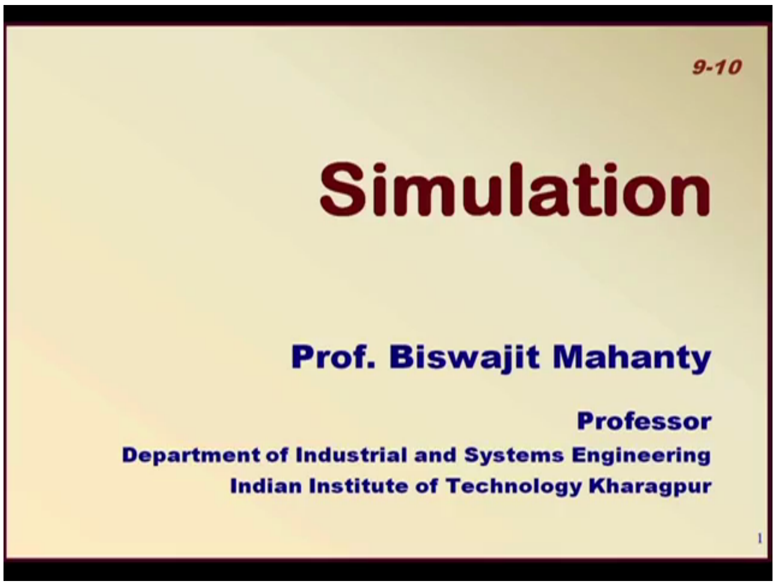
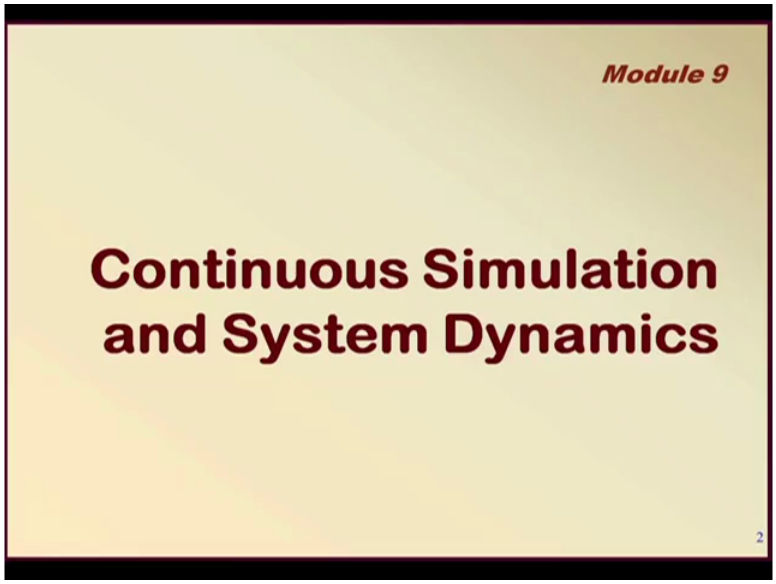
Course on Decision Modeling
By Professor Biswajit Mahanty
Department of Industrial and System Engineering
Indian Institute of Technology Kharagpur
Lecture No 29
Continuous simulation and System Dynamics

(Refer Slide Time 0:25)

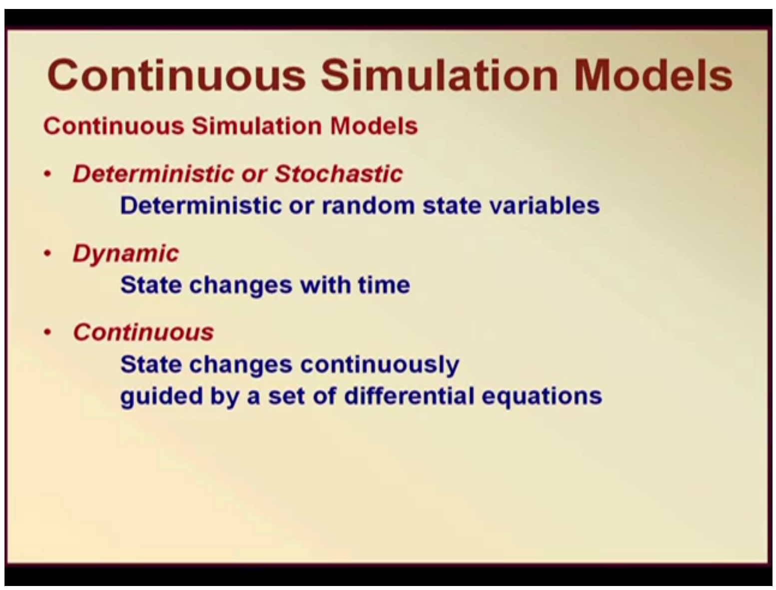


We are discussing simulation and in this particular topic we have already discussed discrete event simulation system in detail and additionally we have also discussed Monte Carlo simulation, we have also discussed an discrete event simulation language by the name Arena and we are now into our last topic in the simulation that is continuous simulation and system dynamics.

(Refer Slide Time 0:57)



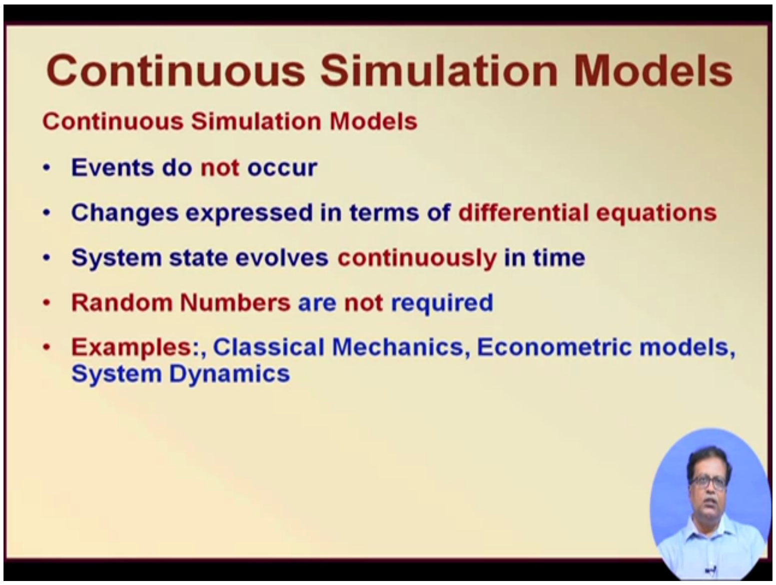
(Refer Slide Time 1:02)



See unlike the discrete event simulation the continuous simulation is different, in continuous simulation as you see that we have first of all both deterministic or stochastic variables are there, you know what is stochastic? Is that probabilities are involved and this probabilities themselves are dynamic in nature. the second thing about this model that there is events are not occurring, however the state or the system state through which the continuous system that we are talking about is represented those variable values are changing with time, so it's dynamic.

