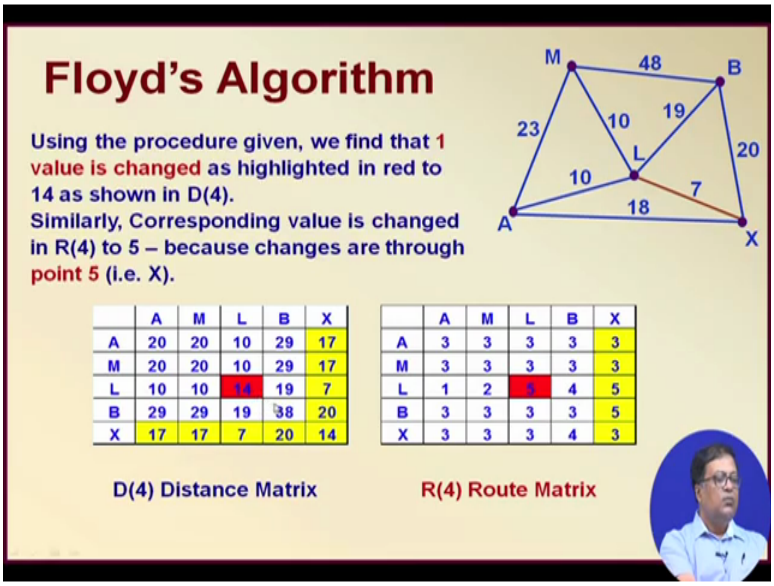

So now we come to our last point that is point number 5 that is X, right. So we highlighted the rows and the columns and here that particular column. So now tell me can you find any change? Look at all the distances 17 plus 17, what about this 22, see 7 plus 7, 14 it will change right 48 does not change, 37 37, 24, 34 27. So not much changing, but may be some changes would be there. So actually only one value is changing so that is this becomes now earlier it was 20 through 1, now it will become 14.

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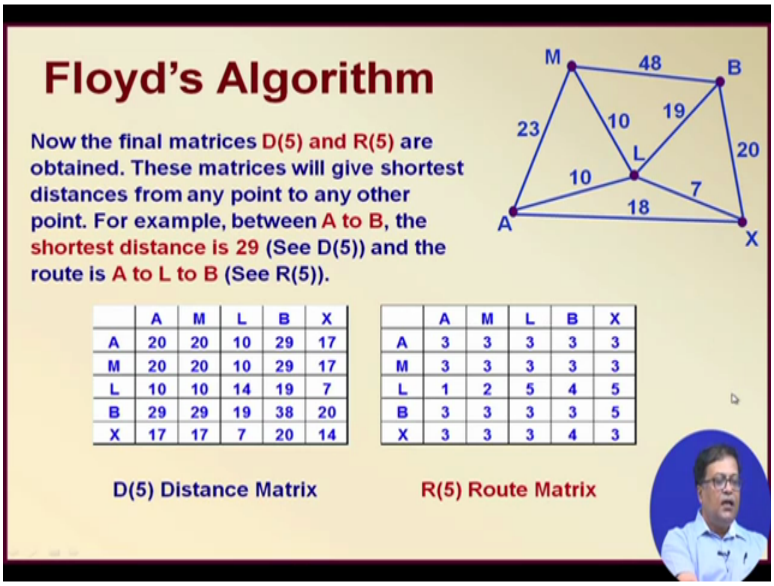

So that now completes our matrix. So we have got this D (5) and R(5) right. So let us look at how to read this particular matrix. Suppose we need a distance from B to L. So B to L is distance as showing as 19. So shortest distance is a direct path that is 19 and through point number 3. So B to L and L is the 3rd point. So basically it is just through L only B to L through point number L right that solve. So another distance what is a distance from A to B? A to B is 29 and it is through point number 3 that is M, sorry what is the point number 3 that is L. So A to B the shortest distance is A to L and L to B right. So you can study this.

