Decision modeling
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Lecture 03
Decision making under risk

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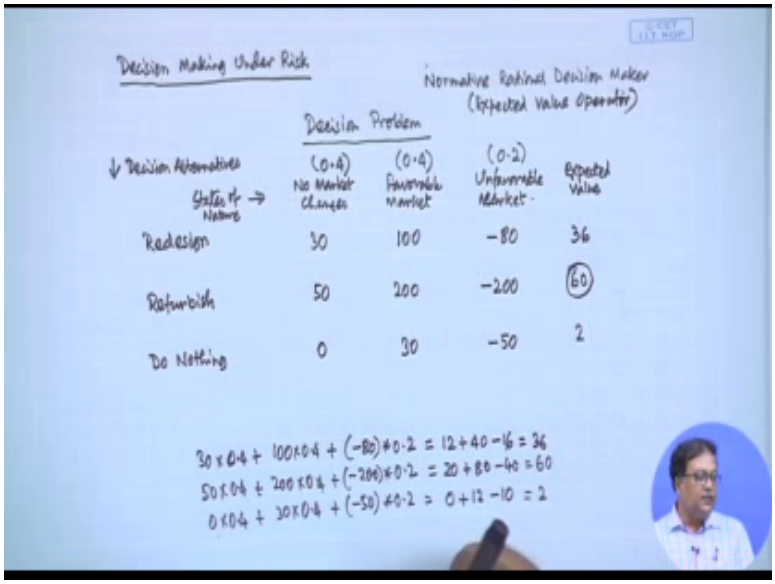

So we were discussing what is known as decision making under risk and under decision making under risk specifically we have last time seen the different kinds of decision maker particularly the maximin, maximax, Hurwicz and the Laplace decision makers,

So all of these different decision makers they had really evaluated a particular decision situation essentially without considering probabilities but specifically the decision making under risks situations.

Are situations where we can actually assigned to a decision problem what is known as the probabilities, probabilities for different states of nature, so let's reconsider the decision problem that we had already taken up earlier basically this side we have the decision alternatives which is under the control of the decision makers, decision alternatives, so we had consider three decision alternatives, one is the redesign, the second one is refurbish with maybe regard to a small shop.

And the third is do nothing, so these are the three decision alternatives and this side we have what is known as the states of nature, so we had consider three states of nature, the first one is

the no market changes and we had considered favorable market and unfavorable market, so these are the three broad situations we have considered right, so now the various payoffs are also given for redesign the payoff are thirty, hundred and minus eighty.

For refurbish the payoffs are fifty, two hundred and minus two hundred and for do nothing zero, thirty and minus fifty, so these are the different payoff, now additionally because this is a situation where we are considering a normative rational decision maker, who can also be called an expected value operator right, so an expected value operator would assign certain probabilities with the states of nature.

Let me repeat once again the states of nature are those on which the decision maker has low control, the decision alternatives are those on which the decision maker has a control basically the decision maker can choose one of the decision alternative but it cannot choose a state of nature, one of the state of nature will occur from an expected value operator would really assume or will have certain probabilities associated with the different states of nature right.

Say let's put some probability values here, let say that point four is the probability for low market changes again another point four for favorable market and point two for the unfavorable market situation, so these are the three different probabilities which will be associated with the different states of nature and what we have to really do, we have to calculate the expected value for these different decision alternatives, so if we have these three different probabilities.

Forty percent, forty percent and twenty percent, let us look what would be our decision for redesign, redesign would be that thirty into point four plus hundred into point four plus minus eighty into point two right, and this will come out to thirty into point four is twelve plus hundred into point four is forty and minus eighty it will come to minus sixteen, so it will boil down to fifty two minus sixteen equal to thirty six.

So that therefore thirty six would be our expected value if the person really goes for redesign, what would be the value for refurbish, it will be again fifty into point four plus two hundred into point four plus minus two hundred into point two equal to fifty that will be twenty, two hundred into point four it will be eighty and minus forty, so it comes to sixty so sixty would be the expected value for this refurbish decision.

Finally for do nothing again zero into point four plus thirty into point four plus minus fifty into point two, so it comes down to zero plus twelve minus ten equal to two, so two would be the expected value, so if we really look at all these calculations.

The expected value would be the thirty six for redesign option, sixty for refurbish option and two for do nothing option, again since the sixty is the highest the person would really go for refurbish and would expect a particular expected value that is the payoff that would be sixty,

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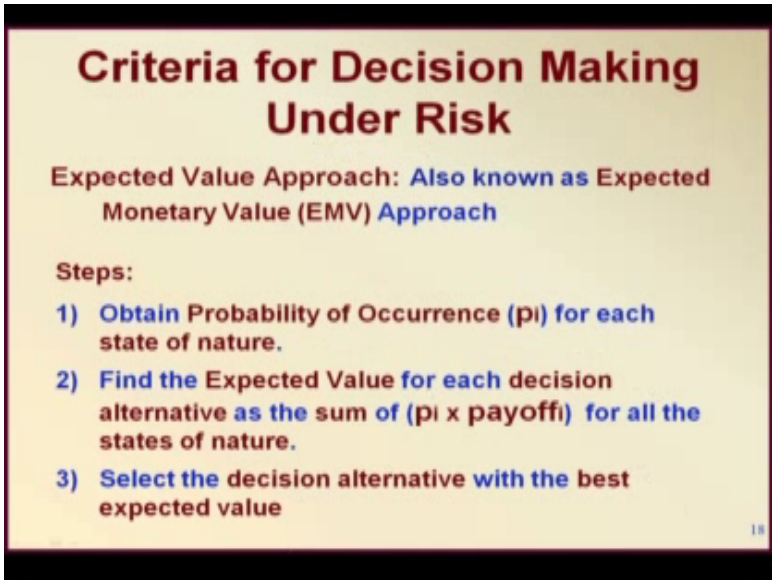

So let's go back to the slides again and let us look that exactly same thing is mentioned here that expected value approach also known as the expected monitory value approach, now first of all obtain the probability of occurrences for the states of nature, so we had three states of nature and we have assume point four point four and point two.

