Course on Decision Modeling
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Lecture 35
Module 7
Cutsets

Right, so we are discussing the network models and in our previous class we have really seen the tree and the spanning tree concepts and from there now let us move ahead to another concept that is known at the cutsets.

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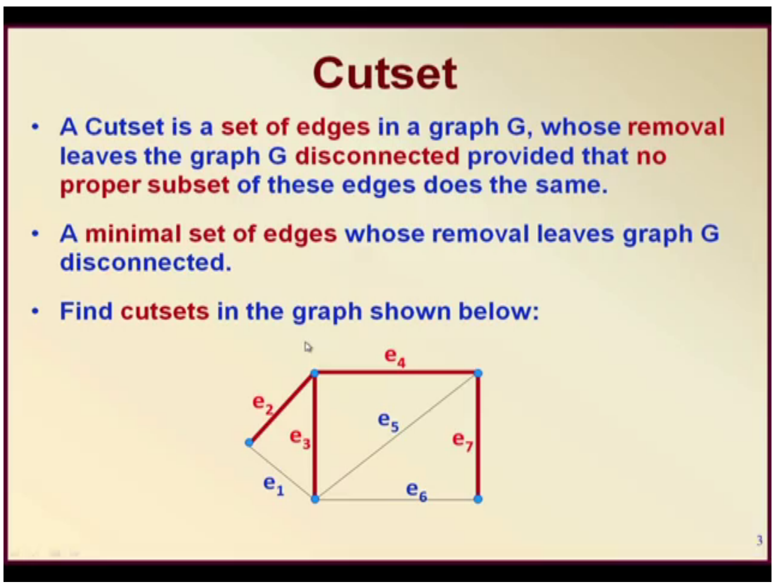

So the cutsets is a very important concept that basically a cutset you know you can say it is a set of edges in a graph G if you remove them then the remaining graph should be disconnected, but the question comes that how many such edges should I remove? You see an extreme case is you remove all the edges and the graph is disconnected so is it a cutset? The answer is no, it should be a minimal set but then this minimal set is also a you know something very important to understand it is minimal set in the sense that a subset of these many edges should not be a cutset that is a essential idea.

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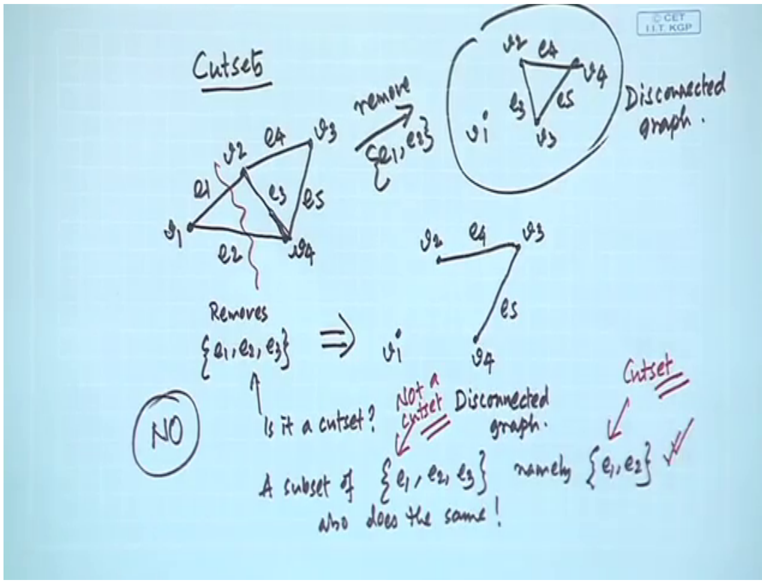

So let us look at some examples to really understand what it exactly means. See cutsets suppose I have a graph of this type so this is a graph, is it alright? So let say this is a small graph so this is a graph and this graph has got 4 vertices v1, v2, v3, v4 and there are edges like e1, e2, e3, e4, e5. So now if you remove let us say out of this graph e1, e2 and e3 a set of edges so remove these three edges, what is the resulting graph? The resulting graph would be v1 would be here, v2, v4 and v3 e4, e5, v2, v4, v3 so this is the resulting graph because we have removed these 3 edges, so is it a cutset?

