Economics, Management and Entrepreneurship
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Lecture – 05
Production

Welcome to the fifth lecture on Economics Management and Entrepreneurship. If you recall, we had covered demand supply, market equilibrium in the first 2 lectures and thereafter we covered demand elasticity, and in the last lecture we covered demand forecasting. In a way, in the last few lectures, we have covered more on the demand aspects of economics of a firm. Today, we shall discuss on production, the supply aspects of the firm.

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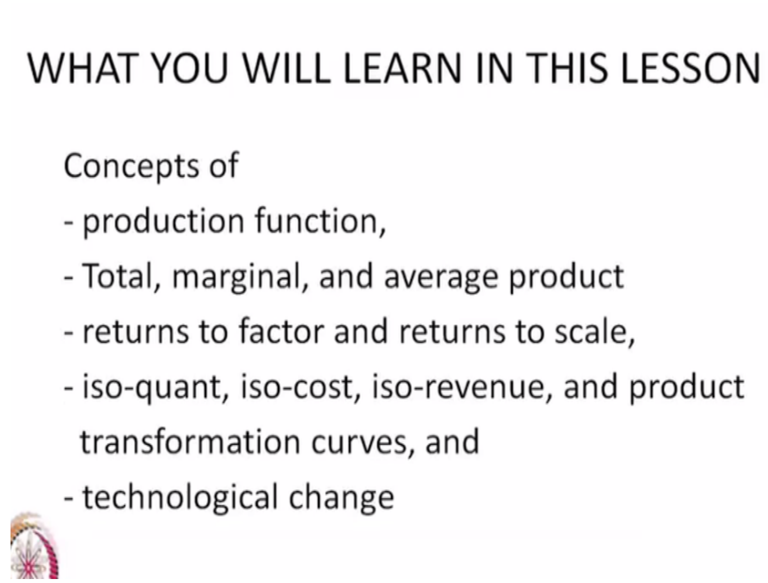

In this lecture, we expect you to have the concepts of production function, total, marginal, and average product, returns to factor, and returns to scale, various curves such as iso-quant, iso-cost, iso-revenue, and product transformation curves, and the effect of technological change on production. Each of these concepts we will discuss hereafter.

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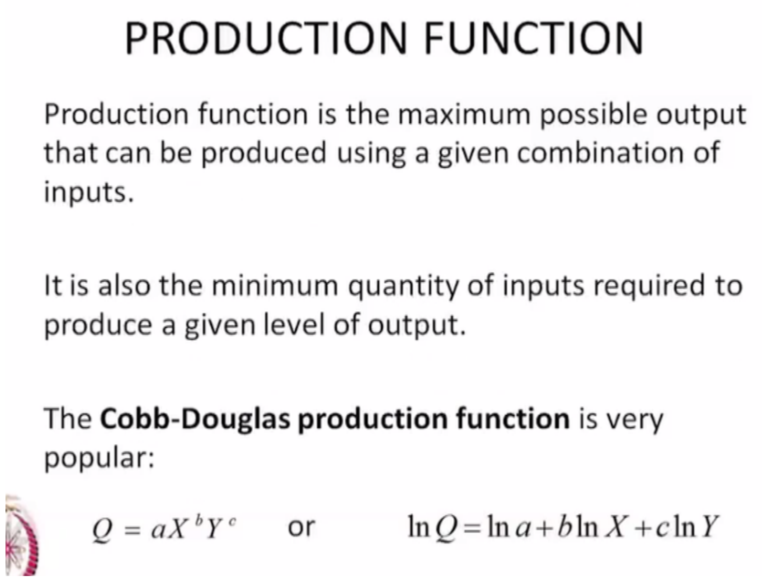

To start with we discussed about production function. By production function, we mean the amount of production that is produced amount of products that is produced because of certain factors, certain inputs, such as material, labour, capital, and so on and so forth. Here, we defined production function as the maximum possible output that can be produced using a given combination of inputs.

It is also the minimum quantity of inputs required to produce a given level of output basically it maps or it defines a relationship between the inputs used and the outputs produced. A very popular production function is Cobb-Douglas production function. It is basically a power function that defines the quantity produced Q as a function of in this case I have given example of 2 input materials, 2 inputs X and Y.

The relationship that defines Q with X and Y is a power function relationship Q = a * X to the power b * Y to the power c and if I take a log transformation of both sides this becomes ln Q, natural logarithmic of Q = ln a + b ln X + c ln Y. Cobb and Douglas 2 economist had suggested that such a production function covers a very wide range of relationships between products produced and the input materials inputs given to a firm.

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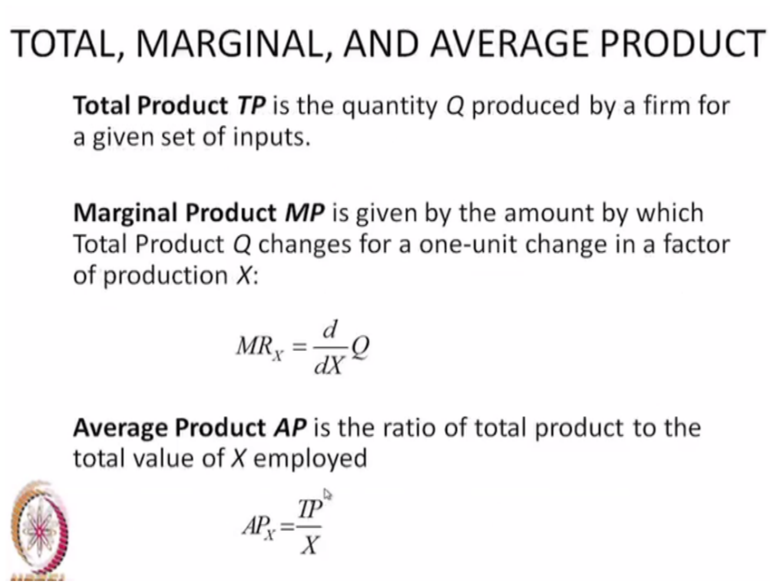

Then, we define 3 types of products: Total product, marginal product, and average product. By total product, we mean the quantity Q produced by a firm for a given set of inputs, naturally in a specific period of time that is called total product. Marginal product on the other hand is the first differentiation of total product with one of the factors. In this case, I have given an example of only one factor let us say x is a factor. There can be many other factors. Then the marginal product is given. I am sorry this would have been MPx rather than MRx.