So it could be some other probability in that case the values will be different, then find the expected value for each decision alternative as a sum of P I into payoff I for all the states of nature and finally select the decision alternative with the best expected value, so we got those thirty six, sixty and two and we choose sixty as the best possible payoff that means we assume

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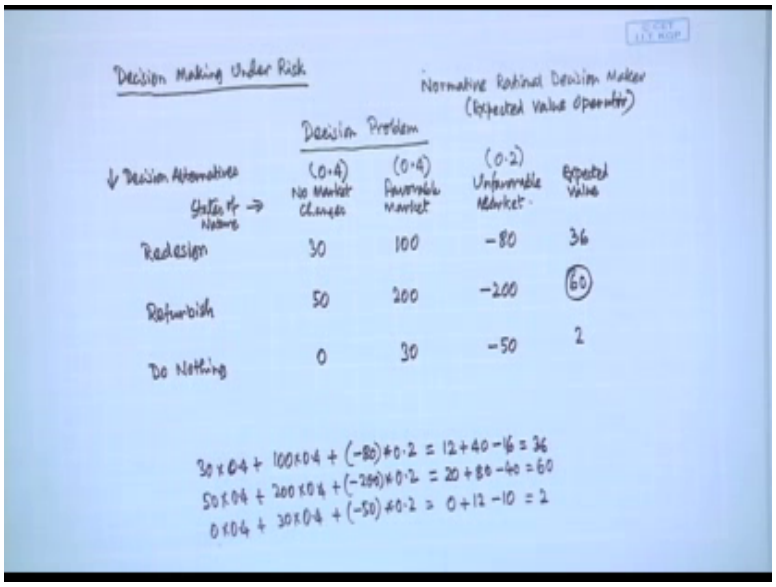

In this case that the expected value operator will go for refurbish. Because that is what is showing the maximum payoff of sixty,

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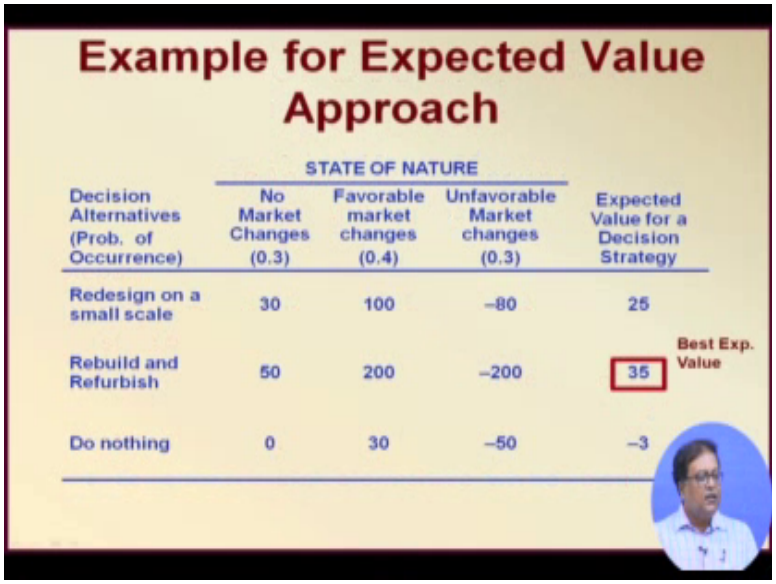

So now here is an another example look at another example which is also given in the slide so you can see suppose the probabilities where thirty percent, forty percent and thirty percent then

the evaluation will be slightly different the expected value for a decision strategy would have been thirty into point three that is nine, hundred into point four that is forty.

Forty nine minus eighty into point three that is minus twenty four and it will become twenty five, so anyway these values will differ based on the probabilities that you have and based on which you can have a different kind of decision.

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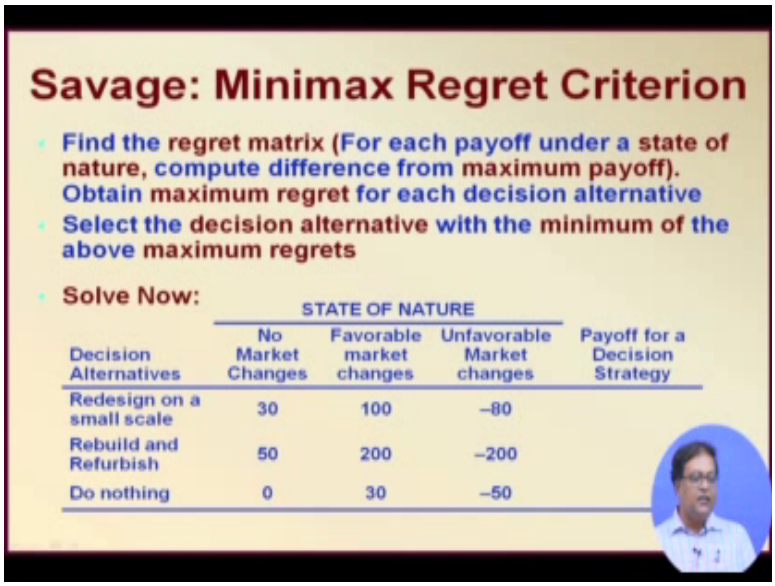

An alternate to this is actually let's look at is what is known as the savage or the minimax regret criteria.Actually find the regret matrix for each payoff under a state of nature compute difference from maximum payoff.