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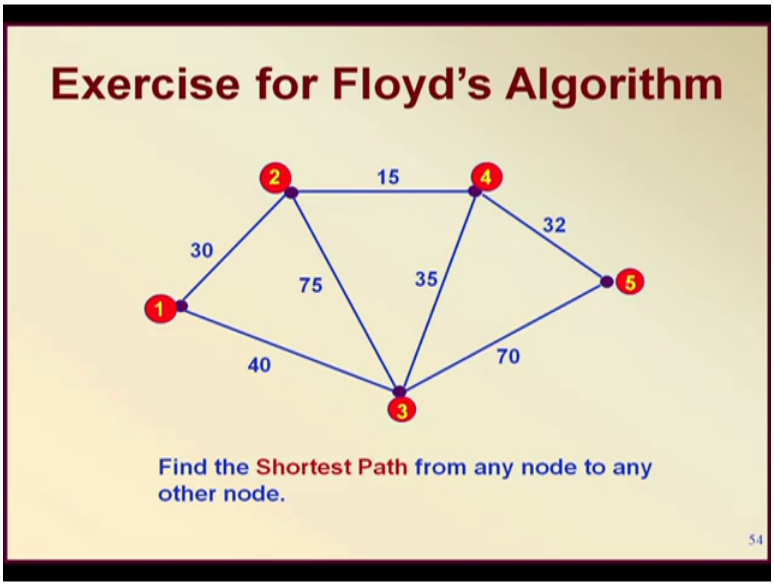

And while doing that please solve this problem also right. please solve this problem also we have 5 points 1,2,3,4,5 and these are the distances use Floyds algorithm, starts with D(0) and R(0) and use this method in a suitable manner and come out with the distances.

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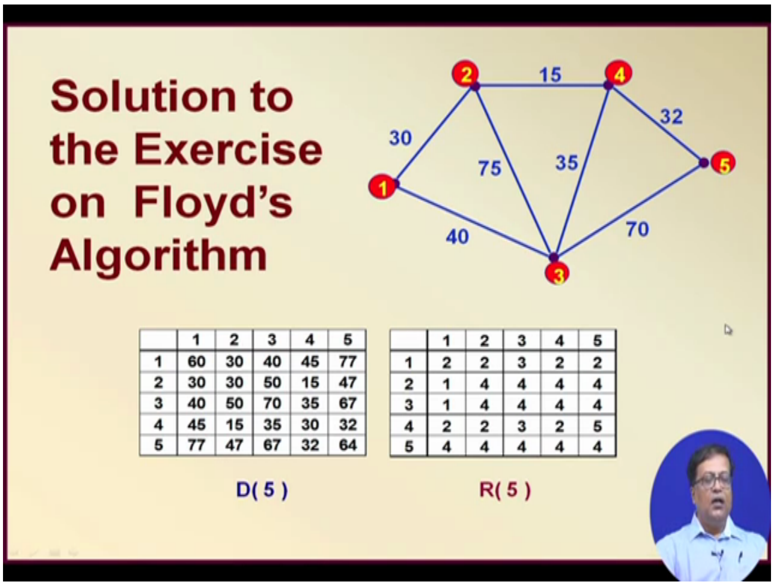

For your reference I am giving the answer here. This is a answer. Now if you look at that answer one interesting thing will be found out which was not found in the previous example. What is the shortest path from 1 to 5? Now 1 to 5 the shortest path is 77 right 1 to 5, 77. How? Through 2**,** but if I see through 2, I find 1 to 2 is 30, but what is a shortest path 2 to 5. So we have to look at 2 to 5 also. So 2 to 5 is 47 and that is through 4. So you have to read like this, see 1 to 5, shortest path is 77, it si through 2 then we have to see 2 to 5 also. 2 to 5 shortest distance is 47 and it is through 4. So therefore 1 to 5 shortest distance is 1 to 2, 2 to 4, 4 to 5. Is all right? So that is exactly how you can work out Floyds algorithm and we have really studied 3 important algorithm, Dijkstras algorithm, Floyds algorithm and although I have did not say but I just mentioned the Bellman-Ford algorithm for negative distances, right. So thank you very much and that concludes our network portion.