This should have been MPx = dQ/dX is the amount by which the total product Q changes for a 1 unit change in a factor of product of production X. So if it is for another factor of production Y then we write MPy = dQ/dY. Basically, this says the amount by which the output Q changes for a unit change in a particular factor of production X. Next is average product AP. This is simply the ratio of the total amount produced by the value of A particular factor X employed and this is called APx. Similarly, if we have another input Y, then APy would be total product TP/Y. So these are the 3 things that we shall be using hereafter.

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RETURNS TO FACTOR
(or FACTOR PRODUCTIVITY)

Returns to a factor (or **factor productivity**) is the magnitude of percentage change in the output of a firm when a particular factor of production undergoes a given percentage change.

We next talk about returns to a factor are also known as factor productivity. Returns to a factor or factor productivity is the magnitude of percentage change in the output of a firm when a particular factor of production undergoes a given percentage change.

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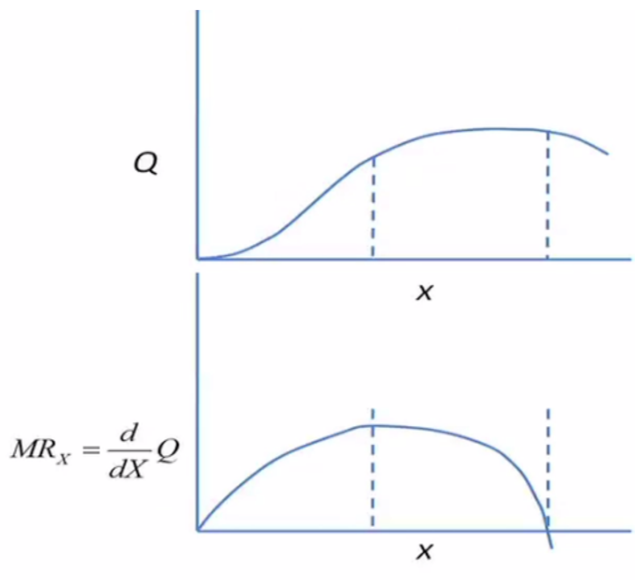

We show it in this case that as X increases a particular factor x increases how the total product Q changes as x increases. Normally, as x increases initially the rate of change of Q is high, but thereafter it shows a decline and the rate of increase in Q so the decline becomes 0 and then it becomes negative again. This rate of change is basically the marginal product. This would be MPx and not MRx. It should be dQ/dX.

So the slope of this line is positive and is growing so the value is also growing marginal productive value is growing till a point of inflection comes where the value decreases so hereafter the value decreases, but still positive and somewhere here the slope can be negative meaning that the marginal product becomes negative. So this shows that as a factor that is input to production increases the amount of production may raise to certain extent becomes stagnant for some time, and then may actually decline.

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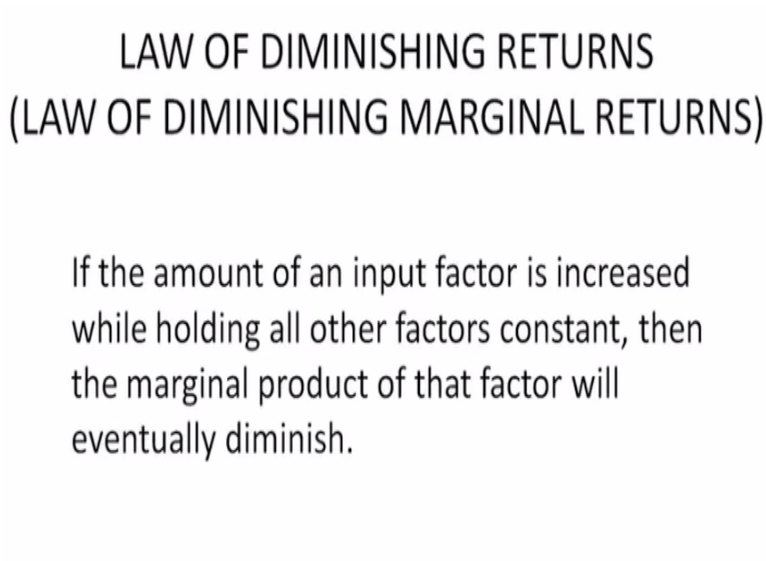

This is known as the law of diminishing return or law of diminishing marginal returns. If the amount of an input factor is increased while holding all other factors constant, then the marginal product of that factor will eventually diminish. This is basically shown in this particular slide that keeping all other factors at certain value. If one factor x increases, then the amount of quantity produced increases for some time comes to a steady state value and thereafter it declines. This is the law of diminishing returns.

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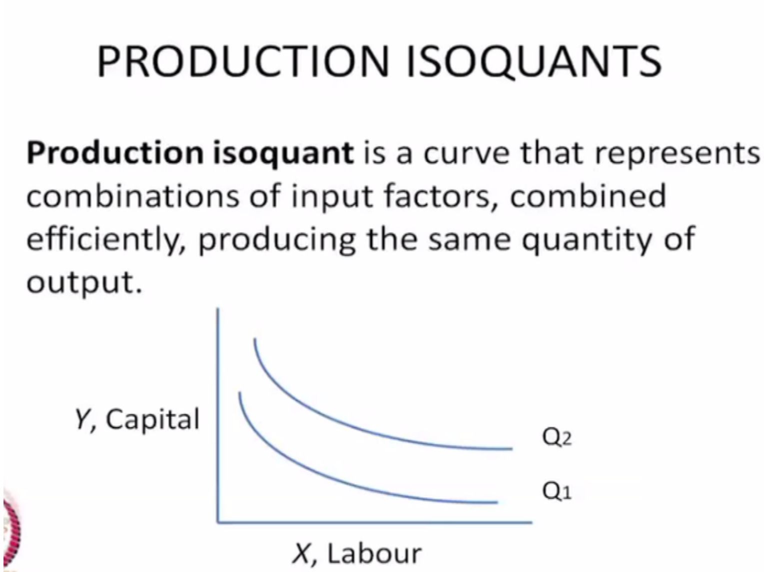

This we give the concept of production isoquant. Basically, isoquantity produced that is same quantity produced for what combination of the input materials. We have in this example considered labour and capital as the 2 input materials for production and this particular 1 curve for Q1 let say and this curve is for Q2, it means to produce a quantity Q1 different combinations of labour and capital could be used.