Obtain maximum regret for each decision alternative, select the decision alternative with the minimum of the above maximum regret, so you can also think of what is known as savage criteria which is essentially another way of looking at the expected value operation but then really looking at it in a slightly different way.

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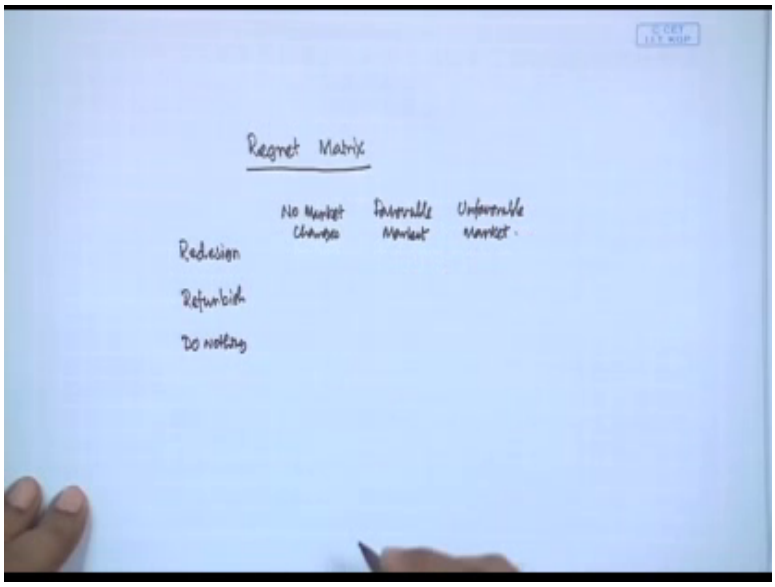

So suppose we have this thing here and let us construct what is known as the regret matrix right, so this is a regret matrix. So again we have three decision alternative redesign, refurbish and do nothing, these are the three decision alternatives and we have three decision states of nature, no market changes then favorable market and unfavorable market, what are the regrets,

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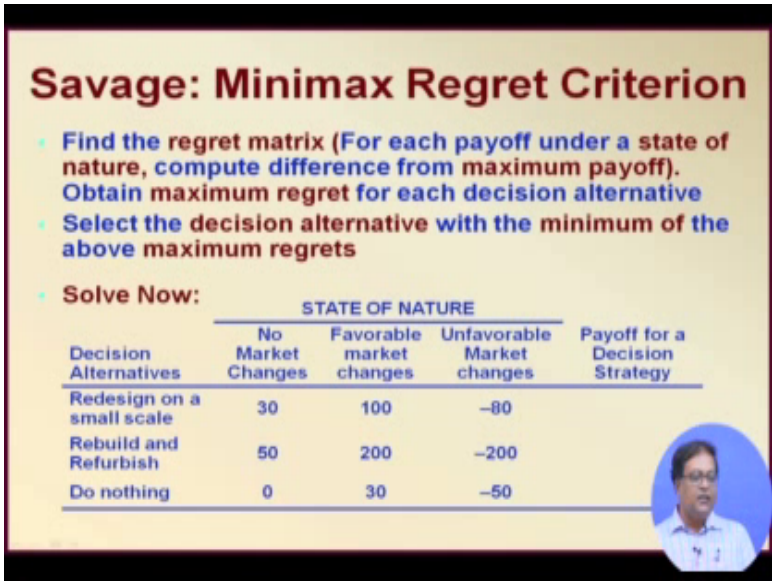

Now first of all you look at the slide. In the first case under the no market change if the states of nature are thirty, fifty and zero, these are our payoff, that means suppose if you would have gone for rebuild and refurbish.

You would have got fifty and that is the best possible payoff that you could have got under the no market changes, that means if you are going for rebuild and refurbish and under the state of nature no market changes you have no regret, but if you would have gone for redesign you have a regret of twenty, fifty minus thirty, similarly what is the regret for do nothing it will be fifty because you could have got fifty, but you got zero that means your regret is fifty.

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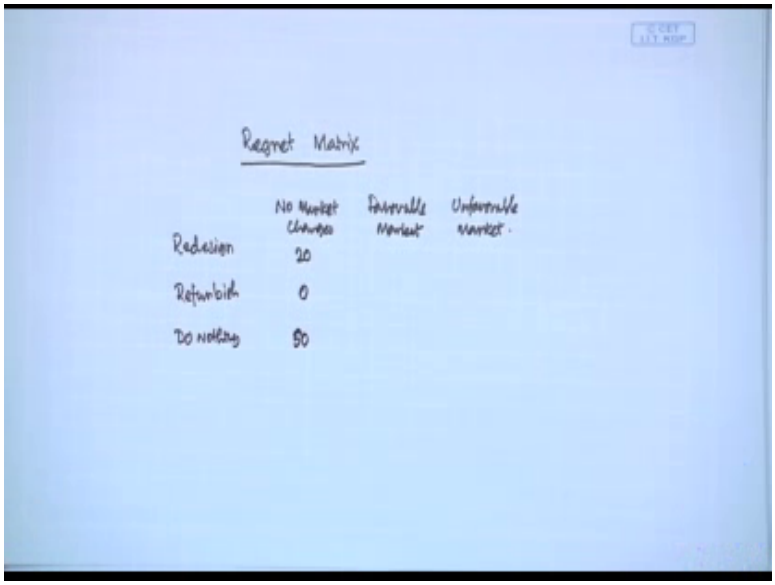

So this is where we can write the regret matrix twenty, zero, and fifty, right? So these are the regrets and one would have had if one would have got for decision other than the best possible decision under the situation that is rebuild and refurbish,

