Course on Decision Modeling
Professor Biswajit Mahanty
Department of Industrial and Systems Engineering
Indian Institute of Technology Kharagpur
Module 05
Lecture No. 24
Simulations Examples

Morning we were discussing so far the simulation and specifically the discrete event simulation. In the previous lectures we have already seen that particularly for discrete event simulation we must have simulation scheme, we must generate a random variation from the random numbers and from there with an objective in mind we carry out the simulation to obtain some specific results.

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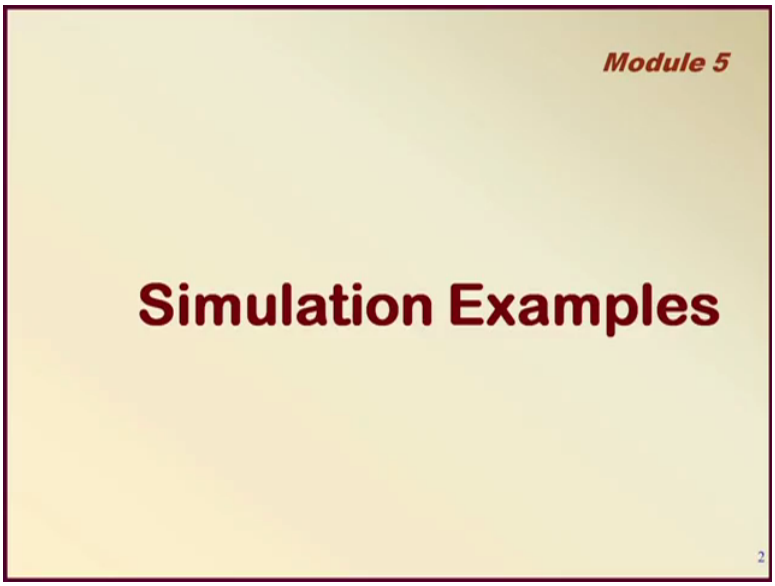

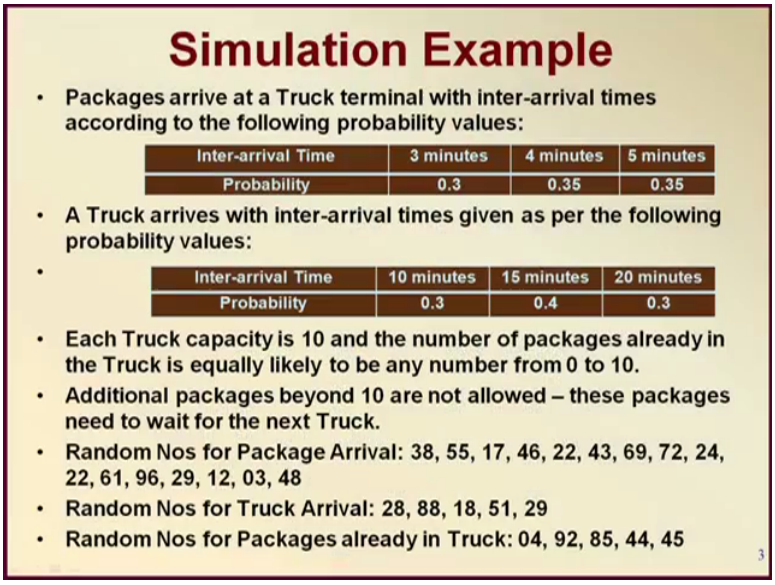

Today let us do some simulation examples, right, based on the knowledge that we have already cleaned in our previous classes. So the kind of things that we shall discuss let us look at a given problem in this particular problem the packages arrive at a truck terminal with inter-arrival times according to the following probability values. So you see mostly the kind of problem that we have done earlier in the queuing problems there we have specified our distribution, but if the specified distribution is uniform then we can take a random numbers themselves.

But suppose the distribution is not uniform or say Poisson distribution or exponential distribution like inter-arrival times mostly they comes from exponential distribution. When the distribution is specified then we have to translate the random numbers through a mathematical operator to a set of random variations, but suppose we do not have such a thing that is we have not really converted the random number to a set of random variates then we must have a set of values and translate those set of values into separate clusters or categories and you know we allocate random numbers to this specific clusters.

How to do that? We shall see a little while from now, but in this given problem the interval time are given as it could be 3 minutes, 4 minutes or 5 minutes with a certain probability values. On the other hand the truck arrives with inter-arrival time could be 10 minutes, 15 minutes or 20 minutes based on 30 percent, 40 percent, 30 percent probability.

Now here it is not like a dedicated truck is given to a set of packages not like that the concept is the packages are arriving, trucks are also arriving and trucks are also carrying packages from other stations, so you know the amount of space that is available that will be filled up by

packages from this particular station. So in this case each truck capacity is 10 and number of packages already in the truck is equally likely to be any number from 0 to 10.

So additional packages beyond 10 are not allowed and this packages need to wait for the next truck. So what we need to do? We need to have a set of random numbers and random numbers for 3 kinds of events to be generated. What are those events? Number one first event is at arrival of the package, so we have given a set of random numbers. The 2nd set of random numbers are given for arrival of the truck and the 3rd set of random numbers are for the number of packages that are already available in the truck.