We could use this labour, a low capital, but high labour, or high capital requiring less labour we may also get the same quantity produced. So this is the isoquantity or isoquant curve meaning that for different combinations of Y and X are here Y and X we could get different, we could get the same value Q1 whereas this is an example of another quantity produced which is Q2 and Q2 can be achieved for different capital and labour, different capital.

But these 2 curves are parallel to each other. This is called production isoquant curves. This is a curve that represents combinations of input factors. In this case, X and Y combined efficiently to produce the same quantity of output. This production isoquant is a very useful technique graphical technique and we shall be using it very frequently hereafter.

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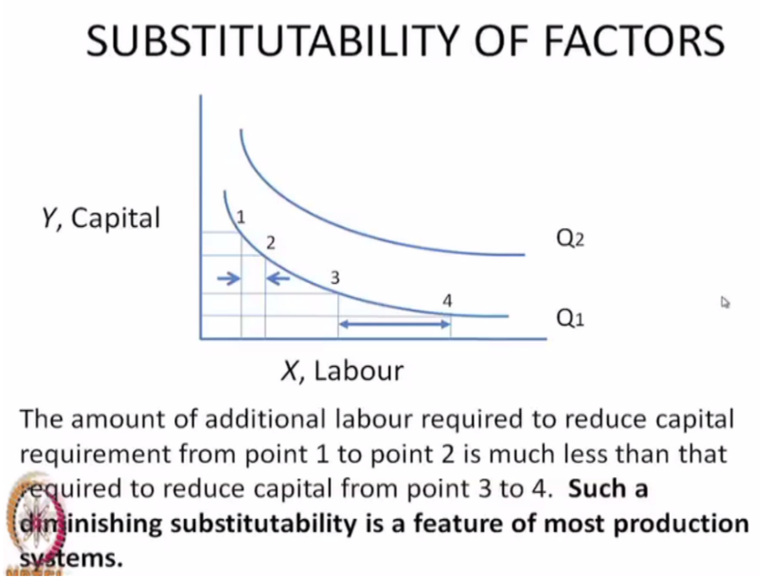

As you can see if you consider any one isoquant say for example this point on the isoquant Q1 indicates that you need high amount of Y and low amount of X and point number 4 requires more labour X, but less capital Y. Now you will see that suppose you move from 1 to 2 it means you are using more labour, but less capital. Now consider points 3 and 4 from 3 to 4 you are also using more labour and less capital, but we realize that the extent of labour force or labour are required to move from 1 to 2 is much less than when you move from 3 to 4.

The amount of labour required here is much larger whereas the amount of labour required here is much less compared to the same reduction in the value of capital that is 1 to 2 the level of capital requirement is reduced by this amount and the level of capital requirement reduced by this amount, but correspondingly the requirement of labour is much more. This is called substitutability of factors.

The amount of additional labour required to reduce capital requirement from point 1 to 2 is much less than that required to reduce capital from point 3 to 4. Such a diminishing substitutability is a feature of most production systems. Firstly, that factors can be substituted that means you can use more labour and less capital. This is substitution but as you move along you will see that the amount of substitution required is much more than when it is operating at these points.

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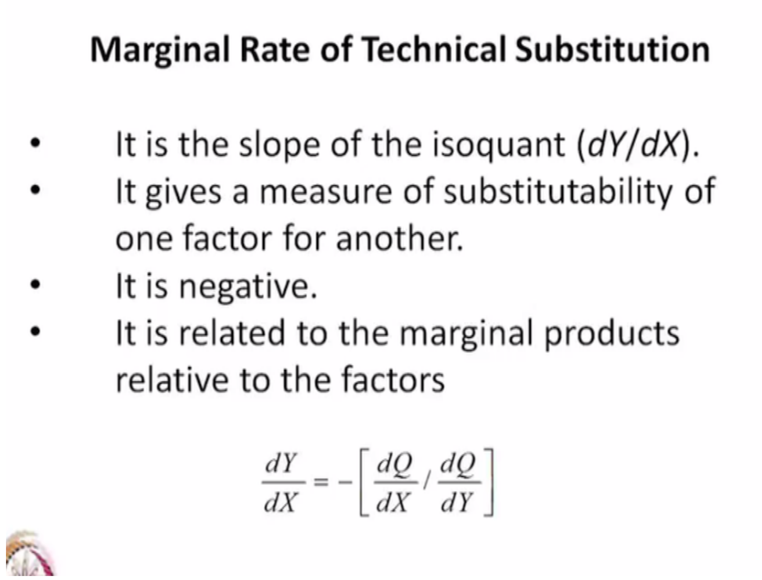

Now we come to the concept of marginal rate of technical substitution. Now here basically we say that it is the slope of the isoquant dY/dX. Now just look at this isoquant, this is an isoquant. Now it is at any point it has a slope defined by a tangent at any point so we would like to that is nothing but dY/dX at that vertical point and this is the slope of the isoquant. It gives a major of substitutability of 1 factor for another for producing the same quantity Q as you can see it has a negative slope that is what we are writing, it is negative.

Basically, dY/dX is the slope of the isoquant curve and that is negative. dY/dX as you can see = can be written as = dQ/dX/dQ/dY. So dQ, dQ cancels out, dY goes to the numerator. dX remains in the denominator. So dY/dX can be written as dQ/dX/dQ/dY, but dQ/dX by definition is the marginal product, the change in Q for a unit change in the value of X. So this is MPx and this is MPy. So therefore, the rate of technical substitution is - because the slope is negative - the marginal product of X/marginal product of Y.