You see what we have done we have a graph from this graph we have removed you know 3 edges and this is the resulting graph question is it a disconnected graph? Yes, why? Because v1 is on one side v2, v3, v4 on the other side and there is no path from v1 to those edges so this is a disconnected graph. So the question comes that e1, e2, e3 is it a cutset? The answer is no, it is not a cutset. Why it is not a cutset? Because a subset of e1, e2, e3 is a subset of e1, e2, e3 namely e1 and e2 also does the same, same means what? If you remove say look here remove e1, e2 then what will be the resulting graph? Resulting graph will be this v1 here v2, v3, v4 is here e3, e4, e5.

So this one is it a disconnected graph? Yes, it is a disconnected graph. So you see therefore you can say that e1 and e2 is a cutset this is a cutset but not a cutset. Obviously we should additionally check that as I remove e1 and e2 is the subset of that also does, does e1 disconnects the graph? No, e2 disconnects the graph? No, so e1 and e2 is a cutset so that is how you have to understand what is a cutset.

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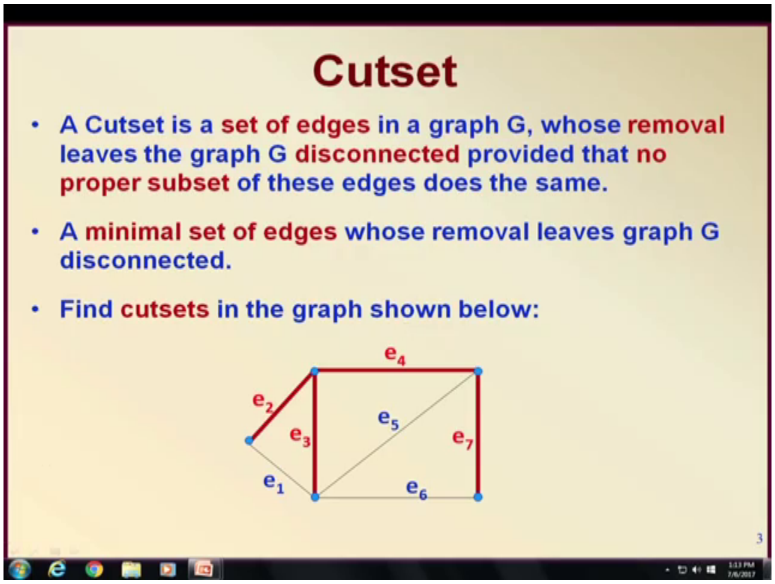

So look at the theory now that a cutset is a set of edges in a graph G whose removal leaves the graph G disconnected provided that no proper subset of these edges does the same.

So this point we must recall and remember. You can alternately say it is a minimal set of edges whose removal leaves the graph G disconnected, is it alright?

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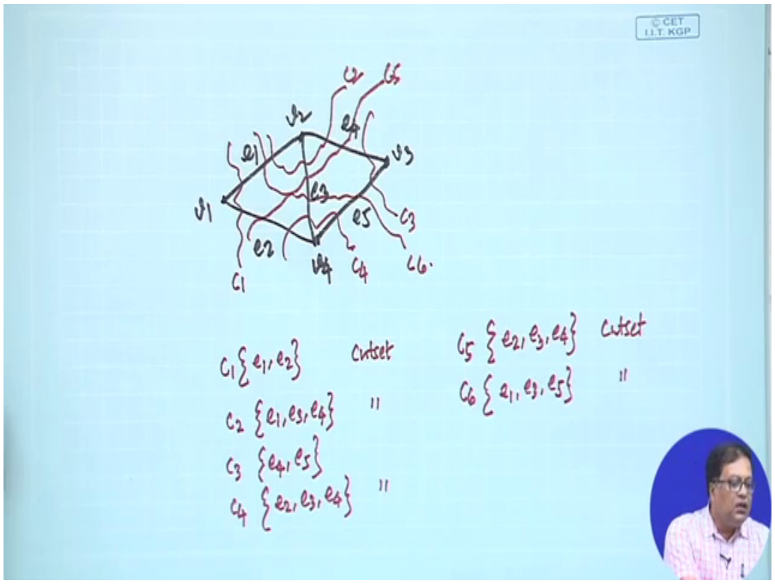

So that we must remember, now let us look at the same graph once again and find out all the different cutsets so we had this graph and we had e1, e2, e3, e4, e5 v1, v2, v3 and v4 so we had these particular graph and what are the different cutsets that you can think of.

You see you can let us look at some cutsets, suppose is it a cutset c1? So c1 is e1, e2 which is a cutset already we have seen that. What about this one, is it a cutset? c2 this is c2 c2 is e1, e3, e4 this is also a cutset, is it a cutset c3? This is also another cutset e4, e5, right? This is also another cutset c4 that is e2, e3, e4 is also a cutset like that we can have this one which may be call c5 so c5 which is e2, e3, e4 this is also a cutset, right?

