Decision Modelling.
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Lecture-10.
Decision Problem Example.

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In our previous class we have seen how to handle on how to carry out decision problems with experimentation with the help of decision tree. So since it is a difficult topic, what we shall do in this class, we shall take another example to really get insight into such problems. So this is another problem that we have taken, let us say a company has 2 decision alternatives, number-one either to restructure business or to carry on with the normal business. And there are 2 states of nature, 1st of all it could be smooth, that is no risk or troublesome, that means high risk.

And these are the kind of payoffs, 1st of all that if we restructure, then and there is no risk than there will be payoff of 800 and if it is troublesome, then it would be -100. On the other side if it is a normal business, then it would be payoff would be 150 and if it is troublesome or high risk, then it will be payoff of 100. In normal business the - is not there, the idea here is since not much additional expenditure is there, even with high risk the business is going to get certain amount of profits. And those are our prior probabilities, 25 percent and 75 percent.

Decision Making with Experimentation

- Consider the Decision Tree Problem on Company Decision with Appointing a Consultant.
- The Consultant Opinion is an experimentation.
- The company now has two decisions to make:
  1) Whether to go for the Consultant
  2) If Consultant Opinion is assertive, then what company decision Else what decision?
- Here the second decision dependent upo[n] outcome of the first.

Decision Making with Experimentation

Alternatives	State of Nature	
	Smooth (S) No Risk	Troublesome (T) High Risk
Restructure Business (R)	800	-100
Normal Business (N) (As Usual)	150	100
Prior Probability	0.25	0.75

- Before arriving at the company decision, the company has option to seek a consultant opinion at a cost of 50.
- Information from consultant could help in deciding which alternative to pursue (restructure or Normal Business)

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Let us 1st see that what kind of experiment we can do for this kind of decision problem also. So consider the decision tree problem on company decision with appointing a consultant. Let us assume that the company can actually appoint a consultant and the consultant opinion in this case is the experimentation, right. So company therefore has to make 2 decisions, 1st of all whether to go for the consultant and if consultant opinion is assertive, means positive, then what company decision, else what decision.

Then what should be the company decision is we have got assertive consultant opinion or if it is not assertive, then what should be the decision. So here the 2nd decision is dependent upon the outcome of the 1st. So once again before arriving at the company decision, the company has an option to seek a consultant opinion at a cost of 50. Information from consultant could

help in deciding which alternative to pursue, that is restructure or normal business. Now 1st of all let us look at the problem from you know simple decision problem point of view.

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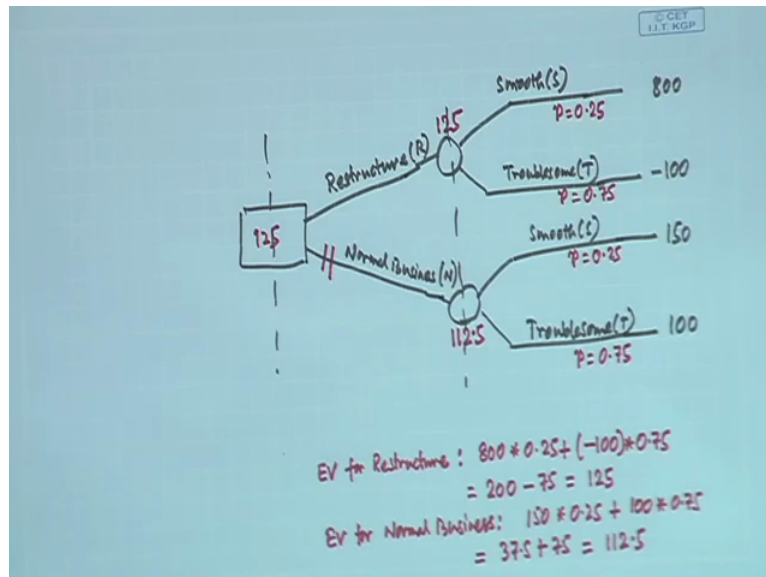
So 1st of all there is a decision point and there are 2 decisions, what are they, 1st one is restructure, one can go for restructure and it can go for normal business, right. Then there are for each case there are 2 states of nature, one is the smooth business situation, there is not much problem in this. But the 2 is troublesome, there is a risk involved, so T. So restructure R and normal business N. So here is our state of nature, so there are 2 states of nature and again it could be smooth S or troublesome T. And what are the payoffs, payoffs are, the 1st case, the payoffs at 800, here the payoffs are - 100, here the payoff is 150 and here the payoff is 100.