And finally they are actually continuous in a state because the state changes continuously as it is already known because it is dynamic and this state changes are guided by a set of differential equations. Now not in all cases the equations may be differential we have those econometric model where we really do not use differential equation but we use a variant of that. So more or less conceptually things are similar, only thing the forms of the equations are slightly different.

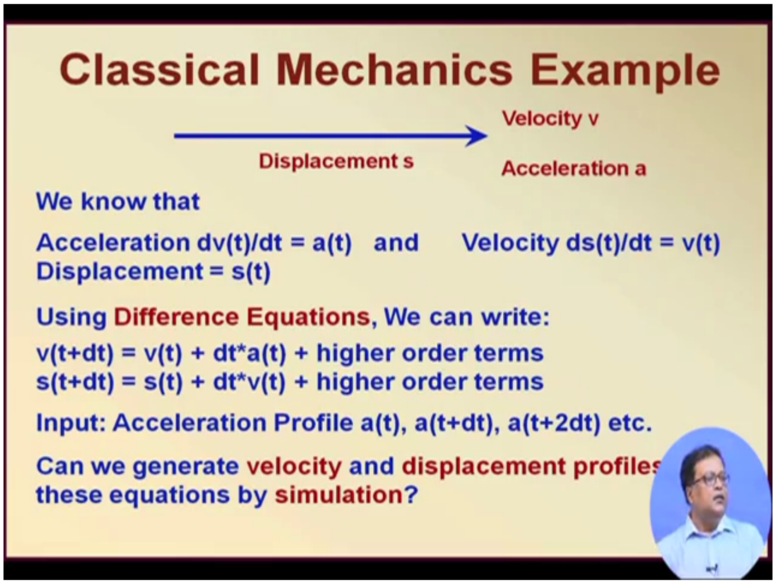
(Refer Slide Time 2:24)



Now here are some specific things that in continuous simulation model there are no events changes expressed in terms of differential equations and system state evolves continuously in time because there are no events random numbers are not required, right? So that is one major thing that we are really not talking about distributions random numbers, random variates these things are not required.

Examples, the examples could be first of all there are classical mechanics then there are econometric and then finally the system dynamics which is a full-fledged theory in itself, how can we model what is known as suicidal systems or socio-economic systems or even sometimes physical systems these are the 3 broad examples that we shall see in our subsequent course.

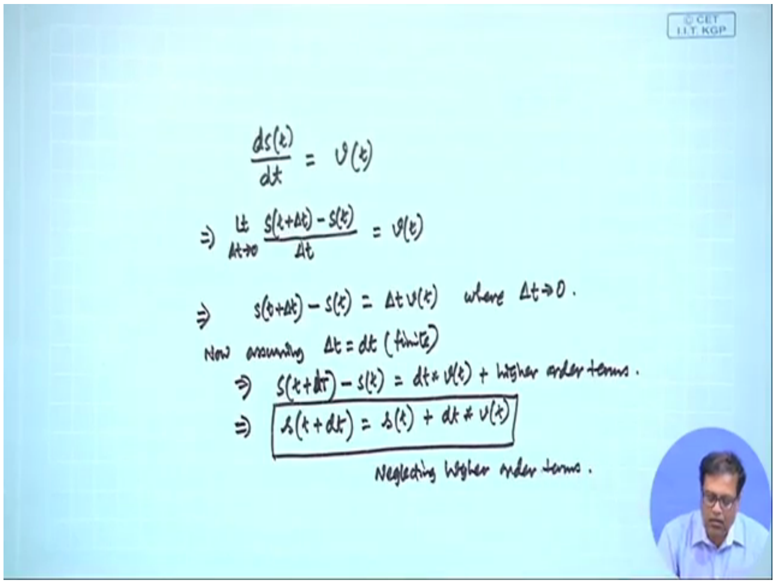
(Refer Slide Time 3:27)



First of all let us look at an example of the classical mechanics you know in classical mechanics what happens? First of all when a particle moves the particle is moving because it has a velocity, right? And because it has a velocity v, it is having a displacement, additionally it can have acceleration and because of this acceleration the velocity also changes, so these are the equations.

What are the equations? Look at those equations first of all the acceleration dv d(t) equals to a(t), a is for acceleration and velocity also itself is a differential equation that is ds(t) dt is v(t) and displacement we know s(t). Now instead of keeping those equations in the differential form the can be also expressed in the difference equation form. , how let us see that?

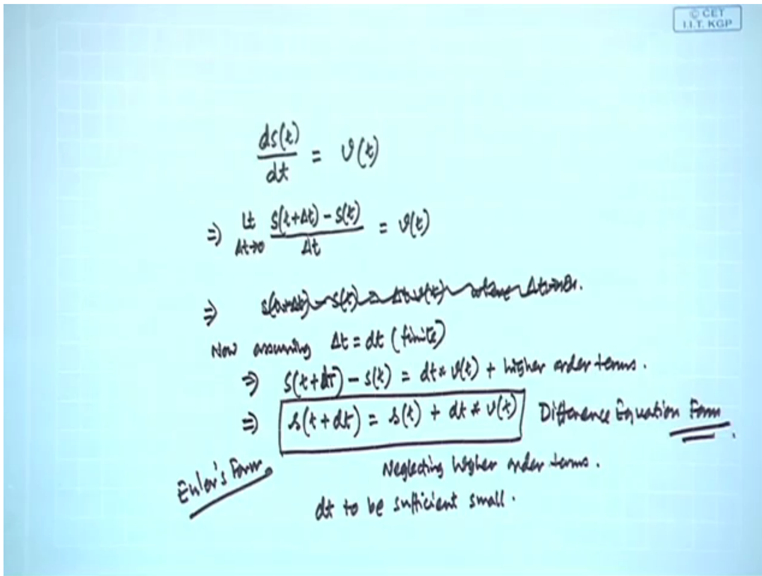
(Refer Slide Time 4:39)



Supposing we have let us say ds(t)dt equals to v(t) because you know that is the velocity equation. we know ddt of s(t) is basically nothing but limit of Delta t tends to 0 is t plus delta t minus s(t0 divided by delta t, so equal to v(t). So we know that, that you know this ddt of s(t) is nothing but limit of Delta t tends to 0, s that is displacement, t plus delta t minus s(t) by delta t equal to v(t).

