Course on Decision Modeling
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Lecture 26
Module 6
Simulation Examples (Continued)

Morning, today we are going to discuss from our simulation discussion, some more examples. So earlier we have discussed some examples and today further additional knowledge that we have had particularly the random variate system. So what should we do in this particular section we shall discuss some simulation examples particularly on queuing systems and first without employing the random variates and then second example by employment of random variates these are the two things that we do first and thereafter we take up some problems on Monte Carlo Simulation.

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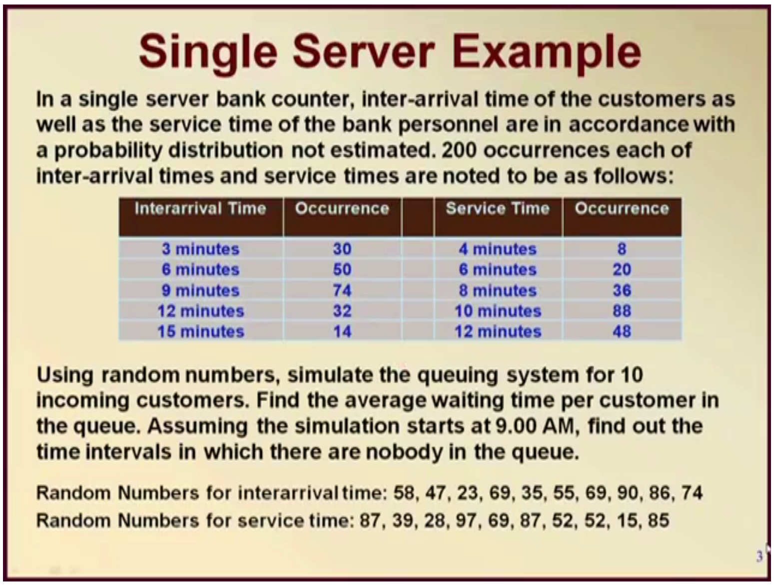

So first let us take this particular example of a single server queue queuing system. Let us say there is a bank counter and people are arriving and there are some service that is also going on and what we have done we have found out the frequency distribution of the different arrival and the service times, right. So if you look at the chart then 200 occurrences of each have been taken and it was found that the arrival could be inter arrival time could be 3 minutes, 6 minutes, 9, 12 or 15 whereas the service times could be between 4 to 12 and their frequencies are as per given below.

So in order to simulate this now we are asking you to really simulate for 10 incoming customers. So one of the thing that we require is a set of random numbers and these random numbers will be one set for the inter arrival times and another set for the service times. Really speaking the kind of random numbers we are going to use here they are all two digit random numbers if you really want random numbers between 0 and 1 then simply divide each by 100 you will get a random number between 0 and 1.

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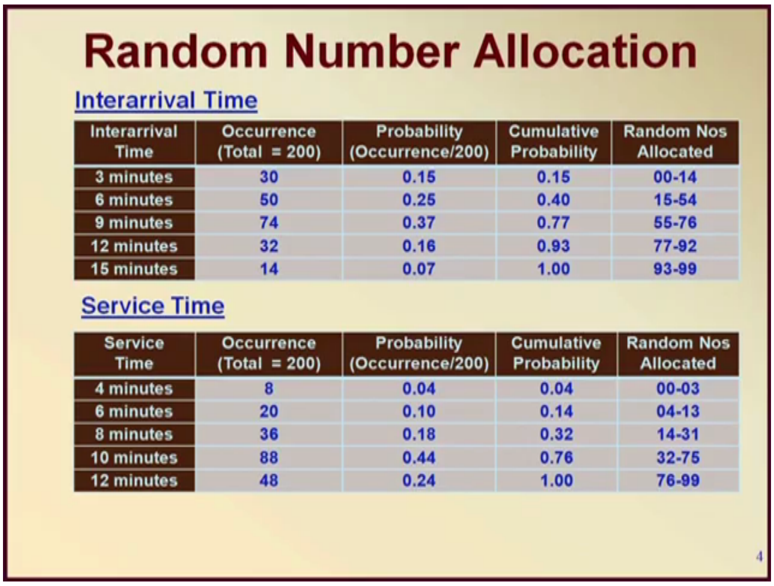

