

**Commodity Derivatives and Risk Management**  
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**Lecture 6**  
**Commodity Options and Commodity Spreads**

Welcome to the next session on Commodity Derivatives and Risk Management ah. If you recall we were discussing about we were discussing about minimum variance hedge ratio and we calculated minimum variance hedge ratio using the formula method as well as the regression method for 3 commodities that is your black pepper, Isabgol seeds and Raw jute which these three contracts are traded at national multi commodity exchanges traded and listed at national multi commodity exchanges. Now let us if I recall I posed a question to each of you that can the minimum variance ratio be greater than 1. So to what is your answer to my question?

Ok, so had it been a regular class I would have come to know your answer, so this is the negative side of any asynchronous communication. So this is something which probably as a teacher none of us enjoy but given the constraint I am trying to make use of the most of it, so I do not know what is your answer so now I will go back to I will start this session by giving answer to your question to the question I posed. But before I go to our discussing whether minimum variance hedge ratio can be greater than 1 or not, let us summarise what we had found out for the 3 contracts commodity contracts listed at MMC, so let us focus on the details which is given in the on the screen.

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Minimum Variance Hedge Ratio				
	BLACK PEPPER	ISOBGUL SEED	RAW JUTE	
Correlation	0.58	0.73	0.35	
Std. Dev- Delta ( spot)	576.30	124.03	15.45	
Std. Dev- Delta (futures)	958.36	91.08	25.72	
Minimum variance hedge ratio	0.35	0.999	0.21	
Coefficient of Determination	0.34	0.54	0.12	

- Empirical Validation of Minimum Variance Hedge Ratio for Black Pepper:

Hedge Ratio	0.35	0.5	0.20	0.80	1
Portfolio Risk for Pepper	467.8	489.0	489.7	635.4	778.1

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So if you recall black pepper had minimum variance of hedge ratio of 0.35, Isabgol seed had 0.999 or equivalent to 1, raw jute has 0.21 and coefficient of determination respectively given. Now again I will pose another interesting question to you, if minimum variances hedge ratio calculated by using then this formula method or the regression method is really the minimum variances hedge ratio. Can there be any other combination of spot and futures combination which will result in a lesser amount of a portfolio risk. So to do to test that or to do a sensitivity test I took just random combination of portfolio both spot as well as a futures combination.

So this particular second panel of this second table in this slide indicates the various combination of hedge ratio and portfolio risk for the black pepper, so if you recall if you the hedge ratio had 0 point minimum variances hedge ratio is 0.35 and the portfolio risk for the pepper that is the standard deviation of portfolio returned daily return is 467.8. So let me take you to the let me take you to the link file, so let me take you, so let us go to this test this is the second worksheet of the excel file, this particular table shows when the spot price is this future price is this.

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Spot	Delta (Futures)	Delta (Spot)	0.35 Hedge ratio	0.50 Hedge Ratio	0.20 hedge ratio	0.80 hedge ratio	1:1 hedge ratio
65850							
66050	981	200	-144.6	-290.5	3.8	-584.8	
65283	697	-767	-1011.8	-1115.5	-906.4	-1324.6	
65200	433	-83	-235.1	-299.5	-169.6	-429.4	
65000	-438	-200	-46.2	19	-112.4	150.4	
65300	257	300	209.7	171.5	248.6	94.4	
65800	-265	500	593.1	632.5	553	712	
66800	830	1000	708.5	585	834	336	
67300	242	500	415.0	379	451.6	306.4	
68000	-660	700	931.8	1030	832	1228	
68100	-669	100	335.0	434.5	233.8	635.2	
68060	-293	-40	62.9	106.5	18.6	194.4	

So when you have a Delta 0.35 please focus on the column G, when the hedge ratio is 0.35 what is going to be my portfolio risk. So portfolio risk is 467.8, similarly when the hedge ratio is 0.5 as given in column H, the portfolio risk is coming to 489. I just took another random number so portfolio hedge minimum variances hedge ratio is 0.2 that means for every one unit long on asset, the producer is entering in to 0.2 units of futures contract with this and the portfolio risk is 489.7, 0.80 as hedge ratio, portfolio risk is 635.4 and like one to

one hedge ratio which is a naive hedge ratio that is one unit long on asset you will be hedged by one unit of a one unit of under futures contract.