What are the prior probabilities, the prior probabilities are for smooth business, the probability is 25 percent, that means P equal to 0.25 and troublesome, P equal to 0.75, again same thing here, 0.25 and 0.75. So what we need to do, we need to compute the expected values, expected value for restructure decision, expected value for restructure decision would be 800 into 0.25 + - 100 into 0.75. So 800 into 0.25, that would become 200 and this one would be - 75, so it would be 125.

Similarly expected value for normal business would be 150 into 0.25 + 100 into 0.75, so this would be 150 into 0.25, that means 37.5 + 75 equal to 112.5. So these are the different payoffs and we can have here is 125 which is the payoff for restructure and 112.5, that is a payoff for normal business. So since 125 is higher, so for the decision point we have 125 as a payoff and we go for restructure, that means normal business is giving us low payoff. So

basically what we look here we get is that it is the decision that we are obtaining is restructure.

And if we go for restructure, then we are going to get a payoff of 125. So this is the 1st cut decision that we have. But the question is that should we really go for it, should we or obtain consultant opinion to get a better value about the market because in this case, okay we will go for restructure but there is a 75 percent probability that the market could be troublesome and we may actually get a - 100 payoff which is very risky indeed, right. So we although, the expected value is 125, that is because that is the market comes out to be smooth, only then an 800 payoff is obtained and if it becomes troublesome, then the whole thing may actually lead to very low payoff.

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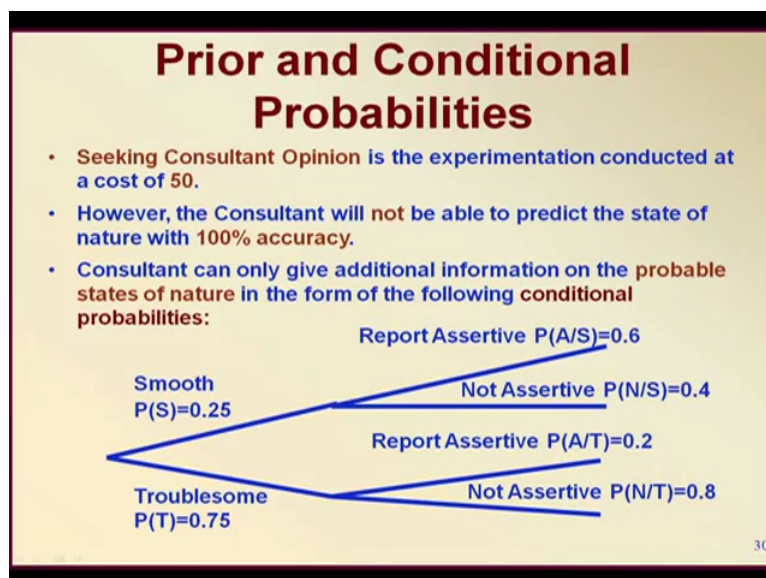
So what is the company can do, company can seek consultant opinion is the experimentation conducted at a cost of 50. However the consultant will not be able to predict the state of nature with 100 percent accuracy, the consultant can only give additional information on the probable states of nature in the form of following conditional probabilities. So what is they can do, the consultant can report assertive with 60 percent probability and not assertive 40 percent probability if the market is smooth.

On the other side if the market is troublesome, then still it can be assertive, maybe 20 percent time and it is not assertive, maybe 80 percent time, right. So these are the different possibilities that you know it can have. So there is the different to know a priori probabilities

and the conditional probabilities which are put here. So after this particular kind of thing, the next thing is calling out the Bayesian analysis.

