Economics, Management and Entrepreneurship
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Lecture – 07
Cost - Volume - Profit Relationships

Good morning, welcome to the seventh lecture in the subject economics, management and entrepreneurship. In the last class, we had solved some exercises and today we are going to formally introduce the concept of cost, cost driver, the classification of costs into fixed and variable costs and then we shall use these concepts to show how the revenue and different costs can be equal at breakeven point.

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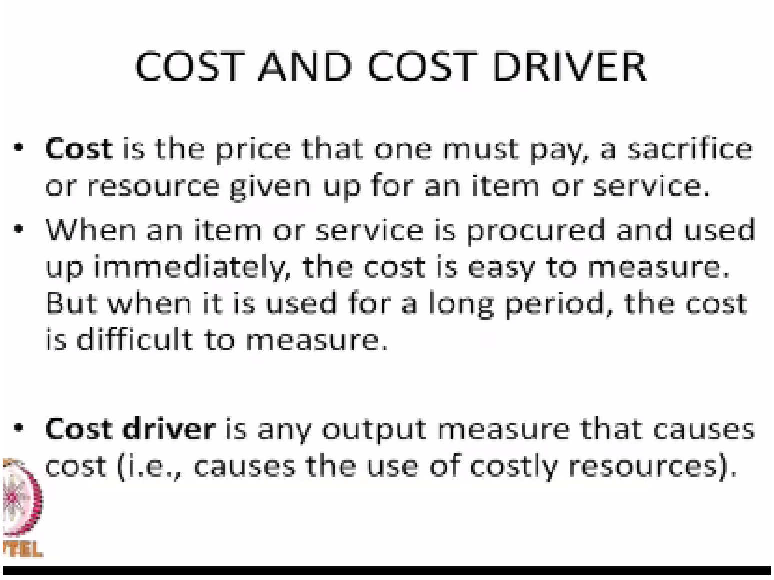

This is a very important topic and is useful in many applications in a particular form, so today's topic is cost, volume, profit relationships also known as CVP relationship; cost volume profit relationships or CVP relationships. To start with, we first define what we mean by cost? Cost is basically the price that one must pay a sacrifice or a resource given up for an item or service. So, basically this is something in live of; something that one sacrifices in live of an item or a service.

And one will see that when an item is; or a service is procured and used up immediately, the cost of this item or service is very easy to define and measure but when this particular item is procured kept for a while and then used up later, it is difficult to define and measure. So, these concepts will be clear in subsequent lectures but today, we are going to discuss how we can use the classification of cost into fixed and variable and how to use them.

Before we proceed further, let us understand the meaning of cost driver; cost driver is basically any measure of output that causes cost that it is; that means it causes the use of costly resources. We shall give some examples to illustrate what a cost driver means in a particular context.

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Value Chain Function & Examples of Cost	Examples of Cost Driver
R & D	
• Cost of market survey	Number of new project proposals
Design of products, services, and processes	
• Cost to develop and test prototype	Number of parts per product
Production	
• Labour wages	Number of labour-hours
• Material cost	Quantity of material used
• Cost of energy expended in machining	Machine hours
Marketing	
• Cost of advertisement	Number of advertisements
Distribution	
• Transportation cost	Weights of items
Customer service	
• Cost of supplies, travel	Number of service calls

When we look at the value chain in a manufacturing function, we will start with R and D research and development which is basically concerned with market survey and design of products, services and processes. The next in the value chain is the actual manufacturing process or production. Once the products are manufactured, it is distributed and sold and finally when these products are in use by the users, the service must be given to see that it functions as desired when it is in service.

So, these basic values and aspects, R and D design production, marketing distribution and customer service in every particular item in the value chain that adds value to the product, as it reaches from an idea stays in the R and D to the final stage of item being used up by the customer. Now, every such; every such values and function one can identify a particular cost and then one can find out a cost driver.

For example, suppose that we would like to find out what is the cost of market survey, we can say that as the new project proposals are generated then we make a market survey to know how will these projects proposals are going to be useful or relevant to a particular market, so this is the cost driver and it causes this cost. Similarly, the number of parts per product that are designed will cause the cost of developing and testing the prototype in the design states.

Number of labour hours is the cost driver that causes labour wages to rise, quantity of material used is the cost or that is the cost driver for material cost, more the machines are used for more number of hours a machine is used, the cost of power increases, so machining energy cost is high in machining, more the number of advertisements; more is the cost of advertisement. So, this is the; these are examples of cost drivers.

And these are actual costs, more the item weighs more would be the cost of transportation. So, basically a cost is caused by a cost driver and later we will you use these concepts whenever, we will try to make an estimate of cost of something we would first relate it to a cost driver and that means that we have to identify what is the cost driver for that particular cost and then find out a relationship between the cost driver and the cost.

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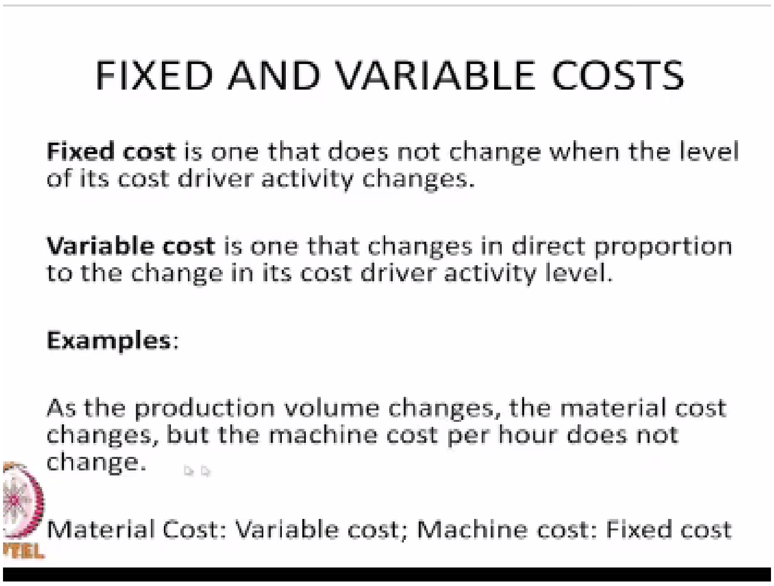

So, this is; these are examples of cost drivers and examples of costs. As I said, there are different classifications of costs but one classification of cost is whether a cost is fixed cost or a variable cost. Now, that classification is based on whether there is a relationship between the cost and the output or the volume of activities that is performed. So, fixed cost is one that does not change when the level of cost driver activity changes.

