

**Decision Modeling**  
**By Professor Biswajit Mahanty**  
**Department of Industrial and Systems Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 02**  
**Payoff Matrix**

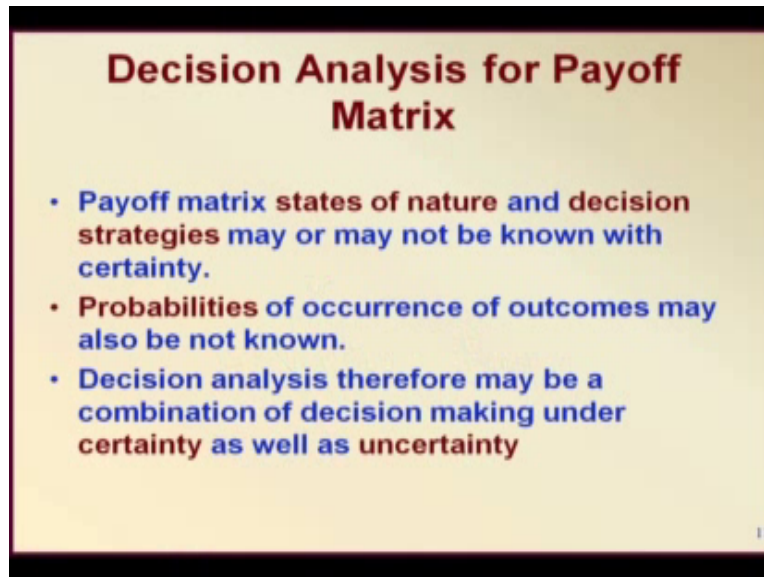
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		<u>Payoff Matrix</u>		
		No market changes	Favorable Market changes	Unfavorable Market changes
Decision Alternatives ↓	Redesign	30	100	-80
	Refurbish	50	200	-200
	Do nothing	0	30	-50

States of nature → Not under the control of the Decision Maker  
Decision Alternatives → Under the control of the Decision Maker

So last time we have discussed the payoff matrices and we really explain, What is a payoff matrix which has one side the state of nature on which the decision maker has no control, on the other side the decision alternatives which the decision maker can actually control, now in order to evaluate the payoff matrices we have to really know the different types decision maker,

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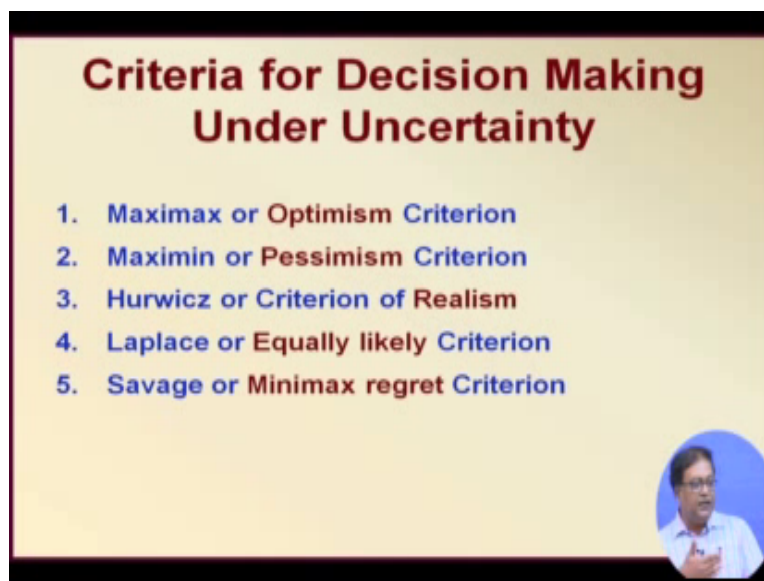
### Decision Analysis for Payoff Matrix

- **Payoff matrix states of nature and decision strategies** may or may not be known with certainty.
- **Probabilities of occurrence of outcomes** may also be not known.
- **Decision analysis therefore** may be a combination of decision making under **certainty as well as uncertainty**

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You see that first of all this is really explanation of the payoff matrices. The states of nature and the decision strategies may or may not be known with certainty, the probabilities of occurrence of outcomes may also be not known and decision analysis therefore maybe a combination of decision making under certainty as well as uncertainty.