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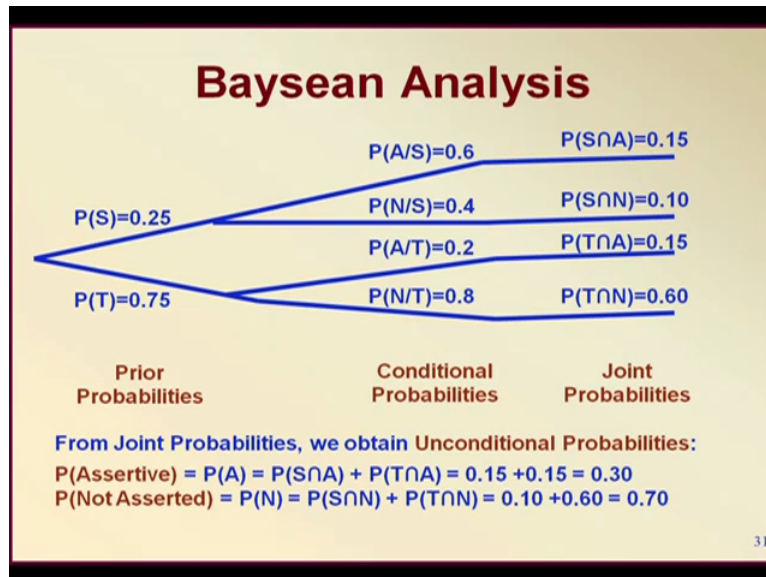
So again following the procedure we have the prior probabilities, we have the conditional probabilities, when you multiply, then we get what are known as the joint probabilities. So 0.25 into 0.6, that is the joint probability will be 0.15, that is the probability of the you know assertive and the smooth, the smooth market and the consultant is assertive, here smooth market but consultant is not assertive. So these are the probabilities. So this is now again from the joint probabilities we can also obtain what is known as the unconditional probabilities by adding the corresponding values.

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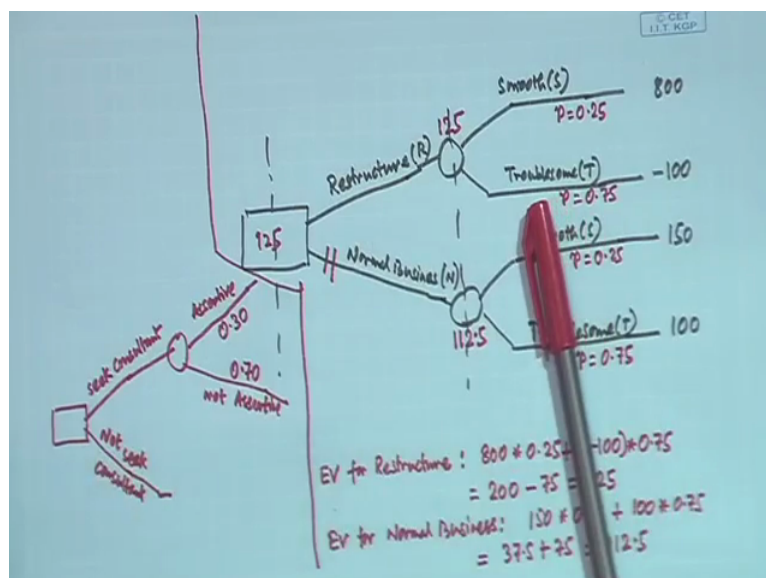
So what is the probability of assertiveness, that both smooth and troublesome, they should be added, so 0.15 and 0.15, so probability of assertive 0.3, probability of not assertive is 0.7. So look here, like we have done earlier, so again we have to think of this that we have a decision point, from the decision point whether to seek consultant or not seek consultant. So this is the decision point that either you seek consultant or not seek consultant. If you do not, then this case becomes nothing but this case, right. So this case, this branch we need not seek.

But if you seek consultant, then the result could be assertive because these are the states of nature, what are they, one is assertive and other is not assertive. And what should be the probability for assertive and not assertive, look at those unconditional probabilities. So they are 0.3 and 0.7, right. So you have to see, that is how this situation will you know really work out. So and then thereafter whether you go for assertive or not assertive, again there would be the corresponding probabilities which are nothing but you know your posterior probabilities.