So you see you can still think of some more like e1, e3, e5, is it alright? Maybe I do not know maybe we have covered more or less all of them so there could be c6, c6 this is c6 which is e1, e3, e5 like a cutset. So you see we have found out 6 cutsets here may be we can find even more cutsets. So you can see that this is becoming really rather complex task to identify all the cutsets of a particular graph, right? There could be so many different cutsets that are actually you know disconnecting particular graph.

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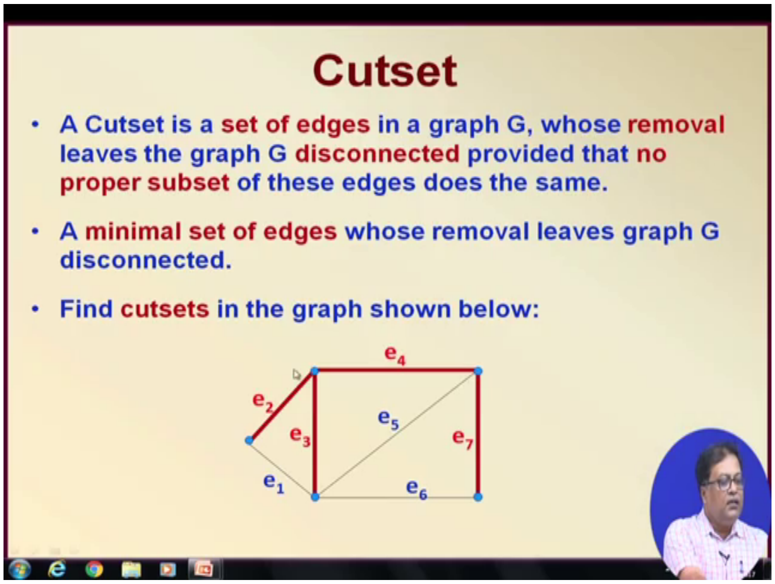

So if you look at the nodes once again here as you can say that in this particular graph if you have to find all the cutsets can you think and identify some of them. What about e1, e2, is it a cutset? Answer is yes, it is a cutset, can you think of something like e1, e3, e4 look if you draw this e1, e3, e4 you know it separates the graph into two parts these two vertices on one side these three on the other side. What about e4, e5, e6? Or e4, e5, e7 what about e6 and e7 you see like that you can find so many different cutsets and even look here some of them are identified here, right?

So cutsets of the graph e1, e2, e1, e3, e4, e1, e3, e5, e6, e1, e3, e5, e7, e2, then e3 and then e4, e2, e3, e5 and e7, then e4, e5, e6, e4, e5, e7 and e6, e7 so you see like that all the different cutsets one can find out and question still remains how to find all the cutsets of a graph. As you can see the process is very cumbersome and the took just for a very simple graph, suppose the graph is little more complicated than what would be the cutsets and how do you find them it could be much more difficult and involved.

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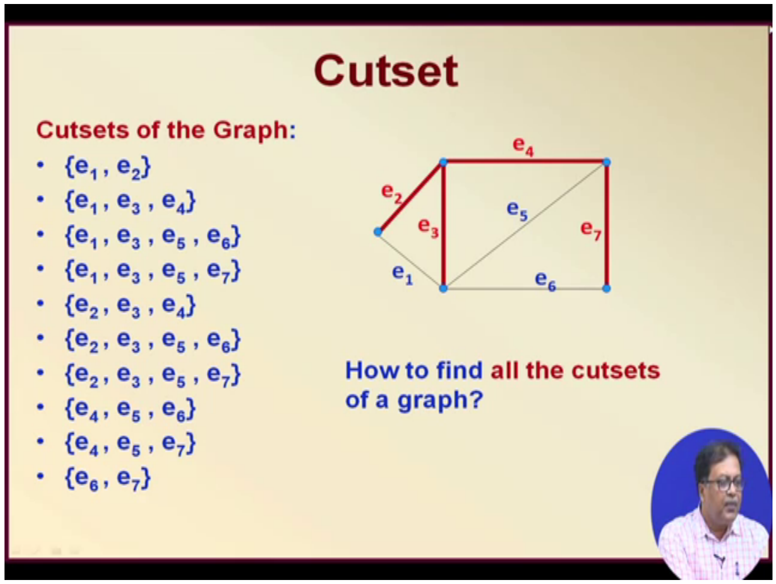

Now let us understand some more important concepts regarding this cutsets and other sort of things one such concept is called the Eccentricity of vertices, right? First of all we must understand that there is something called distance, a distance is the length of the minimum path. Now question is that is it an weighted graph or a not a weighted graph usually particularly for communication circuits sometimes the length is not the important thing the junctions are more important how many nodes it pass through, right?