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### Criteria for Decision Making Under Uncertainty

1. **Maximax or Optimism Criterion**
2. **Maximin or Pessimism Criterion**
3. **Hurwicz or Criterion of Realism**
4. **Laplace or Equally likely Criterion**
5. **Savage or Minimax regret Criterion**



So these are the different types of decision makers you can tell or you can tell the decision making criteria and we shall explain each one of them. But in the very first slide let us see what are the different criteria about decision making the first one is the maximax or the optimism criterion, the second one is the maximin or pessimism criterion,

The third one is a combination of both optimism and pessimism that is called the Hurwicz or the criterion of realism, the fourth one is Laplace or equally likely criterion and finally the savage or minimax regret criterion. So all of these five different decision making criterion.

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**Maximax: Optimism Criterion**

- Find the maximum possible payoff for each decision alternative
- Select the decision alternative with the maximum of the above maximum payoffs
- Solve Now:

Decision Alternatives	STATE OF NATURE			Payoff for a Decision Strategy
	No Market Changes	Favorable market changes	Unfavorable Market changes	
Redesign on a small scale	30	100	-80	
Rebuild and Refurbish	50	200	-200	
Do nothing	0	30	-50	

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let us evaluate and see the details, the first one which is known as the maximax or the optimism criterion, You see a maximax operator or a kind of decision maker who actually is really goes for optimism, what are the conditions which makes a person really go for the optimism criterion, if I really try to look at the different kinds of decision makers we shall find that there are certain kinds of decision makers.

Who really have very little options, the example can be taken let say from the transport business, a transport business is such a business where the profit margins are high maybe twenty to twenty five percent but the uncertainties are also high, when you have a certain number of trucks, you know it is imperative that you are also should be ready with uncertainties or disruptions in the business that means there could be an accident.

The truck may not be operating due to certain situations or there could be certain maintenance problems or even there could be some other kind of problems like some legal problems or any such problems, all of these really makes the truck business a little uncertain, suppose a person who has bought only one truck, he has just one truck and as long as the truck is moving properly there are no accidents, there are no difficulties everything is fine.

Then he does a very good business, but moment the truck faces with some eventuality that means there are some accidents or the truck is under maintenance then the business is not doing well, so in this situation the person probably has very little option but to become an optimist, the optimist because he thinks that nothing is going to happen everything is alright and therefore the business will go on, putting in a different language.

Let's look at these at this particular problem once again, and if you look at the various states of the nature there are three possible changes, one is no market change, favorable market change, and unfavorable market change, here the maximax operator who really believes in optimism would like to think that the best state of nature would occur for whatever decisions he or she may take, for example if the person is going to really take the redesign as an option,

Then what would be the state of nature that he would really assume, he would assume really that the favorable market changes would occur and therefore his payoff,

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Decision Alternatives ↓	Payoff Matrix			Maximax Payoff
	No Market Changes	Favorable Market Changes	Unfavorable Market Changes	
Redesign	30	100	-80	100
Refurbish	50	200	-200	200
Do Nothing	0	30	-50	30

States of nature → Not under the control of the Decision maker  
 Decision Alternatives → Under the control of the Decision maker

If you really look at here the payoff maximax if I write here, the maximax operator payoff would really become one hundred,

Why one hundred because the maximax operator thinks that the best state of nature would happen for a particular decision, so if he goes for redesign the best possible state of nature is favorable.

Because out of this thirty hundred and minus eighty, hundred is the maximum so maximum payoff the person may get is hundred so by similar logic if we really go for the second decision alternative that is if the person really goes for refurbish then again similar kind of thing may happen, the person again would believe that favorable market changes would happen because that is about the optimism or maximax criteria.