We have already done a similar problem in the past. So please look the first task is make an assignment of the random numbers as per the cumulative frequencies, right. So first of all you have to find out the probability figures so those values which were given the occurrences 3 minutes was 30 and if you divide by 200 then the probabilities 0.15. Similarly all the probabilities you can calculate 0.15, 0.25, 0.37, etc and when you add them up then you will get the next table that you can call the cumulative probability table. So we have 0.15, 0.4, 0.77, 0.93, 1 that is the cumulative probability for inter arrival time.

Similarly we can also have the cumulative probability for the service times 0.04, 0.14, 0.32, 0.76 and 1. Now as a repeat of what I said in the previous example the random number that is to be allocated see we have 100 random numbers between 0 to 99, so if we divide them the 15 percent probability would be obtain if we allocate 15 random numbers. So counting 00 also 00 to 14 these are the 15 random numbers which can be allocated to the first one, what does it mean? It means that suppose you get a random number say 12 then it will return as an inter arrival time of 3 minutes, is it alright.

So similarly all the allocations are done for 6 minutes, 9 minutes, 12 minutes and 15 minutes for inter arrival time and similar times for service time as well so this is the first task the allocation of random numbers.

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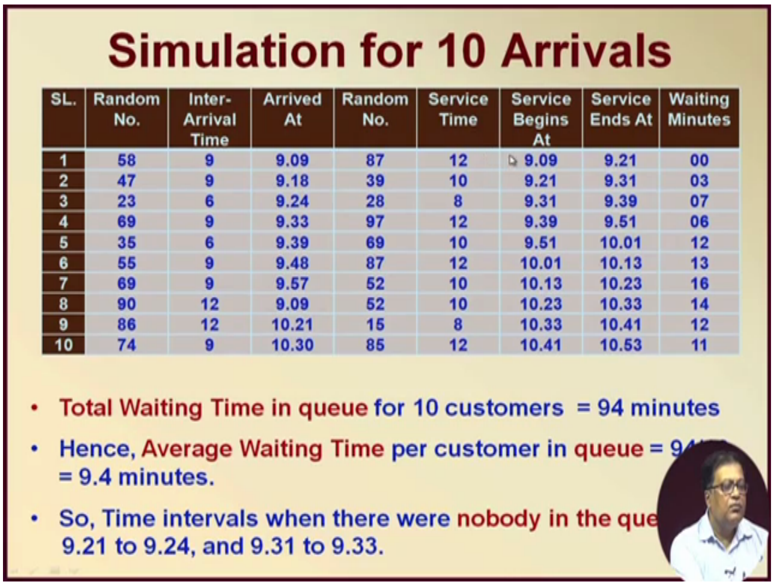

After the random numbers are allocated we come to the next step, right what is that next step? Next step is that we have to now compute the you know service when it begins and when the person arrives, is it alright. So look here what we have done the first person arrives at 99 because arrival times is inter arrival time is 9 minutes, right. So then if the first person is arriving at 99 and then service begins at 99, service ends at 921, is it alright.