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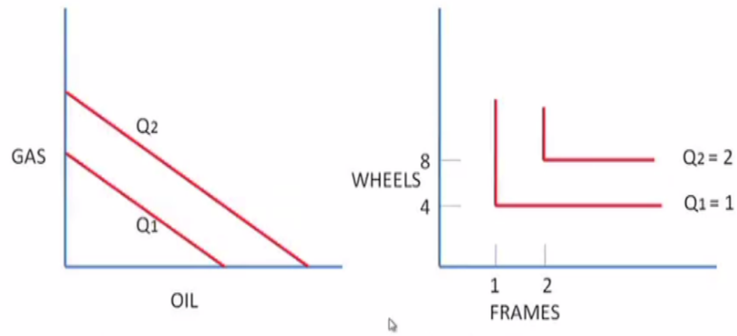

Perfect Substitution in Electric Power Generation

Perfect Complements in Auto Industry

Now let us see whether substitutions how substitutions happen in complements and in other cases. Now this is a case of perfect substitution. An example that we have taken is electric power generation. In electric power generation it can use either oil or it can use gas. Now here the isoquant curve is basically a straight line and for a higher value of electricity (()) (18:52) it is Q2, it is parallel to the Q1 isoquant line. So here we see that if I draw a line here like I did in the case of in this case.

This requirement is much less than this requirement when it is a straight line it will be exactly the same. So this is a case of perfect substitution by gas with oil or vice versa. But, if we consider the case of complements, please recall that the complement is 1 which says that this is a case of a car. In a car if we need 1 frame we will need 4 wheels. This is a perfect complement. If we use 2 frames that is for 2 cars we need to have 8 wheels.

The quantity produced is 1 car, here and the quantity produced is 2 cars here. So this we see there is no substitution here and here is a perfect substitution.

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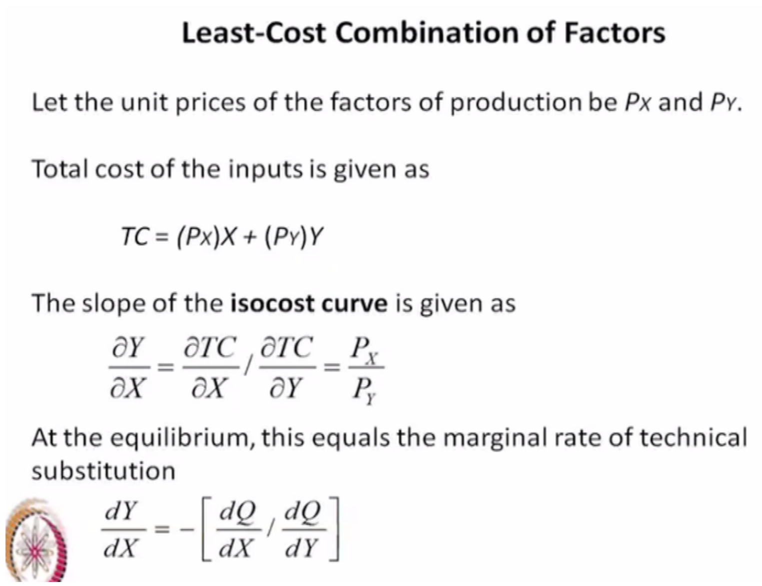

Now we bring in the concept of isocost curve. For that this will be required to know the least-cost combination of factors that means we can find out at what price combination of factors the quantity produced is maximum. Let the unit prices of the factors or production be Px and Py. Px is the unit price of input X, Py the unit price of input Y therefore the total cost of the inputs will be unit price Px multiplied by the amount of X used + the unit price of unit of input Y * the amount of input Y that is the total cost of input.

Now, we can draw a isocost curve and whose slope is actually it should be dY/dX which is nothing. But del TC/del X partial differentiation of TC with respect to X and partial differentiation of TC with respect to Y and that is nothing but if I take personal differentiation of TC with respect to X I get the unit price Px and partial differentiation of TC with Y I get unit price Py therefore dY/dX is nothing but the ratio of the unit price of x to unit price of y.

Incidentally, at the equilibrium dY/dX is nothing but dQ/dX/dQ/dY which is same as dY/dX and this is the marginal product of X MPx and this is MPy. Therefore, the isocost curve at the equilibrium will be = the marginal rate of technical substitution. This is shown here.

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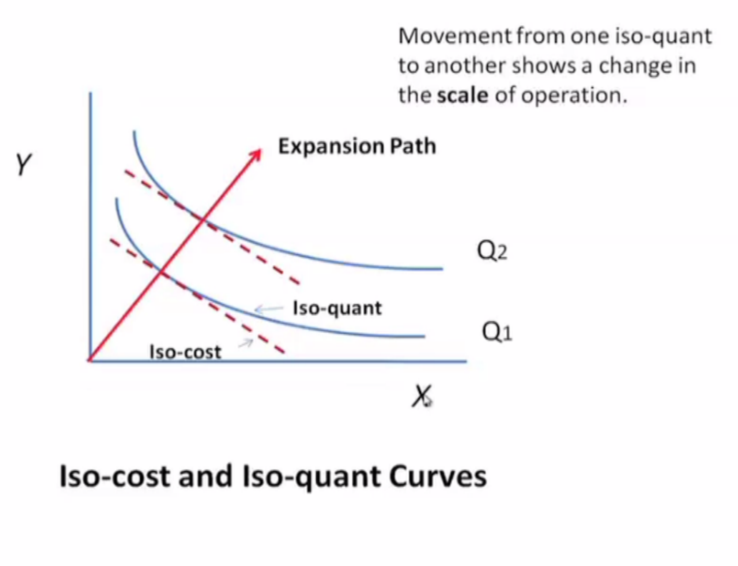

In this curve. As before we have the input X here and the second input Y here. This is the iso-quant curve for quantity Q1. This is iso-quant curve for quantity Q2 and we are assuming that Q2 is higher than Q1. Now this is a particular iso-cost curve having a slope = unit price Px is basically dY/dX and at this particular point the slope of the iso-quant curve is also dY/dX therefore at this point the slope of the iso-quant curve and equals the slope of the iso-cost curve.

So this is the iso-cost curve and for higher price, higher amount. Higher price this is another iso-cost curve and (()) (25:10) they are parallel. So this is Px/Py and basically dY/dX. So iso-cost curve is tangent to iso-quant curve. This is the point of equilibrium. Now, movement from 1 iso-quant that is from Q1 to another iso-quant Q2, so the change in the scale of operation. So when we increase from 1 particular amount of production Q1 to a higher level of production Q2 it means that our scale of operation has increased or that the company has expanded.

