

Economics, Management and Entrepreneurship
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Lecture - 42
Inventory Management

Good morning. Welcome to the 42nd lecture on economics, management and entrepreneurship. In our last lecture, we had seen how production is to be planned and controlled. In that we had basically considered different functions of planning. The functions namely were programming, scheduling, authorization, follow-up and control.

We had seen how different programming models can be formulated for static, dynamic, deterministic and stochastic demand cases. We had of course taken only a few cases. In scheduling, we had considered different types of situations. If one or 2 machines but more jobs or only one or 2 jobs under large number of machines, how scheduling and sequencing have to be done.

Then, we have discussed why and how the production planning control department passes on the authority to different departments of the company to procure goods, to procure tools, jigs and fixtures, to start the operation, how it authorizes the operators to start the work and these are done with the help of route chart under the things and then we had said that the work, the plan when it is implemented has to be overlooked by chasers who have to physically go and see the progress of the work.

And if there is any difficulty, they have to report to the production planning and control department for corrective action. This complete cycle is called planning, making a plan, passing on the authority to other departments to execute the plan, following up, finding out the shortfalls or discrepancies or deviations from the plan and taking corrective action is what is control.

Now that we have considered production planning and control, today we shall discuss inventory control. To plan production, the most important thing is to get proper amount of material and have it in the store, so that when the production starts the inventory is available

in the stores and it can be requisitioned by the factory to start the production. Now as production starts, inventory is requisitioned, the inventory level in the stores comes down.

And therefore there is a need to ask for replenishment that means the stores must place order with the supplier for a fresh supply so that before the inventory comes down to 0, the supply takes place for replenishment. This is inventory control. Now we shall study today and probably in the next lecture, the topics on inventory control, what are the different considerations and what are the different approaches that have been taken in the past to control inventory.

First of all, let us understand that having good amount of inventory is good in the sense that the production does not suffer but there is lot of capital invested in the inventory when it is acquired and it becomes an idle resource and that is a cost. When lot of money is blocked in the form of inventory lying in the stores, it does not give any revenue at that time. Therefore, there is a potential loss of revenue to have a large amount of inventory.

Therefore, a trade-off must be made between having good amount of inventory so that production does not suffer and as less an inventory as possible so that capital is not idling. So these are the principle considerations that are made in inventory control. Now a company may have a large number of materials. In that case how to exercise selectively control on these items?

It is humanly probably not possible to control the position of inventory of each and every item that the company maintains. So selectively it can decide how to exercise control over different types of items. So these are the things that we shall study today. So the topic for today is inventory management.

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Inventory – Definition

Inventory is an idle resource that has an economic value.

In manufacturing organizations, it is the stock of items stored for future use.



First, a general definition, inventory is an idle resource that has an economic value. Now this is a very general definition. It says that any resource if it is idle but it has a potential value then it is an inventory. In that sense not only materials lying idle in the stores is an inventory but also human employees who are idling, not contributing to the company is also an inventory.

Machines that are not being put to use but are idling is also an inventory but in our lecture we shall concentrate on materials inventory. So in manufacturing organizations it is the stock of items stored for future use, which we shall consider as inventory for the purpose of our lecture.

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Why Keep Inventory

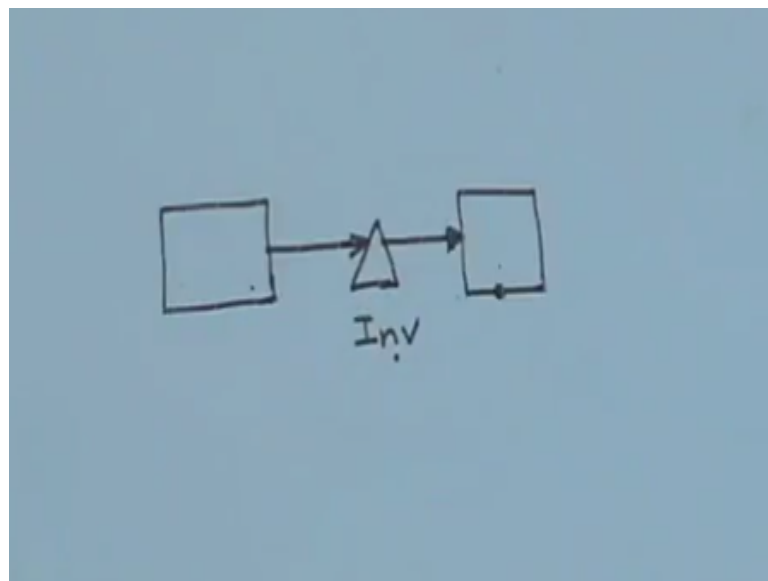
- **Transaction motive** – ensure synchronization of inflow and outflow (i.e., decouple them)
- **Precautionary motive** - A cushion against uncertainty
- **Speculative motive** – store in anticipation of future price rise
- **Economic motive** – take advantage of economic lot size