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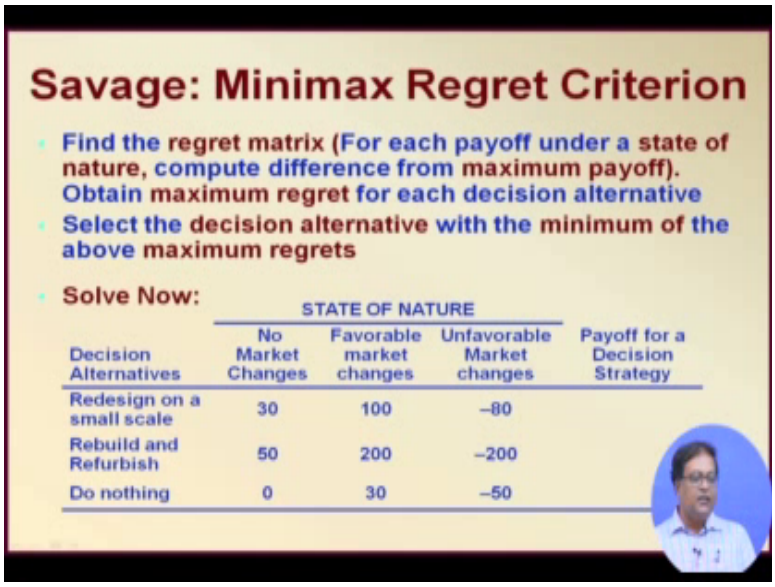

Again go back to slide let us look at under the favorable market changes. If the favorable market changes would have happen then one would have got the best possible payoff that is two hundred.

So if you would have gone for refurbish you wouldn't have regretted anything, but if you would have gone for redesign you would have regretted by hundred, and if you would have got for do nothing you would have regretted for one seventy right, So these are the sum of the regrets that we have and let us put those regrets here as well.

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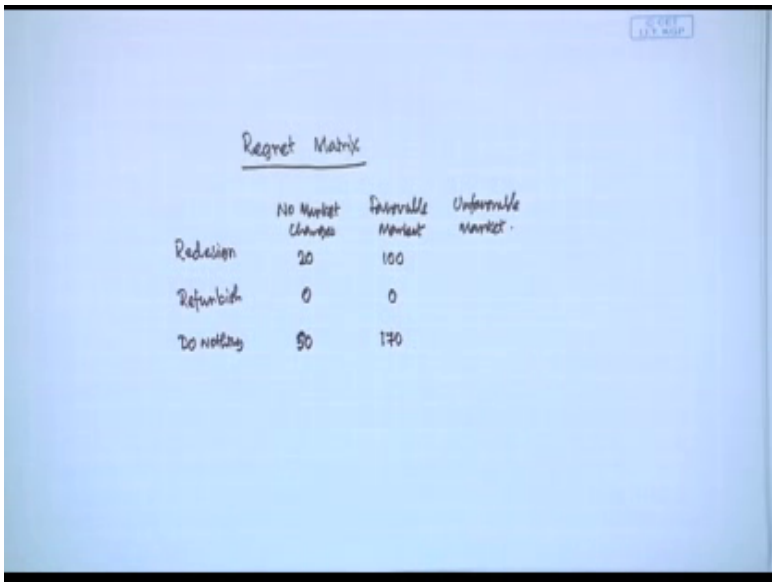

So here the regret is hundred, here is the regret is zero and here the regret is one hundred and seventy right. So let us also calculate the regret for what is known as unfavorable market changes,

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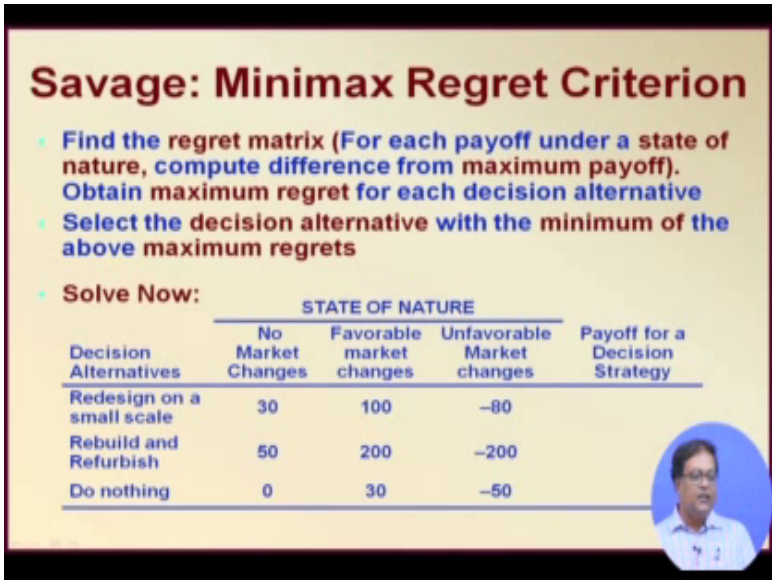

We see that the payoffs are minus eighty, minus two hundred and minus fifty, which one is the best minus fifty that means under unfavorable market changes if one goes for do nothing then

there is no regret right, so it's a zero regret but here the regret will be one fifty and here the regret would be thirty, right?

