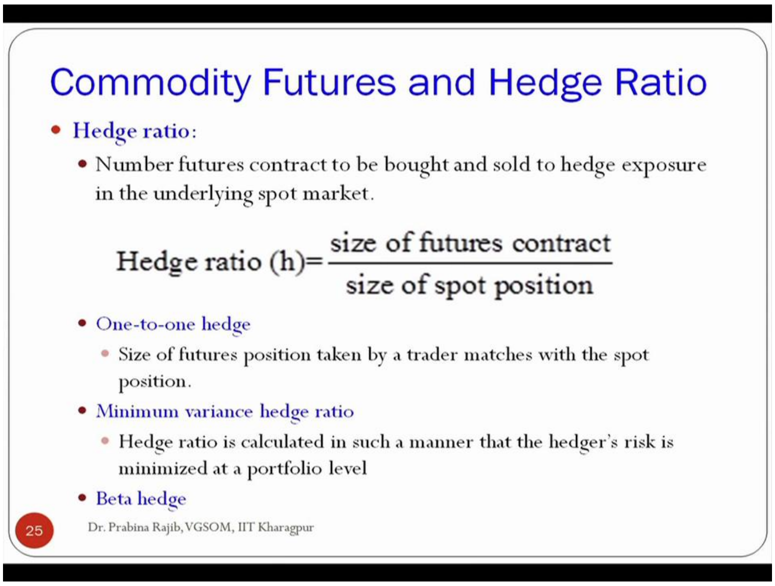
**Commodity Derivatives and Risk Management**
**Professor Prabina Rajib**
**Vinod Gupta School of Management**
**Indian Institute of Technology Kharagpur**
**Lecture 12**
**Minimum Variance Hedge Ratio (Part 1)**

Good afternoon all of you, we are going to discuss more on the concept of hedge ratio and minimum balance hedge ration. Now let us start let me ask a question what could be the definition of a hedge ratio? So if it is a ratio, it means it has to be a numerator and it has to be a denominator so what should be there in the numerator and what should be there in the denominator?

(Refer Slide Time: 1:00)




Okay, now Ahem let me give the definition so the hedge ratio is a ratio which is calculated by number of futures contract to be bought or sold to hedge the exposure in the underlying spot market. Let us say (a) let us go back to our soya oil producer so let us say the soya oil producer is interested to mitigate the price risk of underlying spot position for let us say 10 metric ton and so size of the spot position is going to be the 10 metric ton and the number or the amount of futures in a position it is going to take, it is going to be the numerator. So suppose it if it is going to take let us say 9 metric ton worth a futures contract then the hedge ratio is going to be 0.9.

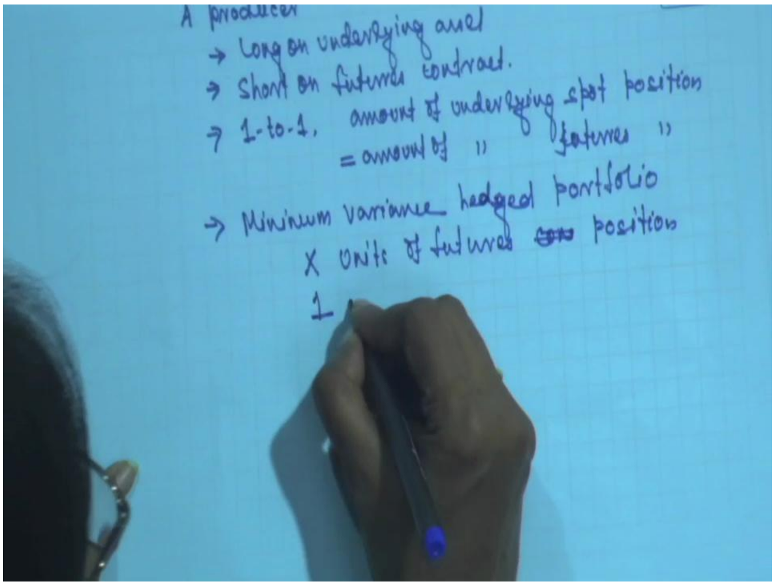
Now, the next question, all of you or next concept all of you are quite aware of, so if a consumer is short on asset, he or she would like to mitigate that price risk by entering into a long futures contract so consumer's price up here his price is going to go up unless he does something, he would be interested to entering the long futures contract or vice versa, if a producer is long on asset, he would be litigating that risk by entering into a short futures contract. Now, in case of a one to one hedge ratio, the ratio is one so in the sense, if the denominator is size of the spot position is 10 metric ton, the numerator is also going to be 10 metric ton so that is how we define it as a define a hedge to be 1 one hedge ration so the underlying to be hedged is equal to the underlying of the futures position taken by the hedger.