So, you can see the volume of activity that is the cost driver activity, even if it is changing that particular cost does not change. Now, this we say is a fixed cost, now variable cost on the other hand is one that changes in direct proportion to the change in its cost driver activity level that means, as the cost driver changes the variable cost changes in direct proportion in a linear fashion, we call that cost a variable cost.

It is not necessarily that it has to be always linear but mostly in the short term, we will assume that the cost changes in a linear fashion in the short term. There are various examples that we will take, one example that we are citing here is production volume is the cost driver, as production volume changes, the material cost changes. Suppose that we go for more production, we require more material and therefore the material cost is a variable cost.

But the machine cost per hour does not change because machine has been procured long ago and it is used for production for many years. So, to produce a particular volume of production, the machine cost will not change, so this is an example of a fixed cost; machine cost per hour is a fixed cost whereas, the material cost is a variable cost. Material cost is variable cost and machine cost is fixed cost.

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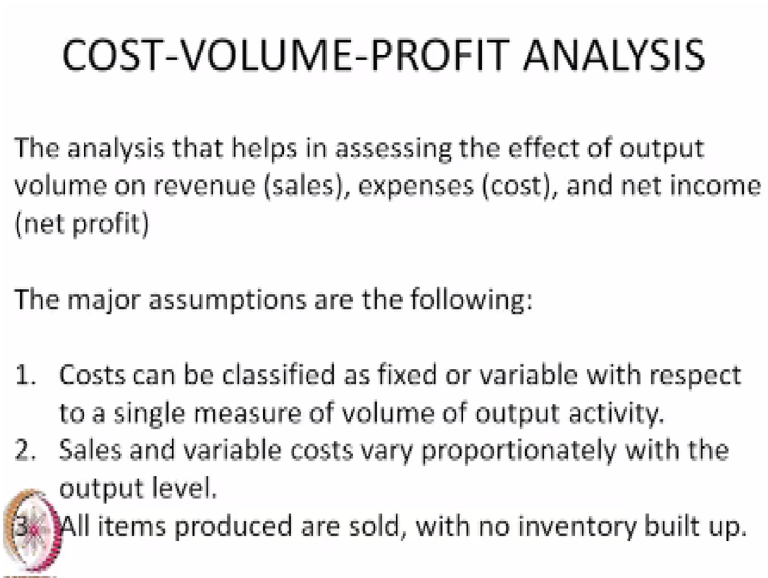

So, this one example of each type; fixed and variable cost are cited here. Now, we make the CVP analysis; cost volume profit analysis. The analysis that helps in assessing the effect of output volume on sales, revenue that is sales, expenses that is cost and net income that is net profit. This analysis; session analysis is called CVP analysis. Here, we make certain measure assumptions; one that the costs can be classified purely as either fixed or variable with respect to the single measure of volume of activities.

The second assumption is that the sales and variable costs vary proportionately with the output level, sales and variable cost and all items produced are sold with no inventory built up, so these are the 3 basic assumptions, whatever we produce we sale and there is a fixed cost that does not change with the production volume and a variable cost and also revenue, they change proportionately with the volume of production and sales.

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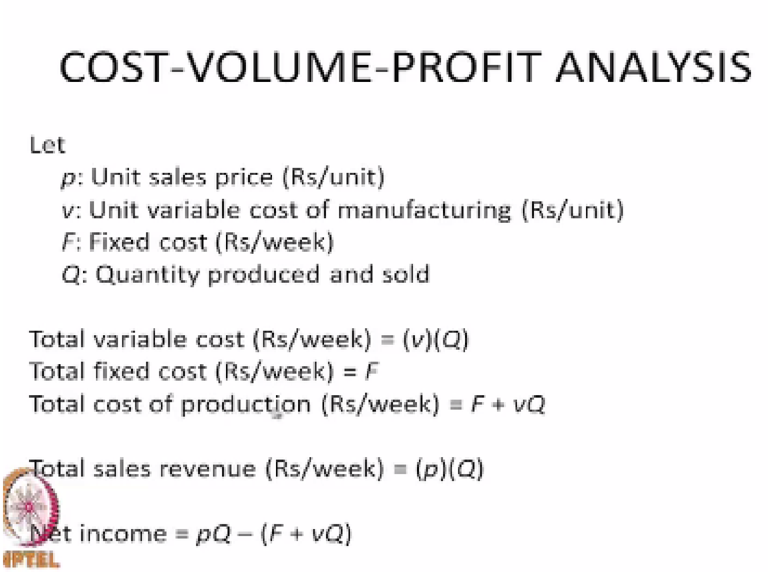

Now, we make the formal mathematical analysis, we first assume that the unit sales price p is rupees per unit; unit variable cost of manufacturing is small v rupees per unit, capital F is taken as fixed cost rupees per week or year some time period and Q is the quantity produced unsold, which is also unit per week or year. Now, with these definitions of the symbols, we now write the relationship for various costs and revenue.

First, the total variable cost; if the unit variable cost is small v and if the quantity produced is capital Q, then the total variable cost will be = small v * capital Q that is what we have written here, the total variable cost is small v * capital Q. The total fixed cost is capital F, it does not change with Q therefore, the total cost of production is the sum of these 2 costs; variable cost and fixed cost which is equal to F + small v * capital Q.