This is also called expansion part. It has shown a different or any increased scale of operation.

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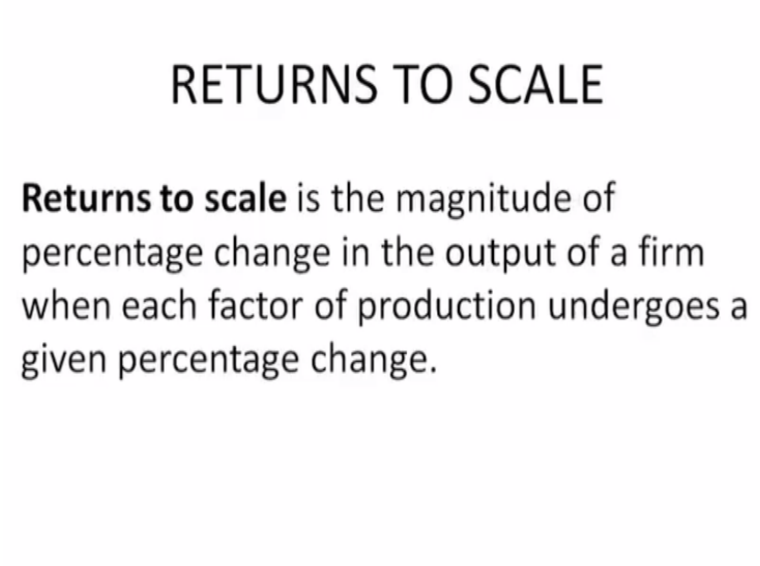

This is what we are now talking next, which is returns to scale. Returns to scale is the magnitude of percentage change in the output of a firm when each factor of production undergoes a given percentage change that means if both X and Y are given equal percentage change then what is the change that happens in the output of the firm. This is given by returns to scale.

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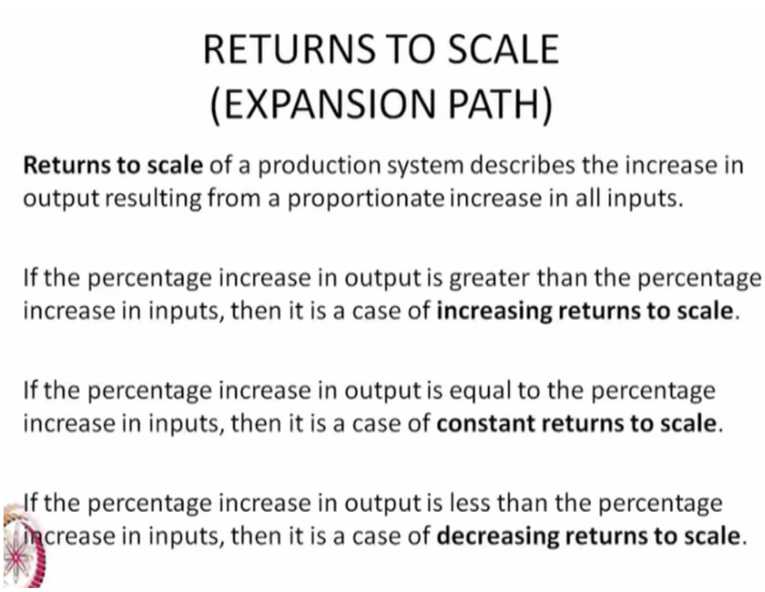

Returns to scale is also called expansion path. Returns to scale of a production system describes the increase in output resulting from a proportionate increase in all inputs just as I had told proportionate increase in all inputs. Now if the percentage increase in output is > the percentage increase in inputs, then it is a case of increasing returns to scale. That means let us say we increase all inputs by 10% and we see that the output rises by 15%.

This is a case of increasing returns to scale, whereas if the percentage increase in output is equal to the percentage increase in inputs, then it is a case of constant returns to scale if by giving a 10% rise in the value of all inputs we see that there is a more or less 10% rise in the output then we say it is a case of constant returns to scale and lastly if we see that by giving a 10% increase in all input factors the output reduces by < 10% say for example 5% or 7% then it is a case of decreasing returns to scale.

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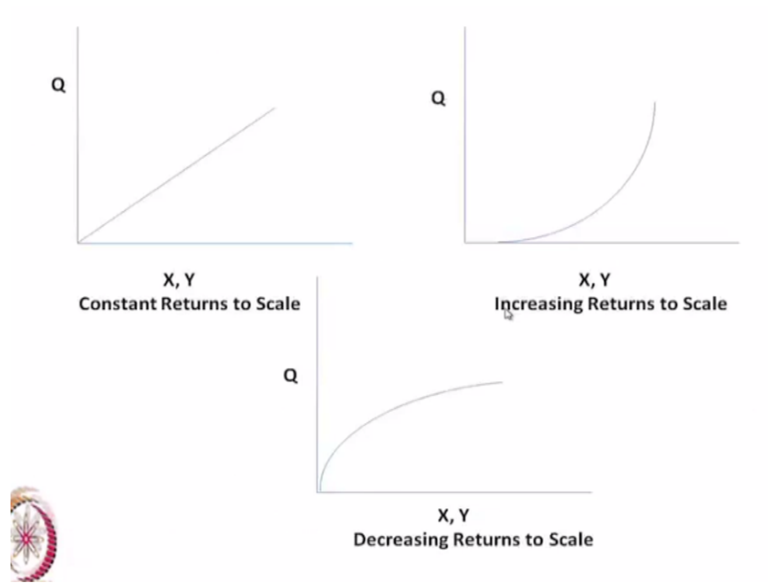

Now this returns to scale can be described from the (()) (28:56) firstly graphically. Graphically this is our expansion path for a constant return to scale. That means if X and Y are given equal percent change then Q rises in the same proportion. So this is a straight line. That is the constant returns to scale. Increasing returns to scale for change in X and Y Q rises in a much bigger fashion whereas here it is less.

This is a case of decreasing returns to scale. So, graphically one can depict returns to scale the 3 types constant, increasing, and decreasing returns to scale in the front wheels. Of course it is possible that one can have for a firm can experience what is known as variable returns to scale which means that.