So therefore the essential idea there is you know what is the number of edges. So if it not a weighted graph we will mainly think of the number of edges at the distance.

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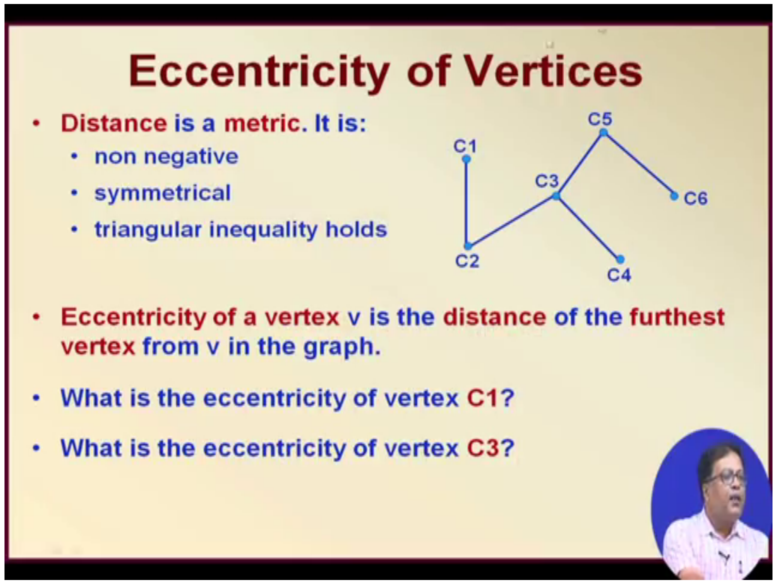

So here that you know that distance is usually a matric because it is a non-negative thing it is a symmetrical because c1 to c3 and c3 to c1 both are same and there should be triangular inequality that means if we have c1 to c5, right? You know should be less than c1 to c5 should be less than c1 to c4 and c4 to c5.

So if you add the distance from c1 to c4 which is 3 and c4 to c5 which is 2 then we get 6 that should be higher than c1 to c2, c3 to c5 which is 4 that is triangular inequality. So distance is a matrix so anything that is a matric has got these 3 properties and since distance is a matrix we use that matric to identify some basic ideas one such idea is an eccentricity of the vertices, why eccentricity of vertices is so important? Because if you can find out the eccentricity of vertices we can know you know the design of the graph can be possible very simply and easily we will see that little later.

So eccentricity of a vertex v is the distance of the furthest vertex from v. So what is the eccentricity of c1 then in this tree? What is the furthest vertex?

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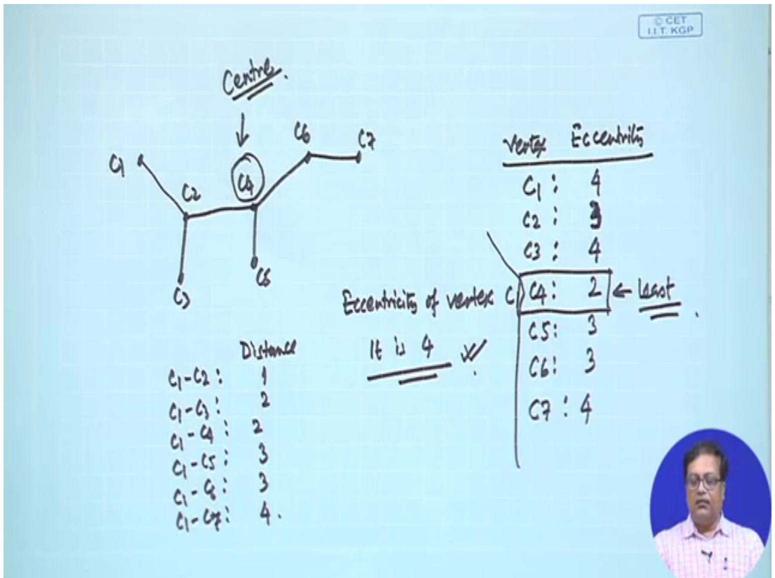

So we can work that out you know supposing we have a particular tree let us say, say suppose this is a tree say c1, c2, c3, c4, c5, c6, c7 now what is c1 to c2 the distance is 1, c1 to c3 2, c1 to c4 2, c1 to c5 3, c1 to c6 3, c1 to c7 4, so what is the eccentricity of vertex c1? It is 4, why 4? Because that is the distance of the furthest vertex from c1. So like this we can actually find out the eccentricity of vertices and usually the pendent vertices are having higher eccentricity, right? So can you find out the eccentricity of vertex c3? See c3, c3 would be this side 2, this side 3, this side also 1, 2, 3, 4, right? So c3 also 4.