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Minimum Variance Hedge Ratio			
	BLACK PEPPER	ISOBGUL SEED	RAW JUTE
Correlation	0.58	0.73	0.35
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So you have a portfolio risk is coming to 778.1, so going back to our slide, so if you see this particular PPT this particular table second panel shows the various hedge ratio combination with risk with respect to the portfolio risk for black pepper and if you can see this with 0.35 as a hedge ratio, the portfolio risk is lowest. In fact with the excel file which is available to you, you can try out other combination of hedge ratio and check whether really minimum variance hedge ratio is holds true with 0.35 sorry portfolio risk is the lowest with hedge ratio of 0.35 or any other combination.

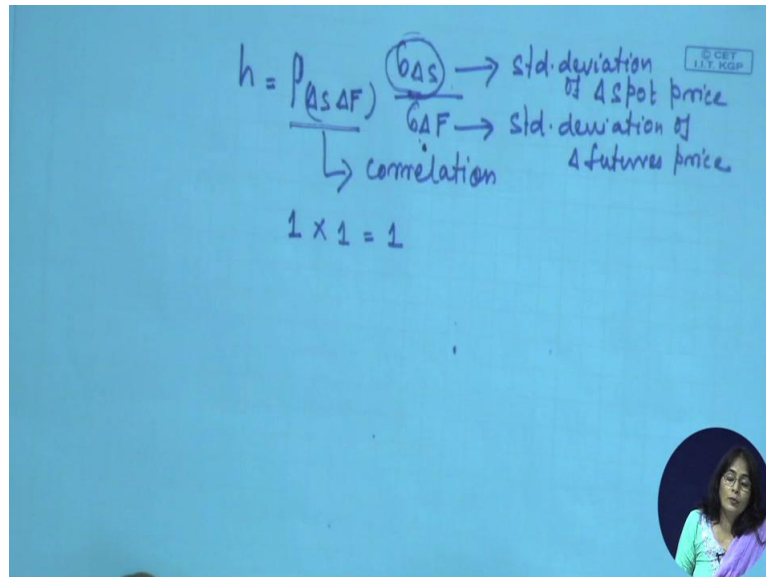
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Minimum Variance Hedge Ratio	
• Can this ratio be $> 1$ ?	
• This ratio will be 1 if prices of spot & futures prices are correlated (maximum value of 1) and variance of both series are same.	
• This ratio will be higher if variance of spot market is higher than futures market – cross hedge.	
• Can this ratio be negative?	

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Now going to my question on can hedge ratio be greater than 1, the answer to this question is yes, hedge ratio can be greater than 1, so why hedge ratio can be greater than one let us take let us discuss how do we define our hedge ratio using your formula method.

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Handwritten formula for hedge ratio  $h$ :

$$h = \rho(S, F) \frac{\sigma_S}{\sigma_F}$$

Annotations:

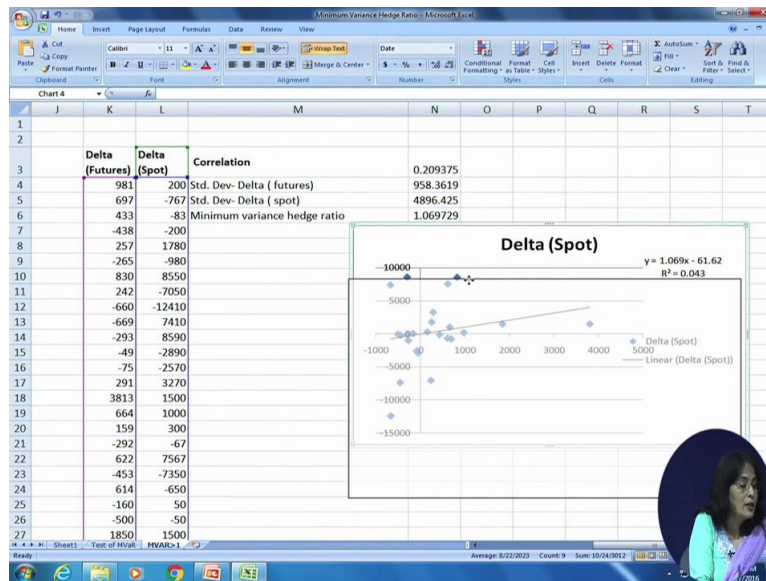
- $\rho(S, F)$  → correlation
- $\sigma_S$  → std. deviation of spot price
- $\sigma_F$  → std. deviation of futures price

Below the formula, it is written:  $1 \times 1 = 1$

If you remember hedge ratio is nothing but the correlations between Delta S Delta F and sigma Delta S is by sigma Delta F, so this is the correlation between price change and sigma S is the standard deviation of change in spot price and this is the standard deviation of change in futures price. Let us say correlation between spot price and future price can be maximum it can go to one, so this factor can be one and if both are standard deviation is same then you can have this ratio as 1 so 1 is to 1 will be 1.

Now the next question is when it can be greater than one, it can be greater than one when you have the standard deviation of Delta S can be significantly higher than the standard deviation of Delta F that is spot price has a greater volatility with respect to the futures price. So this is the situation and where you can have a minimum variance hedge ratio can be greater than one, so I have again taken a hypothetical situation, so let me go back to black pepper May 2016 data. So I have the same data however I have done some modification random modification to the futures price and the spot price without doing any change to the price prevailing on the contract squaring up date.

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So this part beginning a price combination and the last price combination spot futures price combination I have kept unchanged and by doing changing some of the values what I have found out is that if you see the correlation is correlation between spot futures have come down to 0.20 from the earlier correlation of 0.53 and standard deviation of Delta futures is 958, standard deviation of Delta spot is 4896.

So just to give you an example the standard deviation of Delta spot is almost 5 times the 5 times the standard deviation of Delta futures. So taking into consideration all these factors you can have minimum variances hedge ratio as 1.06. So this gives an this answers my question that yes, there could be situation where you have where you have minimum variances hedge ratio as greater than 1 and this could be with respect to a higher volatility in the spot market (( ))(11:17) the futures volatility.

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### Minimum Variance Hedge Ratio

- Can this ratio be  $> 1$ ?
- This ratio will be 1 if prices of spot & futures prices are correlated (maximum value of 1) and variance of both series are same.
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- Can this ratio be negative?



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Now my so as I said this ratio will be higher if variances of the spot market is higher than the future market and this normally happens in case of a cross hedge. So if you if a particular commodity producer is hedging its price risk using the similar kind of or same commodity, let say let say soybean oil producer is hedging its price risk by entering into a futures contract on soy oil, the chances that the hedge ratio going to be more than 1 is very limited. However, if you recall in the last session, last to last session we had discussed that many a times commodity futures contract the number of commodity futures contract available for trading is limited and all commodities futures contract on all types of underlings are not available.

So many a times commodity producers and consumers enter into a cross hedge that is the commodity which is very similarly behaving like their underlying exposure but not the same commodity. So it could be situation in case of a cross hedge there could be there will be a possibility of, possibility of hedge ratio minimum variances hedge ratio to be greater than 1, otherwise in normal situation we know the cost of carrying model will ensure that the spot and futures price move in some similar fashion. So over a sufficiently long period of time if we take to calculate the minimum variances hedge ratio, so the likelihood of minimum hedge ratio beyond 1 and if the underlying asset and the futures contract underlying of the futures is same, there is very little likelihood of minimum variance hedge ratio to be greater than one.