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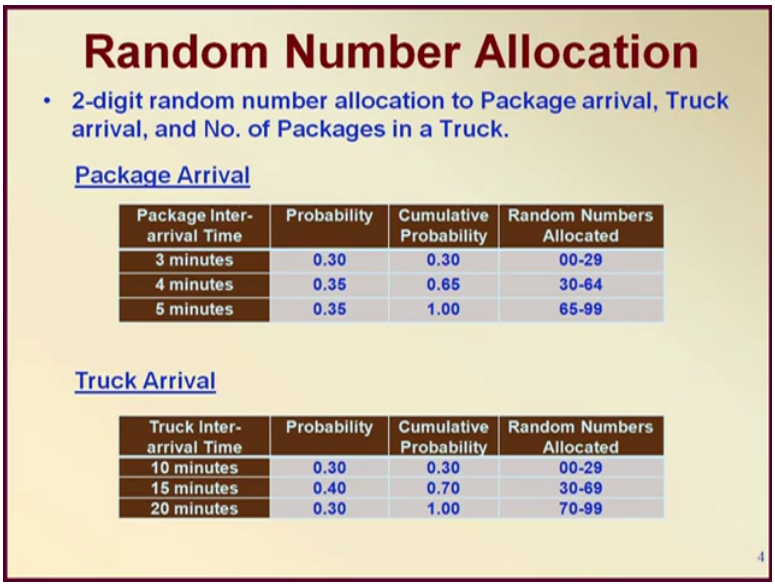

Now let us see how we go ahead, so the first thing would be the allocating random numbers and let us allocate random numbers first to the package arrival. So you see it was given that this probabilities could be 3 minutes, 4 minutes and 5 minutes and the probabilities could be 0.3, 0.35 and 0.35, so what we do? We convert them first to cumulative probability.

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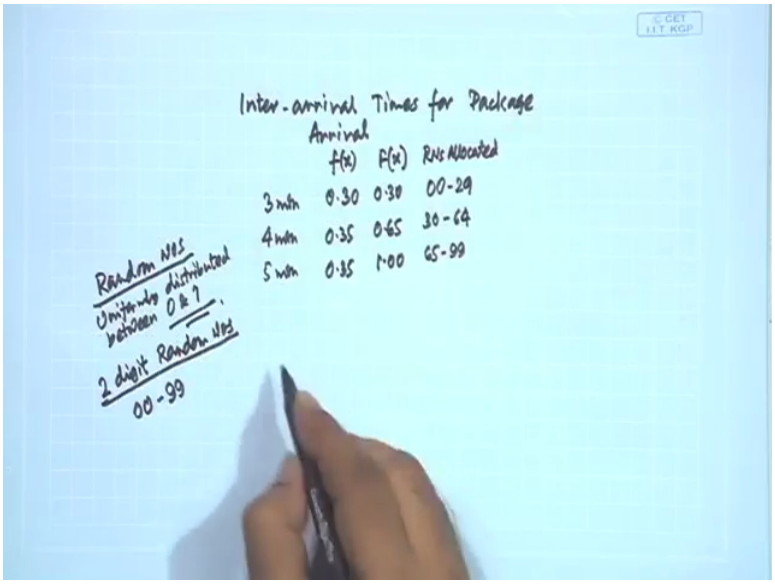

So if we write them down let us also write it that is these are the inter-arrival times for package arrival, right. So you see first of all the probabilities could be 3 minutes, 4 minutes or 5 minutes with probabilities of 0.3, 0.35 and 0.35. You see these are our like fx the probability density or in this case probability mass functions and what would be just CDF the cumulative distribution function that will be 0.30, 0.65 and finally 1, right, because the entire probability is within that.

Now we know that 2 digit random numbers a little bit they must also discussed here. The 2 digit random numbers are usually you will find them in various books and tables, the 2 digit random numbers are from 00 to 99, is it alright. Now you see this 2 digit random numbers which are basically from 00 to 99, but you remember on previous classes we have said the random number is numbers sometimes they are called pseudorandom number if they are generated by computers these random numbers are uniformly distributed between 0 and 1, right, so between 0 and.

So what exactly we should do? If we have to really convert this 2 digit random numbers, 2 random numbers between 0 and 1, we have to divide by 100, so this fine point has to be remembered, is it not. So sometimes we might use such kind of 2 digit or 3 digit or even 4 digit random numbers but if we have to convert them to random numbers between 0 and 1 then we have to divide by appropriate base, right. So we have this you know mass function and we have this cumulative function, the point is how do we allocate numbers.

Look here if the total numbers are 100 between 00 to 99 and then how many numbers should be allocated to really cater to 0.3, right. So random numbers allocated should be here 00 to 29, right, so 30 numbers are located because out of 100 the probability is 30 percent. Next probability is 65 that is another 35 percent. So these numbers are already exhausted, so we go further from here and we allocate random numbers 30 to 64, right and what about the 3rd set, the 3rd set are 0.35 that is at 0.65 to 1 that is 35 percent, so all the remaining numbers 65 to 99, right.