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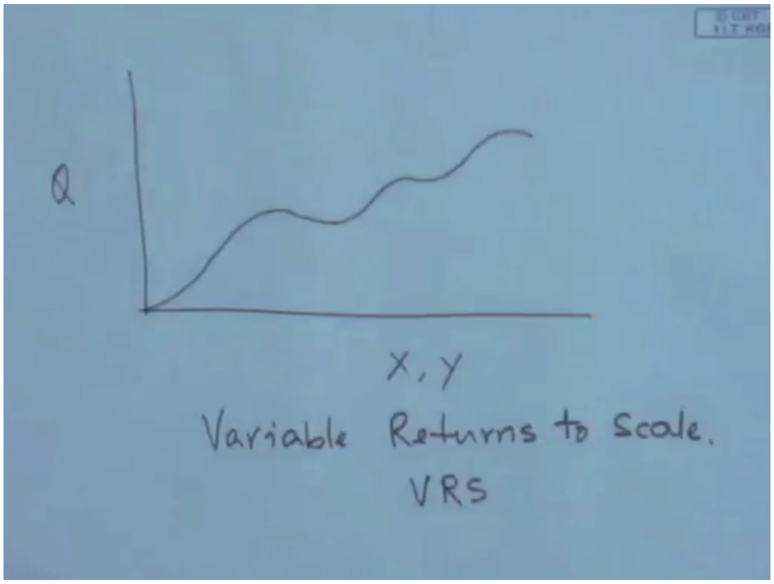

As X and Y changes Q may initially rise in an increasing and thereafter decreasing and then may go down becomes like this, then we say that it is a case of variable returns to scale or VRS. CRS for constant returns to scale, IRS for increasing returns to scale, and DRS for decreasing returns to scale.

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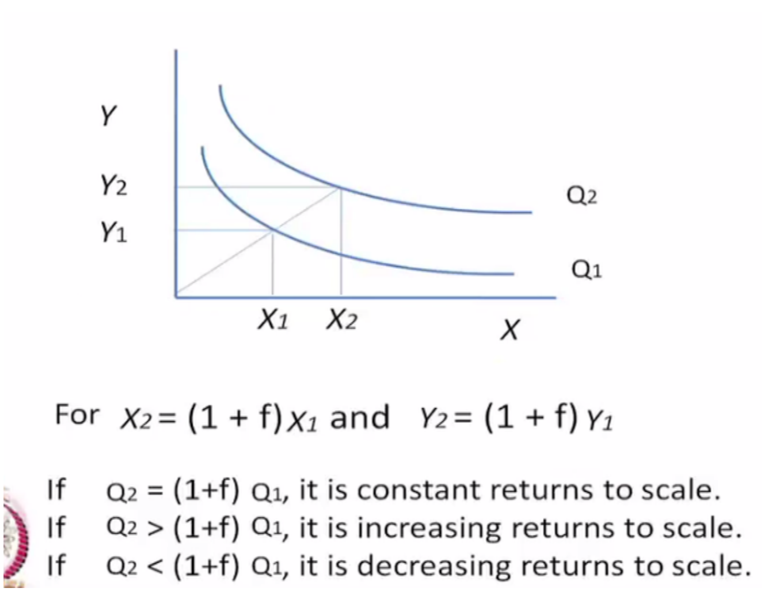

Now this is explained here in another way. It says that as before this is Y and this is X axis. These are isoquants for quantity Q1 and Q2. We are assuming Q2 higher than Q1 and here we are saying suppose that the firm is operating at this point giving an input X1 and the other input Y1 getting Q1 and suppose that X1 rises by a fraction f meaning X2 = 1 + f * X1 and Y2 Y also increases by a fraction f meaning Y2 equals 1 + f * Y1 then the value of production Q2 how it is related with Q1.

Suppose we find that Q2 also rises by this same fraction f that is Q2 = 1 + f * Q1 that means the rise in Q2 is f of Q1 by the same fraction f as the input. Each input was raised then we say that it is a case of constant returns to scale. Whereas if we find that Q2 is higher than 1+ f * Q1 then it is a case of increasing returns to scale, whereas if we find that Q2 is < 1 + f * Q1 then we say that it is a case of decreasing returns to scale.

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Assuming a Cobb-Douglas production function

$$Q = aX^{b}Y^{c}$$

$$\ln Q = \ln a + b \ln X + c \ln Y$$

$$\frac{dQ}{Q} = b\frac{dX}{X} + c\frac{dY}{Y}$$

Since all input factors are increased by the same proportion, say *f*, the following holds:

$$\frac{dX}{X} = \frac{dY}{Y} = f \qquad \frac{dQ}{Q} = (b+c)f$$

If (b+c) = 1, it is a case of constant returns to scale.
If (b+c) > 1, it is a case of increasing returns to scale.
If (b+c) < 1, it is a case of decreasing returns to scale.

This we now illustrate with the help of Cobb-Douglas production function. Recall that Cobb-Douglas production function is a power function Q = a * X to the power b * Y to the power c taking natural logarithmic we get ln Q = ln a + b ln X + c ln Y. We can write from here dQ/Q = b dX/X + c * dY/Y. Now this dQ/Q is the fractional change of dQ, dX/X is the fractional change in X and dY/Y is the fractional change in Y.

Now since all factors are increased by the same proportion f the following will hold that is dX/X and dY/Y will be = f. If it is so then what is dQ/Q? dQ/Q will be = b * f + c * f which is nothing but b + c whole * f. So this also gives another way to define whether the returns to scale is constant or increasing or decreasing by comparing b + c with 1. So from here we say that if b + c

= 1, then dQ/Q = f same as the proportional change or fractional change that we are given to each of the inputs.

Then it is a case of constant returns to scale. Whereas if b + c > 1 then this whole quantity is > f. It means that for a change in f, change in X and Y by amount fractional change in X and Y by an amount f leads to a higher than fractional higher value to the fractional change in Q by more than f. So this is B + C > 1, it is a case of increasing returns to scale. b + c < 1 it is a case of decreasing returns to scale. Please note that b and c are nothing but exponents in the production function therefore barely by looking at or summing the values of b + c.