Now why keep inventory? Principally, there are 4 motives that guide keeping inventory. One is to decouple different operations in the shop floor. It synchronizes the inflow and the outflow. Say for example, a machine is to be used if it is fed with input material then the outflow will automatically come out if every other thing remains same but if the raw material is not available the outflow will be affected.

And if this outflow goes to another machine and if the outflow is not there, the other machine is also affected. So in such a case, you can consider this.

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Suppose that this is a machine and this is another machine. The outflow of this machine goes as inflow to another machine. Now if we have an inventory here something like in-process inventory within 2 processes, then these 2 processes are decoupled from each other, it does not depend so much on this machine. If this machine is not functioning for any reason, this machine's activity or operation is not affected.

Because it gets a supply from the in process inventory, so this is the decoupling function of an inventory. Precautionary motive, cushion against uncertainty. Naturally, the demand for let us say finish product, customer demand may change, it is not constant, it varies. If it varies and if one does not like to lose sales one must have sufficient inventory so that the demand even if it is uncertain is made from the inventory.

There has to be sufficient amount of stock. This is usually called buffer stock to guard against uncertainty in the variation of the demand. So this is the precautionary motive. The third

motive is the speculative motive. Speculative motive is basically hazing against non-availability or high price in the future.

So one would like to store sufficient material so that in the future the price rise which is anticipated now will not affect the profitability of the company and in the future the company may anticipate a shortfall in the supply for which it would like to build up certain inventory at its store. Now this is the speculative motive and of course the 4th motive is completely economic.


Economic motive means that the company may like to order in a lot instead of ordering only one item or 2 items at a time, if it orders 10 items at a time then probably there are certain advantages associated. One advantage is the order preparation cost, the clerical processing cost, the mailing cost, the telephone costs. The other advantages could be that there could be a quantity discount given by the supplier.

So these benefits are obtained when the order quantity is larger than one or 2 units only. So this is purely an economic reason why more inventories stocked at a point of time. So these are the 4 different motives why inventory is kept.

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Types of Inventory

- **Raw material**
- **In-process (WIP)** – materials and components being worked on or waiting between operations in the factory
- **Finished products** (ready-to-stock or ready-to-ship goods)
- **Components** (parts and subassemblies ready to go into final assembly)



Now there are different types of inventory. We already know about raw material that is purchased from outside agency, the vendors or the suppliers. Then we have just talked about in-process inventory. In-process inventory, we already know between machines or between

operations and materials and components being worked upon both constitute the in-process inventory.

Materials that are being worked upon and materials that are waiting for operations between 2 different machines. These are in-process inventory. The finish products inventory is ready to stock or ready to ship items and then finally the components of parts and sub-assemblies that are ready to go into the final assembly. So there are different types of inventory.

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Multi-Item Inventory Management – ABC Analysis

- Classification of items on the basis of annual cost of usage (= unit cost x annual usage)
- Arrange the items in decreasing order of the annual cost of usage.
- Cumulate the annual cost of usage.
- Define the cut-off cumulative annual cost of usage for the A, B, and C classes. Often they are taken as 80 %, 15 %, and 5 %.
- Group the items as belonging to A, B, and C classes.
- Plot the ABC curve – Percent of total annual cost of usage vrs. percent of number of items.