So this will give us that you know s(t) plus delta t minus s(t) equals to Delta t v(t) where Delta t tends to 0. Now assuming you know Delta t equals to dt finite then we have really s(t) plus dt equal to minus s(t) equal to dt times v(t) plus higher-order terms or in other words st plus dt equal to st plus dt times v(t) neglecting higher-order terms, right?

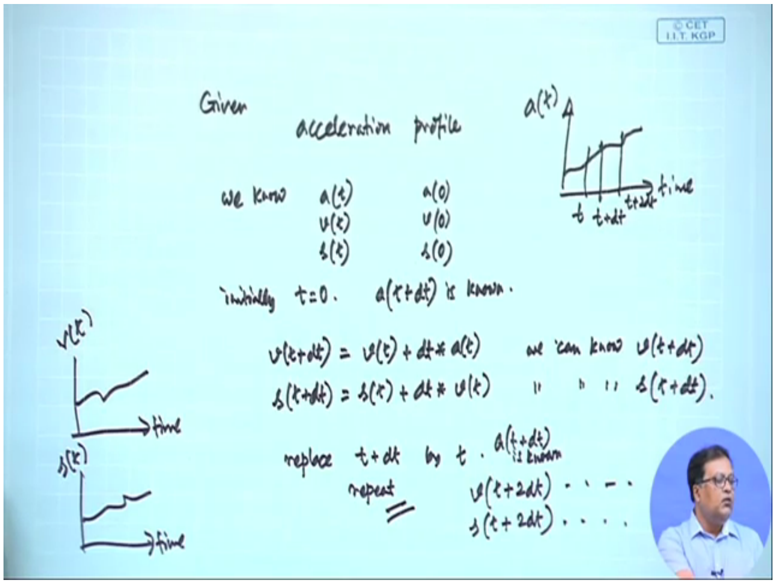
(Refer Slide Time 7:10)



Basically you know you can even delete this equation because from here itself, if you straightaway right Delta t equal to dt then you can get this form, right? So this form may or may not be possible to write, so once you have this then you can get this particular form and this form is called the difference equation form, right? Obviously here you must have dt to be sufficient small and this is called Eulers form, right? So this is an Eulers form.

So you know, look at the slide now, so basically same thing as been written used Euler's form and you can right the difference equation in these forms. So can we generate velocity and displacement profiles from these equations by simulation? So how can you generate the velocity and you know displacement profile?

(Refer Slide Time 8:34)



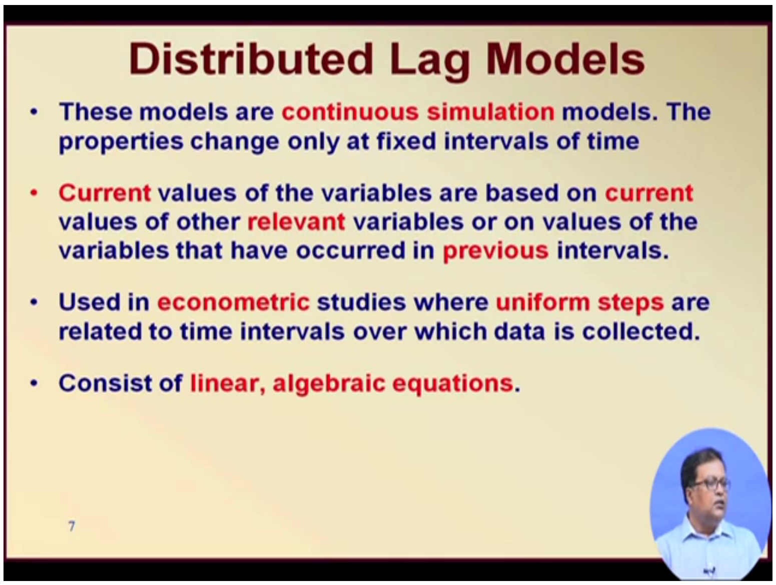
So given acceleration profile, so we have to have something, we have acceleration profile, what does it mean? It means that if this is time and this is acceleration then this plot is given, right? So if this is t, this is t plus dt, this is t plus 2dt etc. So this graph is known, so if this graph is known supposing we know a(t), we know v(t) and we know s(t). A Special case we know a(0), we know v(0) and we know s(0) like that 0 I mean assume t equals to 0 initially.

Initially t equals to 0 and a(t) plus d(t) is known, all these a(t) values are known. So we know vt is an integration, so v t plus dt equal to v(t) plus dt times a(t). So we can we can know v t plus dt. Similarly since st plus dt equals to s(t) plus dt times v(t) we can know st plus dt, alright? So now you know Uh if we replace t plus dt by t and repeat then what will happen you can know v t plus 2dt, s(t plus 2dt), why? Because a(t plus dt) is known. So like this we can know all of these values. So we know vt, we know vt plus dt, we know vt plus 2dt, so we can generate the velocity and displacement profiles, right?

So we can generate these profiles. So that is the real thing about this classical mechanics that we can generate velocity and displacement profiles, so same thing is written here that is we know a(t), v(t), s(t) values for t equals to 0 and for small d(t) and neglecting higher-order terms we can find out subsequent values and generate the profiles, right? So this is our, we can actually continue simulation by solving essentially, what are we doing?

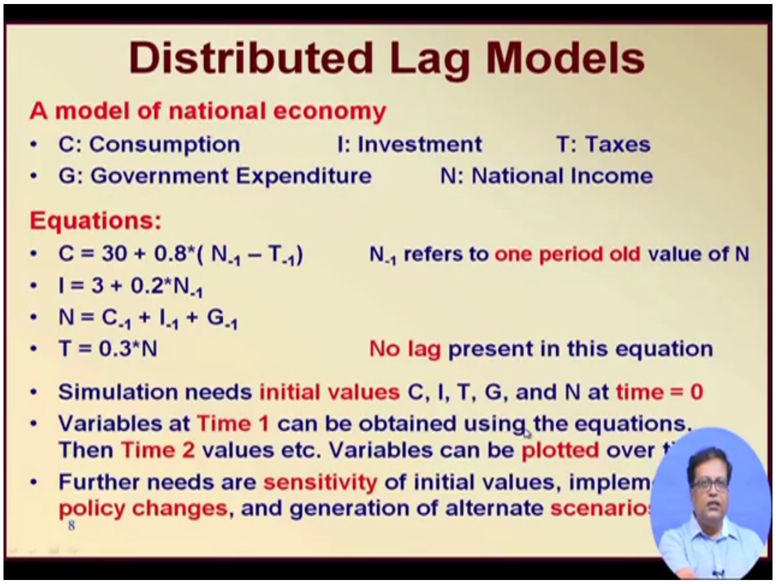
We are solving those differential equations numerically; one point must be remembered while we do like this there could be some errors due to the neglecting higher-order terms. How to reduce this error is a very important thing? But one must, we may remember that if we take sufficiently small d(t) these errors will be small and it will may not affect that much in ever simulation, is it all right? or what we can do? We can use better equations then difference equations those are available in the numerical analysis theory, right?