Now, many times the one to one hedge ratio does not give us a optimal result to the hedger. When I am using the word optimal so what do we mean by optimal or how optimality can be defined? So when we are talking about the optimality, optimality is a situation where the combination of the portfolio risk, that is a hedger who is suppose like say short asset and goes for long futures so without future without the future contract the consumer has only exposure to the underlying asset with the future. the consumer has exposure to the underlying asset as well as to the futures contract.

So when we are talking about the optimality that means the consumer's or the hedger's total risk is minimum. Without the futures position, only risk he is facing is the risk of the fluctuating underlying asset price with the futures contract. The hedger is exposed to the volatility of the underlying asset as well as the volatility of the futures contract and when we are talking about the optimality, we have to find out a combination of spot and futures such that the risk is minimized and the portfolio risk is minimized, not a individual component spot price risk or future price is minimized. It's a combination of portfolio risk is minimized.

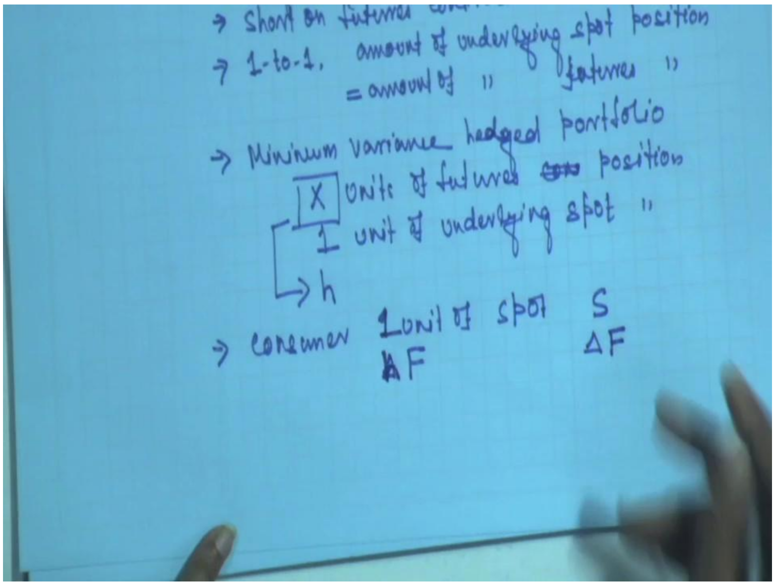
So that position or that situation is known as your minimum variance hedge ratio, so optimality leads to a selection of number of futures contract that is going to give the lowest risk of the portfolio. Now, let us go to little more understanding of how this minimum variance portfolio can be achieved so let us let me start with suppose a a producer who is long on underlying asset, he would be able to mitigate this risk by going short on futures contract.

(Refer Slide Time: 6:16)



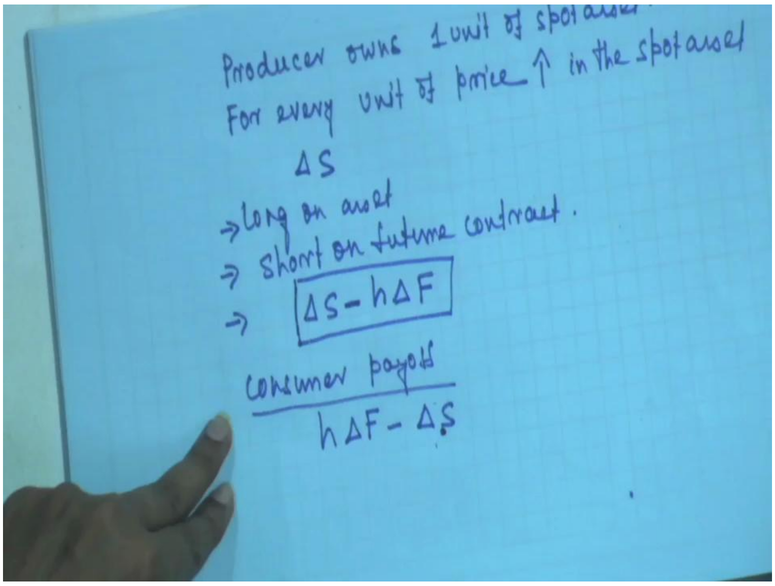

In case of a one to one hedge, so amount of underlying spot position will be equal to the amount of underlying futures position, but in case of a minimum variance hedged portfolio, we have to find out X unit of futures contract or contract position for every one unit of underlying spot position.

(Refer Slide Time: 8:24)




So let us represent this X unit as RH, so if a consumer has one unit of spot, he would like to he would like to mitigate that risk by entering into H unit of futures. So you have one unit of spot position so let us put it as a S and this is your Delta unit of futures position, H unit of futures position. Let me rephrase, the one unit of spot is equal to S, and H units of futures is going to be HF.