Now to start with we shall consider a case of multi-item inventory management. As I said, a company maybe procuring 100s and 1000s of different items depending on the size of the company and the type of products and the variety of products that the company is manufacturing. Now it is difficult to keep track of each and every item, although it is not impossible.

So usually one makes a selective inventory management or selective control of inventory of items. This is normally called ABC classification of inventory, multi-item inventory management ABC analysis. What is basically done here is that all these items that are stored in the company in the stores, they are classified into 3 categories or classes. A class, B class and C class items.

This classification or categorization is made on the basis of annual cost of usage. That is the annual cost of usage as defined by the unit cost of the item and the number of units used in that year. It is the average value of course average usage rate in a year*unit cost. So this is the

conjunction of the item on average in a year. So for all the items, the annual cost of their usage is found out.

And then what is done, arrange the items in decreasing order of their annual cost of usage. Now for every item we have found out the annual cost of usage and then arrange them in the decreasing order, the highest coming first and the lowest coming the last. Then we cumulate these costs and then define the cut-off cumulative annual cost of usage for each class. Often, they are taken as 80%, 15% and 5%.

We can say 80+15 is 95+5 is 100% and group the items as belonging to A that caters to 80% of the cumulative annual cost of usage, B class items to 15% annual cost of usage and C to 5% annual cost of usage and then plot the ABC curve showing percentage of total annual cost of usage versus the percent of number of items. Now these steps which are illustrated with the help of an example.

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Item	Annual Usage (unit/year)	Unit Cost (Rs/unit)	Annual Cost of Usage (Rs/year)
I1	10	20	200
I2	30	30	900
I3	20	80	1,600
I4	40	10	400
I5	50	40	2,000
I6	20	50	1,000
I7	40	20	800
I8	60	70	4,200
I9	25	30	750
I10	10	40	400




Let us say that we have 10 items although in practice, there could be 100s of items. So we are assuming here that there are 10 items, i1 item 1, item 2, 3, 4, 5, etc item 10 and their annual usage this is average value, average annual usage is recorded as 10, 30, 20, 40, 50, 20, 40, 60, 25, and 10 units per year and their unit cost rupees per unit is 20, 30, 80, etc. Now if you multiply them, you get the average annual cost of usage, 10*20 is 200, 30*30=900, 20*80=1600 and finally 10*40=400.

So first of all, collect data for each item and then multiply them to find out the annual cost of usage. Now the second step was arranged them in decreasing order of sequence. Now here the highest is 4200 occurring for item number 8 followed by item number 5 followed by item number 3 etc and the last is i1 that is what we have done here.

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Item	Annual Usage (unit/year)	Unit Cost (Rs/unit)	Annual Cost of Usage (Rs/year)	Cum. Annual Cost (Rs/Year)	Percentage of Total Annual Cost
i8	60	70	4,200	4,200	34.0
i5	50	40	2,000	6,200	50.6
i3	20	80	1,600	7,800	63.6
i6	20	50	1,000	8,800	71.8
i2	30	30	900	9,700	79.2
i7	40	20	800	10,500	85.7
i9	25	30	750	11,250	91.4
i10	10	40	400	11,650	95.1
i4	40	10	400	12,050	98.0
i1	10	20	200	12,250	100.0


A: i8, i5, i3, i6, and i2; B: i7, i9, and i10; C: i4 and i1

The last is i1, 200 is the annual cost of usage, but the i8 has the highest annual cost of usage of 4200 followed by i5 2000 etc. So this we have arranged in decreasing order of the annual cost of usage. Next, we cumulate or accumulate. When we accumulate for 1200 is this, this+this becomes 6200, this+this+this is same as this+this which is 7800 and then+1000 is 8800 etc. It continues and the highest is 12,250.