Now, this is the cost aspect, now we come to the income aspect; the revenue aspect, the sales revenue is small p multiplication capital Q, small p is the unit sales price and if Q items are produced and sold, the total sales revenue will be p multiplied by Q. Therefore, the net income will be sales revenue, which is pQ - the total cost, which is fixed cost F and variable cost; total variable cost of v * Q.

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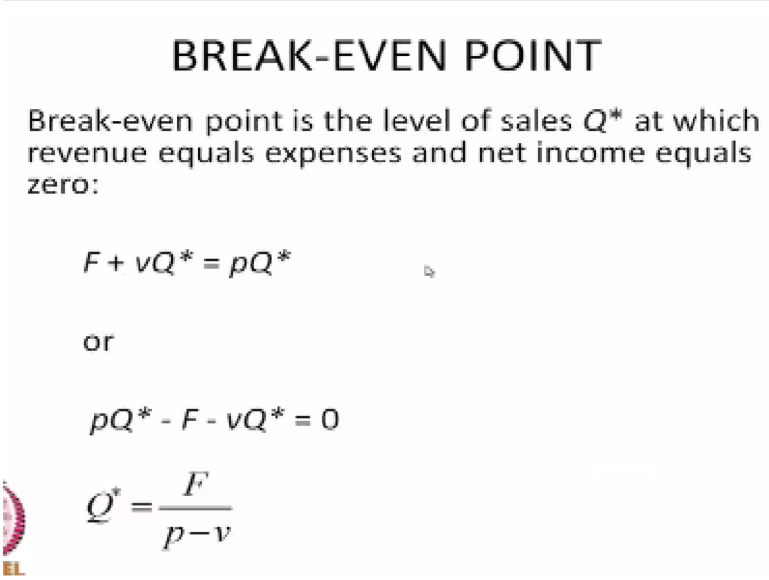

So, F + v * Q is subtracted from sales revenue, so that will be the income, when the volume of reduction is equal to capital Q. Now, we define breakeven point has that level of sales at which a revenue equals the costs. Now, revenue is p * Q and cost is fixed cost F + the total variable cost v * Q, so if we subtract the cost from the revenue, we get the income and we say that at Q equal to Q star, the total cost F + v * Q star is = pQ star.

So, the level of sales Q star at which this total cost and total revenue equal that value of Q is called the breakeven sales, so we are interested to find out the value of Q star, the breakeven sales or production, so if this is so, then pQ star - F - vQ star is = 0 and therefore Q star is = F/ p – v, so the breakeven production or breakeven sales because we are assuming production as equal to sales with no inventory built up.

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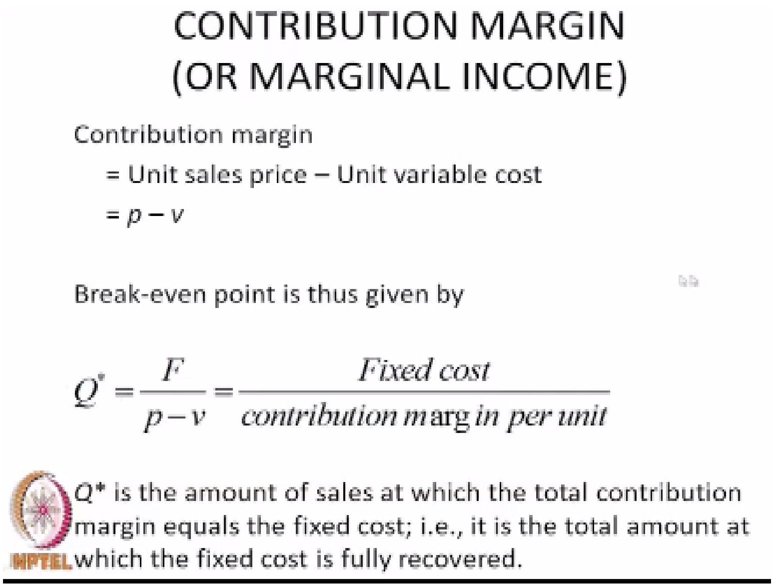

Therefore, Q star is = F/ p – v, now we can now interpret the meaning of F/ p – v, first of all let us understand the p is the unit sales price and v is the unit variable cost, this difference between price and the unit variable cost is known as contribution margin. It says if one item is produced and sold; produced it will give a revenue of p and sold the variable cost is v, then the contribution to profit is contribution margin which is p – v.

Now p - v comes in the denominator, this is the contribution margin per unit, F is the fixed cost, so the interpretation for breakeven analysis is that Q star is the amount of sales at which the total contribution margin equals the fixed cost that is p - v * Q star; p - v is the unit contribution margin multiplied by Q star is = F that is the total amount at which the fixed cost is fully recovered.

So, for Q star to be positive, p has to be higher than v, if p is lower than v, then it is not feasible, it becomes negative or if even p equals v, Q star becomes infinity. So, p has to be > v for a feasible value of Q star.

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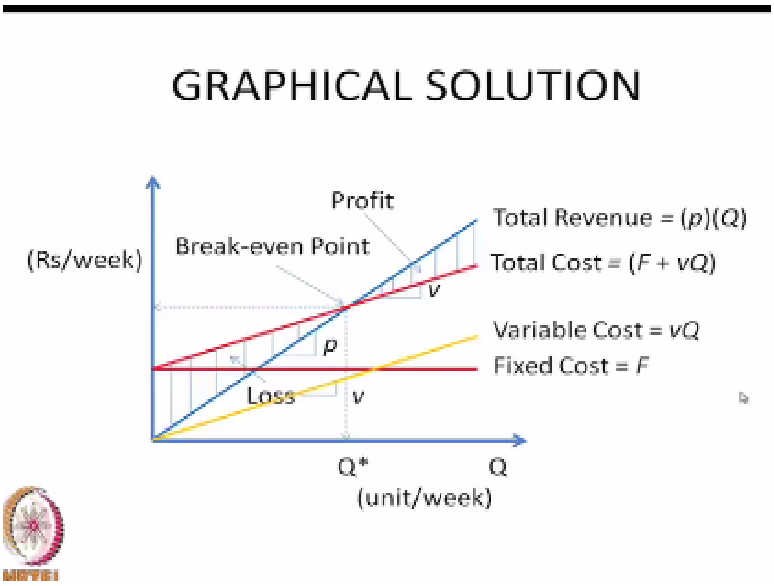

Now, whatever we had shown in the analytic form can be shown very nicely in the graphical form. Let us see the graphical form, here in the x axis we have Q star that is Q, the amount of products production or amount; quantity produced Q and this side is Rupees per week, now the horizontal line is the fixed cost F, it is not a function of Q and therefore, it is a horizontal line parallel to the Q axis.