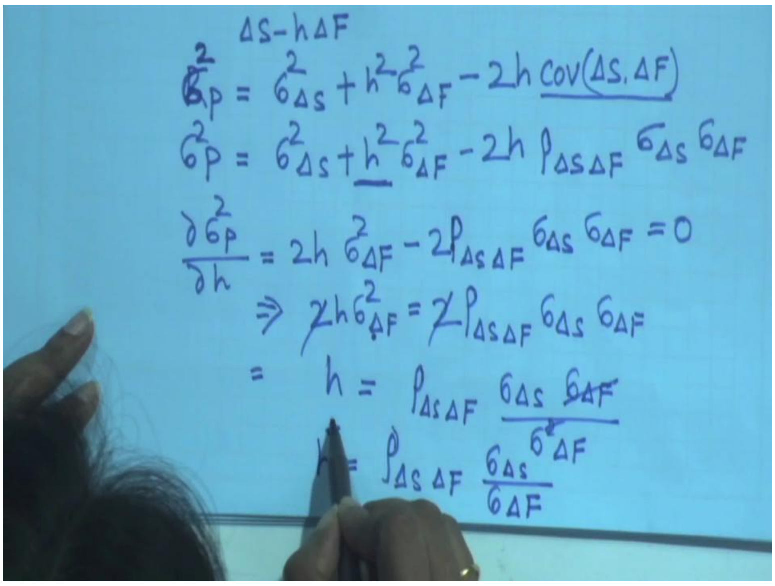
(Refer Slide Time: 9:17)




Now let us say let us say the producer owns one unit of spot asset and if for every for every unit of price increase in the spot asset, he is going to benefit Delta S so his benefit is going to Delta S Delta S. And the producer is producer is long on asset, he would mitigate the risk by going short on futures contract so his total benefit from the both long on asset and short futures contract is to be Delta S minus H Delta S. Why are we using the why are we saying minus because whenever the spot price will increase and future price increases, he will gain from the spot position and he will be incurring loss in the futures position so his net pay off is going to be governed by Delta S minus H Delta F.

So similarly you can have for a consumer, consumer pay off a consumer benefit is going to be H Delta F minus Delta S so if price increases, consumer is going to incur loss in the spot position however he is going to be gaining in the futures position, so his benefit or pay off from the price change is going to be governed by H Delta F minus Delta S. Now, let us if this is doing to be the daily pay off, let us say that is going to be the portfolio risk.

(Refer Slide Time: 11:59)



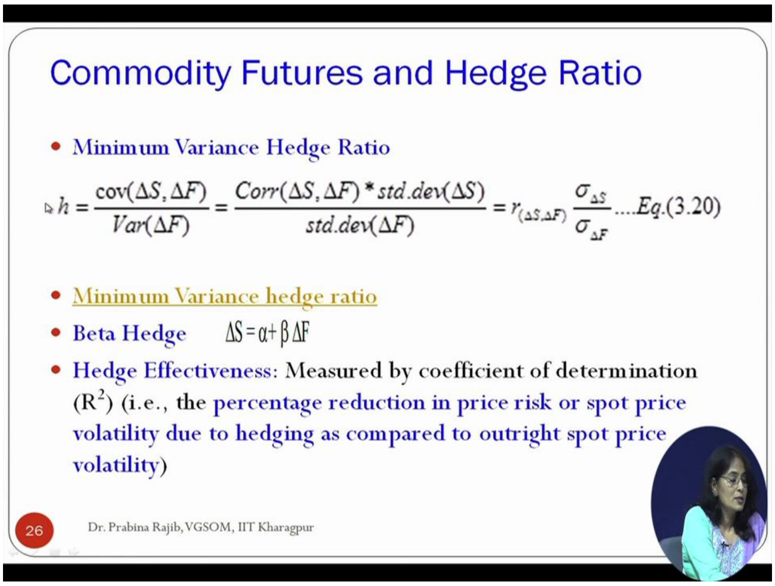
This portfolio risk will be coming from portfolio risk from producer point of view who has taken Delta S minus H Delta F so the portfolio risk, let me put it as RP. Portfolio risk is going to be governed by sigma square Delta S. A sigma square sigma square P will be sigma square Delta S plus H square sigma square Delta F minus 2 H covariant between Delta S and Delta F. Or sigma square P that is portfolio risk is going to be Delta square S plus H square Delta square sigma square Delta F minus 2 H and covariant between Delta S and Delta F can be represented as co relationship between Delta S Delta F and sigma Delta S and sigma Delta F.

And when we are using the word the sigma square, this is the variance of the return of the underlying asset, sigma square is the variance of the return of futures a futures combination and you have the how the both Delta S and Delta F is moving with each other, it is governed by the co variance factor, so this is the our sigma square P so now if we have to find out that combination of H for which combination of H Delta square P is going to be minimum. So we have to take the derivative of this portfolio with respect to Delta H and equate to 0 so going by that we will have two H into sigma square Delta F minus 2 H, 2 H will be H will not be there so co relationship between Delta H Delta F and sigma Delta S and sigma Delta F, so this will be equated to zero.