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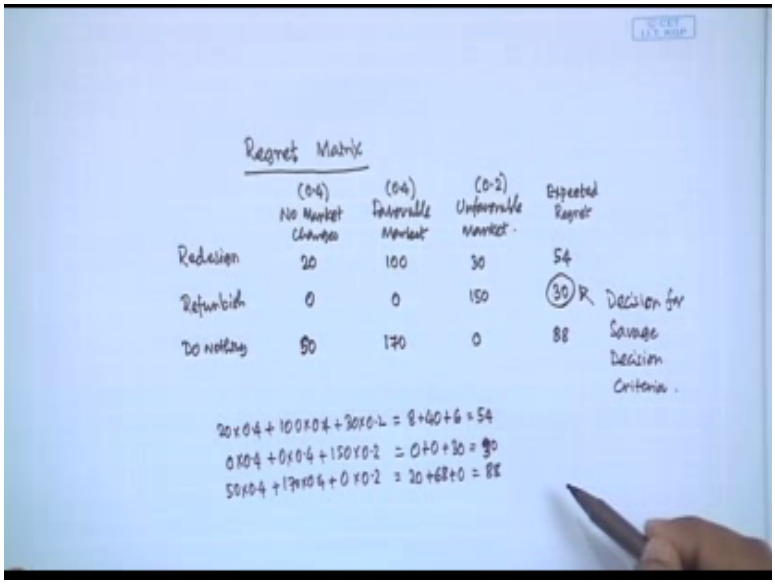

So we can write all of these here that is thirty, one fifty and zero right. So under unfavorable market situation we have the three different regrets that is thirty, one fifty and zero, so if we really look at this then what we can put is that these are the total regrets out of the three different situations,

Now out of the states of nature we have the values that is point four, point four and point two these are what we thought are the probabilities associated with the different states of nature, forty percent or point four is the probability.

That no market changes may occur, point four is the favorable market situation may occur and point two unfavorable market situation may occur, so out of all these we can calculate what is known as an expected regret value, so what are the expected regret values so like we have calculated the expected payoff we can also calculate what is known as the expected regret, if one goes for redesign what is the expected regret.

It will be twenty into point four plus hundred into point four plus thirty into point two that means eight plus forty plus six it comes to fifty four, so fifty four would be the expected regret, similarly for the refurbish this expected regret will be zero into point four plus zero into point

four plus one fifty into point two why, because the expected regret was zero, zero and one fifty under the three different states of nature and each one is having probabilities of forty percent.

Forty percent and twenty percent, so it will be zero plus zero plus thirty that comes to thirty, and for the third case it will be fifty into point four plus one seventy into point four plus zero into point two that means twenty plus sixty eight plus zero equal to eighty eight, so these are my expected regret values, fifty four, thirty and eighty eight right so that is what is the savage or minimax regret criteria, now out of all these which one shows the minimum regret.

It is thirty right, so this is going to be our decision for savage decision criteria right, this will be our decision for the savage decision criteria why, because this particular decision would like to go for the expected regret and take that particular decision alternative which minimizes the expected regret is it, alright?

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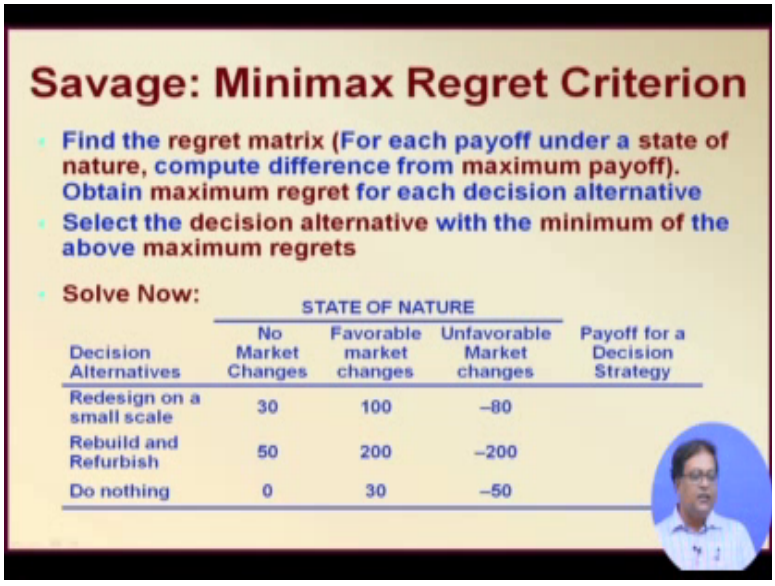

So that is our next criteria that was minimax regret criteria. But once we have obtained this let us look at the previous slide once again.

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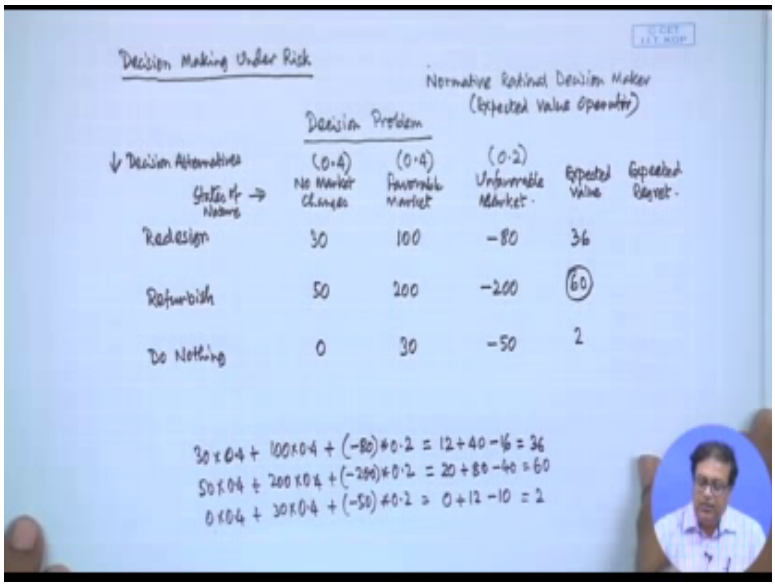

And we had our expected value operator and we have already calculated those values. Now once again let us put back what is known as the expected regret values, please revisit what was there in the regret matrix.