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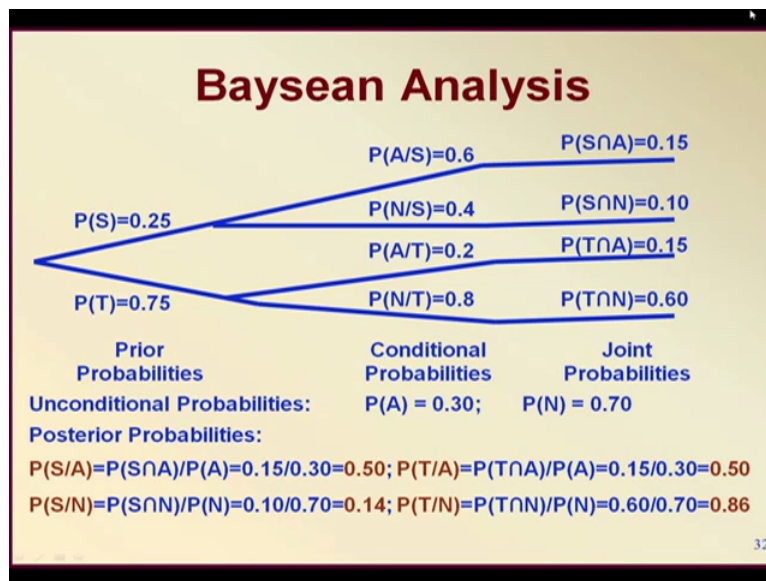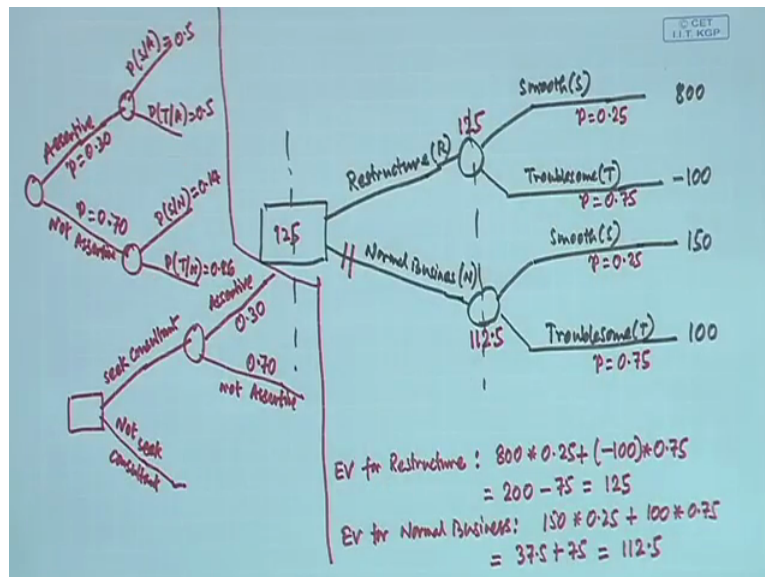
Let us look at the posterior probabilities 1st. So thereafter once we obtain the joint probabilities, then we can actually obtain the posterior probabilities, the posterior probabilities of if the consultant is assertive, then the smooth would be you know 0.15 by 0.3 because this 0.15 and this 0.15, they are both for assertive. So 0.15 by 0.3, that is 50 percent and this also will be 50 percent, whereas other 2 if we see, because they are 0.1 and 0.7, they will come to 0.14 and 0.86 respectively. So basically if it is the assertive route, then let us draw here once again.

So this side, this is assertive and this is not assertive, right. Here the P equal to 0.3 and here is the P equal to 0.7 and again there are 3 options because there are decision options but here we are just seeing the different options. So if it is assertive, then again the probability of smooth given assertive and probability of troublesome given assertive and here probability of smooth given not assertive, and probability of troublesome given not assertive. So these are the different probabilities, so this will be 0.5, this will be also 0.5, this will be 0.14 and this will be 0.86, right.