So when we equate this one to 0 so this gives rise to 2 H sigma square Delta F = 2 co relationship between Delta S Delta F minus sigma Delta S sigma Delta F so 2, 2 cancels out so H is going to be correlation between Delta S Delta F into sigma Delta S sigma Delta F by sigma square Delta F so sigma square sigma square Delta F, sigma Delta sigma square Delta F cancels out. So H is nothing but correlation between Delta S Delta F and sigma of Delta S by sigma Delta F. So depending upon the co relationship between the change in the spot price and change in the spot price, change in the spot price and change in the futures price into the standard deviation of the change in the spot price divided by standard deviation of the futures price is going to give us the minimum variance hedge ratio that is the value of H.

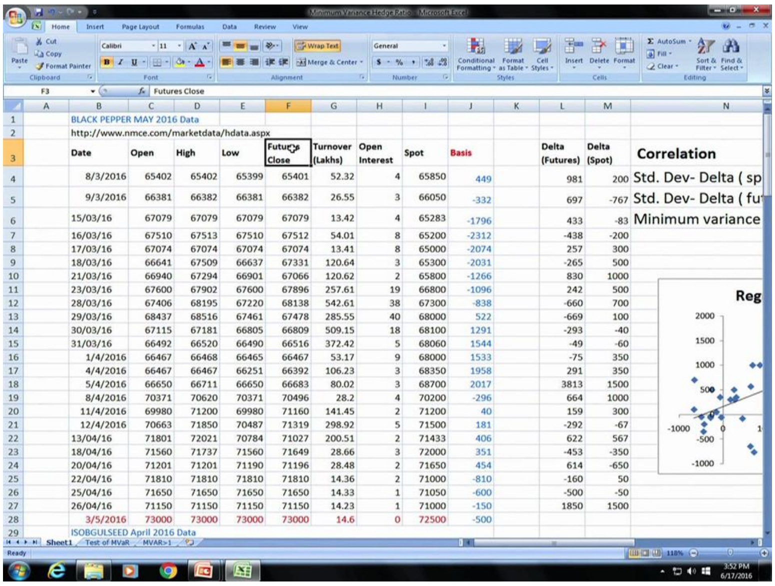
Let us say we are going to get a value of 0.6 that means every one unit of asset a producer is owning, he must be using or he should use 0.6 units of that underlying commodity for buying a or for taking futures position. Now let us take some real life example to find out how this H is exactly calculated.

(Refer Slide Time: 16:55)




So now this is the formula as it is mentioned here. H minimum variance hedge ratio is nothing but the co variants of Delta S by Delta F divided by variance of Delta F, so this is what is what we have discussed just now that is co-relation R Delta S Delta F divided by Delta sigma S by Delta sigma F.

(Refer Slide Time: 17:50)