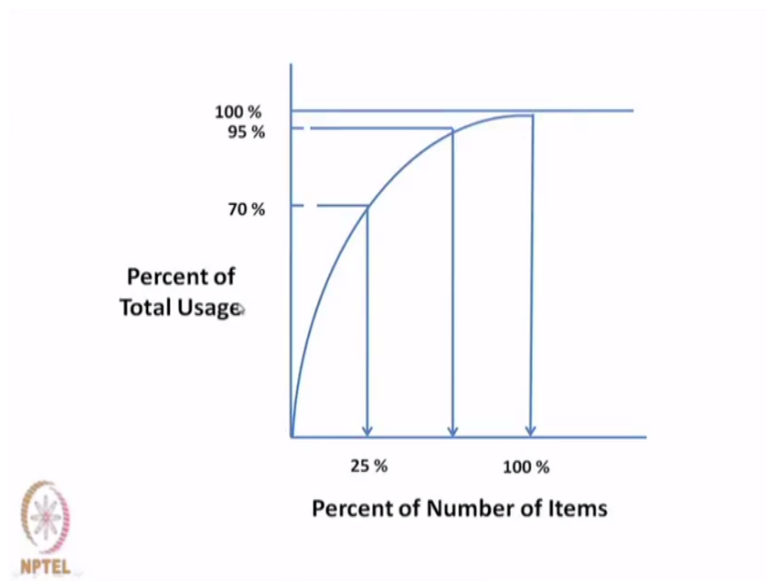
Now if 12,250 is 100%, then how much percentage is 4200? It is close to one third. It is 34%, so similarly we have calculated corresponding to this it is about 50%, 50.6, 63.6 etc. This is what I said. If you take this final one as 100% then express this in terms of percentages. Now you can see the management had decided that ABC classes the cut-off values for the cumulative annual cost of usage would be 80%, 15% and 5%.

So 80% comes up to about this up to i2, so highest is this, so highest item this item alone caters to 34%. These 2 items together are responsible for 50.6%, these 3 items together are responsible for 63.6% of the annual cost of usage. So 80% meaning up to i2, 79.2 nearly 80% these 5 items are responsible for that. Then up to 95, 80+15 is 95 that means up to this these are B class items.

So i7, i9 and i10 and i4 and i1 are C class items that is what I have written down here. A class items are 8, 15, 13, and 16. Of course, 12 could have been also included here because we have taken 80%. If we take 70%, then of course 12 would not come. So this is a management's decision as to whether it should take 80, 15 and 5 or 70, 20 and 10 or whatever.

So if it is 80% then I think 12 should have come here, yes I think I would rather make the change here. So because we have taken 80% then i2 should also be included there. So i2 should be included in the A class items.

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So when we plot it, so we are now plotting percentage of total usage, oh we have taken here 70%, 25% and 5%.

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Multi-Item Inventory Management – ABC Analysis

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- Plot the ABC curve – Percent of total annual cost of usage *vrs.* percent of number of items.

So I think what we need to do is redefine them as 70% and 25% and 5% in that case i2 would come here yes i2 would come here that means our original thing was alright.

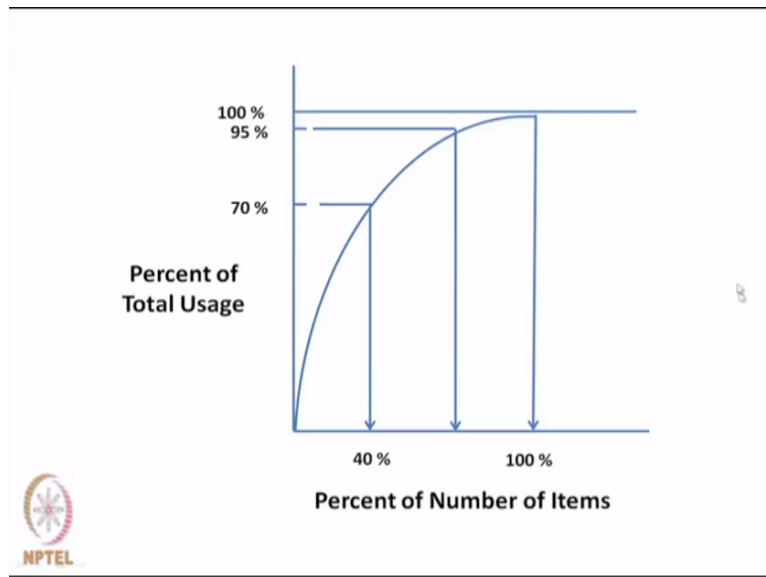
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Item	Annual Usage (unit/year)	Unit Cost (Rs/unit)	Annual Cost of Usage (Rs/year)	Cum. Annual Cost (Rs/Year)	Percentage of Total Annual Cost
18	60	70	4,200	4,200	34.0
15	50	40	2,000	6,200	50.6
13	20	80	1,600	7,800	63.6
16	20	50	1,000	8,800	71.8
12	30	30	900	9,700	79.2
17	40	20	800	10,500	85.7
19	25	30	750	11,250	91.4
110	10	40	400	11,650	95.1
14	40	10	400	12,050	98.0
11	10	20	200	12,250	100.0