(Refer Slide Time 12:56)



Now we take up the other one that is an econometric model which we can call the distributed lag models. So these models are continuous simulation models and the property change only at fixed intervals of time. So current values of the variables are based on current values of other relevant variables or on values of variables that have occurred in previous intervals.

(Refer Slide Time 13:24)



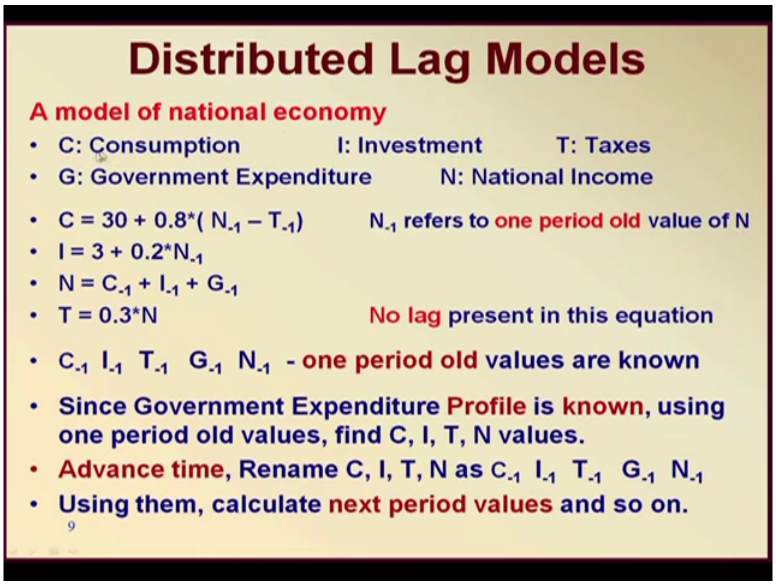
So used in econometric studies where uniform steps are related to time intervals over which data is collected and consist of linear algebraic equations. so here is an example, so suppose we have a model of national economy where C is consumption. I is investment, T is taxes, G is government expenditure and N is national income, right? So we have certain equations and these equations are C equals to 30 plus 0.8 star N minus 1 minus T minus 1, right?

N minus 1 and T minus 1 they are one period old values, similarly I equals to 3 plus 0.2 N minus 1, N equals to C minus 1 plus I minus 1 plus G minus 1 and T is 0.3 N. Obviously these are all econometric equations, we can understand easily because tax is assumed to be around 30 percent of national income, right? National income is last period consumption investment and government expenditure that is called National income and investment is equal to 3 because it is some kind of equation, what was the national income?

So 20 percent of the national income plus say constant value that is taken as the investment and consumption is you know national income minus taxes. So that is the money that is available and 80 percent of that let us say is consumption plus a constant value 30. So these are some equations, now assume that you know we really want to simulate, simulate means we have one set of values using that one set of values, can you compute the next set of values?

That means we have all the previous values and using those previous values, can we calculate all the next term values? So here let us see we have all the minus 1 values, right? We have C minus 1 value, I minus 1 value, N minus 1 values and T national this thing but we do not have let us say the government expenditure, so what we can do? We can take government expenditure as a kind of independent variable and suppose we know the government expenditure variable profile, right?

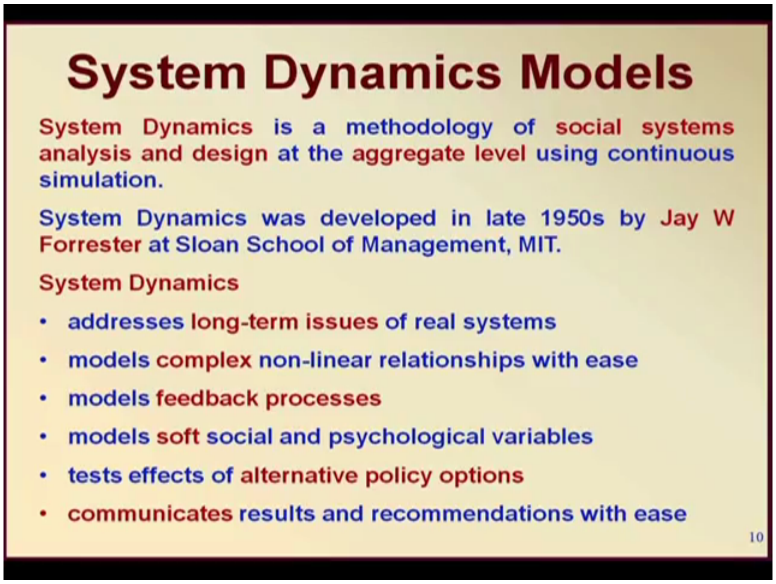
(Refer Slide Time 16:06)



So if we have that then probably we can calculate all the others, let us see how? So you see the same equations are written here. So you see C, last 1-year-old values, I 1-year-old, T G N all the one period old values are known. So we also should know what is known as the government expenditure profile? So that means we know all G values, right? Starting from G0, G1, G2, G3, G4 and G5.