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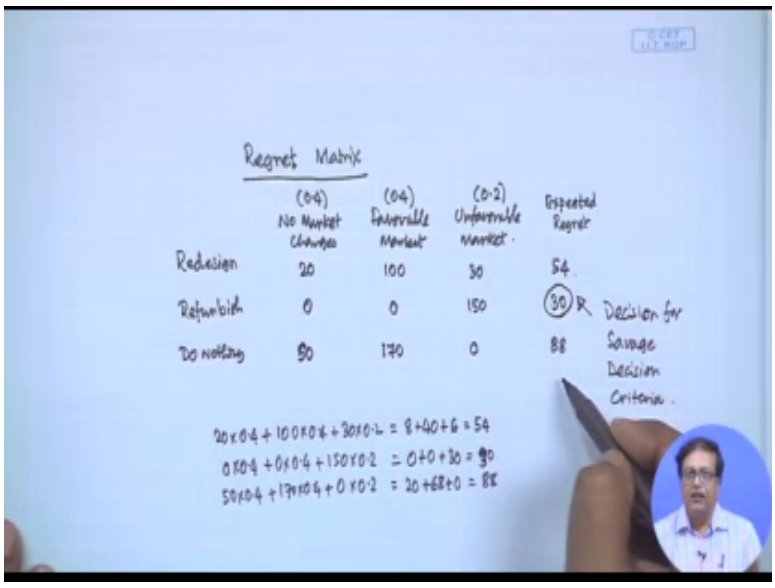

The regret matrix, the expected regret values for the same probabilities were fifty four, thirty and eighty eight,

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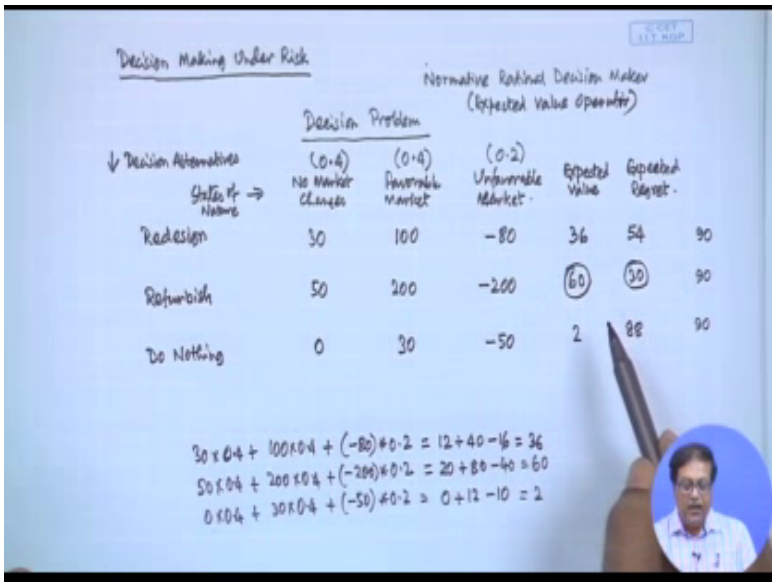

So what we do we put the same values here the fifty four, thirty and eighty eight and we note that this thirty is the expected regret right. One very interesting thing you note that the decision that we have been able to make for the expected value operator that is sixty and the decision that we could make for the expected regret both of them have the same value,

I mean the same value of the decision alternative, the decision alternative that comes out of the expected value operator and the decision that has come out of the expected regret operator it has come same.

Even interesting thing actually to note further that if I add the expected value and the expected regret then what is the total the total comes to what is known as ninety here, ninety here and ninety here, it is very interesting that the total is exactly same right,

So in a way you can say that expected regret is a kind of a complimentary value to the expected value, why not we, we can even guide that the expected value is nothing but ninety minus the expected regret isn't it.

So something more must be there and what is that something more, why the expected value and expected regret value when we add we get exactly the same number, you see we have made some calculation for the expected value and we have put those probabilities,

We have multiplied by the payoff with those probability values and then sum them up right, and we got those expected regret values. Now question is that what is it or is it going to happen only for this problem or for all problems that is also another very interesting question let us try to get an answer.

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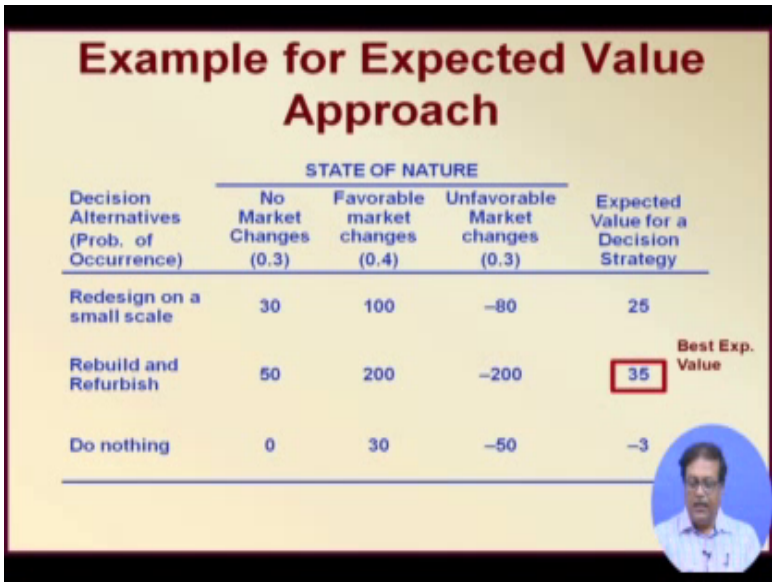

So please look at the slide once again, you see this is a slightly different probability values that we have ahead, look at the slide and you see that we have thirty, hundred and minus eighty and we had this twenty five, thirty five, and minus three and those are our expected value for a decision strategy.