So like that we can think of look at c4, what is the eccentricity of c4? The c4 is eccentricity is this side is 2, this side is 2, this side is 1, this side is 2 so is there any other vertex which is having lower eccentricity let us find them all. So here let us write down eccentricity and vertex c1 is we have found out 4, c2 also 4, what is the eccentricity of c3? c3 would be sorry c3 is c2 is 3, c2 is 4 because c2 is further from here, what about c4? c4 would be 2 because you see the furthest vertex is only at a distance of 2. What about c5? c5 would be 3, c6 c6 will be 3 again because from this side, and what about c7? That will be 4.

So you see the least eccentricity is that of c4 and, what is the meaning how what is the advantage of that? Then we call it the center, right? So center is the and that eccentricity of the center is called the radius of the graph. So any how these concepts are important really to know that how connected a particular graph is and what is the furthest vertex from those and that can be measured in terms of eccentricity.

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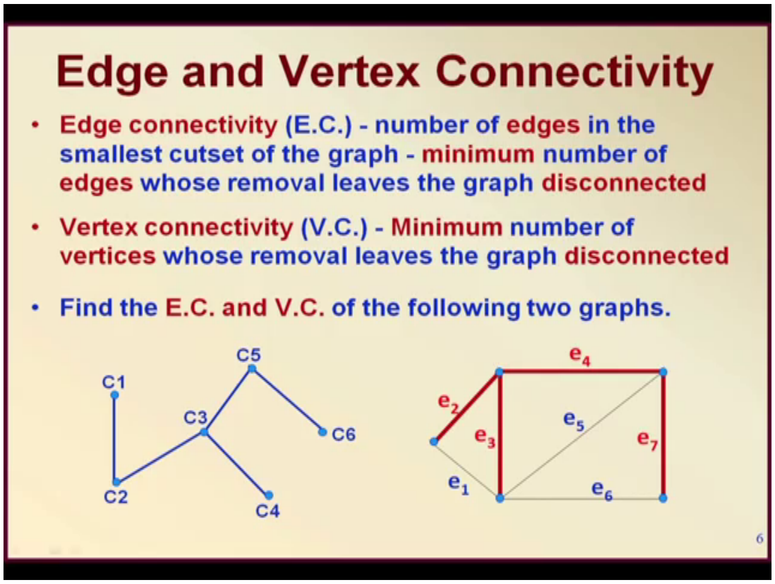

And this eccentricity once we know you know after that we can actually look at two very important concepts they can be called as edge and vertex connectivity EC and VC edge connectivity EC and the vertex connectivity VC.

Let us see how are they important, you see edge connectivity is a number of edges in the smallest cutset of the graph, right? Smallest cutset of the graph.

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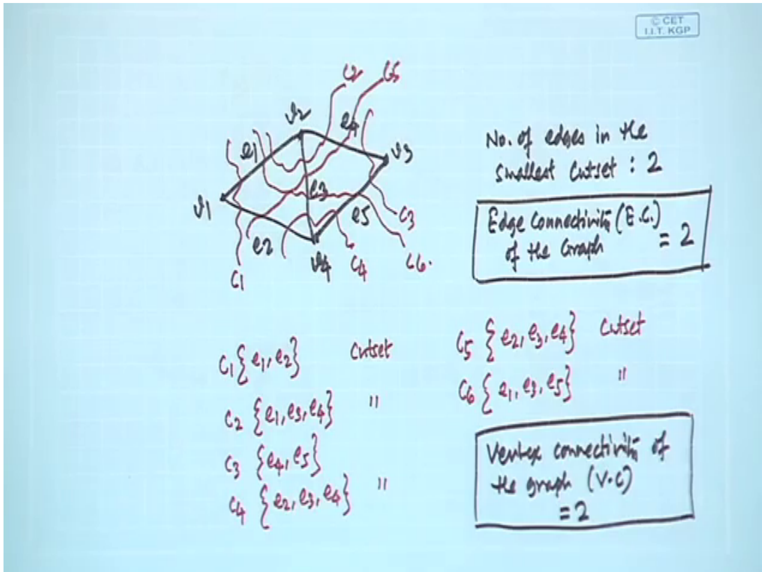

So let us look at this one you know what is suppose look at this particular slide you know here we have seen the different cutsets, so what is the smallest cutset in the sense what is the number of edges in the smallest cutset, is it alright? The smallest cutset here is number of edges in the smallest cutset, can you tell me the number? That number would be 2 because 1, 2 there is no other.