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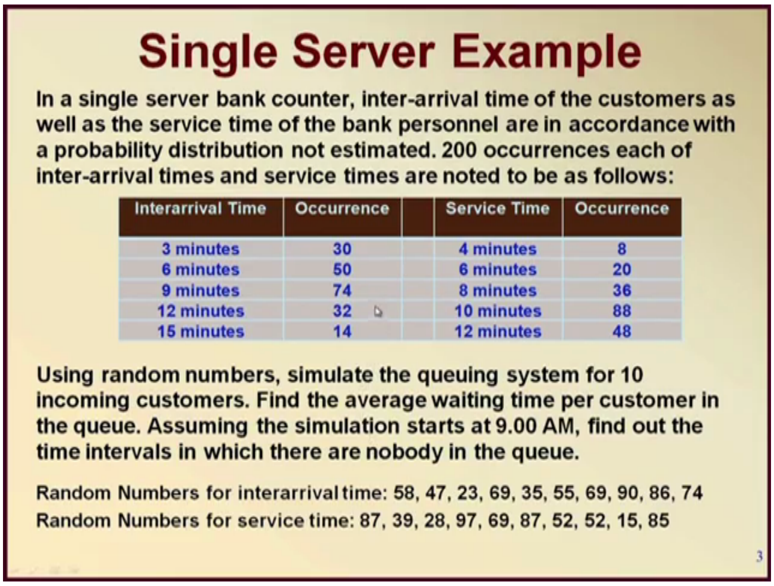

So what we have now question is that why it is 9 minutes, right let us understand this that this one just a minute, right. So you see what was the random number the random number allocated was 58, right so if you see the random number 58 the 58 falls here, look here that between 55 to 76, right. So since the random number falls here between 58 to 76 so the inter arrival time here we get as 9 minutes.

So that means the first arrival is at 9 inter arrival minutes and all these are the different inter arrival times 9, 9, 6, 9, etc and these are the times when the people are arriving that is at 99, 918, 924, 933, etc. Now similarly the service times are also obtained from the random numbers so 12 minutes, 10 minutes, etc, etc. Now look at how the calculation is done? The first person arrives at 99 and the first person leaves at 921 because there were nobody in the system and he is the first person to come so exactly 12 minutes he will take and he will go out of the system without waiting for anytime.

But what is happening to the second person look at very carefully, the person has arrived at 918 but what he finds he finds that one person is already in the service because his service is not over yet. Look at this 921 that means a person is going out at 921 so only at 921 the second person can get into service and it takes 10 minutes the person goes out at 931 so waiting minutes 3, 918 to 921.

The third person has already arrived at 924 but we will find the service busy and we will be able to join the service only at 931, right so it has to wait for 7 minutes, is it alright it has to wait for 7 minutes this is the waiting time in the queue, right. So this is not really the waiting time in total waiting time, the total waiting time will be for this person will be 7 plus 8 that is that is 15 minutes, right. And this person total waiting time is 0 plus 12 that is 12 minutes because waiting time in the queue 0 minutes, waiting time in the queue 12 minutes, alright.

So like this if we compute for all the 10 incoming customers then you will find that all these people have waited for different amount of times in fact it takes some time for queue to stabilize so initial values will be low and then later on values will be higher. So it is a good thing sometimes to remove some of the earlier data because it takes a little time to stabilize the queue we have not done it but we can definitely do that.

So then we can look at all the simulation results and we can carry out some computations like total waiting time in the queue for the 10 customers are 94 minutes, is it alright. So 94 minutes therefore average waiting time in queue is 9.4 minutes, right and what was the time interval when there was nobody in the queue 921 to 924, why? Because look here 921 the first person you know goes out of service and the second person comes and joins the service, right and at that time when the second person was in the service the third person came only at 924, so these are the three minutes when there was nobody in the queue, right.

So that is the one point and at the second point at 931 look here the second person has gone out and the fourth person the third person has come and join the service and the fourth person arrives at 933, so this is the another time when there was nobody in the queue.

So you can see that basic idea is that have a set of random numbers, use those random numbers to obtain from the frequency table the different arrival times and the different service times use them really to work out when the arrival takes place and for the service only note the times and then do a computation really see whether the service is free or not that means the previous person has gone out or not and only then the service can be started and then completed and then from the data that you have generated compute the various parameters.

So this is the essence of simulation, really speaking you are not using any equations as such you are really generating from certain distributions and let it happen. So that is a essential idea of simulation.