Now let us go to the next question can this ratio be negative. So I am not going to answer this question at this point of time, this is a food for thought for each of you find out or think intuitively whether this ratio can be negative. So if negative then what would be the strategy, how a hedger will be able to hedge its risk using a negative hedge ratio if at all negative

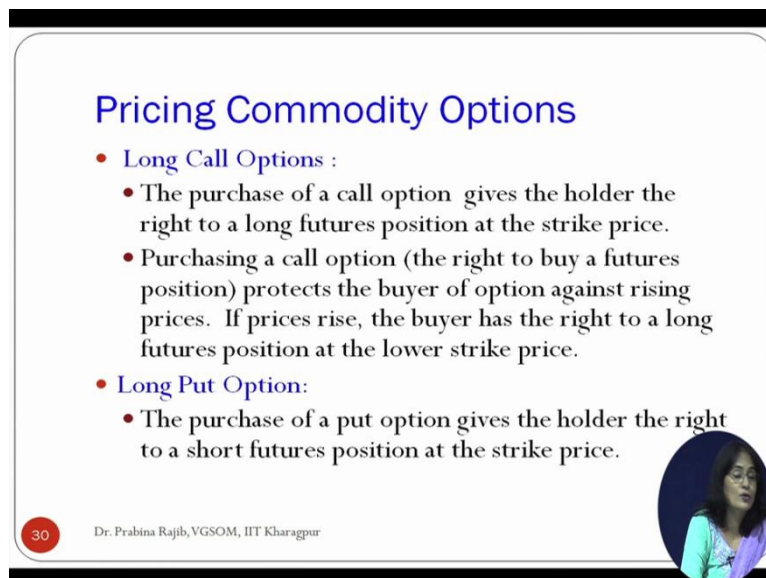


hedge ratio is possible. Now if you recall we had discussed about commodity options in some earlier part of the lecture though commodity option contracts are not rebate, today I will be spending some very briefly sometime on how do you go about value commodity options because substantial amount of time we have they spend on valuing futures contract and pricing an valuing futures contract.

And today we will be spending not much of in greater detail but some basic aspect of how commodity contracts commodity options can be valued and I will just take you through the basic concepts of commodity options if you recall, the exchange listed commodity options do not have the commodities underlying but they have commodity futures as the underlying and the long call option position holder, whenever he exercises he gets to take a long position in a underlying commodity futures contract.

Similarly, whenever the count whenever a long put option position holder exercises its option, and that that party takes a short futures position. So let me repeat, a long call option holder when it exercises its option, he takes a position long futures position on the underlying at a strike price and a long put position holder, when he exercises the option he takes the a short futures position at the exercise price.

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**Pricing Commodity Options**

- **Long Call Options :**
  - The purchase of a call option gives the holder the right to a long futures position at the strike price.
  - Purchasing a call option (the right to buy a futures position) protects the buyer of option against rising prices. If prices rise, the buyer has the right to a long futures position at the lower strike price.
- **Long Put Option:**
  - The purchase of a put option gives the holder the right to a short futures position at the strike price.

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Now let us focus on whatever is mentioned here, let me let me just again read out long call option, the purchase of a call option gives the holder the right to long futures position at the strike price and purchase of a call option that is the right to buy futures position, please note here when we are talking about purchasing in the call option, it gives the buyer the right to

buy a futures contract at a later date. In a regular options you have the buyer has the right to buy the underlying but in case of a commodity futures the long call position holder or the buyer has the right to buy the futures contract and at what price, at a at the exercise price and by taking a long call option it protects the buyer of the option against the rising prices.

So if prices rise the buyer already, buyer can exercise its option, it is a long position in the long futures position in the underlying and if the underlying price increases and commensurately the future price also increases, the buyer of the long futures position stand to gain because he has bought a futures at a lesser price and subsequently the futures price has gone up and he will be able to make profit for from the long futures position. Similarly the long put option gives the buyer the right to sell the futures contract at a later position. A later time so the long put position takes a short futures position when he exercises its option and when he will exercise his option, he will be exercising his option when the underlying price is has is going down and commensurately the futures price has gone down.