And the variable cost has; this is the variable cost, it has a slope v, so the variable cost is = v * Q, therefore the total cost which is the fixed cost + the variable cost is this line; fixed cost + the variable cost is this line, which is the total cost which is = F + vQ and now we look at the revenue line, revenue line is this blue line, its slope is p as I said for a feasible solution, p has to be > v, so it has to be steeper than v, which it is in this case.

And the total revenue is p * Q therefore, this line passes through the origin and its steeper than the variable cost line. Now, this is the total revenue line and this is the total cost line, this is the point of intersection, which is called the breakeven point, so this is where the total revenue breaks even with the total cost; this point has got corresponding value of Q as Q star, the breakeven production or breakeven sales.

And this is the breakeven amount in terms of cost or rupees, they are the same value, here it is pQ star or it is F + vQ star, both give rise to this value, we will realize that this zone is basically the profit zone at this point, the firm does not make any profit or income but if it produces more than Q star, it makes this amount of profit; revenue – cost. If it produces this much, the profit is this much.

If it produces this much, the profit is this much, basically total revenue - total cost, so this is the profit region or profit zone but if it for some reason, the firm produces here then it incurs a loss because the cost is higher than the revenue. Suppose it produces only this much, the cost of production is much higher here than the cost of than the revenue that it can (()) (21:25) by selling it in the market.

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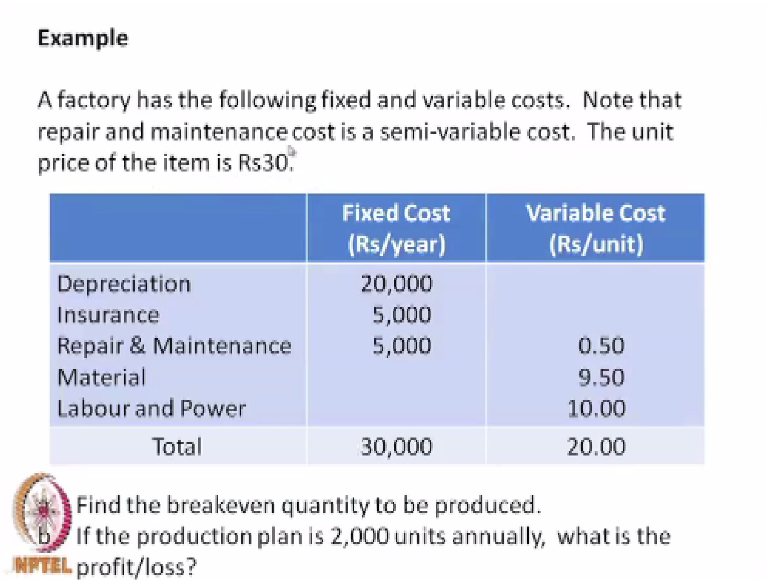
Example

A factory has the following fixed and variable costs. Note that repair and maintenance cost is a semi-variable cost. The unit price of the item is Rs30.

	Fixed Cost (Rs/year)	Variable Cost (Rs/unit)
Depreciation	20,000	
Insurance	5,000	
Repair & Maintenance	5,000	0.50
Material		9.50
Labour and Power		10.00
Total	30,000	20.00

a) Find the breakeven quantity to be produced.

b) If the production plan is 2,000 units annually, what is the profit/loss?

So, this is the loss job therefore, the company is interested to know what is the breakeven point such that it should plan to produce more than this breakeven quantity to be able to make profit. Now, we take a small example, in this example we are considering a factory to have 2 types of costs; fixed and variable costs but there is only one cost item called repair and maintenance cost, which has got both a fixed component as well as a variable component, such costs are called semi variable cost.

Now, in this example depreciation of the machine that is a fixed cost that does not change with the volume of cost driver activity, insurance paid for the machine or the plant is also constant, it does not vary with the production volume, repair and maintenance has got a fixed part maybe some maintenance screw, there anyway they have to be deployed, they are permanent workers with the company and that is a fixed cost.

And the; but once the repair action takes place on the machines, there is some variable cost associated in the form of power, in the form of tooling, in the form of various spare parts, so there is a variable part. The material cost obviously is a variable cost more the production, more is the material cost, power predominantly is a variable cost, whereas labour may or may not be always a variable cost if there are constant level force.

Fixed lever force, then it is a fixed cost but if labour is higher or if the labour force is paid on a production volume basis following and unwage incentive scheme then it is a variable cost. So, basically what is done in this simple example, we have considered different cases excepting for repair and maintenance, all other cost aspects are either fixed or variable, only repair and maintenance are is having both the fixed cost component as well as the variable cost component.

The total fixed cost therefore, comes to 30,000 per year and the variable cost, this is unit variable cost as you can see it is Rupees per unit, it is 20 Rupees per unit, so this is v, this is capital F and the unit price of the item is given as = rupees 30, therefore p is rupees 30. There are 2 parts to this question; what is the breakeven quantity Q star and the second part is if the production plan is 2000 units annually, what is the profit or loss?

That is irrespective of what the breakeven quantity is, if the manager decides a plans to produce only 2000 units whether, it will give a profit situation or a loss situation and if so, what is the amount of profit or loss?

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Solution:

a.

Given
F = 30,000 Rs/year, v = 20 Rs/unit, p = 30 Rs/unit

The breakeven production quantity is given by

$$Q^* = \frac{F}{p-v} = \frac{30,000}{30-20} = 3,000 \text{ units/year}$$

b.