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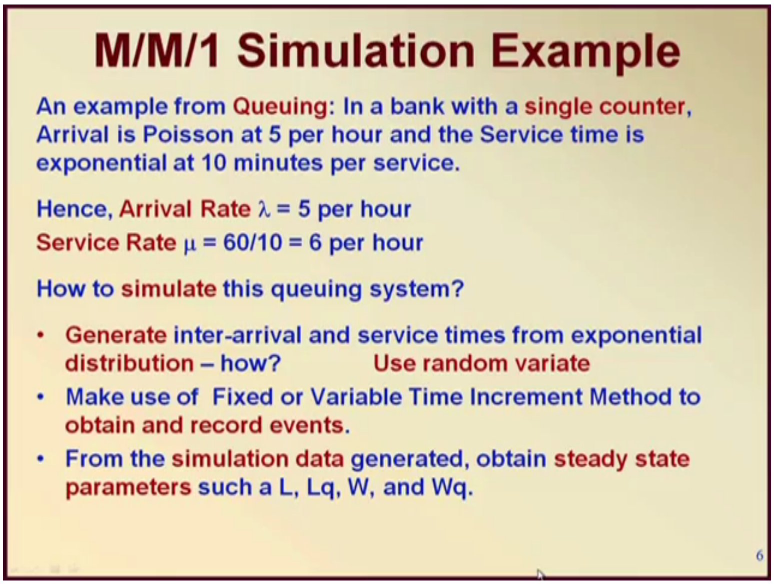

Now let us see another example and in this example really we shall use what is known as random variates, there was the earlier system the random variate was obtained from the random numbers by converting them to a frequency table and allocating random numbers. But if you really know the distribution you can use the appropriate random variable formula.

So in this example is an M/M/1 simulation queue in the bank there is a single counter, the arrival is at 5 per hour and service is exponential at 10 minutes per service arrival is Poisson distributed and here arrival rate is lambda at 5 per hour, service rate Mu at 6 per hour, alright and how to simulate this queuing system what should we do? Again we have to generate the inter arrival time and the service time from the exponential distribution, how? Use the random variate.

Then use either the fix time or variable time method to really record all that is happening in the queuing system, right why? Because so that you can actually calculate the different parameters. Then finally obtain all these different parameters and then interpret how the queuing system has performed. So that is the essential idea.

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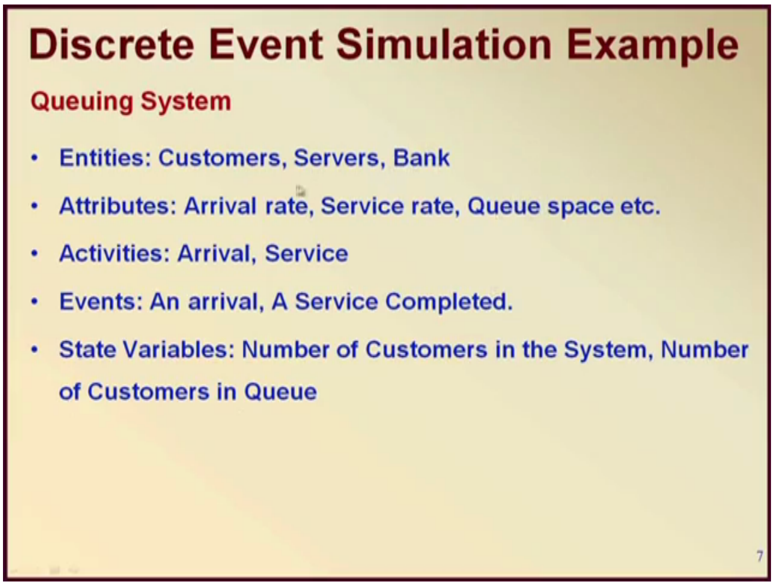

Let us see now some of the you know entities, attributes, activities, events and state variables. You know entities very obvious customers, servers, then bank. The attribute could be the arriving rate, right the service rate and serving facility, how much facility, queue space, etc, etc. All these things that describe those entities they can be called as attributes, right. Then we have what are the different activities the activities are the arrival and service and what is the events that are occurring two events one is arrival and another is the service and how do you measure all those? L lq, etc. So those are the state variables, is it alright.