BLACK PEPPER MAY 2016 Data

http://www.nmce.com/marketdata/hdata.aspx

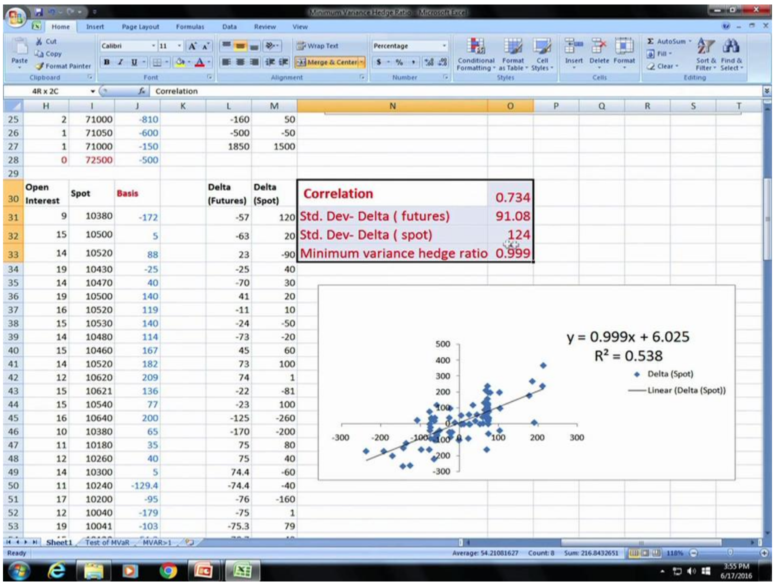
| Date | Open | High | Low | Futures Close | Turnover (Lakhs) | Open Interest | Spot | Basis | Delta (Futures) | Delta (Spot) | Correlation |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8/3/2016 | 65402 | 65402 | 65399 | 65401 | 52.32 | 4 | 65850 | 449 | 981 | 200 | Std. Dev- Delta ( sp |
| 9/3/2016 | 66381 | 66382 | 66381 | 66382 | 26.55 | 3 | 66050 | -332 | 697 | -767 | Std. Dev- Delta ( fu |
| 15/03/16 | 67079 | 67079 | 67079 | 67079 | 13.42 | 4 | 65283 | -1796 | 433 | -83 | Minimum variance |
| 16/03/16 | 67510 | 67513 | 67510 | 67512 | 54.01 | 8 | 65200 | -2312 | -438 | -200 | |
| 17/03/16 | 67074 | 67074 | 67074 | 67074 | 13.41 | 8 | 65000 | -2074 | 257 | 300 | |
| 18/03/16 | 66641 | 67509 | 66637 | 67331 | 120.64 | 3 | 65300 | -2031 | -265 | 500 | |
| 21/03/16 | 66940 | 67294 | 66901 | 67066 | 120.62 | 2 | 65800 | -1266 | 830 | 1000 | |
| 23/03/16 | 67600 | 67902 | 67600 | 67896 | 257.61 | 19 | 66800 | -1096 | 242 | 500 | |
| 28/03/16 | 67406 | 68195 | 67220 | 68138 | 542.61 | 38 | 67300 | -838 | -660 | 700 | |
| 29/03/16 | 68437 | 68516 | 67461 | 67478 | 285.55 | 40 | 68000 | 522 | -669 | 100 | |
| 30/03/16 | 67115 | 67181 | 66805 | 66809 | 509.15 | 18 | 68100 | 1291 | -293 | -40 | |
| 31/03/16 | 66492 | 66520 | 66490 | 66516 | 372.42 | 5 | 68060 | 1544 | -49 | -60 | |
| 1/4/2016 | 66467 | 66468 | 66465 | 66467 | 53.17 | 9 | 68000 | 1533 | -75 | 350 | |
| 4/4/2016 | 66467 | 66467 | 66251 | 66392 | 106.23 | 3 | 68350 | 1958 | 291 | 350 | |
| 5/4/2016 | 66650 | 66711 | 66650 | 66683 | 80.02 | 3 | 68700 | 2017 | 3813 | 1500 | |
| 8/4/2016 | 70371 | 70620 | 70371 | 70496 | 28.2 | 4 | 70200 | -296 | 664 | 1000 | |
| 11/4/2016 | 69980 | 71200 | 69980 | 71160 | 141.45 | 2 | 71200 | 40 | 159 | 300 | |
| 12/4/2016 | 70663 | 71850 | 70487 | 71319 | 298.92 | 5 | 71500 | 181 | -292 | -67 | |
| 13/04/16 | 71801 | 72021 | 70784 | 71027 | 200.51 | 2 | 71433 | 406 | 622 | 567 | |
| 18/04/16 | 71560 | 71737 | 71560 | 71649 | 28.66 | 3 | 72000 | 351 | -453 | -350 | |
| 20/04/16 | 71201 | 71201 | 71190 | 71196 | 28.48 | 2 | 71650 | 454 | 614 | -650 | |
| 22/04/16 | 71810 | 71810 | 71810 | 71810 | 14.36 | 2 | 71000 | -810 | -160 | 50 | |
| 25/04/16 | 71650 | 71650 | 71650 | 71650 | 14.33 | 1 | 71050 | -600 | -500 | -50 | |
| 26/04/16 | 71150 | 71150 | 71150 | 71150 | 14.23 | 1 | 71000 | -150 | 1850 | 1500 | |
| 3/5/2016 | 73000 | 73000 | 73000 | 73000 | 14.6 | 0 | 72500 | -500 | | | |

ISOBGULSEED April 2016 Data

Now let me give you this real life example. This particular excel file shows you the hmm this excel file has got 3 worksheets so we are just talking about the work sheet number one. If you can see if it is visible to you this is black pepper main futures contract available from NMC website and this column is the futures price and column I is the spot price. Based on the futures prices a futures price and the spot price, we have calculated the Delta. This Delta is nothing but your F1 minus F0, F2 minus F1, so and so forth so Delta futures is the change in a futures price on a daily basis that is tomorrow’s futures minus today’s futures price.