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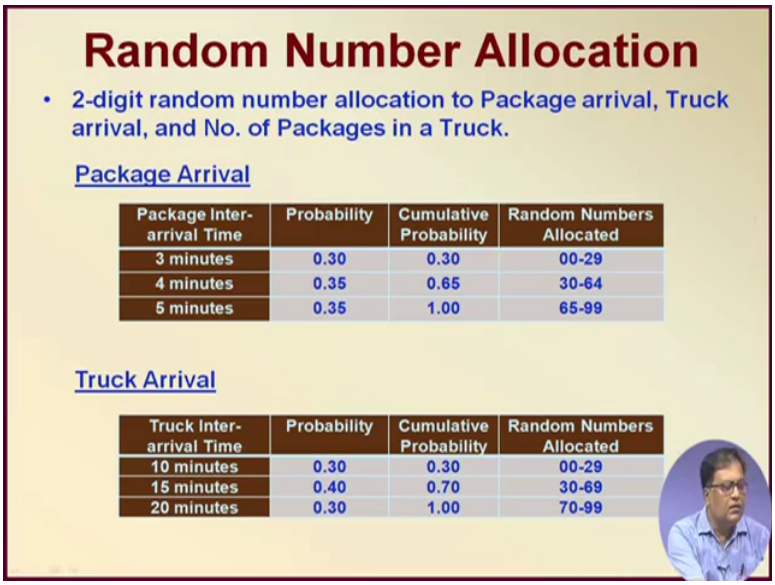

So this is exactly look at what we have done, so these are the probabilities that we assign,. So in this case for 2 digit random numbers are allocated, 3 minutes 0.00 to 29, 30 to 64 and 65 to 99.

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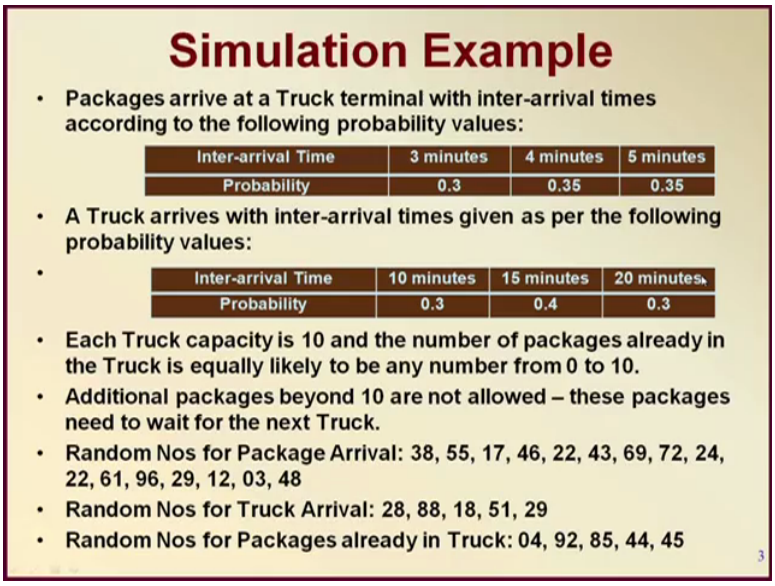

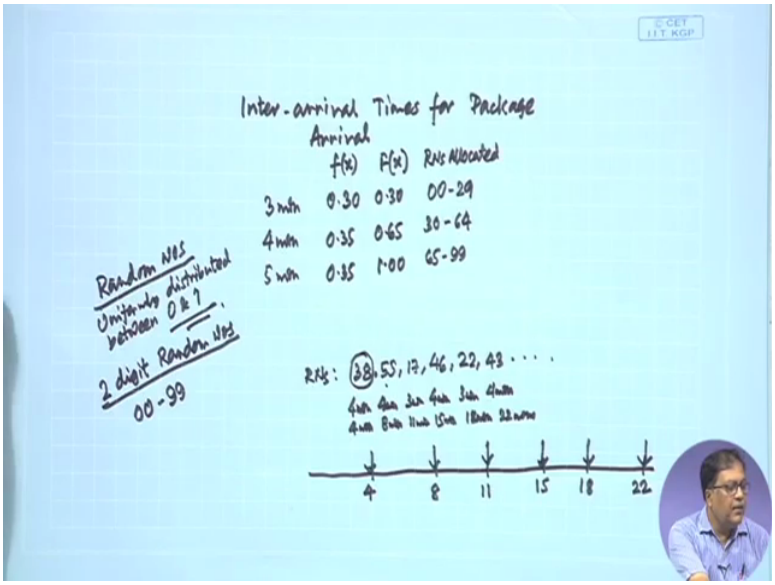

Let us look at some of the other things, now you know what are some random numbers? Let us write down those random numbers also. Look at this particular line random numbers for package arrival 38, 55, 17, 46, 22, etcetera. So let us write down some random numbers these are 38, 55, 17, 46, 22, 43, etcetera, so these are some random numbers which are already available to us.

So you see the first number that is 38, so this 38 corresponds to which one, you see you look this is 00 to 29, 30 to 64, 65 to 99. So if your random number is 38, what would be the inter-arrival time for package arrival? It should be in this category. So you can get 4 minutes, what this 4 minutes mean? Is that the first package will arrive at 4 minutes, I hope you understood. So 55 will also be there for 4 minutes, the 17 will be how much? It will be 3 minutes, 46 will

be 4 minutes, this will be 3 minutes and this will be 4 minutes, that means one, 2, 3, 4, 5, 6 , 6 package arrivals we have generated.

In fact if you put it on a timescale then 8, 11, 15, 18, 22, so cumulative will be 4 minutes, 8 minutes, 11 minute, 15 minute, 18 minute and 22 minute. So first will come in 4, 2nd will come in 8, 3rd will come at 11, 4th will come at 15, next one will come at 18 and then next one will come at 22nd minute, right. So now if the simulation starts at 9 AM like it was given you know additional packages and right okay whatever time it starts, suppose it starts with a given time then maybe in 904 that is the time that first package arrival the arrival will take place.