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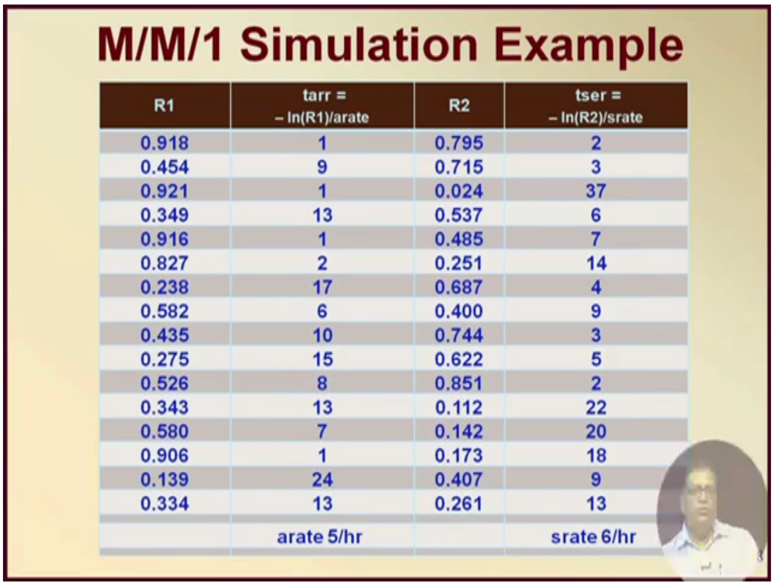

R1	tarr = – ln(R1)/arate	R2	tser = – ln(R2)/srate
0.918	1	0.795	2
0.454	9	0.715	3
0.921	1	0.024	37
0.349	13	0.537	6
0.916	1	0.485	7
0.827	2	0.251	14
0.238	17	0.687	4
0.582	6	0.400	9
0.435	10	0.744	3
0.275	15	0.622	5
0.526	8	0.851	2
0.343	13	0.112	22
0.580	7	0.142	20
0.906	1	0.173	18
0.139	24	0.407	9
0.334	13	0.261	13
	arate 5/hr		srate 6/hr

Now let us see how this particular simulation is taking place. Again in this case you will see that we are going to have a set of random numbers, right in fact this example that you are seeing these are generated by you know Excel software.

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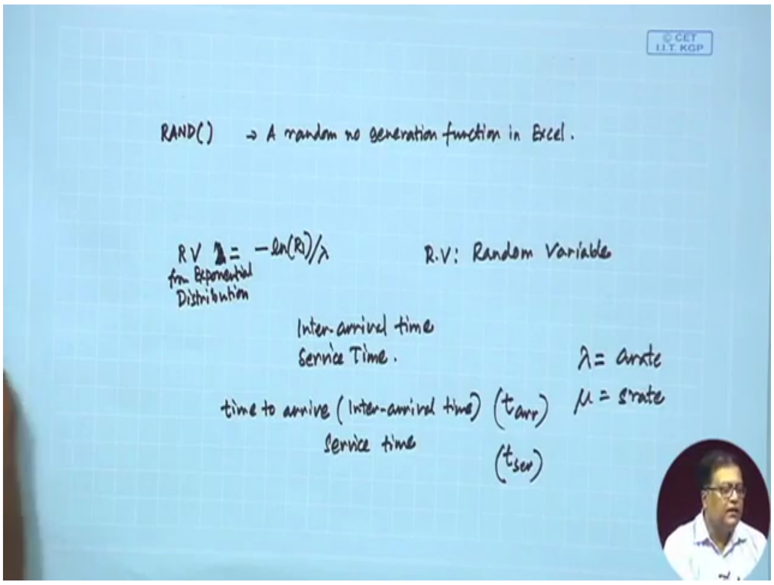

And in Excel there is a function simple function that is called RAND, right so there is a RAND function. So if you use the RAND function then a random number is generated, this is a random number generation function in Excel, right. So you you can actually use this RAND function and you can generate this random numbers. So we have generated all these random numbers R1 and a series of random numbers R2 and using them and then look here what is the formula you have used you know the formula is lambda equal to minus lnR1 that is sorry that is the lambda that is the random variate not lambda this is the random RV from Exponential Distribution, right.