A: 18, 15, 13, and 16; B: 12,17, 19, and 110; C: 14 and 11

Yes, now it is okay so what we have done basically that we have defined this figures as 70%, 25%, and 5%. So if you do that then only up to this it is coming. So percentage of total usage is 70%, 95% and 100% and in this case how many percent number of items are responsible for 70%? So here again I think I should be more realistic. In this case, 4 out of 10 items are responsible for nearly 70%.

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So this value should not be 25% but it should be 40%. So basically it is saying that these 4 items i8, 5, 3, and i6 out of 10 items are responsible for nearly 70% of the cost. This is shown here and these are A class items, less number of items are responsible for a high percentage of the total usage and these items are B class items and these items are C class items and this is called normally a Pareto distribution of the items.

Pareto is an economist who suggested that 20% of the population in a country are holding 80% of the total capital of the country. So similar thing is happening here in inventory control. Here in the ABC classification says that A class items are few in this particular example 40% but sometimes only 20% or 15% items constitute the A class items. Now in this case we are saying that 40% of the items are responsible for 70% of the annual cost of usage.

So if so much of money is being used or invested in the A class items are understood by their usage, we should control the inventory of A class items more closely than we do it for B class items and for C class items we can be quite loose in controlling their inventory level. This is the basic idea that we are getting from this ABC analysis.

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Degree of Control

For A Items:

Exert tightest possible control, detailed records, real-time updating, accurate control, regular review, small lot size, small buffer stock, and close follow up,

For B Items:

Moderate control, batch updating of records, medium lot size, moderate buffer stock, and occasional follow up

For C Items:

Loose control, no record, large lot size, large buffer stock, infrequent and visual review of physical inventory

Now we can exercise different degrees of control. For A class items, exert the tightest possible control, keep detailed records, do real-time updating of the inventory level, go for accurate controls, regularly review and normally instead of giving large lot size go for small lot size, will talk more about how to decide on the economic lot size but more frequently you do review.

And therefore you order for less number of items and small buffer stock. Safety stock is normally small because you are ordering more frequently. It is not necessary to have larger buffer stock or safety stock and closely follow-up the inventory level. For B class items, go for moderate control, go for batch updating rather than real-time updating. The lot size need not be very small neither very high, it can be medium.

Buffer stock can be moderate and do occasional follow-up. For C class items, you may not even have any record. You can have a loose control, you can order large lot size you can ask for, you can keep large buffer stock and you can go for infrequent and visual review of physical inventory because you are not keeping any record, you may not keep any record and therefore it can only visually see the position of the inventory and place the order.

So you can see that selectively you are controlling. So if you do more control, you need more effort to be spent here for A class items, less for B class items and much less for C class items. Now this classification has been made on the basis of average annual cost of usage but it is quite possible that if a particular item is not available, then there is a large stock-out cost. Now such an item is a critical item.

Now there is always a possibility to divide or classify the items in terms of the stock-out cost or criticality of the items as expressed in the form of stock-out cost but what we have shown here is an ABC classification, which is normally done not on the basis of stock-out cost but on the basis of average annual cost of usage but as I said it is possible to also consider stock-out cost along with the average annual cost of usage to make a classification and then selectively apply your controls.