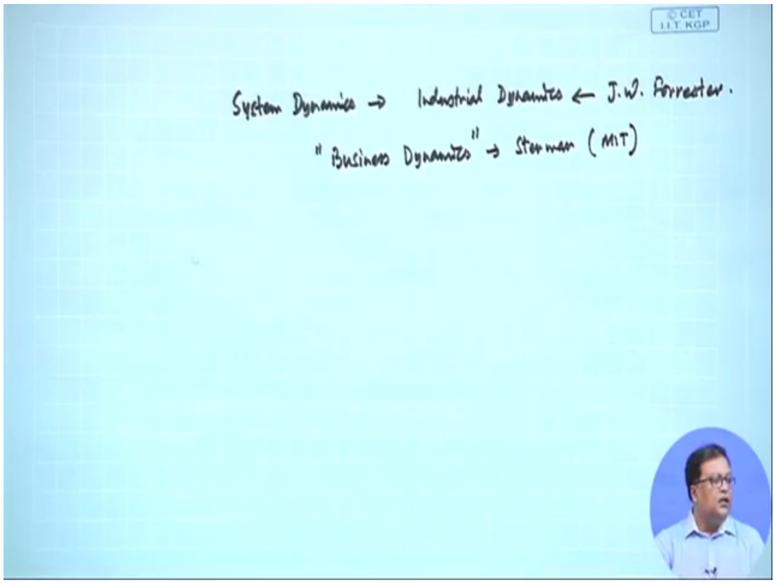
So using all the 0 values we can calculate all the 1 values using these equations then using all the 1 values we can calculate all the 2 values. So like this we can keep on calculating and we can generate the profile of CINT that means how they vary over time? All of these variables can actually be obtained. So that is essential idea of such distributed lag models and such models are used in econometric, right?

(Refer Slide Time 17:16)



So our third one that is we can look at the system dynamics model. You see the system dynamics model we can discuss these in detail and these are some models which were originated by JW Forrester in late 1950s in the Sloan school of management at MIT.

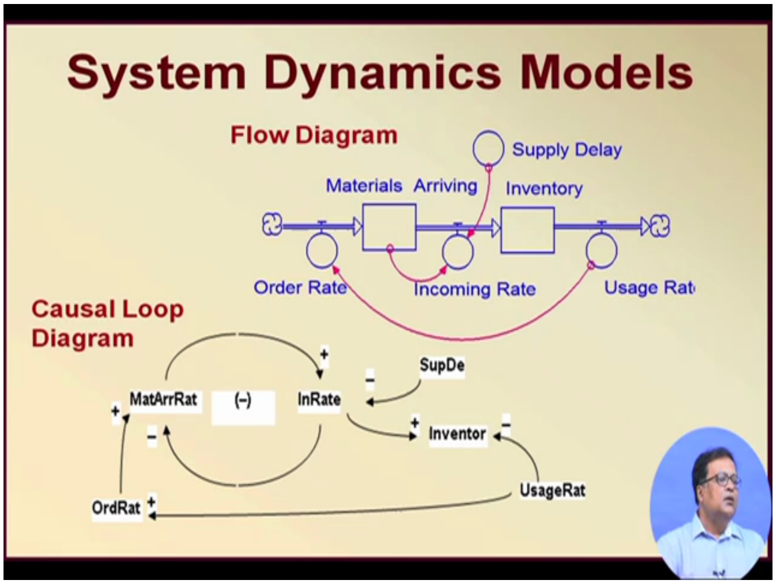
(Refer Slide Time 18:00)



So system dynamics originally this was known as industrial dynamics and that was name given by JW Forrester at MIT, right? So that was how it originated but after it has originated it has passed you know quite a bit of transformation and in fact today one of the best books on system dynamics is known as business dynamics, right? So this business dynamics book written by Star man of MIT, right?

So you can look at this particular topic this book business dynamics to know further on this topic system dynamics. Now what is system dynamics? You see first of all there are several things you know that are required that if does first of all it addresses long-term issues of real systems, models complex non-linear relationships with ease, models feedback processes, able to model soft social and psychological variables, tests effects of alternative policy options and communicates results and recommendations with ease, right? So it's a methodology of social systems analysis and design at the aggregate level using continuous simulation. How it does?

(Refer Slide Time 19:28)



Let's look at an example, right? You see first look at the flow diagram, what is really happening is that supposing a particular material is required and when you require this particular material you know we keep a store or an inventory for it then as we order the material, the material is in the transit then after sometime after a delay period the material arrives. As the material arrives it is taken into the stock and it is consumed. So as you are consuming the store reduces, right?

So suppose we are trying to simulate, we have also seen how such models are modelled in discrete events simulations. The difference here, there it was done through a random number through an event-based exercise here we shall do the same thing but with a set of differential equations. So look at this diagram once more you see there are 2 types of variables mainly in the main physical flow line is one type of variable which are called rates.

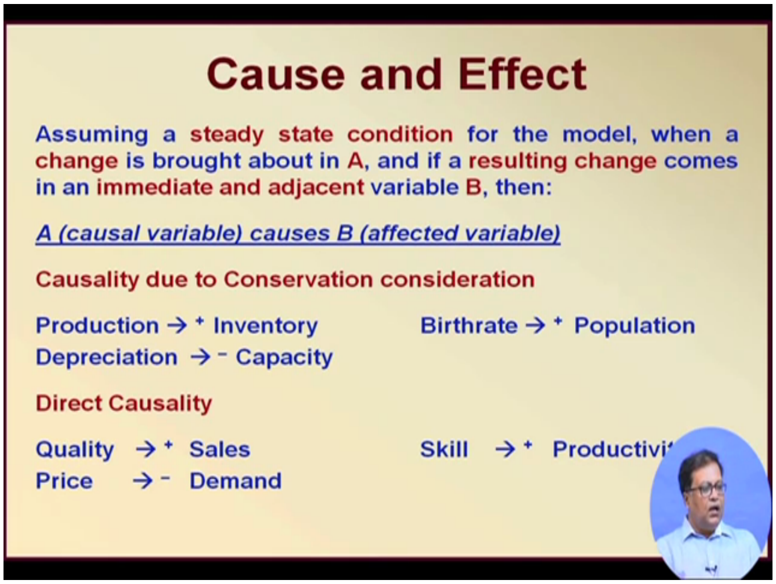
So we have the order rate, we have the incoming rate and we have the usage rate, the other types of variables which are called accumulations they are like materials arriving and inventory. So these are called rate variables and these are called state variables and they happen in a physical flow, right? A material is flowing, so here we are ordering the material, here the material is coming and here the material is in order form, here the material is actually coming after a delay time and that delay is called a supply delay.

So let's say some delay takes place and the supply delay is taking place and then the material joins the inventory and finally after usage the material goes out, is it all right? But the question is that how much should I order? And let's say a very simple policy is followed by the company that whatever we are using same thing we are ordering. Suppose that is what we do, is it all right?