And what is RV, RV is random variable random variable. So we see that you know we we have this formula which we have derived earlier that the random variate for Exponential Distribution and what are exponentially distributed inter arrival time you see you know it that if the number of arrivals in a given time is Poisson distributed then the inter arrival time is exponentially distributed and also service time that is also exponentially distributed.

So here we wrote that there are two things one is we call the time to arrive time to arrive or inter arrival time inter arrival time and the other is service time so there are two times that we have calculated so which we are calling tarrival and tser, as a short form we are calling lambda equal to arate and Mu equal to srate, so these are the things that we have used in this particular slide.

So look at the slide once again, so these are the random variate formulae, right. So taar equal to minus lnR1 by arate and tservice is minus lnR2 by srate and what are R1, R2 these are some random numbers between 0 to 1, how to generate them? Very easy go to Excel use the RAND function, right. So using them we have been able to convert these random numbers into a series of you know exponential, here one thing must remember these arate is 5 per hour, right.

So 5 per hour really means that you know a 5 then 12 minutes, right inter arrival time lambda is basically 60 by 5 the average inter arrival time is 12 minutes, right. So this has to be remembered and average service rate is 10 minutes and all this 1, 9, 1, etc all these are written in minutes, right. So basically here arate is taken exactly at 5 it is taken as 5 by 60. So when you use this formula you will get all these minutes arrival minutes and the service minutes that are generated.

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Generating Arrival and Service

R1	tarr = – ln(R1)/arate	R2	tser = – ln(R2)/srate	Arrived at	Service at
0.918	1	0.795	2	1	3
0.454	9	0.715	3	10	13
0.921	1	0.024	37	11	50
0.349	13	0.537	6	24	56
0.916	1	0.485	7	25	63
0.827	2	0.251	14	27	77
0.238	17	0.687	4	44	81
0.582	6	0.400	9	50	90
0.435	10	0.744	3	60	93
0.275	15	0.622	5	75	98
0.526	8	0.851	2	83	100
0.343	13	0.112	22	96	122
0.580	7	0.142	20	103	142
0.906	1	0.173	18	104	160
0.139	24	0.407	9	128	169
0.334	13	0.261	13	141	182
	arate 5/hr		srate 6/hr		

What is the next task? Next task is after generating these you know these arrival and service time then we have to take you know this chart has to be prepared and again it requires an involved calculations what is that involved calculations? First of all look here the arrival is 1, service at 2, so service at 3, there is a first person goes out at third minute. The second person is arriving after 9 minutes that means the second person comes at 10th minute and 3 minute service time, so 13th minute the second person goes away.

But the third person comes 1 minute later that is at 11th minute so in this case you know we have to really do a very simple calculation look at this 11 and look at this 13, the previous person goes out at the service time to both put the higher value, why? Very simple because 11th minute the service is not free this is a single service system and the server will be free only at 13th minute when the second person goes away, alright and then it takes 37 minutes so 50 at 50th minute the third person goes out.

So like that we have taken 16 such arrivals and for each of these arrival such computations are done. Let us do one more the 4th person takes another 13 minutes to arrive that means the 4th person has arrived at 24th minute and it takes 6 minutes of service. So 24th minute obviously could have gone by 30th minute but really speaking it is not that the person is not able to go out because the service is busy till 50th minute just imagine till 50th minute the service is busy.

So all those people who have arrived 24th minute, 25th minute, 27th minute, 44th minute they all will join queue, is it alright. And only at 50th minute that person goes out and this person goes into the service and goes out at 56th minute. So one has to really account for all of these and look at that the arrival and the service computations have to be carried out.