So you know you can see the graph also you have to remove at least two edges from this graph to make the graph disconnected, is it alright? So therefore the edge connectivity of the graph that is EC equal to 2, edge connectivity of the graph is 2. On the other hand the vertex connectivity there is what is the number of vertices whose removal would leave the graph disconnected, how many vertices do I have to remove to leave the remaining graph disconnected? If I remove v1, will the graph be disconnected? Answer is no, because if you leave v1 all the other things are on the other side.

Please remember if I remove v1 the e1 and e2 will go also, is it alright? Now if I remove v2 is the remaining graph disconnected? No, because everything else is on the other side, if I remove both v1 and v2 then also no, but if I remove v2 and v4 then the remaining graph will be disconnected, is it alright? So interestingly we can see that vertex connectivity of the graph VC is also 2. Now question is between edge connectivity and vertex connectivity you know there is a relation and what is that relation we shall explore that little later but at this point of time just remember that edge connectivity of a graph is the removal of the minimum number of edges whose removal will leave the graph disconnected and same thing about the vertex connectivity.

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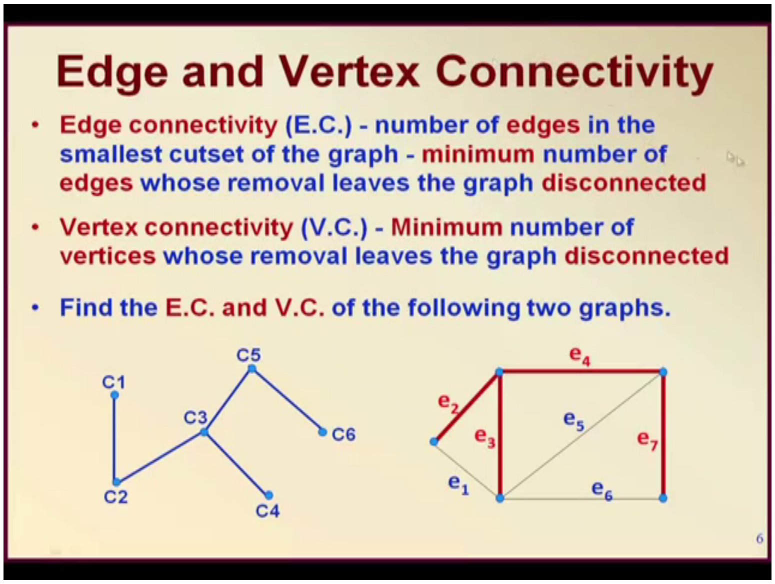

So here just look at the definitions EC is the number of edges in the smallest cutset of the graph, minimum number of edges whose removal leaves the graph disconnected. Vertex connectivity minimum number of vertices whose removal leaves the graph disconnected, alright? So find the EC and VC of the following two graphs that is if I have to find, now look at this tree now tell me what is the EC and VC of this tree? How many edges do I have to remove minimum number to make the graph disconnected, is it 1 only? Because it is a tree it is a minimally connected graph there are no circuits.

So removal of any edge will leave the remaining graph disconnected, is it alright? So if I remove this particular edge then c1 will be on the other side, if I remove this edge then this portion and this portion will be on the two different sides and there will be two components, right? So therefore for a tree the edge connectivity is 1 and what is vertex connectivity? You know you just remove say c3 look here the remaining graphs are disconnected.

So you can see that the VC for the tree is also 1, so EC equal to 1, VC equal to 1. Now look at this particular graph what is its EC and VC? How many edges do I have to remove to make the remaining graph disconnected? Is it 2 again because if I remove e1 and e2 then remaining graph is disconnected so it is 2 here. And what is the vertex connectivity how many vertex do I have to remove? If I remove only 1 vertex just see the graph is still connected, alright? And if I remove two let us say these two or these two then the graph is disconnected.