The production plan is Q_p = 2,000 units/year.
Total revenue = (2,000)(30) = 60,000 Rs/year
Total cost = 30,000 + (2,000)(20) = 70,000 Rs/year
Loss = 10,000 Rs/year

Solution is very simple, first of all given F rupee is = 30, 000 rupees per year, the variable unit variable cost is 20 rupees per unit and the unit price is 30 rupees per unit, p is > v and the breakeven production quantity is simple, F/ P - v which is 30, 000/ 30 – 20; 3000 units per year that means, the firm should produce 3000 units per year to be able to make up the cost to breakeven at this point, no profit no loss situation.

But the second part says for some reason, the company has decided to produce only 2000 units per year. If this is the production plan, then the total revenue is 30 * 2000 which is 60, 000 rupees but the total cost is the fixed cost 30, 000 + the variable cost * 20, which is 40, 000, so 40 + 30 is 70, 000 rupees is the cost whereas, the revenue is only 60, 000, so the difference is therefore a loss, so this is in the loss zone.

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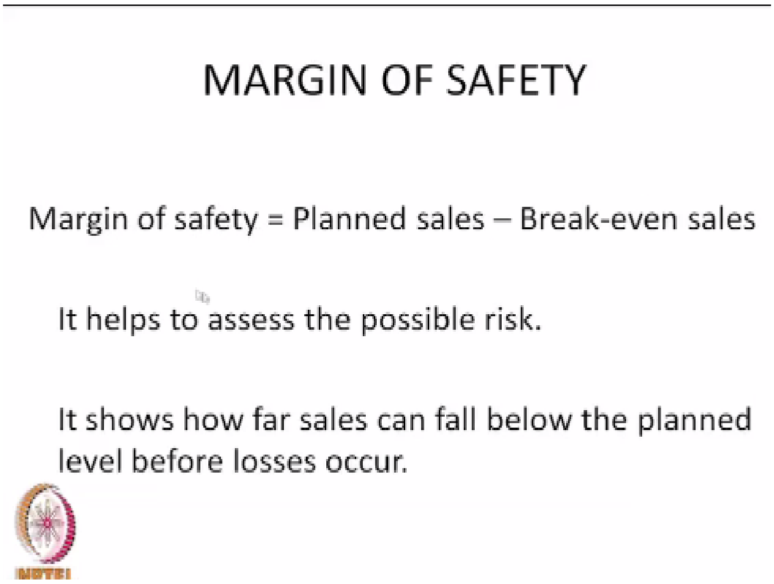

Now, you can see from here that knowledge of breakeven point is useful in planning the production. Now, this gives rise to a concept of margin of safety; planned sales - the breakeven sales is called the margin of safety. It helps to assess the possible risk; it shows how far sales can fall below the planned level before losses can occur.

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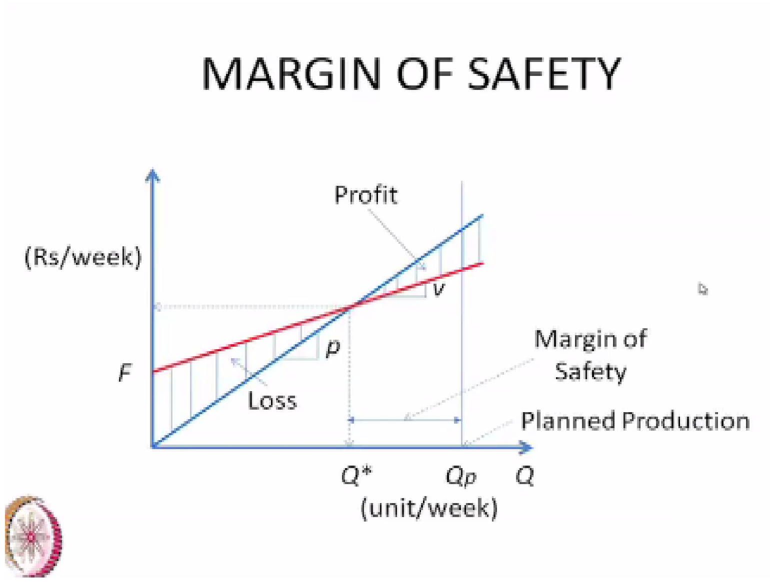

This is illustrated with the help of this diagram, this is very simple, this is the total revenue line and the red one is the total cost line, this is the breakeven point and the breakeven production is Q star. Suppose, the company has decided to produce Qp. then this difference between Qp - Q star is known as margin of safety, if for some reason, let us say material is not available and therefore Qp cannot be realized, one cannot produce Qp, it will produce less.

Still we have; we are operating the firm is operating in a zone, which is higher than Q star; it can still make profit, so the margin of safety is here. The higher the margin of safety, the lower is the risk of making a profit; this is the concept of margin of safety.

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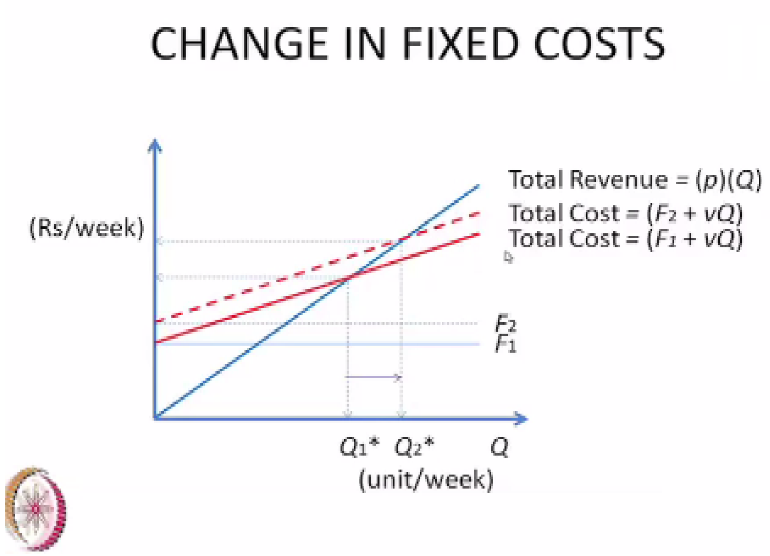

Now, we consider some, how Q changes; Q star changes, when F, p etc changes? Now, this is a case when we are showing, if fixed cost changes then how the breakeven point changes? We are assuming that revenue curve remains the same, the variable cost remains the same, only the fixed cost changes from F1 to F2, so when it was F1, then this was the breakeven point and this was the breakeven quantity to be produced Q1 star.