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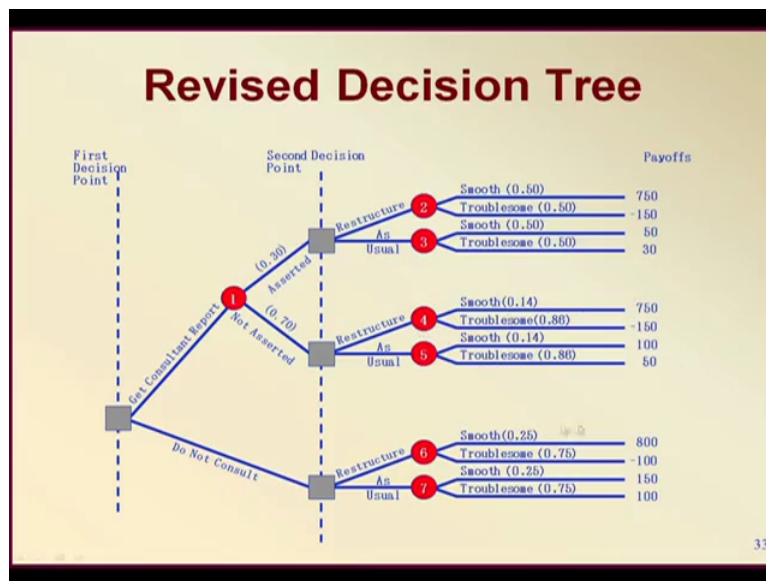
So this will be the broad structure, that is a broad structure that we can actually think of and then together the whole thing into our the decision tree. So that is the revised decision tree that we shall get. Now look at this revised decision tree, so how, what exactly we have, in the 1st decision point we can get consultant report or we do not consult. If we get consultant, do not consult, then again we can do either restructure or normal as usual, then it could be smooth or troublesome, smooth or troublesome, 0.25, 0.75.

Similarly here, if we go for restructure and as usual, again there could be smooth and troublesome, smooth and troublesome. But if we not asserted, then restructure and as usual it could be smooth and troublesome, smooth and troublesome. So look at this picture here. Here we saw that 0.3, 0.7, these are the unconditional probabilities, 0.5, 0.5, 0.14, 0.86, these are the posterior probabilities, right. So one hand we have the unconditional probabilities, other side we have the posterior probabilities.

So here what we have done, we have put the unconditional probabilities here and the posterior probabilities here because that is how they get converted. So if you look at, the smooth market was only 25 percent but when you get consultant report and if the consultant asserts, then these probabilities become 0.5 and 0.5. On the other side of the consultant do not asserts, then those probabilities become 0.14 and 0.86. And since the consultant charges 54 carrying out this analysis, then the payoffs also get modified to 750 from 800, - 150 from -100, 50 from 150 sorry 100 from 150 and 50 from this thing.
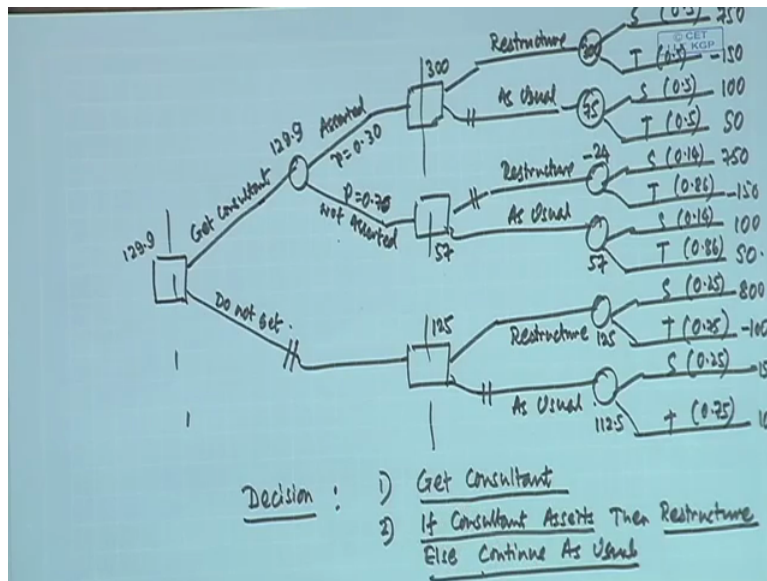
So these 2 are wrong, so they should be 100 and 50, right, so those calculations have to be corrected. So anyhow, so what really happens then, 750 to 0.5, so I think they can also do the calculations, let us do that. So basically at the 1st decision point, this is the 1st decision point, so this is that you know get consultant and this side is do not get. So as you get consultant and do not get, so here it could be asserted and not asserted, right. And then there is a 2nd decision point, this is the 1st decision point, this is the 2nd decision point, at this point there are 2 decisions, one is restructure and this is as usual.

Right, similarly here for each one there could be 2 decision points, restructure and as usual, right. And based on restructure as usual, here these are the states of nature, it could be smooth or troublesome, right. So here smooth, troublesome, smooth, troublesome, smooth, troublesome, smooth, troublesome smooth, troublesome, smooth, troublesome. But these probabilities they are you know 0.75 and 0.25, here 0.25, 0.75 and here these are 0.14 and 0.86 because these are not asserted. So for not asserted, these are our posterior probabilities.