That if you go for favorable market changes then the maximum payoff is going to result, so if he goes for refurbish then the decision really would be two hundred that will be our payoff and therefore that would be the payoff for the second decision alternative that is the refurbish, the third one when it comes to do nothing again by same logic the payoff would be thirty because that is the best possible payoff horizontally that the person would expect.

Because the person thinks that the possible states of nature would occur for this particular decision alternative, so we have got this three maximax payoffs hundred, two hundred, and thirty and since two hundred is maximum we would assume that this would be the payoff, why this would be payoff because the maximax decision maker would then go for refurbish, why refurbish because out of the different payoffs that the maximax operator can have.


That is hundred, two hundred and thirty, two hundred is maximum and therefore a maximax operator would really like to go for the refurbish option,

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### Maximin: Pessimism Criterion

- Find the minimum possible payoff for each decision alternative
- Select the decision alternative with the maximum of the above maximum payoffs
- Solve Now:

Decision Alternatives	STATE OF NATURE			Payoff for a Decision Strategy
	No Market Changes	Favorable market changes	Unfavorable Market changes	
Redesign on a small scale	30	100	-80	
Rebuild and Refurbish	50	200	-200	
Do nothing	0	30	-50	



Now let's come to the second one which is known as the maximin operator, the maximin operator would find the minimum possible payoff for each decision alternative, select the decision alternative with the maximum of the above maximum payoff, so I am sorry this is select the decision alternative.

With the minimum of this because it should be the minimum of the above maximum payoffs right, so instead of this one that select the decision alternative maximum of the above minimum payoffs right, so find the minimum possible payoff for each decision alternative and select the decision alternative with the maximum of the above not maximum but minimum payoffs.

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Decision Alternatives ↓	Payoff Matrix			Maximin Payoff	
	No Market changes	Favorable Market changes	Unfavorable Market changes	Maximin Payoff	Maximin Payoff
Redesign	30	100	-80	100	-80
Refurbish	50	200	-200	200	-200
Do nothing	0	30	-50	30	-50

States of Nature → Not under the control of the Decision Maker  
Decision Alternatives → Under the control of the Decision Maker

So let's go back and let's try to see what is there for a maximin operator payoff. So maximin operator what will be the payoff, so for a maximin operator the person is pessimist because of pessimism it would be like thinking that for every decision alternative the worst state of nature is going to happen, so let's say the person goes for redesign then what are the payoffs, the payoffs are thirty, hundred and minus eighty, thirty for no market changes, hundred for favorable market changes and minus eighty for unfavorable market changes.

Out of these the worst payoff is minus eighty, that means if the person is a pessimist who thinks that worst is going to happen he or she would really go for the redesign alternative and expect a payoff of minus eighty, why because that is the worst payoff, in similar logic if the decision alternative is refurbish again the worst state of nature is unfavorable market changes and in this case the payoff to expect is minus two hundred.

And when it is do nothing out of the three zero three and minus fifty the expected payoff would be minus fifty, now obviously it is a maximin that means maximum of this minimum payoffs so which one is the maximum of this minimums that is minus fifty, because others are even lower that is minus eighty and minus two hundred, so in this case the decision of the decision maker would be do nothing for maximin and the payoff expected would be minus fifty, so this is for the maximin operator.


So we have the maximax operator where the operator would think the best possible situation will result and whereas for maximin the expectation is the worst possible payoff would result, however in real world we really don't always get an optimist person or an out and out pessimist person we get a person who is a combination of the maximax and maximin or in other words the person who is optimist.