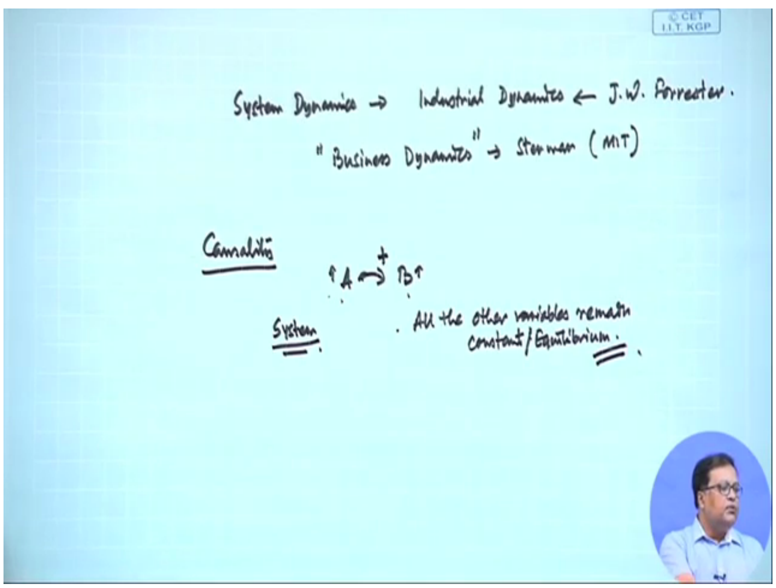
That is the model, kind of model we draw in a flow diagram, alright? We shall see that in detail and here is the same thing shown in another diagram which we are calling a causal loop diagram. So you see if you look at the causal loop diagram then as a material arriving great you know material arriving great and that is nothing but the order rate. the material arriving, this is the material arriving.

The material arriving is you know increases as you order and as you order that the material arriving increases the inrate, so this is this part of the diagram and you know then as the material, incoming material you know then it just goes to the inventory and then through the usage rate and order rate it is actually modelled.

(Refer Slide Time 22:55)



(Refer Slide Time 23:31)

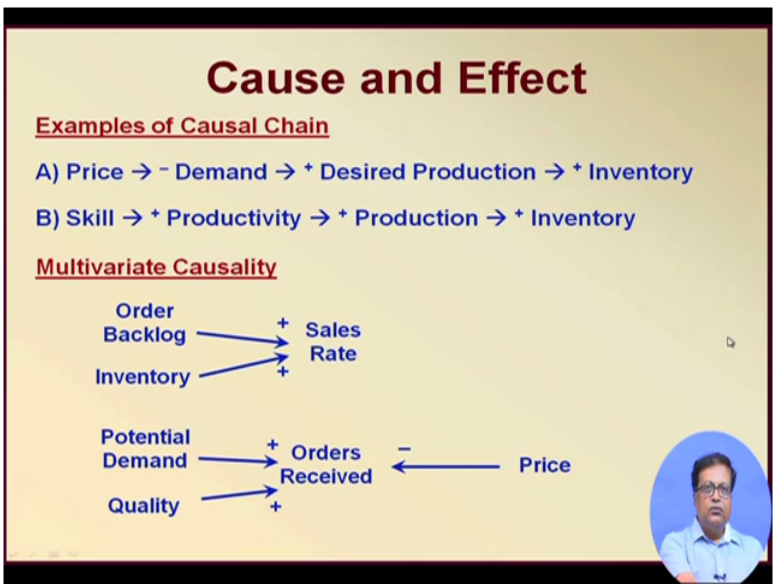


So now all these models you know we have our steady-state conditions for the model when a change is brought about in A and the resulting change comes in an immediate and you know just variable B. So that is essential meaning of causality. So let us see what is causality? Causality is we say that A causes B, right? So let's say in a plus manner, what does it mean?

You see A and B are taken from a system A and B are taken from a system and in this particular system you know what we have is that as we increase A the B also increases but there is a particular requirement that all the other variables remain constant, right? All the other variables remain constant or constant or in equilibrium, sometimes we not be able to keep exact equilibrium so they will be in equilibrium or near equilibrium.

So then we make a small change in A and if there is a corresponding change in B then we say that A causes B, right? That is essential idea of causality and if A increases, B increases then we put plus if we say A increases B decreases then we put minus. So here are some more examples of causality? So A has caused the causal variable B has caused the affected variable, so here are some examples as you increase production inventory increases. As you increase the appreciation capacity decreases, as you increase birth rate population increases, as you increase skill productivity increases, as you increase quality sales increases, as you increase prize demand decreases, more price less demand.

(Refer Slide Time 25:26)



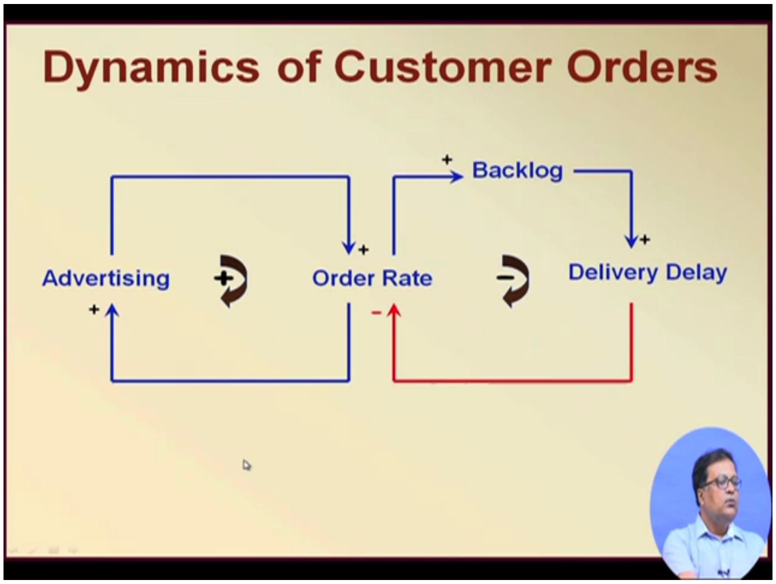
So you know like but there would be a chain also you know like more price less demand, less desired production less inventory because you have to read you know you know when you read please be careful that what it means that you know when you read demand and desired production you read only these 2, don't see anything else. So more demand, more desired production.

Similarly more desired production more inventory, is it all right? Inventory means stores, so similarly look here, more skill more productivity than look only this more productivity more production, more production more inventory but when you have a chain the chain may behave friendly. You see more prices less demand but if you have less demand will it be more desired production, no.