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Waiting Time Computation

R1	tarr = – ln(R1)/arate	R2	tser = – ln(R2)/srate	Arrived at	Service at	Waiting Time	Waiting Time in Queue
0.918	1	0.795	2	1	3	2	0
0.454	9	0.715	3	10	13	3	0
0.921	1	0.024	37	11	50	39	2
0.349	13	0.537	6	24	56	32	26
0.916	1	0.485	7	25	63	38	31
0.827	2	0.251	14	27	77	50	36
0.238	17	0.687	4	44	81	37	33
0.582	6	0.400	9	50	90	40	31
0.435	10	0.744	3	60	93	33	30
0.275	15	0.622	5	75	98	23	18
0.526	8	0.851	2	83	100	17	15
0.343	13	0.112	22	96	122	26	4
0.580	7	0.142	20	103	142	39	19
0.906	1	0.173	18	104	160	56	38
0.139	24	0.407	9	128	169	41	32
0.334	13	0.261	13	141	182	41	28
	arate 5/hr		srate 6/hr		Sum	517 min	343 min
					Avg	57.44	38.11

After doing this the next task is you know to really calculate the waiting time and waiting time in the queue, right. So the first person total waiting time is 2 minutes and waiting time in queue is 0 minutes. Second person waiting time 3 minutes, waiting time in the queue 0 minute. A very simple computation can be done because you can just simply calculate the service time and arrive time and difference will be nothing but the total waiting time.

And what is the waiting time in queue this waiting time minus the service time. So if let us say the 4th customer total waiting time is 32 minutes and out of this 32 minutes the person was you know service time is what is known as only 6 minutes so that means the person was waiting in queue for 26 minutes, is it alright. So not that difficult to really understand because the person came at 924, right 924 and it requires only 6 minutes of service so waited till 50th minute and then from 50th to 56th minute the person got service, is it alright.

So what happened then all this for 16 customers if you really sum them up then you will get total waiting minutes 517, total waiting time in the queue 343 minutes and since there are 16 customers you can divide them by 16 and you can get an average waiting time 57.44 and average waiting time in the queue 38.11 minutes. So this is how really we can simulate these kind of systems.

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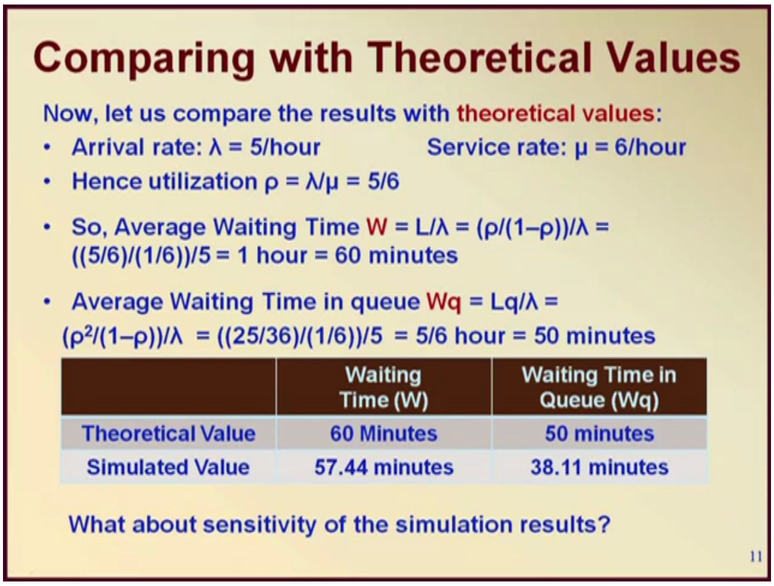

Let us now look how these values compare with the theory. So already in the queuing class we have understood how these calculations are to be done, the arrival rate is 5 per hour and the service rate is 6 per hour. So what is the you know utilization that is equal to Rho lambda by Mu 5 by 6 and what is the average waiting time then? It will be L by lambda where L is Rho by 1 minus Rho, alright so Rho by 1 minus Rho will be how much? Rho is 5 by 6, 1 by Rho is 1 by 6.