But here these are 0.5 and 0.5, here 0.5 and 0.5, right. So these are all the probabilities and what are the payoffs, payoffs here were 800, -100, -150 and 100.

Here what are the payoffs, 750, -150 and here also 750, -150 and since 50 is chosen, that it will be 100 and 50, this also 100 and 50, right. So now since 100 and 50, so this should be 75, right and here 750, so this should be 300, right. Here again this should be 750 and -150 but these are into 0.14 and 0.86, so this comes out to be -24, that would be the payoff, that is how

the calculation goes. And this one will be 57 and similarly this calculation 125 and 112.5 which we have already done before.

So these are the payoffs and since 300 is higher, we put 300 here and we do not do this, whereas here we do not do restructure, we do as usual, so we get 57. And here since restructure is higher, here we get 125. Now again the asserted probability, this P was 0.3 and this P was 0.7, so therefore 0.3 times 300, 0.7 times 57, the total comes to 129.9. Now out of 129.9 and 125, since 129.9 is higher, so again this is cut, right. So what is the decision here, the decision is 1st of all 1 get consultant and 2 if consultant asserts, then restructure, else continue as usual, right.

So get consultant, if consultant asserts, then restructure, else continue as usual. So this is what is really happening that at the 1st point that consultant but there 2 things can happen, he can assert, he may not assert, when he asserts, then it is higher payoff coming from restructure, so go for it, 300 and supposing this 0 should be 75, but anyhow, if suppose he do not assert, then go for restructure, sorry as usual because that gives a payoff of 57. So all of these are actually put here in the form of this payoff matrix, the same thing that we have done.

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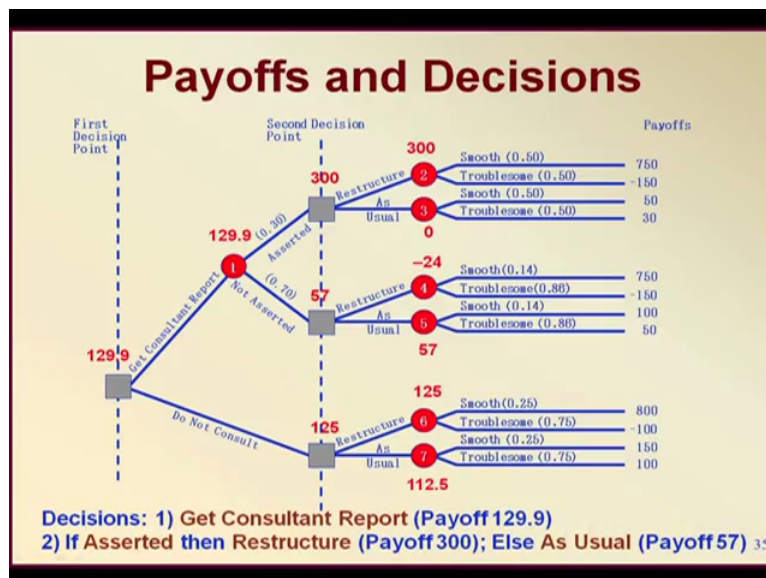
And if we look at this final diagram, then you can see that the decisions once again that get consultant report payoff 129.9, if asserted then restructure, else go as usual, right. So essentially in all these problems, whenever there is a Bayesian analysis is involved, do remember that we have one type of decision structure before the Bayesian analysis is carried

out. After Bayesian analysis is carried out, we get 2 sets of probabilities, one is a set of unconditional probabilities, another is a set of posterior probabilities.

The unconditional probabilities are actually put you know based on whether to go for experimentation, whether to go for asserted, I mean weather, what is the result, then those unconditional probabilities will be put here and based on those results the ultimate, the nature, states of nature, there we should put the what is known as the Bayesian the posterior probabilities. So the posterior probabilities would be definitely an improvement over the probabilities which we had earlier.

For example, here we see that we had probabilities of 25 percent and 75 percent when the consultant asserts, they become 50 percent, when consultant do not assert, however they become only 14 percent, right. So that is what really happens, that we can experiment, our decisions we come better and we are able to come up with better payoffs with the help of such experimentation. This is where we conclude our decision analysis, in our next class we starts with queueing theory, thank you very much.