It will be less because there is a plus, so as demand decreases her desired production will decrease also. Similarly less desired production less inventory. So if you read the entire causal chain then more prices fewer inventories. Similarly between skill and inventory, so we can have multivariate causality also. So like there could be order, backlog and inventory and if you take them together then sales rate may change.

So both the 2 variables are affecting sales rate, there could be another example that potential, demand, quality as well as price they all may change what is known as order received, right? So production potential demand, quality and price they altogether will change the orders received, right?

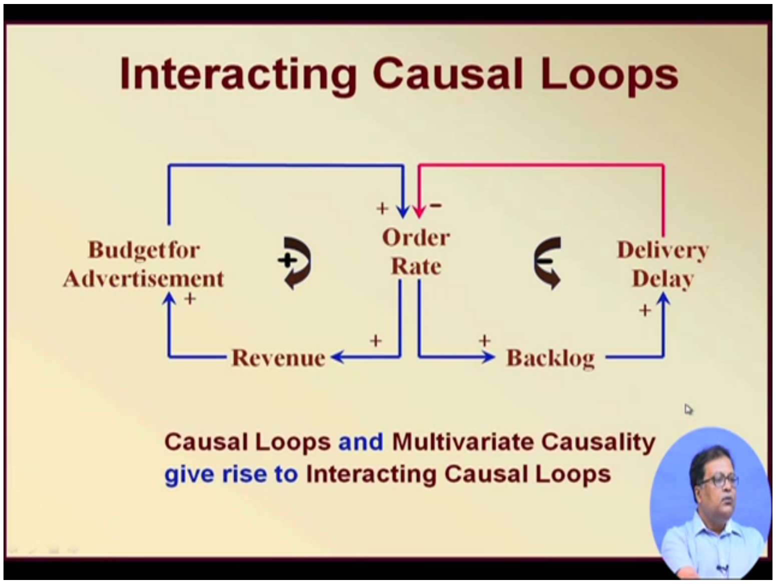
(Refer Slide Time 27:12)



So look at some examples, here is one example that with more advertising more orders, more orders more advertising. So you know it goes and then we have what is known as a feedback loop, so this feedback loop is positive because you see more advertising more orders, more orders more advertising, right? Again more advertising even more order rate. So things are increasing reinforcing, so this is a positive feedback loop.

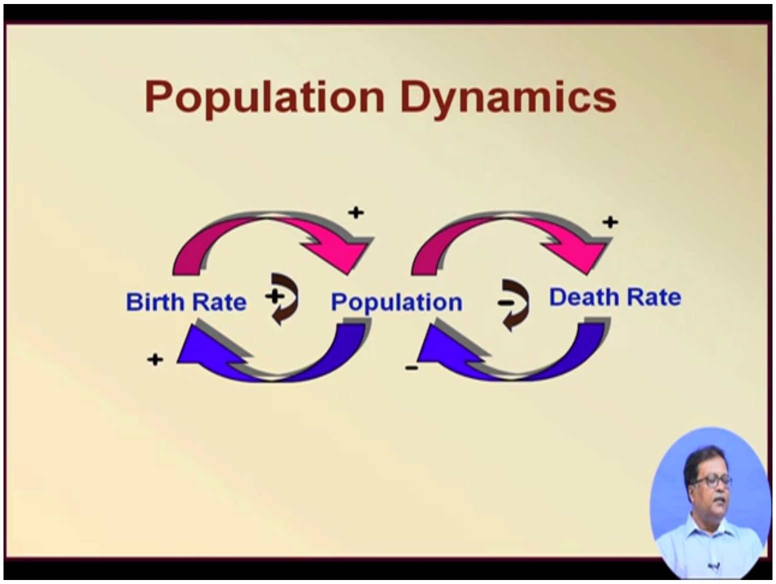
But on the other side more orders more backlog, more backlog more delivery delay and more delivery delay less orders because if you delay delivery what will happen? You may get less and less amount of orders, right? So here is a negative, why it is negative? Let's look at this way, more advertising more orders, more orders even more advertising but more orders more backlog, more backlog more delivery delays, more delivery delays less orders, right? So this is the kind of thing that happens, look at another one.

(Refer Slide Time 28:29)



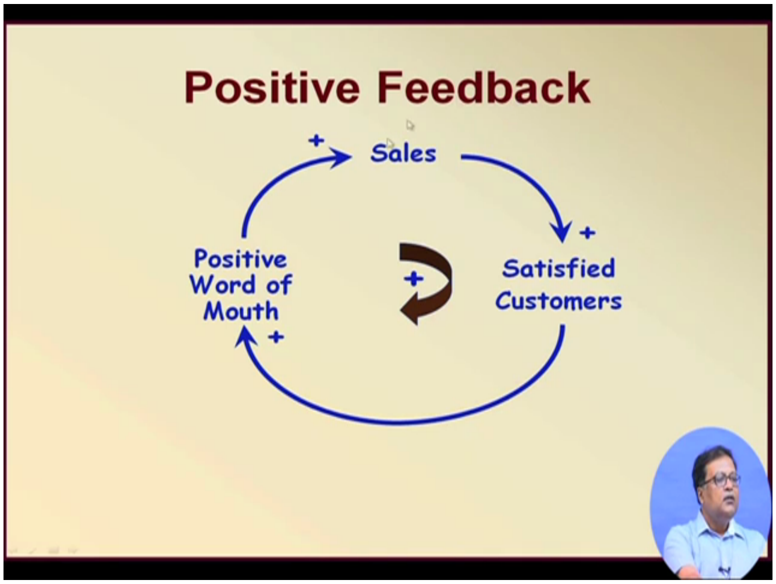
So it is an interactive causal loop, right? So more orders more revenue, more revenue more budget for advertising but more budget more orders but more orders more backlog, more delivery delay more orders, so less orders. So this is a negative, so there is a multivariate causality gives to interacting causal loops because order rate depends on both.