We can say whether the firm is operating in an increasing return to scale or a decreasing returns to scale. If we find that b + c is more than 1 then there is a chance, there is high probability thereby increasing the amount of input we get more than proportionate increase in the value of Q. Whereas if we find that b + c < 1 it means that if we increase X and Y by some equal amount there will be < proportionate increase in the value of Q. This information is very important in deciding how much more input we should give to increase our production.

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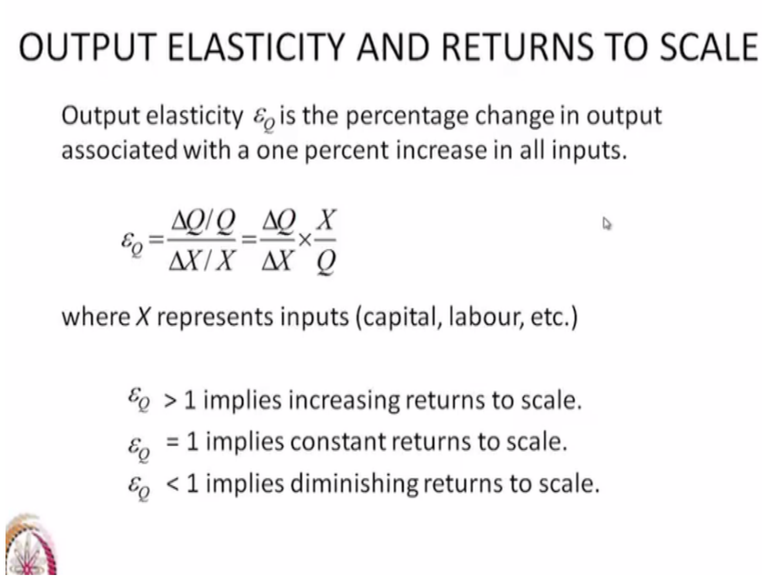

Now we come to output elasticity and relative to returns to scale. Output elasticity is epsilon Q is the percentage change in output associated with a 1% increase in all inputs. We define this as epsilon Q as = del Q/Q percentage change in output/del X/X, where X represents all inputs put together or added together.

So del Q = del Q/del X multiplication X goes to the numerator X/Q. If epsilon Q is > 1 then also it is a case of increasing returns to scale. If epsilon Q = 1 then it is a case of constant returns to scale. If epsilon Q is < 1 it is a case of diminishing returns to scale. This we say merely by understanding that we have defined epsilon Q the output elasticity as the percentage change in or the fractional change in Q for a fractional change in X.

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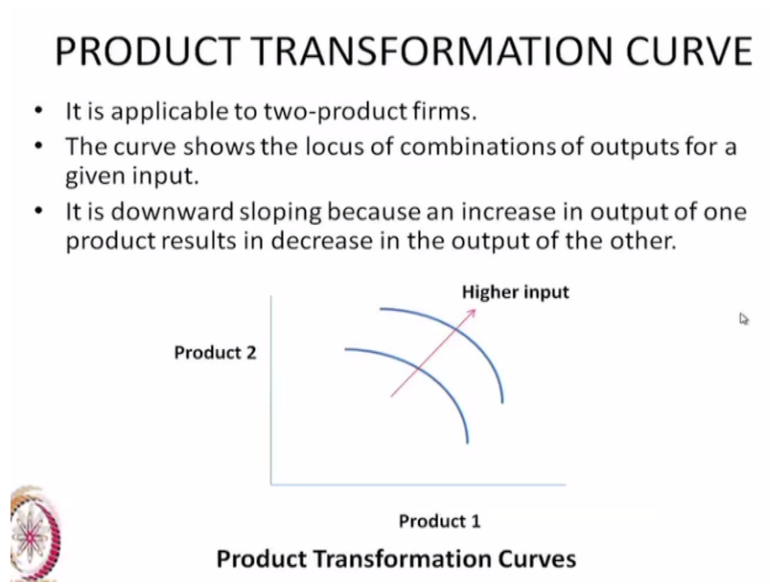

Now we talk about product transformation curve. Here this curve is relevant when the firm is producing 2 or more products. In this case, we are considering of course only 2 products: Product 1 and product 2. We are considering and the product transformation curve basically shows the locus of combinations of outputs for a given input. Suppose that we are spending so much money in our input labour, capital, material, and so on and so forth then we can produce different quantities of products 1 and product 2 for this same input.

So this is using the same input different combinations of product 1 and product 2 can be produced. For example, let us say by using 10 lakh rupees we could produce this amount of product 1 and this amount of product 2 or we could produce more product 2 and less product 1 and if we spend more on input instead of 100 lakh suppose that we spend 200 lakhs then we could produce more of product 1 or more of product 2, but the curve will be a higher curve.

So this is called a product transformation curve for different inputs I have drawn 2 curves. This is for lower amount of input. This is for higher amount of input, but input is constant here. The curve shows the locus of combinations of outputs for a given input. The curve is downward sloping. The slope is negative and becoming more and more because an increase in output of 1 product suppose I increase an output from here to here then it decreases the output of the other so the curve is negative.

The slope is negative and therefore it is moving downward sloping for higher input curves are like this. So this is the product transformation curve.

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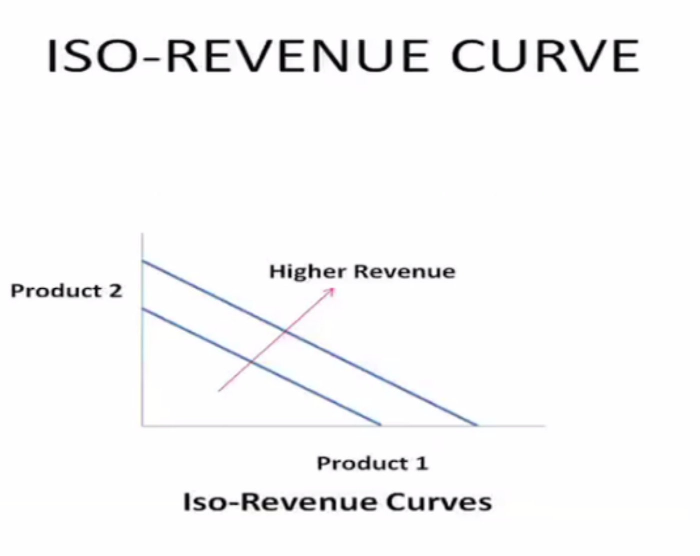

Now we saw iso-revenue curve. Now if each product has got the particular price then we can realize some revenue by selling different amount of material that is if the price is P1 for product X1 or Q1 then P1Q1 + P2Q2 that is the revenue. So this could be 1 revenue and for higher revenue can be realized if we have more products manufacture product 1 and 2.