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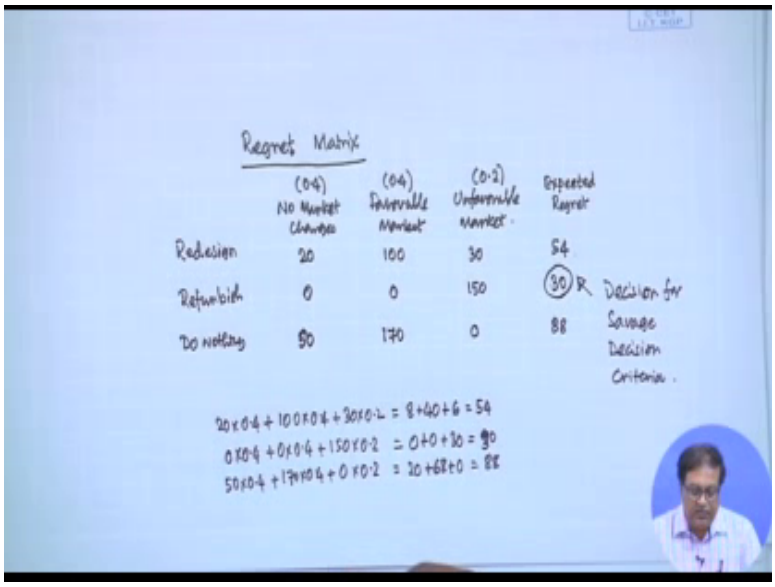

Now look at the calculation once again then let us look at that regret matrix.

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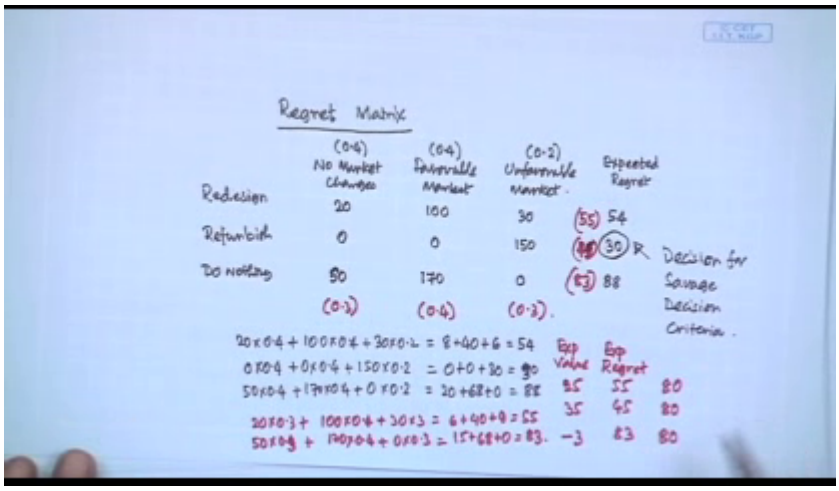

And suppose now if we recalculate these values once again for a different set of probabilities and let me right down the probabilities here with a different color, suppose we had point three, point four, and point three, what would be our expected regret values, the expected regret values would have been then the twenty into point three, hundred into point four and thirty into point three.

So it comes to six, it comes to forty and it comes to nine, so it will come to six plus nine plus forty that it will come to around let's calculate twenty into point three plus hundred into point

four plus thirty into point three it will come to six plus forty plus nine, so it will come to fifty five right, so it will come to fifty five, the second one will come to again it will be straight forward hundred fifty into point two so it will become thirty.

So let's right it down so here it will become fifty five, here it will become thirty and the other case it will become the fifty into one seventy and zero, so it will become fifty into point three plus one seventy into point four plus zero into point three, so it will become fifteen plus sixty eight plus zero, so it will become eighty three and in the second case let's recalculate the second case also ya because there is a different probabilities.

So this is not thirty this will be forty five, why it is forty five please understand that this calculation is here one fifty this point three, so this one fifty into point three because zero into point three is zero, zero into point four is zero and one fifty into three is forty five right, so not point two because point two is was for this case, so fifty five, forty five and eighty three, so let us compare these fifty five, forty five.

So let me write it here once again the expected regret is fifty five, forty five and eighty three, and what was the expected value for this particular case, let's look at this slide, expected values were twenty five, thirty five and minus three, so let put them here twenty five, thirty five and minus three, just add it will become eighty, it will become eighty, it will become eighty right, so again we see that even for a separate problem where the probabilities are different.

The expected value and expected regret they all are bringing out the similar value, is there some significance into it right, there is a very different significance and let us look at why this significance and what is it that are so important in this context right,

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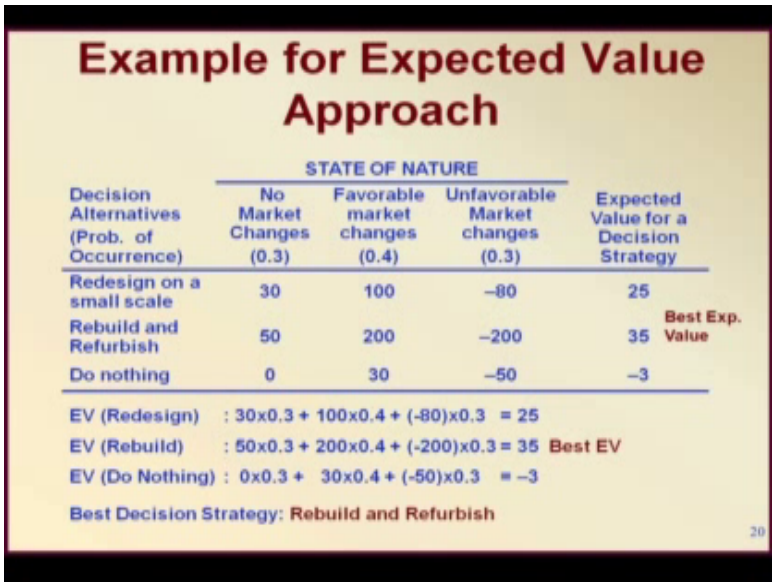

so let's us look at and try to see about those sort of things, so once again that this is what is here what we have that for an expected value operation what we find is that.