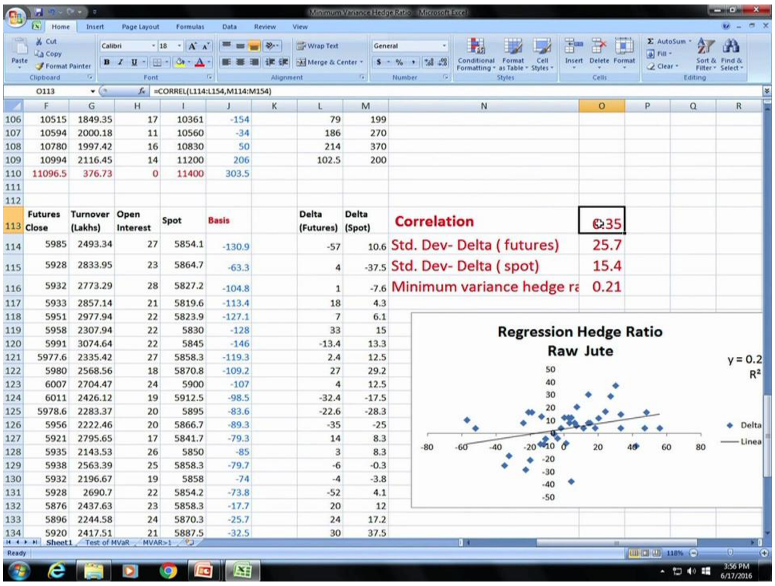
Similarly Delta spot has been calculated and we have just now discussed that minimum variance hedge ratio, it can be calculated with 3 components that is the correlation between Delta S Delta f and standard deviation of Delta spot divided by standard deviation of Delta futures so if I calculate this so if this particular correlation between the black pepper Delta spot and Delta futures indicates 0.58 and standard deviation of spot Delta spot is 576, standard deviation of Delta futures is 958, going by that calculation you have your Delta minimum variance hedge ratio is 0.35. That means every one kg of underlying a commodity producer or consumer is holding or interested to buy, he should enter into a futures contract for 0.35 kg so if he is holding let us say 100 metric tons of sugar sorry black pepper underlying, a producer is holding 100 metric tons, he can go ahead and buy futures contracts for 35 metric ton.

(Refer Slide Time: 20:05)



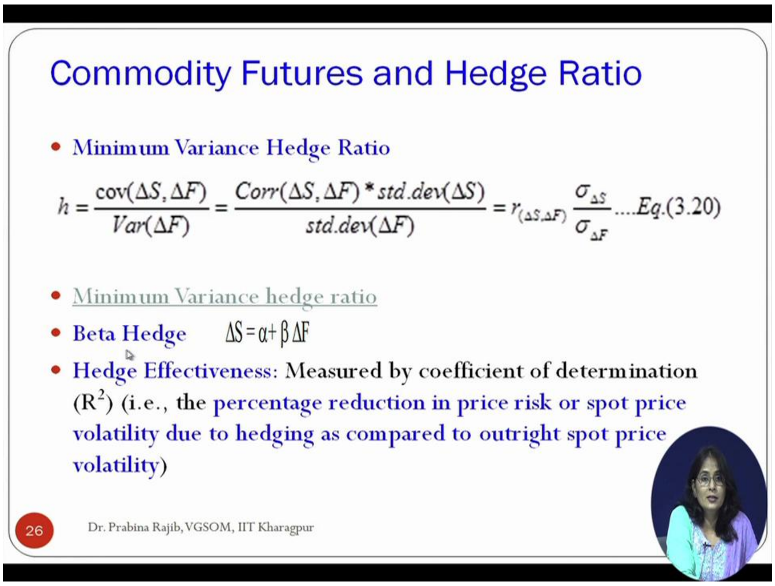

Now let us take other example. Besides the black pepper, I have calculated the Isabgol seed a minimum variance hedge ratio. is particular information which is given in the red font so you have a correlation. Correlation is 0.734 Delta futures is 91.08, Delta spot is 124 and minimum variance hedge ratio is coming to practically 1.999 that means every one unit of Isabgol spot position being held by a consumer or producer, the a consumer producer should take a futures position for the same number of same quantum of underlying so 100 kg of Isabgol should be, a price risk associated with 100 kg of Isabgol should be hedged by entering into futures contract for a quantum of 100 kg.

(Refer Slide Time: 21:15)



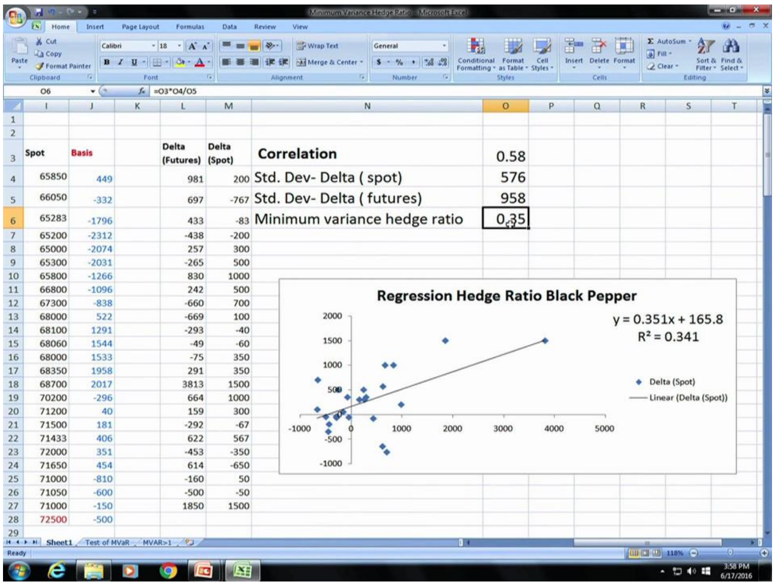
So now let us go to another commodity that is the third commodity which is the Raw jute and in case of a Raw jute, let me increase the font size so if you see the Raw jute, this Raw jute is your correlation is 0.35, standard deviation of Delta futures is 25.7, Delta spot is 15.4 and it is minimum variance hedge is 0.21 so 100 kg of Raw jute if a trader is holding then he should be able to mitigate the risk by entering into 21 kg of futures contract, so this is the minimum variance hedge ratio. Minimum variance hedge ratio can also be found out from graphically that is through our ordinary regression method that is known as your Beta hedge.