Now, when we are assuming that there is a change in F; fixed cost increases for some reason then the line is here and therefore, the total cost line is here and therefore, the point of intersection is here, the breakeven point is here and therefore the breakeven Q star is higher than Q1 star. So, as fixed cost increases, we have to produce more to be able to breakeven the cost.

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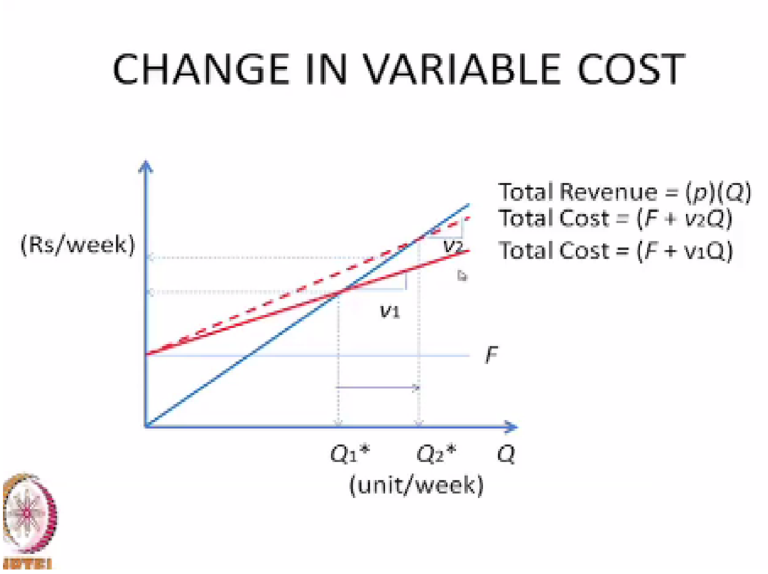

Now, this is a case when we are considering, if instead of fixed cost remaining same, if variable cost is more, this can happen when the slope changes. Suppose that the weights prices increase the labour cost increases or material cost increases or power cost increases, if it happens then the variable cost line will be steeper. Now, when F and when with a less slope the breakeven point was here, Q1 star.

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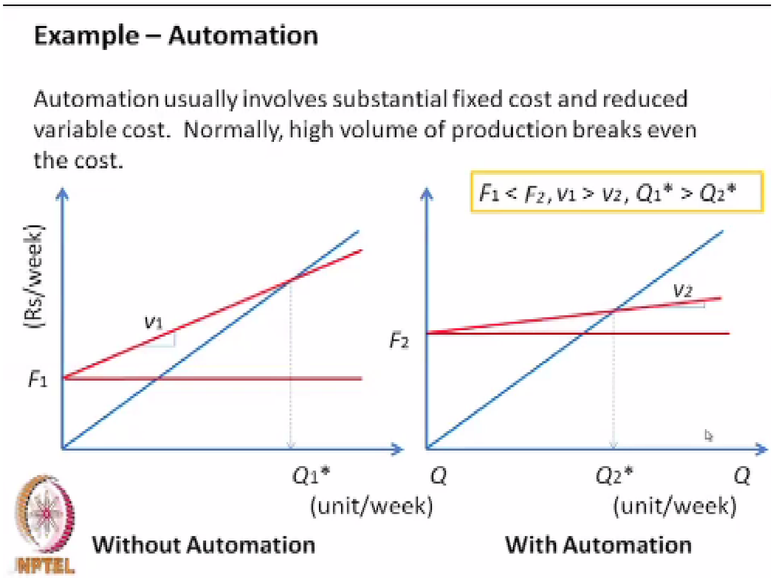

But when it is variable cost rises, the breakeven point goes up that means, if the margin of safety therefore is less, if fixed cost rises or if variable cost rises, so one has to be careful. Now, a good example of both fixed cost and variable cost changing is that of automation. Normally, we know that if a company goes for automation, then the fixed cost rises and variable cost reduces a little because normally, it requires less manpower to operate.

But the power cost can be quite high but in any case normally, we believe that if automation has to take place, the volume of production that can break even with the cost has to be very high otherwise, the fixed cost will not be fully realized but there can be a situation where the breakeven point can actually reduce. We will show this in the form of 2 figures, let us say that this is when it is not automated.

In this case, we are assuming that this is the fixed cost and this is the variable cost, now when we go for automation normally, more fixed cost is required, so F2 becomes higher than F1 but less manpower etc, less power consumption may also be there. So, v2 is the slope of the variable cost line and which is less than the slope of the original case that is v1, so v2 F2 is; F1 is < F2; F1 is < F2, v1 is > v2; v1 is > v2.

Now, depending on the actual values of F1, F2, v1 and v2, the breakeven point during automation can be high; a higher or lower than the earlier, the first case. In this case, we have shown that Q2 star is definitely lower than Q1 star this of course, is not very usual but we have shown that suppose at the fixed cost; rise in fixed costs is not very high but the variable cost is reduced drastically because of automation.