So just now we have seen that how we allocate random numbers if we have the data values available for a particular situation, right. It could be the situation for another case say for example the truck arrival, right. At what time the trucks will arrive? The truck arrival could be at 10 minutes, at 15 minutes and at 20 minutes. And what are those probabilities? 30 percent, 40 percent and 30 percent.

So what we need to do? We have to first convert them to cumulative probability and they are 30, 70, 100 or 0.3, 0.7 and 1, right. And then random numbers allocated 0- to 29, 30 to 69, 70 to 99. Why? Because out of totally random numbers this random numbers are 30 percent, these are another 40 percent, these are the rest 30 percent, so this is how you do.

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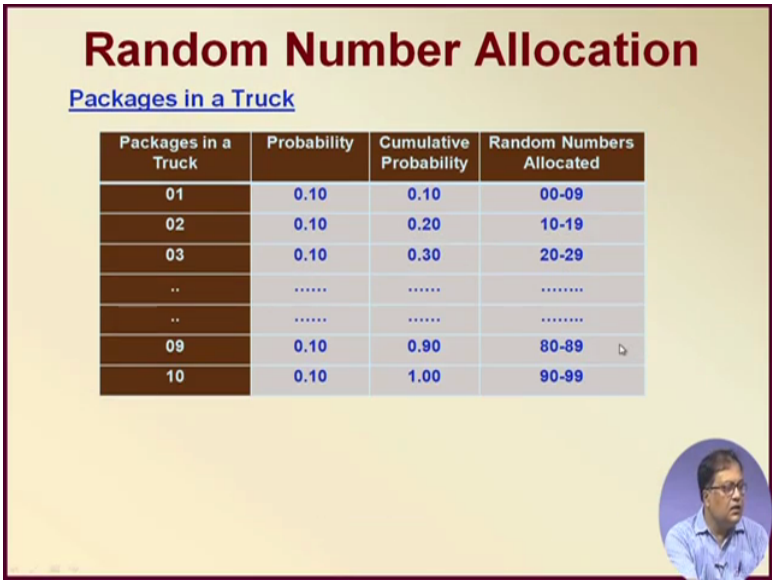

And here the next one that is the packages in a truck that is you know we have said that how many packages are already available in the truck that will be uniformly distributed. So up to

10 could be there and all are equally likely, so that means each will have 10 random numbers. So your 00 to 09 for package one, 10 to 19 for package to, 20 to 29 and finally 90 to 99 for package 10, so all of these numbers are allocated and that is a fast task.

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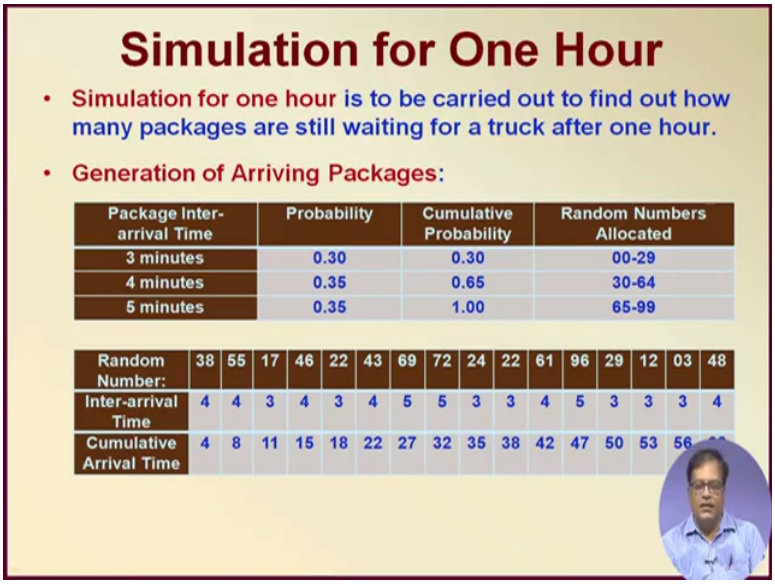

Now we have to generate the arriving, generation of arriving packages. So already if you recall we have generated the first few those are 4, 8, 11, 15,18 and 22, right, so we have already generated them. If we use the remaining random numbers like after 22 the next number was 69, so since that number is 69 you know these numbers will fall into these 3rd category and that number will be 5.

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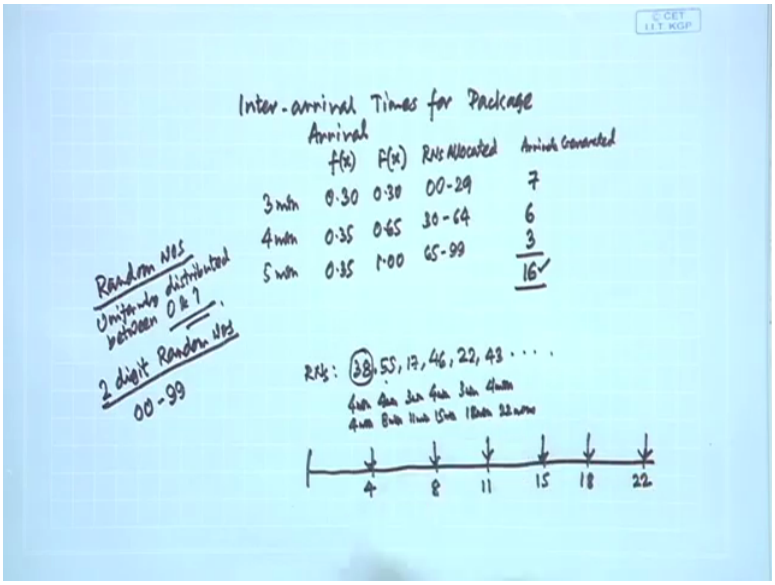