(Refer Slide Time: 21:58)




Let us as you can see from the screen beta hedge so what is a what is beta hedge, beta hedge is the in case of a in this particular regression you have a Delta spot, change in the spot price is taken as a dependant variable and change in the futures price is taken as the independent variable. And using the regression methodology we find try to we find out what is the beta and the beta is the minimum variance hedge ratio. In fact these 2 minimum variance hedge ratio calculated using this formula and this formula gives the same result because this is the beta represents the H.

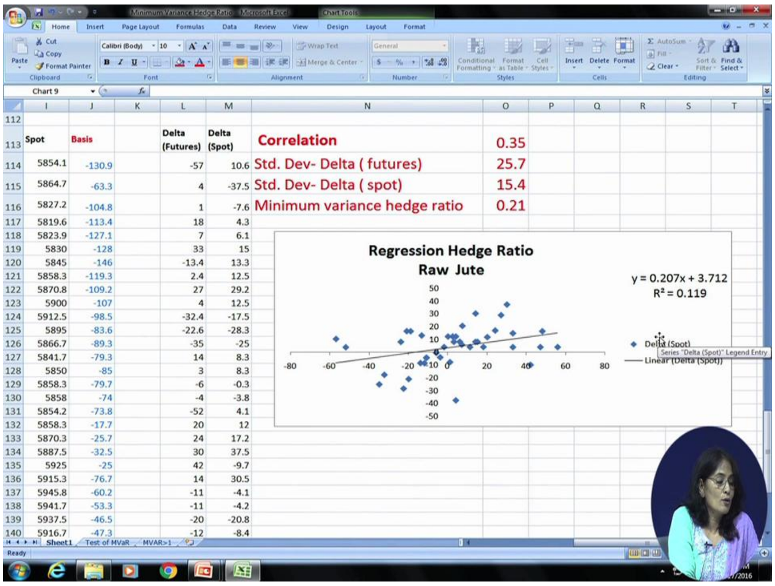
(Refer Slide Time: 23:03)




Now let us go back to our same excel file. I will show you how exactly the regression hedge ratio for black pepper has been calculated so please see this one so you have a minimum variance hedge ratio using our formula method we calculated 0.35 and this is your regression hedge ratio using a scatter plot, so this particular graph has been drawn using a scatter plot and adding a trend line and when we add a trend line and scatter plot, what we get to see equal is here.

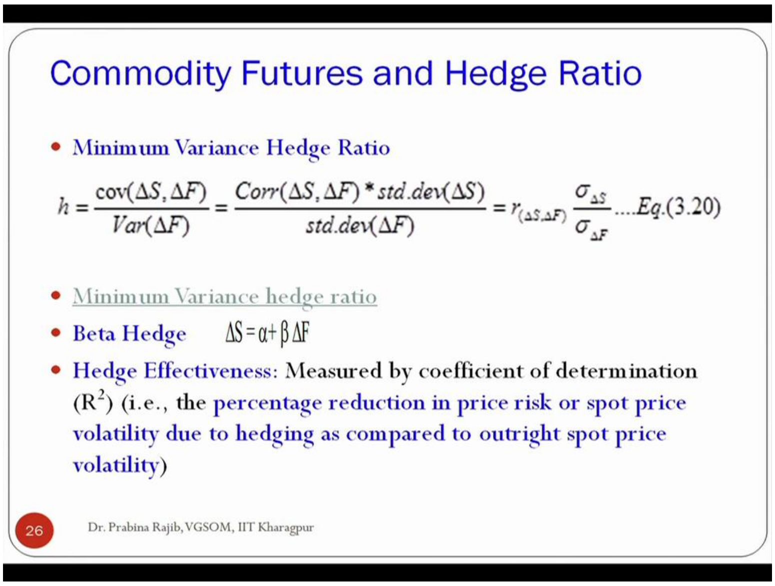
It is Y equal to 0.351 X plus 165.8, so this is what the 0.35 X and this X is your Delta futures, Y is your Delta spot and this point 0.35 is your minimum variance hedge ratio. That is the coefficient of beta is the minimum variance hedge ratio. Now similarly quickly I will take you through the beta for the beta for the Isabgol seed using our formula method, it is 0.999, 0.999 and using your regression method also, if you can see from the equation, it is 0.999X. Y equal to 0.99X plus 6.025.