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### Hurwicz: Criterion of Realism

- Choose a Degree of Optimism (a). Obtain Maximum Payoff (Max) and Minimum Payoff (Min) for each decision alternative. Obtain Hurwicz Payoffs for each strategy as:  $a \cdot \text{Max} + (1 - a) \cdot \text{Min}$
- Select the decision alternative with the maximum of the above Hurwicz payoffs
- Solve Now:

Decision Alternatives	STATE OF NATURE			Payoff for a Decision Strategy
	No Market Changes	Favorable market changes	Unfavorable Market changes	
Redesign on a small scale	30	100	-80	
Rebuild and Refurbish	50	200	-200	
Do nothing	0	30	-50	



But to a degree and that is where we have the Hurwicz criteria, so Hurwicz criteria basically says that choose a degree of optimism let's called it (a), obtain maximum payoff (Max) and minimum payoff (Min) for each decision alternative and then obtain Hurwicz payoff for each strategy as into max and plus one minus a star min, that means select the decision alternative with the maximum of the above Hurwicz payoffs.




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Decision Alternatives ↓	Payoff Matrix		Degree of Optimism $\alpha = 0.4$			
	No Market changes	Favourable Market changes	Maximax	Maximin	Hurwicz	
			Payoff	Payoff	Payoff	
Redesign	30	100	-80	100	-80	<b>-8</b>
Refurbish	50	200	-200	<b>200</b>	-200	-40
Do Nothing	0	30	-50	30	<b>-50</b>	-18

States of Nature → No Market changes, Favourable Market changes, Unfavourable Market changes.

States of Nature → Not under the control of the Decision Maker  
 Decision Alternatives → Under the control of the Decision Maker

$100 \times 0.4 + (-80) \times 0.6 = 40 - 48 = -8$   
 $100 \times 0.5 + (-200) \times 0.5 = 50 - 100 = -50$



So suppose let's take a particular case for Hurwicz that we have to have a certain value for a Hurwicz operator right,

So let's take an A value or say point four right, so what is point four, the point four can be called as the degree of optimism, so we can write it here the degree of optimism A equal to point four, so the degree of optimism is point four and is the Hurwicz operator so basically what it means.

It means the person is forty percent is optimist and sixty percent pessimist, so if that is so then an optimist if he goes for redesign and expects a payoff of hundred, a pessimist would like to have a payoff of minus eighty because the pessimist would believe that worst state of nature would happen but the person who is let's say forty percent optimist and sixty percent pessimist what would be the payoff, the payoff really be forty percent of hundred.

And sixty percent of minus eighty, so forty percent of hundred is forty and sixty percent of minus eighty is minus forty eight right, so minus eighty into sixty means point six is minus forty eight plus forty so the Hurwicz payoff will be minus eight, how we get minus eight hundred into point four that is forty right, we can write it somewhere here hundred into point four plus minus eighty into point six right, so that is forty minus forty eight equal to minus eight.

So this is how we get the Hurwicz payoff, it's a combination of the maximax and maximin, let's for second let us assume that the degree of optimism is point five, if the degree of optimism would have been point five then the value would really be hundred into point five plus minus eighty into point five that means fifty and minus forty right, so let's do once again hundred into point five plus minus eighty into point five equal to fifty minus forty equal to ten right.

So an Hurwicz payoff would have been ten if the degree of optimism would have been point five, that means the person is equally likely to be exactly in between optimism and pessimism, very simple terms hundred plus minus eighty by two that means twenty by two equal to ten right, so I think now it is very clear that if the degree of optimism is low the less it will be to the optimist value and more it will be pessimist value.

Whereas if the degree of optimism is equal then it will be simple average between that two, now the next thing that is second decision refurbish, so again we have seen the maximax payoff was two hundred and maximin payoff was minus two hundred, so again if we give forty percent weight then two hundred into point four it will be eighty and minus two hundred into point six will be minus one twenty, so on one side we have eighty.

And other side we have minus one twenty, so it will be minus forty that would be the Hurwicz payoff for the second decision refurbish, the third decision alternative that is do nothing again we have seen the maximax payoff is thirty and maximin payoff is minus fifty, so again by the same logic minus fifty into sixty percent that is minus thirty and thirty into point four is twelve, twelve minus thirty so it comes to minus eighteen right.

So thirty into point four twelve minus fifty into point six is thirty minus thirty so twelve minus thirty minus eighteen, so these are the Hurwicz payoff we have got minus eight, minus forty and minus eighteen right, so once we have got all these three Hurwicz payoffs we can see again the maximum of them, which one is maximum minus eight, so we go for this particular payoff and the person may rally go for redesign right.