So if we divide it will become you know 5 that is L divided by lambda that is 5 per hour, so you will get 1 hour or 60 minutes, right and what is the average waiting time in the queue? Wq will be Lq by lambda, so again if you compute it will come to 50 minutes. So look here this comparison theoretical value 60 minutes, waiting time in queue 50 minutes, simulated value 57.44 minutes and waiting time in queue 38.11 minutes.

But you know very interesting thing to note that these computation I have done not just once, I have done many times and all of them are called replications each set. The first set that I had done I got this result.

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Another Replication

R1	tarr = – ln(R1)/arate	R2	tser = – ln(R2)/srate	Arrived at	Service at	Waiting Time	Waiting Time in Queue
0.487	9	0.207	16	9	25	16	0
0.177	21	0.965	0	30	30	0	0
0.242	17	0.432	8	47	55	8	0
0.125	25	0.397	9	72	81	9	0
0.325	13	0.504	7	85	92	7	0
0.162	22	0.375	10	107	117	10	0
0.249	17	0.226	15	124	139	15	0
0.037	40	0.001	65	164	229	65	0
0.727	4	0.668	4	168	233	65	61
0.252	17	0.076	26	185	259	74	48
0.775	3	0.231	15	188	274	86	71
0.523	8	0.637	5	196	279	83	78
0.323	14	0.661	4	210	283	73	69
0.760	3	0.164	18	213	301	88	70
0.620	6	0.663	4	219	305	86	82
0.348	13	0.506	7	232	312	80	73
	arate 5/hr		srate 6/hr		Sum	765	552
					Avg	85.00	61.33

And when you do the second set you know another set let me show some results you know this is another replication, what is difference here? The difference here is random numbers are all different, right this is one set of random numbers, this is another set of random numbers for arrival and service the you know arrival inter arrival time and service times are all computed and all these are obtained then waiting minutes are obtained and if you see the average the average comes to 85 and 61 it is on the higher side.

Look compare these results 57, 38 theoretical value 60, 50 and these are on the higher side. Does it mean that it will come on the higher side? No, absolutely not. Actually you know when I computed several other times I got you know waiting minutes even as low as 12 minutes, 24 minutes, 35 minutes all sorts of results are obtained. You know interesting thing to note that you know if you simulate even for only for 16 customers that is not sufficient number to get a meaningful result, to get a really meaningful results we have to simulate for even more time that is the first thing.

And second thing by doing it for many replications you know we have to come up with what is the variance of the results. So suppose the result varies say from 24 to 85 you know that is the spread of the thing and if you take an average that is an estimate of the what is called average waiting time. So these kind of things are required really to simulate for multiple times and really compare with the original results.

Now one more interesting thing let us compare let us look at those values once again some of the values. You see please remember the arrival rate is 5 per hour or every 12 minutes, service rate is 6 per hour or every 10 minutes. But look at some of the values you see there are some arrivals you know it took 24 minutes later, there is some service which took as high as 37 minutes this is the peculiarity of the you know the exponential distribution, is it alright.

Then now values could vary from very low to very high and that is probably the reason you know it may appear a little peculiar to you that arrival is only you know 5 per hour and service rate is 6 per hour and still people are waiting as much as 50 minutes, 60 minutes what is happening. You know this waiting time is high because the service rate and the arrival rate they are so much fluctuating and particularly exponential distribution, just imagine we have done some example where the service time is let us say deterministic for a constant service time you know the Lq becomes half.

So immediately if Lq becomes half these theoretical computations you look the estimated value would come down to say 25 minutes and W will become 35 minutes only. So this is interesting that these times are on the high side because of the nature of the distribution and if you really look into these factors then we can compute them further, right. So what we do? The Monte Carlo Simulation Examples we will give in the next session, right so think you very much.