So now when he has a short futures position and the future price goes down, he will be able to, he will be able to make profit because the short futures position holder gains from the futures contract only when the future price goes down. So he had agreed to sell at a higher price and he will be able to square up or buy the underlying contract at a much lesser price, thus gaining from the decline in the futures price. So this is in a nutshell what is the difference between a regular option contract on financial asset and a option contract for commodities which have a futures as underlying.

Now I am sure all of you must be knowing how Black-Scholes option pricing model is used to calculate regular options and exactly in place of a Black-Scholes option pricing model we do some amount of modifications to value the commodity options having a having a futures as underlying. So in the year black 1976 is a Black in the year 1976 developed model for pricing the commodity options, if you recall your Black-Scholes option pricing model, so let me just briefly write. Black-Scholes option pricing model so you have if you recall the call premium is nothing but your  $S_0 N(D_1)$  area under normal distribution -  $K$  into  $E$  to the power  $-rT$  into  $N(D_2)$



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B&S option pricing model

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$
$$P = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

→ call premium

→ put premium

So I am not going into the explanation of ND 1 ND 2, I am sure all of you must have done this this concepts in your regular derivatives subject. If you have not done then I would advise all of you to read any standard book on option valuation that is John C. Hull and by Sankarshan Basu which is a very seminal book on derivatives or any other book or any other website which you feel comfortable. So this explanation of ND 1 and explanation of ND 2 is beyond the scope of this particular lecture series. So C is S 0, so S 0 is your spot price, K is your exercise price and R is your continuously compounded risk free rate T is your time to maturity.

Similarly P is your K into E to the power - RT N - D 2 - S 0 into N - D 1. So C is your call premium, P is your Put premium and this is the premium which is paid by a long call position holder and a put premium which is given by the, given by the long put position holder to the counter parties that is the short call position holder as well as the short put position holder respectively.

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### Pricing Commodity Options

- Black's (1976) model on commodity option valuations uses the futures contract as the underlying.

$$c = e^{-rT} [F_0 N(d_1) - K N(d_2)] \dots \text{Eq. (3.22)}$$

$$p = e^{-rT} [K N(-d_2) - F_0 N(-d_1)] \dots \text{Eq. (3.23)}$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

Where  $F(0, T)$  is the futures price,  $K$  is the strike price,  $\sigma$  is the volatility of futures price,  $T$  is the time to maturity of the option contract.

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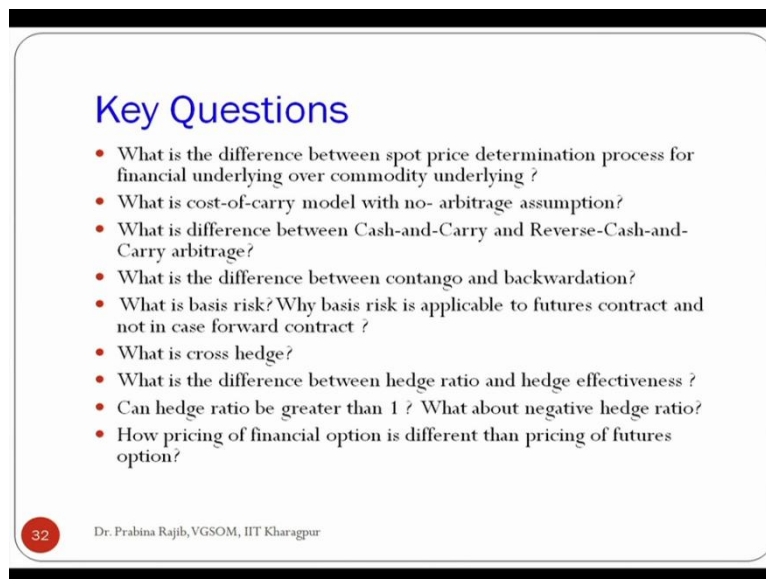
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Now when the underlying is not  $S_0$  but it is a futures contract which is going to mature at a certain point of time in future, we do some adjustment to the formula and in place of  $S_0$  we use the futures price. However, we are pricing the call and put on today's date but this futures contract is going to mature at a later point of time, so to this adjustment this Black 1976 formula so if you see this, you have  $F_0$  is your,  $F_0 T$  is future price prevailing today and  $K$  is your exercise price and you have  $E$  to the power  $R T - R T$ ,  $R$  is your risk,  $T$  is time to maturity, so this your this is the this is the formula for the call premium.