So person if he is a Hurwicz operator ohm alpha equal to point four would really get expect a payoff of minus eight and go for redesign,

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## Laplace: Equally Likely Criterion

- Find the average value of payoff for each decision alternative
- Select the decision alternative with the maximum of the above maximum payoffs
- Solve Now:

Decision Alternatives	STATE OF NATURE			Payoff for a Decision Strategy
	No Market Changes	Favorable market changes	Unfavorable Market changes	
Redesign on a small scale	30	100	-80	
Rebuild and Refurbish	50	200	-200	
Do nothing	0	30	-50	



The next one is which is known as a Laplace operator, The Laplace operator would find the average value of payoff for each decision alternative, select the decision alternative with the maximum of the above maximum payoff, so basically what really happen in a Laplace decision maker.


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Degree of Optimism  
 $\alpha = 0.4$

Decision Alternatives ↓	Payoff Matrix			Maximax	Maximin	Minimax	Laplace
	No Market Changes	Favorable Market changes	Unfavorable Market changes	Payoff	Payoff	Payoff	Payoff
Redesign	30	100	-80	100	-80	-8	$50\frac{1}{2}$
Refurbish	50	200	-200	200	-200	-40	$50\frac{1}{2}$
Do nothing	0	30	-50	30	-50	-18	$-20\frac{1}{2}$

States of Nature → Not under the control of the Decision Maker  
 Decision Alternatives → Under the control of the Decision Maker

$100 \times 0.4 + (-80) \times 0.6 = 40 - 48 = -8$   
 $100 \times 0.5 + (-200) \times 0.5 = 50 - 100 = -50$



The Laplace decision maker really would think that there are some probability associated with the different states of nature and all this probabilities are equal right,

So a Laplace operator would assume equal probabilities, see real probabilities are not known are to be estimated, so initially we are thinking it is like a certain kind of uncertainty situation and since the probability is not known we assume the probabilities are equal.

That means probably point three three point three three point three three around that but then again we did not really write those probabilities because if they are equal probability then what would happen basically if we multiply by the same probability then virtually it become taking an average of the three numbers right, so what would be the Laplace payoff if the person goes for redesign, it will be the average of thirty, hundred and minus eighty.

So hundred thirty minus eighty that is fifty, the Laplace payoff in that case would be fifty by three right, because thirty plus hundred, hundred thirty minus eighty is fifty divided by three that will be the average of the design, it will be the average of thirty, hundred and minus eighty, so hundred thirty minus eighty that is fifty, the Laplace payoff in that case would be fifty by three right, because thirty plus hundred, hundred thirty minus eighty is fifty divided by three.

That will be the average of the three payoffs, similarly if it is refurbish it will be fifty plus two hundred minus two hundred again it will be fifty by three and the last one is zero plus thirty minus fifty it is coming to minus twenty by three, so what would be the decision of Laplace decision maker, the Laplace decision maker would really go for redesign or refurbish both are equally likely, so in this case the Laplace operator would really go for either redesign.

Or refurbish because both are having equal probability, so we have seen that depending on various decision operator that whether it goes for maximax there will be a payoff maximin there will be another payoff, the Hurwicz there will be another payoff, and Laplace is having another payoff, so what is very interesting to note is that even for a simple payoff problem that we have taken up here that the decision is changing.

That means which decision alternative a particular operator would really like to go for is changing based on what is the kind of decision maker, a maximax decision maker goes for refurbish, a maximin decision maker goes for do nothing, a Hurwicz decision maker goes for

redesign, and a Laplace decision maker makes equal choice between redesign and refurbish, so in all of these a basic question comes back that what is known as rational decision maker.

Who is rational decision maker, is a maximax decision maker a rational decision maker is he or she taking a rational decision probably no and what makes someone really go for a rational decision making, it's a very important question and the answer really lies in understanding what is happening in the transport business, let's go back to there one more time, you see a particular person has gone for a transport business and if the person has only one truck.