Before we proceed let us look at another interesting thing see 1, 2, 3, 4, 5, 6, 7, 8, 16 arrivals are generated within one hour, so what is their frequency? How many are generated in 3 categories? That is 3 minutes. Let us say a 1, 2, 3, 4, 5, 6, 7, 7 arrivals generated, the arrivals generated are 7 in the first category. How in the 2nd category? One, 2, 3, 4, 5, and 6. And how many arrivals are generated for 5, right. So 5 only 1, 2 and 3, only 3 arrivals are generated. You use 16 arrivals are generated out of that 7 are only in the 3 minutes category, 6 are in 4 minutes and 3 are in 5 minutes.

But you may question that look here the probabilities were 35 percent and 35 percent and this was 30 percent now how can that we have generated more arrivals for 3 minutes and this arrivals for 4 minutes and 5 minutes, so what about this probabilities? It is an interesting question answer is very simple that here we have generated only 16 numbers. This 16 is an insufficient number for carrying out a simulation, this is an illustrative example and therefore we are working with small numbers.

When you are doing actual simulation you have to to if you really want results with variances, not very high, low variances then you have to stimulate for very long period of time. How long is long? What should be our length of simulation run? Is there a formula? Answer is yes, there is a formula we can actually calculate the length of simulation run also and those things we shall discuss later.

But at this point of time please remember that as we increase the simulation the number of separation not 16 maybe 1600 or even more if we simulates for a very long time only then those probabilities will be actually equal to them. If you take small numbers then the variances will be high, right, so this point must be remembered. You cannot get if you do only 16 data points you take just cannot exactly those numbers will not be getting that does not mean that probabilities are changed, right, it does not mean that. Because variances are high and these numbers will be within the variances, fine.

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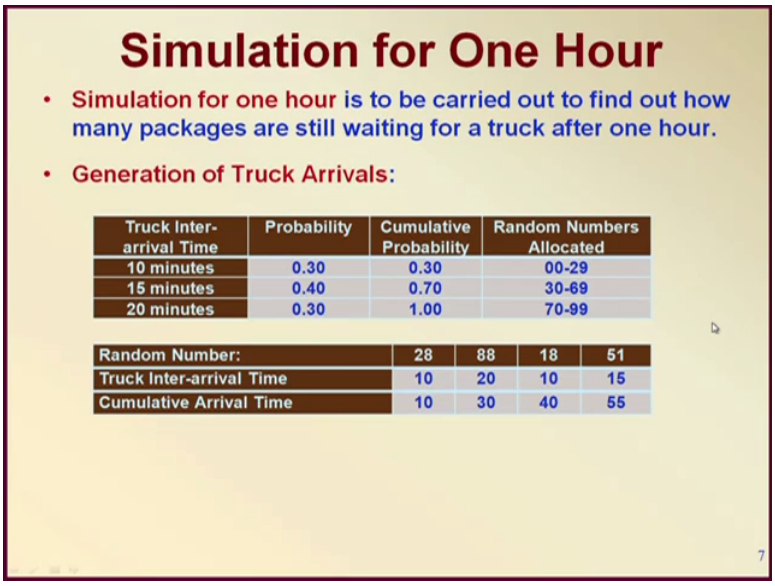

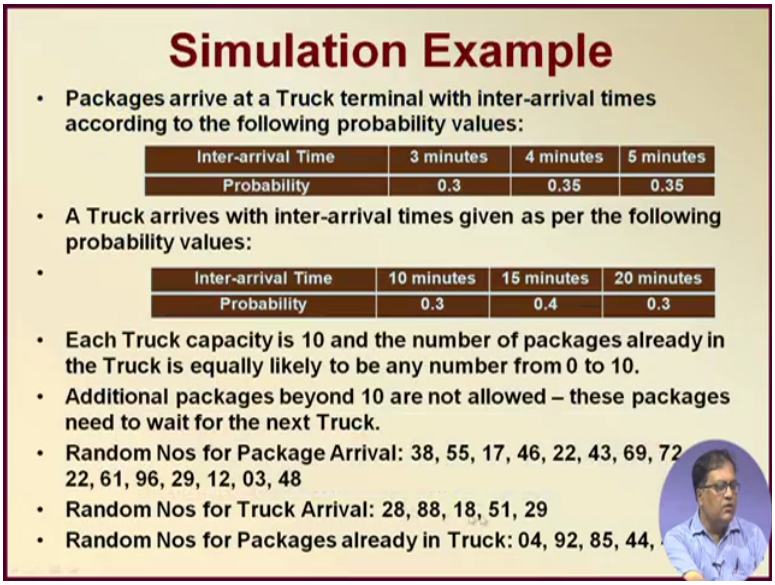

Now that we understood this that how we actually generate the arriving and similarly we also have to generated the truck arrivals, so we have those 4 numbers 28, 88, 18 and 51, In-fact we have 5 numbers but out of only just look at the original problems there were 5 numbers given, but you see only 4 we could use because by the time you know we have generated the 4 arrivals within the 1 hour simulation that we are going to do, is it alright. So what happens that generation of truck arrival, trucks are going to arrive at you know at 10, 20, 10 and 15, these are the numbers and what are the cumulative arrival? 10, 30, 40 and 55.