Accordingly put premium is governed by the second equation that is  $E$  to the power  $-R T$  into  $K$ ,  $N(-d_2) - F_0$  into  $N(-d_1)$ , so if you recall in place of your this is just the same formula with minor adjustment because of we are not using the underlying for valuing the call put option but we are using the spot sorry the futures price prevailing at that point of time as the underlying.

So accordingly your  $d_1$  and  $d_2$  has been defined, so  $\ln(F_0 / K) + \sigma^2 T / 2$  so this formula can be used for valuation of European commodities option with commodity futures as underlying. As I see it because commodity options are yet to be started in Indian exchanges, we are not going much detailed into the valuation more valuation or actual valuation of this commodity option contracts taking some real live data.

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### Key Questions

- What is the difference between spot price determination process for financial underlying over commodity underlying ?
- What is cost-of-carry model with no- arbitrage assumption?
- What is difference between Cash-and-Carry and Reverse-Cash-and-Carry arbitrage?
- What is the difference between contango and backwardation?
- What is basis risk? Why basis risk is applicable to futures contract and not in case forward contract ?
- What is cross hedge?
- What is the difference between hedge ratio and hedge effectiveness ?
- Can hedge ratio be greater than 1 ? What about negative hedge ratio?
- How pricing of financial option is different than pricing of futures option?

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Now I will be summarising what we have discussed in the last couple of sessions, so let me pose this questions to all of you so that you will be able to understand what should have what should be the key take away from last 4 to 5 lecture sessions. So what are the key questions or key takeaways, what is the difference between spot price determination processes for financial underlying over commodity underlying?

So we have futures and options on financial underlying like stocks bonds and dices like Sensex and Nifty and we also have futures on single stock like in your stocks of ITC, L&T etc and we have commodity futures, so what is the difference between the spot price determination process for the financial underlying over commodity underlying. Well if there is a difference does it have any bearing on the calculation of the future price, so that is the first key takeaways.

The second question is or second key question is what is the cost of carrying model with no arbitrary assumption and all of that futures price be it financial futures or commodity futures are price based on the cost of carrying model with no arbitrary assumption. Similarly if this cost of carrying model does not hold true then traders can or arbitrators can take cash and carry and reverse cash and carry arbitrary position, so how exactly it is done that you should be also clear.

What is a difference between Contango and backwardation and why Contango and backwardation happen, what is the logic behind Contango and backwardation? We also spent considerable amount of time understanding what is a basis risk and what is basis and how

what is basis risk and why basis risk is applicable to futures contract and not for forward contract, so this is something also which you must get a clarity on that futures contract have basis risk while forward contracts do not have basis risk. We also spend some time understanding what is a cross hedge and we also last couple of sessions we have discussed what is a hedge ratio, what is a naive or one to one hedge ratio, what is minimum variance hedge ratio and what is a what is hedge effectiveness measured by the coefficient of determination.

I also had asked this question already we have already answered this question can hedge ratio be greater than 1 and my second question what about negative hedge ratio I am yet to discuss. However, I am leaving it to you all to read and get answer to this question whether hedge ratio can be negative, if so than what would be the traders strategy. And finally very briefly we discussed how commodity options with futures as underlying can be priced both call and put option, and by this session you should be also get some clarity on how the commodity options can be priced in a different manner as compared to the regular financial options.

So thank you all of you I am looking forward to interacting with you all in the next session and I hope this concepts you will spend time to assimilate and understand these concepts in detailed manner, Thank you all of you.