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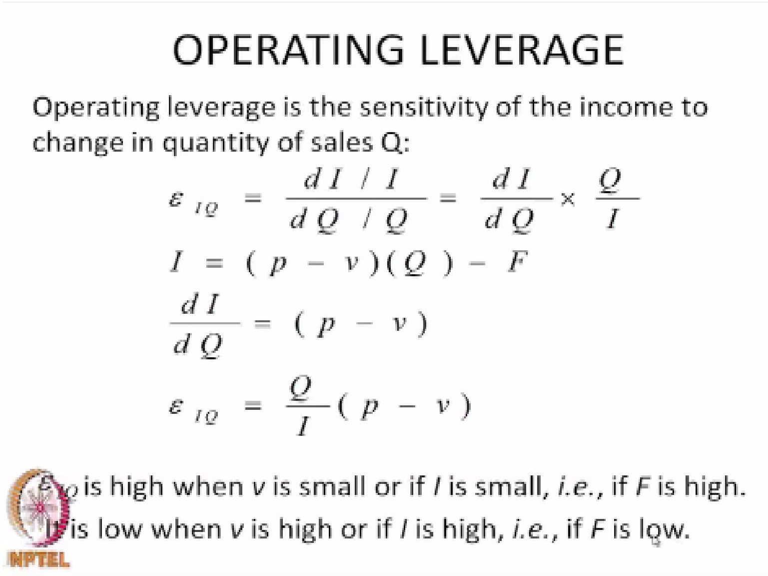

In that situation, it is possible that the breakeven point is less that means, automation is justified even if one does not have to produce very large amount of goods. Now, we introduce a concept known as operating leverage. Operating leverage is the sensitivity of the income; the net income to the change in quantity of sales Q. Now, this is like elasticity that we have already discussed when we discussed demand.

So, this is basically dI/ I, the fractional change in income due to a fractional change in quantity dQ/Q, which is same as di/ dQ multiplication Q/ I, we already know that the total income I is p – v * Q, pQ is the revenue, vQ is the total variable cost - the fixed cost. So, dI/ dQ is nothing but p – v, so this dI/dQ is obtained as p – v, therefore the operating leverage epsilon IQ is nothing but Q / I * dI/ dQ which is p – v.

So, epsilon IQ is Q/ I * p – v, now epsilon IQ, the operating leverage is high when v is small, if v is small this will be high or if I is small then also epsilon IQ is high. Now, I is small from this relationship, I is small if F is high, so epsilon IQ is high, when v is small and F is high alternatively, epsilon Q is IQ is low, when v is high or F is low just the opposite that is what is written here.

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ε_{IQ} is high when v is small or if I is small, *i.e.*, if F is high.

Thus ε_{IQ} is high if the ratio F/v is high.

Such a company is called a **high-leverage company.**
Such a company can be risky; it can give high profit and high loss.

ε_{IQ} is low when v is high or if I is high, *i.e.*, if F is low.

Thus ε_{IQ} is low if the ratio F/v is low.

Such a company is called a **low-leverage company.**
Such a company is less risky.

IQ is high, when v is small or if I is small that is if F is high, alternatively we can say epsilon IQ is high, if the ratio F/ v is high that means, if the ratio fixed cost to variable cost is high then it is a case of high operating leverage or such a company is called a high leverage company. A high leverage company has got a high fixed cost and a low variable cost compared to a low leverage company, which has got a low F and a high v or ratio F/ v adds low.

Now, we can show that the high leverage company can be very risky, it can result in high profit but it has a risk of also high loss compared to a low leverage company, which gives low profit but also low loss.

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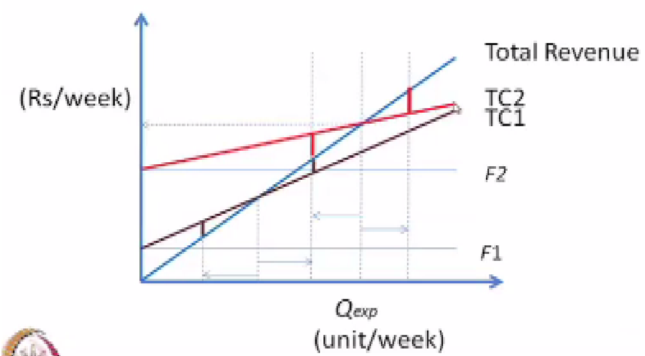
RISKINESS OF LEVERAGED COMPANIES



Now this is shown in this diagram, now in this diagram, we are considering a high leverage company here, a high leverage company has got a high fixed cost is F2 but a low v, so the total cost curve is this red line and the total revenue is here. Now, this is a breakeven point suppose, the company operates or the planned production is here, then the profit is this much but suppose for some reason by the same amount, the planned production falls below, the breakeven point then the loss is also quite high.

So, this is a case of high leverage company with high F and low v, this gives us high profit in the profit zone but is associated or accompanied by also high loss compared to; compare this situation to another company, a low leverage company with a small fixed cost F but a high variable cost, its slope is higher than this, so F/ v is small, so we will say low leverage company.

Now, the breakeven point is here, you will see by the almost the same amount I have taken here the production plan, the profit is only this much, the loss is also that this much, so this is a case of less risky situation and this is a case of high risk situation. So, high leverage companies are associated with high risks and low leverage companies are associated with low risk.

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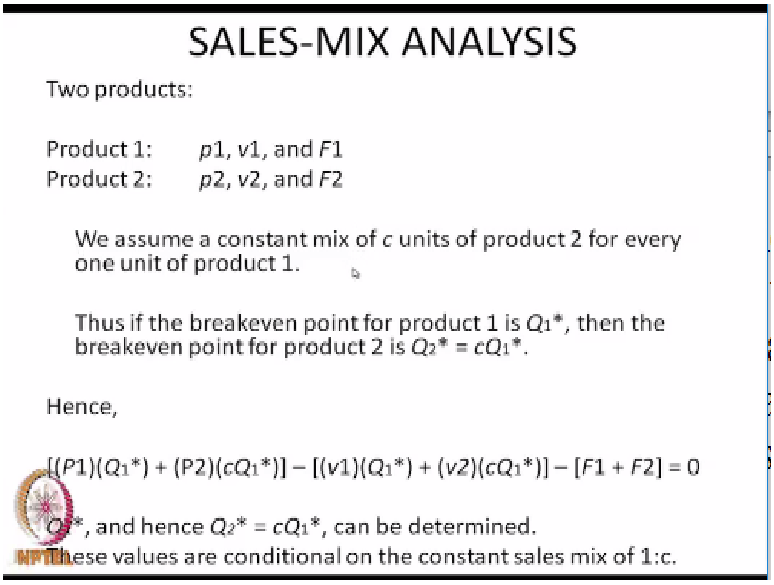

Now, we consider a case, when there are; there is more than one product, let us say that we have 2 products; product 1 and product 2. Product 1 has got prices p1, unit variable cost v1 and fixed cost F1. So, naturally the total revenue is p1 and Q or Q1 and total cost is F1 + v1 Q1, does the total cost. For product 2, the total revenue is P2, Q2 and total variable cost is v2 Q2 and fixed cost is F2.