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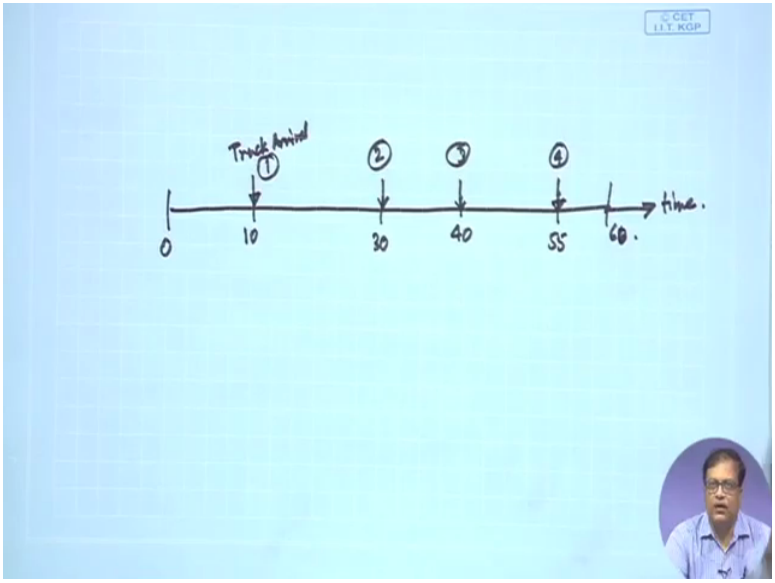

So let us do that, let us draw it and also understand what exactly it is? That is on a timescale, if this is my timescale, in this timescale if this is 60 then somewhere here will be 30, somewhere here will be 10, so 10, 30 then somewhere here will be 40 and somewhere here will be 55, so 0, 66, right. So these are my truck arrivals, so truck is arriving 4 times, right, truck is arriving 4 times at 10 time 10, this side is time at time 10, at time 30, at time 40 and time 55 this is the 4 times the truck is arriving.

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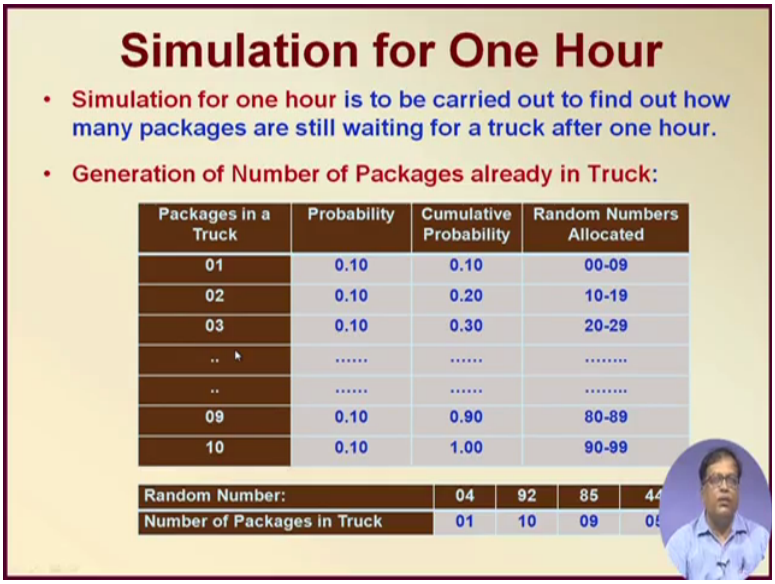

And look at this generation of numbers of packages, the number of packages if you really look at then you will find that packages in a truck 1, random numbers allocated is 00 to 09 all these things you already know and these are the random numbers about the 4 truck has

arrived. So you see these are the 4 random numbers if you really look at, the number of packages in truck, the first case there is only one package, 2nd case there are 10, 3rd case 9 and the 4th case 5.

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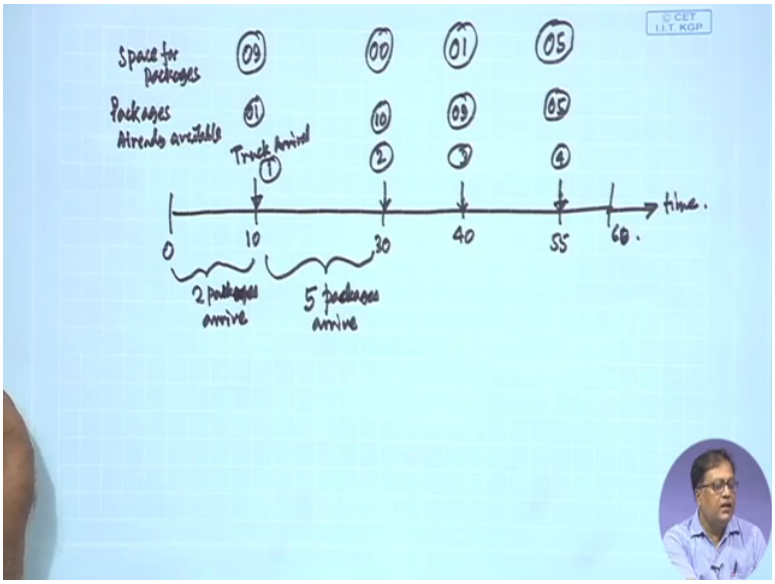

So let us also write down the truck is arriving with one package, here the truck is arriving with 10 packages here the truck is arriving with 9 packages and here the truck is arriving with 5 packages, what are these? These are packages already available, right. So how many it can take? You see it can take that packages there are only one and how many it can take then? It can space for packages, how many are possible to really accommodate, right.