(Refer Slide Time 29:03)



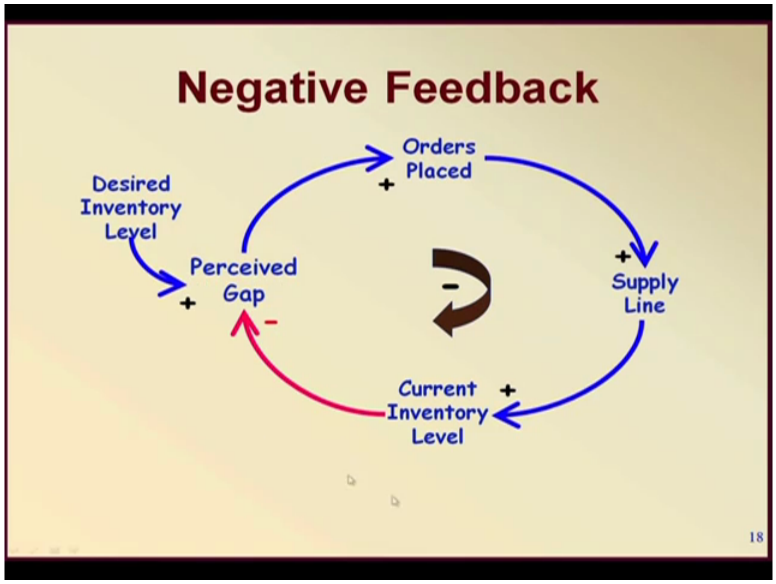
There is a classical model you know about the population birth-rate and death rate. So you know more birth-rates more population, more that's rate less population, right? But the both birth-rate and death rate could be a fraction of population. Assuming that these are happening than birth-rate is a fraction of population, death rate is also fraction of population, in that since both birth-rate and death rate maybe functions of population. So we can have positive loop here and a negative loop here. So that's population dynamics.

(Refer Slide Time 29:45)



Then another example of positive feedback loop that more sales more satisfied customers, more satisfied customers positive word of mouth and even more sales. So this is a positive feedback loop. So overall as you go once around the loop then things are in a reinforcing manner so sales continue to grow resulting from positive word of mouth from satisfied customers.

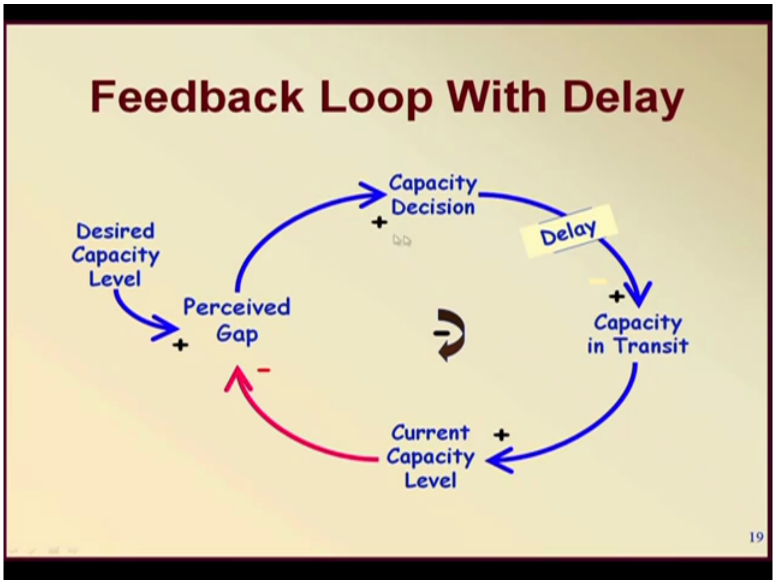
(Refer Slide Time 30:07)



On the other hand a negative feedback loop is what is happening you see as you place order the supply line increases, current inventory level goes up but the desired level and current level the gap reduces, why? Because you know suppose you want 100 in the store and currently with more orders more supply this has gone up let's say 90. So the gap is only 10 then you order less, right? So that is why a negative loop.

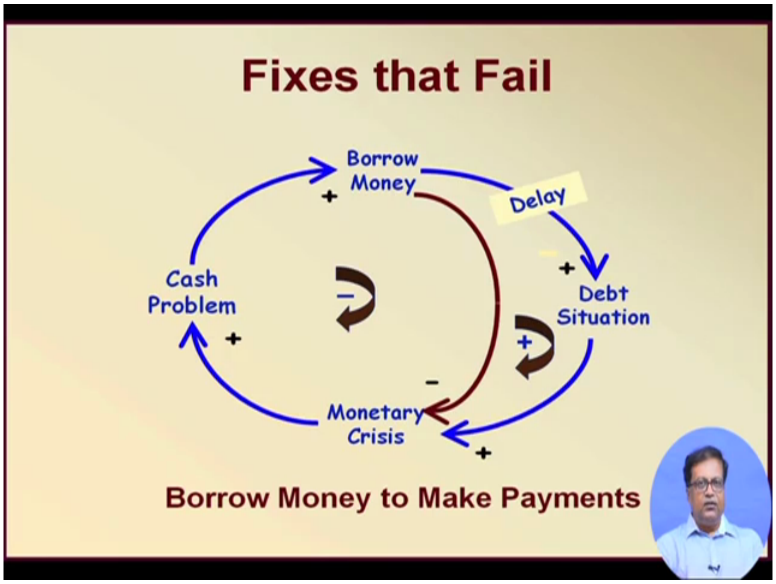
Look if the gap increases more orders so that's why it is plus but what is the total effect as the current supply? Suppose more orders more supply, more supply more inventories but gap more inventory less gap. Now as the gap reduces, less gap fewer orders. So we started with more orders, more orders more supply, more supply more inventory, more inventory less gap and less gap less orders. So we started with more ended with less that is why that negative. That is why this entire loop is called a negative feedback loop, is it all right.

(Refer Slide Time 31:37)



So such kinds of things do happen, there sometimes there could be delays also for example when you take a capacity decision then only after a delay the capacity comes in, right? And we have the current capacity level that goes up and as you have perceived gap then you take further capacity decision. So we have a delay coming here that the loop is taking place its effect is seen but only after a time, right? So that is what, is a feedback loop with delay.

(Refer Slide Time 32:11)



Here is another example where we are calling it the fixes that fail, so what is fixes that fail? Suppose you know we have cash problem, as you have a cash problem you borrow money but as you borrow money you know your monitoring crisis reduces, right? More cash problem more borrow money, more borrow money less monetary crisis you know less cash problem less borrow money. So that's a negative loop but after sometime, so this is like a fix, right?

Borrow money to make payments but what is going to happen? As you borrow money to solve your cash problem after sometime you dept situation will worsened, right? So you will have even worse debt situation, right? So we will put it in a plus in that sense and more is the dept situation means worse is the debt situation more monetary crisis will be in, so there is a bigger positive loop around.

Just look here, borrow money more after sometime your debts go up, monetary crisis go up, cash problem also goes up, right? So I stop here and we continue our discussions in our next class also, thank you very much.