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Inventory Costs

- **Procurement Cost**

- **Independent of order quantity**

- (Clerical processing, mailing, telephoning, reminding, setup in manufacturing)

- **Depends on order quantity**

- Inspecting stores for ordering and inspecting incoming material



$$K + CQ$$

Now that we know this let us go to deciding how to place order for replenishing stocks, that will depend on the cost consideration. Now different types of costs can exist. One is the procurement cost. Procurement cost can be of 2 types, one is independent of order quantity, it does not depend on how much order you are placing and these are clerical processing cost. You have to have a person to type or to word process the order.

So there is a time involved, there is a person who has to spend half an hour or 15 minutes time for every order and in a day there can be 10 or 20 different types of order. So it is just not a question of typing, he has to also see the inventory position, he has to decide how much inventory to be replenished or should be the order quantity and mailing has to be done, occasional telephoning and reminding has to be done.

And if it is a manufacturing situation that means suppose that we are discussing about finish product inventory and the order is placed to the production department or to the factory shop floor to actually produce so much then the production department has to make a set up and

for every order there is a separate setting up of the machine that is required and there is a cost involved there.

All this is not a function of how much we are ordering. So this is one type of cost, we are calling it K , K is a constant and there is procurement cost also depends on the order quantity. Order quantity, inspecting stores for ordering and inspecting incoming material plus of course the cost of the material is there but in addition to that we will have to inspect whether the material that has come is up to this specification that we are asked.

So this is what also we have to inspect the stores for ordering and inspect the incoming material. So this depends on the order quantity that we are ordering. So suppose that it depends on this, C is a proportionality constant so we can say that the procurement cost= $K+CQ$, CQ is the unit cost of the item, Q is the amount ordered.

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Inventory Holding Cost

- Real out-of-pocket costs

Rental, personnel, utilities, watchman, breakage and pilferage, taxes, insurances, handling costs

- Opportunity costs

Loss in return due to capital held up in inventory

Inventory charge, I (Rs/unit time/Rs invested in inventory, %/year)

Normal values: 12 – 35 % per year



A second type of cost is the inventory holding cost. Now here again, there are 2 types of cases, one is the real out-of-pocket expenses, holding inventory in a place for which rental has to be paid, a guard is to be there or somebody has to operate the inventory, so personnel cost, utilities, watchman.

There may be some breakage or pilferage, he may pay insurance or tax and there is a material handling cost, keep taking the inventory from one place to another place etc, putting it in the proper position so these are real out-of-pocket expenses. The other cost which is quite substantial is the loss in return due to capital held up in inventory. When certain amount is

invested, certain money is invested in the inventory; the interest on that capital investment is a loss.


So that is an opportunity cost, it is not under real out-of-pocket expense. Normally, these costs are expressed in the form of what is known as inventory charge, I rupees per unit time per rupees invested in inventory. So if you can find out how much of money we have put in that amount, then how much is lost per year. Normal values vary from 12% to even 35% per year.

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Stock-out Costs

- **Lost sales**
Loss in revenue, loss of goodwill
- **Backorder cost**
paper work, loss of goodwill

Backorder cost = P (Rs/unit backordered/unit time)