So in this case also edge connectivity is 2 and vertex connectivity is also 2, alright? Now question is do you like higher edge connectivity or lower edge connectivity? Do you want lower values of EC and VC or you think an higher value is good? Think of road network, in any road network there could be a some sort of disruption maybe due to some reason or maybe due to disaster. So let us assume this road e4 is not working, right? This road is not working if this road is not working, then what should be how to do I go from say this vertex to this vertex?

It is now possible that I can take this path I can go to here and from here I can go, is it alright? So you see look here since the vertex here the edge connectivity is 2 then even if a particular edge is removed you can still go from by using other edges but vertex connectivity is important too, right? So suppose these two vertices are removed then all these edges are gone all these edges are gone there will be no path that is available, is it alright?

So but suppose this vertex is removed, then will happen all these three paths are not useable (()) (25:09) so vertex is so important it is just a junction point like a bridge suppose this particular bridge is not working, right? You cannot go through the bridge has collapsed then what will happen if you have to go from here to here this path is not useable or from here to here this path is not useable but then you can still use this particular path.

So naturally you can understand that one expects a good value of the edge connectivity and vertex connectivity it is always better that if you can achieve a higher value of edge connectivity and vertex connectivity, right? So that is what is required.

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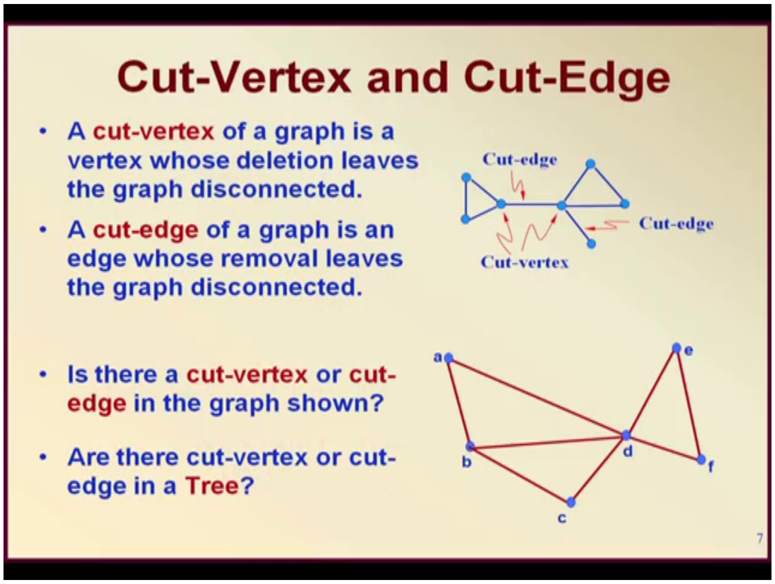

Now let us come to another concept that is about edge connectivity and vertex connectivity you know that is called a cut vertex or a cut edge. Look at this graph you know if I remove a particular vertex say this vertex then the remaining graph is disconnected or if I remove this edge then the remaining graph is disconnected just imagine that this is let us say a particular road and these are the two cities and anything happens to this city or this city or this road you know all the traffic that goes from here to here will be disrupted or imagine that this is one locality and this is another locality and this is the network or communication connectivity. So telecommunication link if there is a failure anywhere between this to this then entire communication link or either this area or this area will be disrupted because if the link is coming through that.

So it becomes so vital and there is no other alternative to really fall back to. So that is the importance about the cut vertex and cut edge. So cut vertex is a vertex whose deletion leaves the graph disconnected, right? Similarly the cut edge concept. Now look at this this is very interesting what is its edge connectivity? How many edges do I remove to make the graph disconnected? Look it should be at least 2 so these two if you disconnect then the graph is disconnected, but what is its vertex connectivity? How many vertex we can remove to really make the graph disconnected? Just see just 1 simply if you remove this particular node then this particular graph is disconnected.

So its vertex connectivity is just 1, so sometime even if the edge connectivity is good but if it is a poor vertex connectivity then you cannot achieve much out of a particular network, so we have to really see that both EC and VC they become good.