That out of all the different things that the kind that put the best expected value that actually becomes our best decision strategy that is the rebuild and refurbish in this case and a similar kind of decision we had also obtained under slightly different probability values that is point four, point four and point two.

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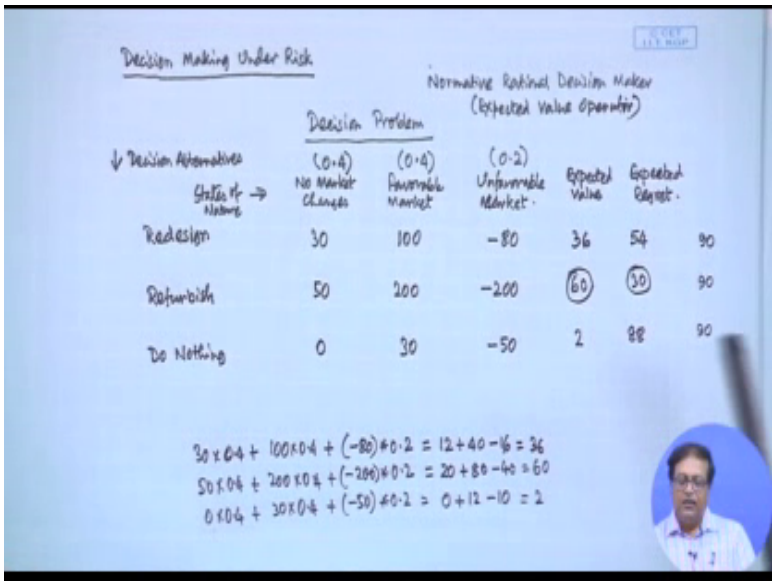

So in fact and very interesting question we have also got that is between expected value and expected regret when you sum them up you get same sum all the time. And it mishaps some specific significance and what is it that we shall see in our next section,

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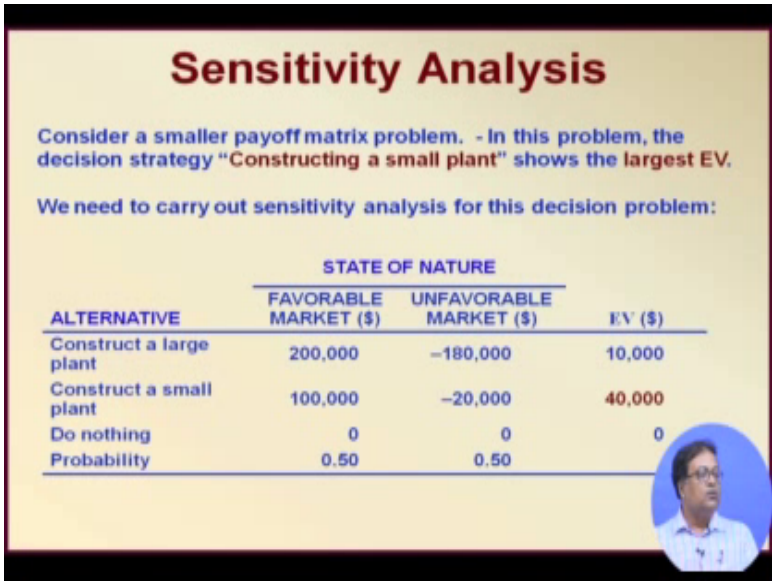

But before that before we come to the next section there is also another very important thing which is known as the sensitivity analysis, so consider a smaller payoff matrix problem.

In this problem the decision strategy is constructing a small plant and shows the largest EV or expected value, so suppose we need to carry out a sensitivity analysis for this decision problem.

Let's look at the problem once again; this problem has got two states of nature favorable market and unfavorable market, for the favorable market the total payoff comes out to be two hundred thousand dollars and the unfavorable market situation the payoff comes out to be minus one eighty thousand dollars, whereas if we go for a small plant then the favorable market would return hundred thousand dollar.

But unfavorable market would return minus twenty thousand dollar, if you do nothing then you get nothing so whether favorable or unfavorable in both the cases it will be zero and assuming an equal probability to the favorable and unfavorable market if we calculate the expected value, the expected value really comes out to be ten thousand and forty thousand, the important decision that is we are speaking about here is that in this case if we calculate.

Then we find the expected value comes out to be ten thousand for constructing a large plant and forty thousand for constructing a small plant right and however there is a important decision that we have to make that is should we straight way go for constructing a small plant or we would like to see a little bit more, what is that little bit more that is what is called sensitivity analysis, see the probability that we have put fifty percent for favorable market.

And fifty percent for unfavorable market they are at best estimates, so since they are estimates the question always arises suppose this probabilities are different, what is the sensitivity, at what point the decision should go for constructing a large plant other than a small plant right, so that is what is about sensitivity analysis, anyway we stop here and in our next section we shall discuss the specific issues about the information particularly the value of information right.

So we can call the next section as our value of information section, after we complete the sensitivity analysis part we shall take up the value of information that as a concept.