In this case 9 packages could be accommodated, in this case 0, in this case 1, and in this case 5, right 9, 0, 1 and 5. But question is how many packages are actually arriving within those time? Let us look here, right, so how many packages are arriving? See look here at 4th minute cumulative arrival up to 10 minutes 2 packages are arriving, is it not upto 10 minutes, after 10 minutes 2 packages are arriving, upto 30 minutes 1, 2, 3, 4 another 5 packages are arriving.

So here let us write 2 packages arrive and here another 5 packages arrive, so that is up to 30 and beyond 30 also say 40 between 30 to 40 again another 3 packages arriving and thereafter between upto 55 another 4 package is arriving. So here 3 and between 40 to 55 another 4 and finally 2, right, so 2 plus 5, 7, 10, 14, 16, so these are.

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Simulation for One Hour

- Simulation for one hour is to be carried out to find out how many packages are still waiting for a truck after one hour.
- Generation of Truck Arrivals:

Truck Inter-arrival Time	Probability	Cumulative Probability	Random Numbers Allocated
10 minutes	0.30	0.30	00-29
15 minutes	0.40	0.70	30-69
20 minutes	0.30	1.00	70-99

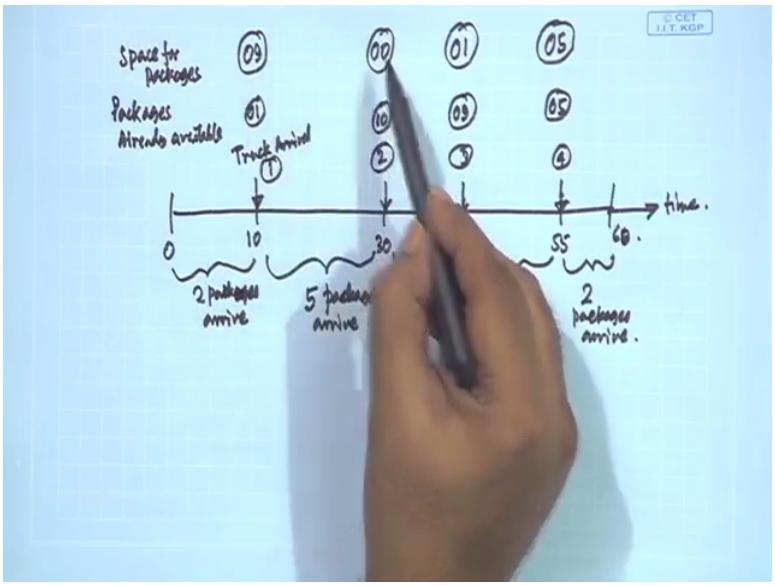
Random Number:	28	88	18	51
Truck Inter-arrival Time	10	20	10	15
Cumulative Arrival Time	10	30	40	55

Simulation for One Hour

- Simulation for one hour is to be carried out to find out how many packages are still waiting for a truck after one hour.
- Generation of Number of Packages already in Truck:

Packages in a Truck	Probability	Cumulative Probability	Random Numbers Allocated
01	0.10	0.10	00-09
02	0.10	0.20	10-19
03	0.10	0.30	20-29
..	.....	.....	.....
..	.....	.....	.....
09	0.10	0.90	80-89
10	0.10	1.00	90-99

Random Number:	04	92	85	44
Number of Packages in Truck	01	10	09	05



So if it is not clear let me repeat the whole thing one more time you see what we have done let us look at this particular thing it is the trucks are arriving at time point 10, time 30, time 40, time 55. The truck is having 1 package, 10 package, 9 package, 5 package, so if we have this pictures then how many packages are possible to put? 9 packages, because 1, 10, 9 and 5 and 9, 0, 1 and 5, right.

And if you really look at the truck arrivals at this slide then you find between the 0 to 10, you know 2 packages arrive, 10 to 30 another 5 packages arrive, 30 to 40, 3 packages arrive and 40 to 55, 4 packages arrive and finally another 2 packages arrive. So all these things if you look at now you know we have now put together, so tell me what happens when the first truck arrives? The truck has arrived it has a space for 9, how many are waiting? 2, right. So what will happen here? 2 packages goes to truck none waits, right 2 packages goes to truck.

What happens here when the next truck comes? There is no space, but again another 5 packages have arrived, so what will happen? None goes to truck all 5 waits, is it alright. Here what happens one another 3 has arrived, so one pack goes to truck. How many will wait now? 5 plus 3, 8, 7 waits. Now another 4 packages arrived and 5 package are going into the truck, because there are 5 vacant spaces, so what will happen? 5 packs goes to truck, so 2 plus 4, 6 waits and here no more bus comes, so 8 wait.

So once again let us see the thing, 2 packages are arrived, free space is 9, so 2 will go to truck none waits. 5 packages are arrived here and none goes to truck all 5 waits, is it alright. Here 3 packages have arrived and 1 goes to truck, so 7 will wait, is it alright. Here 4 packages have arrived, so 5 pack goes to truck 6 will wait. So finally another 2 have come nothing has gone so 8 will wait, is it okay. So that is exactly what is simulation all about and let us see how the whole thing is put together.