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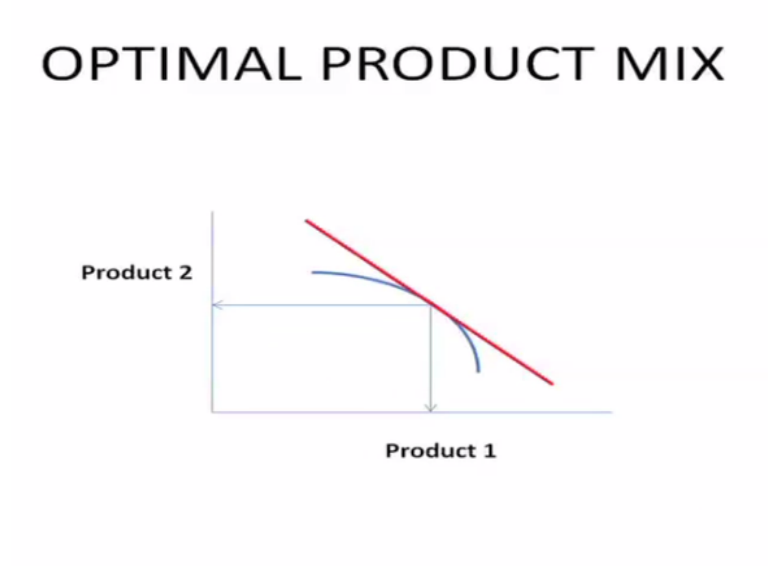

Now this diagram shows that we can decide from a product transformation curve and the iso-revenue curve what should be our optimal mix that means that if this is the iso-revenue curve and this is the if our unit price for product 1 and product 2 are given then their slope will decide the slope of this line and that slope and the slope of the product transformation curve must be equal to decide the point at which the optimal product mix will be achieved.

That means if I operate at this point that means if I produce so much of product 1 and so much of product 2 and then I will realize the maximum revenue out of producing and selling these goods. If however I produce this much product 1 and this much product 2 then the revenue will be much less parallel to this line going here. So this is an example of deciding how or how much of product X1 and how of product 1 and how much of product 2 should be produced given that unit selling price.

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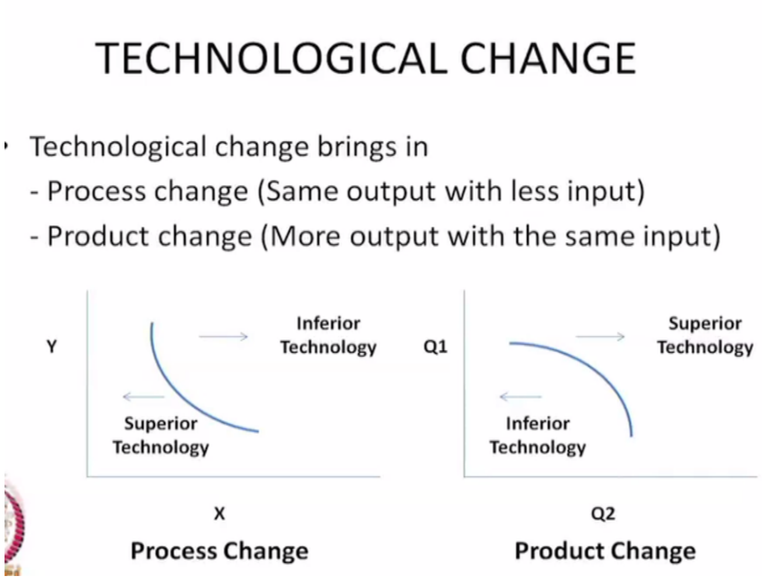

Now we talk about technological change and its effect on production. As you know technology technological change is the order of the day and production is getting affected as time progresses as new technology is coming. Now technology can influence production in basically 2 ways. One by a change in the process or a change in the product when we talk about process change we are saying that we are able to achieve the same output with less input.

That means the productivity is high however when we are bringing in a product change that means with the same input we were able to produce more output. Now this is shown in the form of 2 figures. Now, here we are showing this is a iso-quant for the input material X and input material Y and this let us assume is the present operating condition of a firm using input X and input Y we are able to get the same quantity for different combinations of X and Y.

Now if a superior technology is available, then this curve will shift to its left meaning that we will be able to produce the same output, but with less amount of X and less amount of Y. This curve will shift to the left, whereas if it is a case of inferior technology another firm using an inferior technology we will experience an iso-quant curve which will be on the right side of this meaning that it will be somewhere here for the same quantity Q it will require more X and more Y. This is the case of process change.

Now let us consider the case of product change. Now this is a look at the axis. This is Q1 the quantity of product 1 and this is Q2, the quantity of product 2. So this is the case of product transformation curve. Now in the product transformation curve we for the same input we are saying that given the same input we are able to produce different types of mix of product, different product mix meaning, different values of Q1 and Q2 are produced for the same input.

Now if we are using superior technology, then with the same input the product transformation curve will move to its output indicating that we are able to get more output for the same input, whereas a firm using inferior product technology or product design will have a product transformation curve that will be moving to its left meaning that for the same input, output will be less. Now technological change is a vast area and lot of studies have been made in this area and we have not discussed them here.

So if I summarize what we discussed today to start with is that, the quantity produced in a company is a function of various input factors and our interest was to decide or to find out if we increase a one factor how much change is happening in our quantity produced. If we increase all factors by the same amount how much change is happening to production and we are defining them as returns to a factor or returns to scale.

After that we considered the effect of technological change on production. Well this 5 lectures we have devoted to general economics, microeconomics, and the remaining lectures we shall spend on more operating decision making principals of economics in particular costing, accounting, and similar such things that will be useful in managerial decision making. Thank you.