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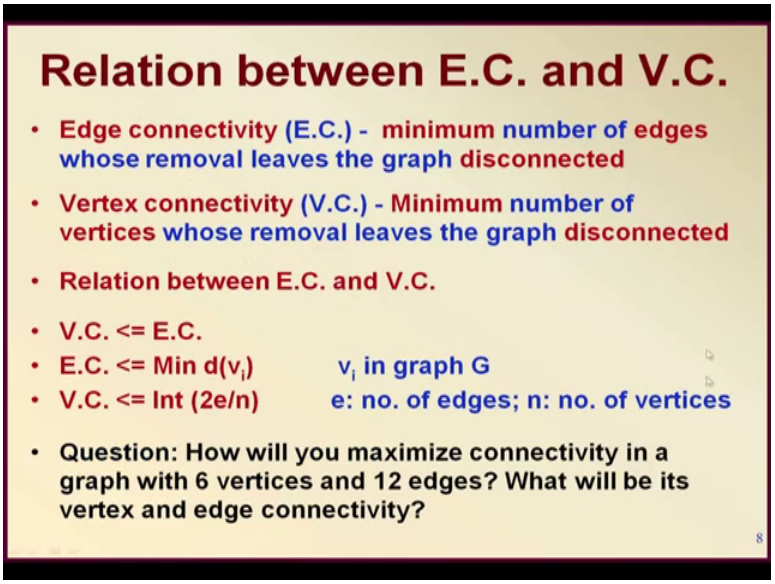

Now some of the relationships that VC should be less than equal to EC, EC should be less than minimum degree in the you know of the vi not should be it is found that it happens and VC is also less than integer 2 e by n, right? Why? Because just look at the degree, what is the degree? The total degree is 2 e. So what is the average degree? The average degree is 2 e by n, right? Every vertex the two edges are connected so total degree in the graph is 2 e.

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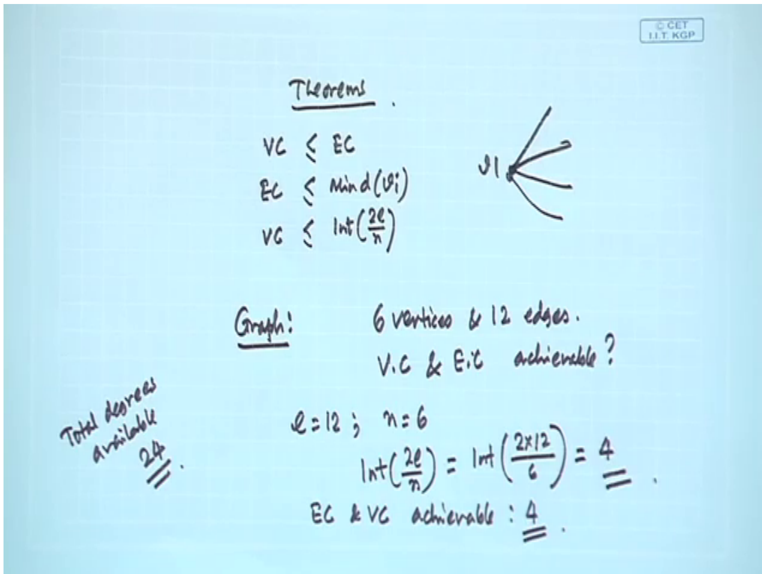

And if there are n vertices so if I make 2 e by n with an integer then we get that the vertex connectivity should be less than equal to so these are the relations we have VC EC then EC is less than minimum degree in a graph and also finally we have seen VC equal to integer 2 e by n. So these are the theorems, so these 3 theorems are there we shall not bother too much about them but just look at this question, how will you maximize connectivity in a graph with 6 vertices and 12 edges? What will be its vertex and edge connectivity, right?

So you see you know we have a graph where we have this much is only given we have 6 vertices and 12 edges, alright? So basically the question is what VC and EC are achievable, so this is the question how much VC and EC we can achieve. So here the e equal to 12 and n equal to 6 because 12 edges and 6 vertices. So what is integer 2 e by n? Integer 2 e by n it will be integer of 2 into 12 by 6 so can you see that this is coming out to be 4, right? So if it is a say more than 4 then we have to take the integer portion and it will become 4.

So basically it says that it is possible to really achieve an EC and VC achievable is 4 only, right? And how many total degrees are available graph? Total degrees available 24 because 2 into 12. So in the 6 different vertices we have to allocate them and we have to see that each particular vertex should be having something like 4 edges, so if this is a vertex v1 we should ensure that 4 things go out of that them and then like this we complete the design, alright?

And you know it therefore helps to really design a network by really creating the highest possible EC and VC if there are 6 vertices and let us say 6 cities to be connected by 12 roads, how do I make maximum connectivity? So that is the kind of question we have addressed here, right? I stop here and we will continue the more about cutsets and other concepts in our next lecture, so thank you very much.