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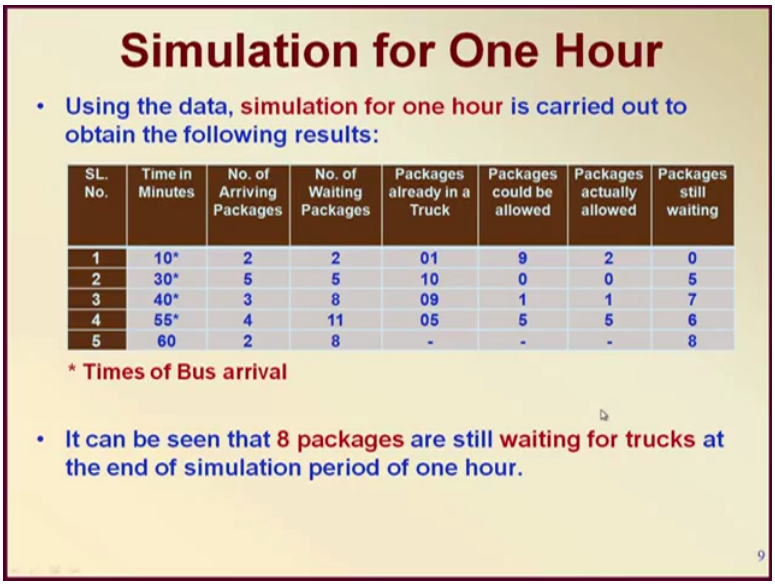

In this particular sheet if you look you know this has been put together into a detail of simulation for 1 hour, so at time at 10, number of arriving package is 2, number of arriving package is 2, number of waiting packages is 2 and packages already in the truck 1, package could be allowed 9, package actually allowed 2, package still waiting 0, right. So this is exactly what happens at time 10 already we have seen this.

At time 30 number of arriving package is 5, waiting package is 5, because all have gone in the previous truck. Packages already in a truck, 10, there is space nothing is allowed, all 5 are still waiting. And then at time 40 another 3 packages have arrived, total waiting packages are 8, already in truck 9, space is only 1, so only 1 is allowed 7 are still waiting. Like this if you do then finally at time 60, 8 packages are still waiting. So it can be seen that 8 packages are still waiting for trucks at the end of stimulations period of one hour, is it alright.

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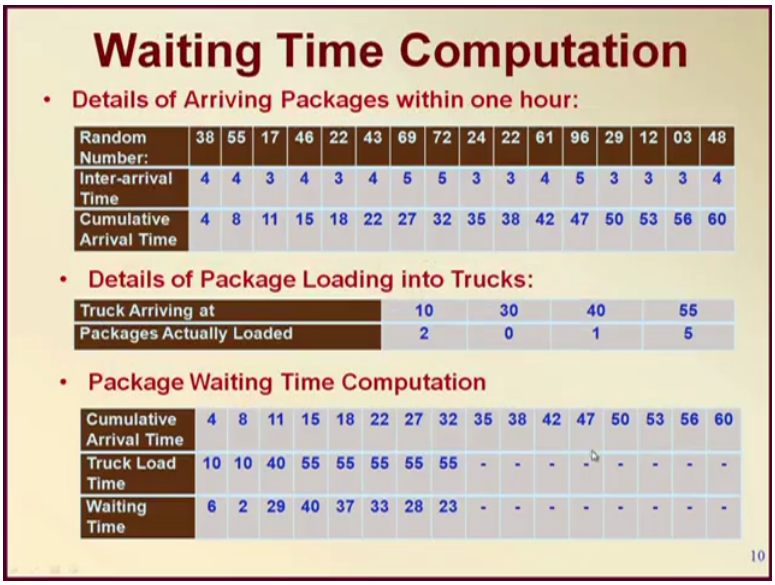

And suppose we want to compute the waiting time also, so what are the different? Look here the arrow here all the arrival times and load times. So truck load time is for 10 and here at 40, one has been taken and remaining 5 has gone at 55. But these particular you know package have come at time 4, so you see all these arrival times have put here and truck load time is 10. So how much time it is rated? It has waited for 6 minutes. 2nd is waiting for 2 minutes, 3rd package is waiting for 29 minutes, 4th is waiting for 40 minutes, 5th one 37, 6th one 33, 7th 28, 8th is 23, remaining you know we have not given because they are not loaded yet, they may be loaded in the subsequent time.

Say for example suppose the next truck comes in another 10 minutes and let us say fortunately there are 8 free spaces. So suppose there are 8 free spaces then what will happen? All 8 will go into that and they will then wait for subsequent times, right. Now anyhow we have to simulation and we have got data for 1, 2, 3, 4, 5, 6, 7, 8, full what you called loading thing and you see so what we can do? We can add 6 plus 2, 8 plus 29, 37 plus 40, 77, another 37 so 84 114 and then 147, 175, 198. So that is the total waiting time is 198 and that is over 8 you know completed packages.

So 198 by 8 that will be the average waiting time for these packages. So you can see the kind of calculations that we actually do in simulation you are actually making it let it happen and as it happens you are also accounting for it. And the accounting that you have done for this kind of things from there you will be able to compute the parameters for the real system and

then you have to take a decision, is this waiting time good enough or you know you have to do something about it.

But let me tell you this one-hour simulation is not sufficient, this is only for illustrative purposes, in real simulation we have to do further analysis, is it alright. So leave it here will see some more examples later on, but in our next class we shall discuss random variates. So thank you very much.