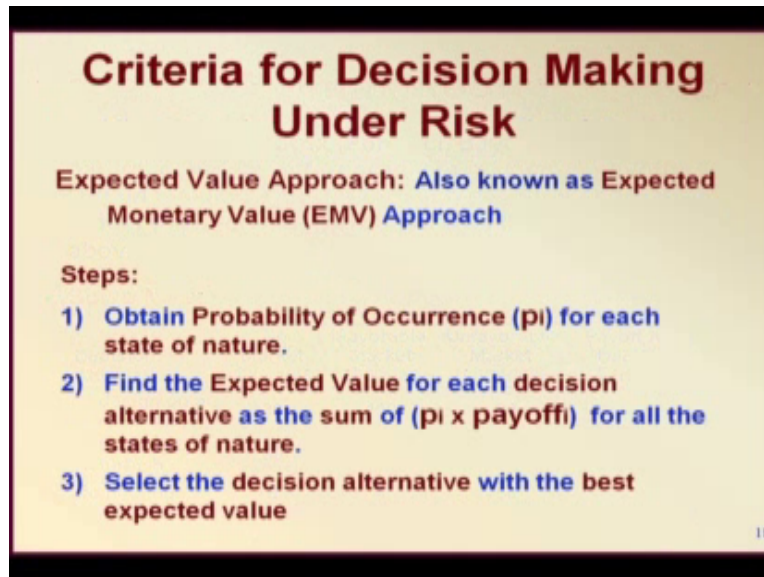
Sooner or later he may go out of business, why because just with one truck sooner or later it will have some kind of eventuality either accident or maintenance problem or legal issues and therefore the business may not continue, but imagine if a person is having hundred trucks, if the person is having hundred trucks there is always an expectation that out of these hundred trucks some will be under accident, some will be under maintenance.

Some will have legal issues but the rest of the trucks maybe around ninety would be operating, so what will happen the person therefore will say that these are the probabilities that ninety trucks would be available, some will be accident, some will be under maintenance, some will be under legal issues and therefore the resultant profit or the resultant payoff would depend on such probabilities right, so basically therefore a person can become rational.

Only when there are opportunities to really cater to all of these, you see alternate situations can result, a person can have one truck but like that hundred people also can have one truck each, they can either go for insurance or they can go for what is known as a cooperate team, suppose hundred people forms a cooperate team then it is like the cooperate team is having hundred trucks and these hundred trucks are basically one truck each for hundred people.

But they have gone for a cooperation and this particular cooperate team is having hundred trucks again ninety would operate but the profit will be shared by the cooperate team even if a few trucks are under difficult situation the cooperate team can still be in business and doing well right, so basic point here is a person will become rational operator only when there are situations where the portfolio is basically obtained right, and that portfolio has to be created.

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**Criteria for Decision Making Under Risk**

**Expected Value Approach: Also known as Expected Monetary Value (EMV) Approach**

**Steps:**

- 1) Obtain Probability of Occurrence ( $p_i$ ) for each state of nature.**
- 2) Find the Expected Value for each decision alternative as the sum of ( $p_i \times \text{payoff}_i$ ) for all the states of nature.**
- 3) Select the decision alternative with the best expected value**

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Let's really look at how exactly these things are happening, so basically when it comes to that kind of situation we call what is known as the expected value approach also known as the expected monetary value approach, so either we call it expected value approach or sometimes we can also call it as a rational decision maker or an expected monetary value, so under all of these situations there would be probability of occurrence for each state of nature.

And find the expected value for each decision alternative as the sum of  $P_i$  into payoff  $i$  for all the states of nature and select the decision alternative with the best expected value, so essentially what is the process, the process is that we put specific probability values for each specific payoffs we calculate the expected value of the decision alternatives and finally we find out which decision alternatives are coming out with a best expected value right.

So all of these things we shall discuss in our next lecture,

Thank you very much.