(Refer Slide Time: 24:24)



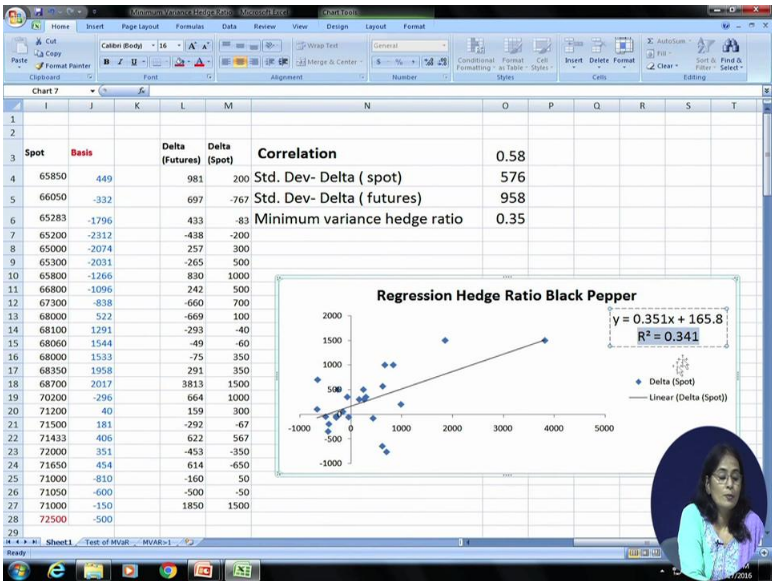

Similarly for your Raw jute, the beta is 0.21 and also in this particular equation, Y equal 0.207X so which is same as your 0.2. We can find out the minimum hedge ratio, variance hedge ratio by using the formula method or by using the regression method. Now, with respect to minimum variance hedge ratio, I would like to discuss a little bit with a concept called coefficient of determination. I am sure each of you have done what exactly is a coefficient of determination within a regression so that is your R square, so R square measures the hedge effectiveness.

(Refer Slide Time: 25:20)



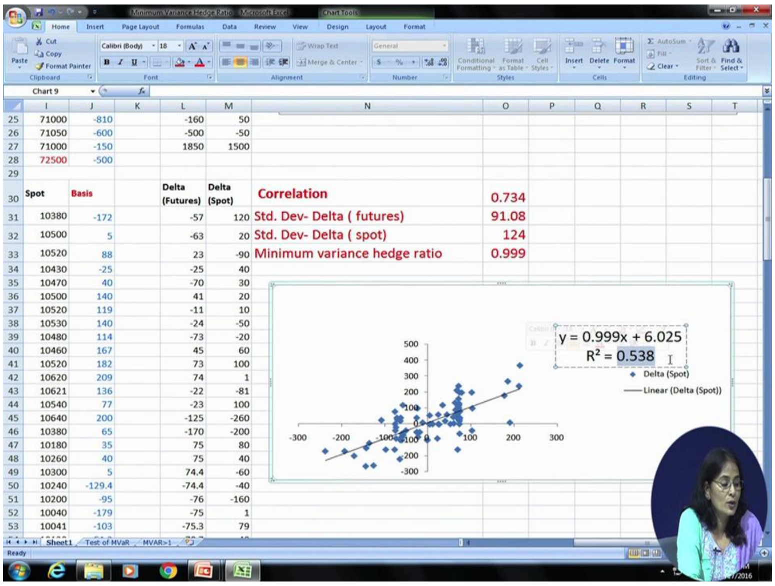

So what how do we define hedge effectiveness, hedge effectiveness is the percentage reduction in price of a risk or the spot price volatility due to hedging as compared to outride spot price volatility. So how much of how much volatility of the spot price has been has been able has been mitigated by adding futures into the portfolio, so hedge effectiveness indicates that ratio. In fact it is nothing but the square of the correlations coefficient between Delta S and Delta F.

(Refer Slide Time: 26:09)




Let us go back to again once again to our particular excel file. If you see R square that is coefficient of determination 0.31, 341 and correlation is 0.58, sorry 0.58 so if I just square it up so if you see this one, so this to the power 2 so if you see this, it is nothing but your coefficient of determination. So what exactly this coefficient of determination explains that how much variability of Y or that is the Delta spot has been able to be mitigated by adding the futures contracting to the portfolio.

(Refer Slide Time: 27:05)




Similarly let us go to our other two contracts.  For Isabgol contracts, the R square is 0.538 and for your Raw jute, R square is 0.119. Now, I will end this session at this point of time by asking a question. Can minimum variance hedge ratio be greater than one? Please think it over and we will discuss this aspect for the minimum variance hedge ratio can be greater than one or not in the next session so thank you all of you. I am looking forward to meeting all of you again once again in the next session.