Therefore, the total cost is F2 + v2 Q2, therefore here the total income is p1 Q1 – F1 – v1 p1 and here the total income is P2 Q2 – F2 – v2 Q2. Now, we would like to know what should be the breakeven quantity Q1 star and Q2 star that will break even the revenues with the costs. Now, here we cannot apply our earlier approach straight away, we have to make a small assumption. The assumption is that there is a constant mix of c units of product 2 for every one unit of product 1.

Now, this is an assumption that says that the ratio of production at whatever level, they produce is constant, it means it says that suppose, product 1; the company produces 10 items and suppose there c is = 5, then 50 items of product 2 will be produced. Suppose that instead of 10, the firm produces 20 items of product 1, then it will produce 20 multiplied by 5, 100 items of product 2.

So, like this there is a constant ratio of the number of items received between the number of items produced for product 1 and that produced in for product 2, with this assumption we can now use our earlier analysis. So, here we say that if the breakeven point for product 1 is Q1 star then the breakeven point for product 2 will be always will be Q2 star, which is nothing but c times Q1 star, we make this assumption.

With this assumption, suppose we can now use the same thing, our total revenue now is = p1 Q1 star + p2 Q2 star but Q star Q2 star is = c * Q1 star, so p2 * c * Q1 star, so this is the total revenue - the total cost. In the total cost, we have the fixed cost F1 and F2, they are subtracted straight away and then, we have the variable cost 1v and Q1 star + v2 * Q2 star now, Q2 star once again is = c Q1 star, so we write here c Q1 star, this is – 0.

So, what is given or what is known is F1, F2, c, v2, v1 and c p2 and p1, so we only do not have knowledge of Q1 star and we can therefore find out the value of Q1 star and once Q1 star is known, the value of Q2 star can also be found out as = c * Q1 star. Now, these values of course are conditional on a constant sales mix of 1 is to c.

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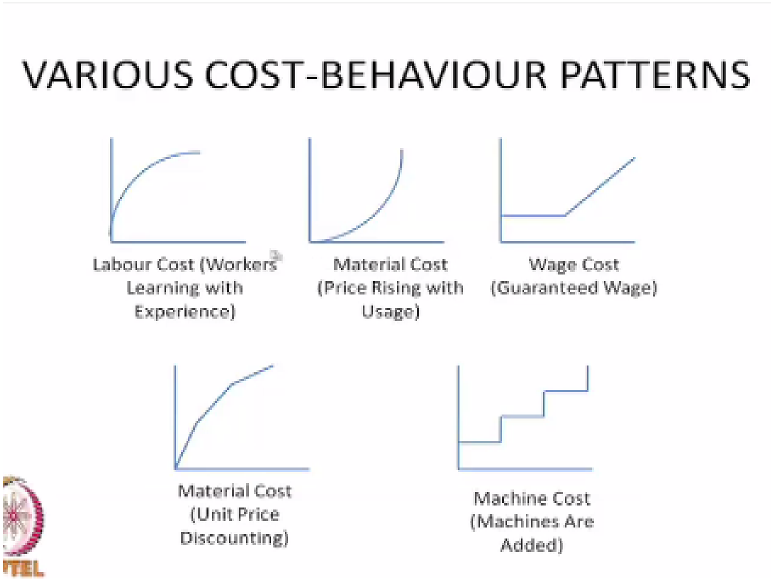

So, friends we have; we have shown here that when there are; when there is more than 1 product, the breakeven analysis is applicable only when we assume that there is a constant ratio between the amount reduced of product 1 and product 2. Now, we take our last slide we would like to tell here that actually in real life, it is very difficult to have a linear relationship between any cost item and its cost driver.

We give examples of various nonlinear costs or piecewise linear costs, this is an example of labour cost, as workers learn with experience, they produce more item and therefore, more item for the same time because of learning, the cost reduces, as they produce more and more number of goods it is no longer straight line but it is, it reduces almost asymptotically. Material cost; suppose that price rises as we go for more and more material, price material is not available.

And therefore they have to be procured with higher price, then material cost can go up higher than linear, now this is another type of cost, where a worker this is a wage incentive scheme, a worker is given a constant amount for some time but if he can produce more, then he is paid more hourly whether it is more, so his cost of labour or wage cost rises that is a guaranteed wage up to this.

Here is an example of material cost, if you go for purchasing more material then because of unit price discounting; you get some discount for the amount higher than this, so the cost can droop like this. So, material cost can have a variation such as this, machine cost; suppose that we had one machine, the cost of the machine depreciation or machine cost was only this, you go for 2 machines.

Because beyond this you cannot produce, is the capacity of one machine is exhausted therefore, if you want to produce this you have to procure another machine and that adds up to the cost, so the fixed cost machine cost which is normally fixed considered fixed is actually varying as volume of production increases. So, these are examples of costs, where you will see that they are not really linear, they are changing.

So, friends in today's lecture, we covered a very simple but a very powerful technique known as breakeven analysis or CPV analysis; CVP analysis, cost volume profit analysis and this is very useful for decision making, we have given some examples here but we will be giving more examples in the coming lectures. Thank you.