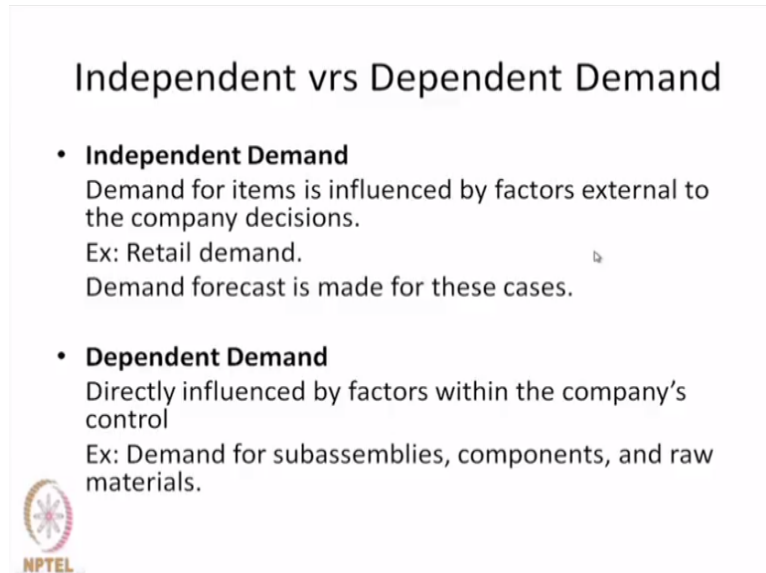


Then, we have a third type of cost, which is called the stock-out cost. If the demand is variable or if the supply lead time, the delivery time after we place the order is normally call the lead time, if the lead time is variable then there are 2 types of possibilities. One the order which we would have got we may not get at all, this is the case of lost sales in which case we are losing certain revenues and losing the goodwill of the customer.

The other cost is that the customer is willing to wait and this customer order remains with the company as a backorder or backlog and if there is a delay then there is a loss of goodwill and because this is an additional operation certain other paperwork has also to be done. So this also is a cost. Now backorder cost is normally given as rupees per unit backordered per unit time or as lost sales.


One really is not very sure whether the lost sale is only an approximation but people have even tried to relate it as rupees per unit lost per unit time. So these are 3 different types of costs.

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Independent vrs Dependent Demand

- **Independent Demand**
Demand for items is influenced by factors external to the company decisions.
Ex: Retail demand.
Demand forecast is made for these cases.
- **Dependent Demand**
Directly influenced by factors within the company's control
Ex: Demand for subassemblies, components, and raw materials.



Now there are 2 cases or 2 situations, one is that the inventory or the demand for the item is independent of demand for anything or of other items. It is quite independent whereas there is a case of dependent demand. Demand of one item is dependent on the demand for another item. So we shall study both the cases. To start with we are studying first of all the independent demand.

Firstly, independent demand for items is influenced by factors external to the company such as retail demand or customer demand. Demand forecast is made for these cases. The dependent demand is influenced by factors within the company's control, demand for sub-assemblies, components and raw materials.

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Inventory Control – Case of Independent Demand

- Deterministic Models
- Models with variable demand and lead time



Now we are studying the case of inventory control, the case of independent demand and we will study both these situations, deterministic model that means where we are assuming the demand to be known and constant and the supply lead time is also known and constant and also we shall discuss case of models where the demand is variable. It follows a distribution and the lead time also is not constant and follows a distribution.

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Basic EOQ Model

Assumptions:

- Deterministic (continuous and constant) demand
- Instantaneous supply
- Single item
- Multi-period infinite horizon
- Constant Price of the item
- Constant setup cost
- No demand is lost or backordered.

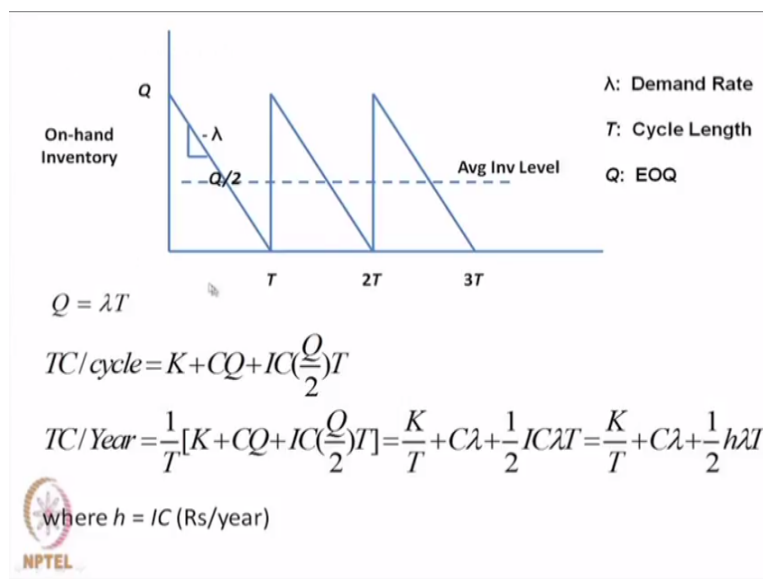


So to start with we are considering the first situation, which is the case of independent demand with deterministic model that means everything is constant. That is normally known as an economic order quantity model EOQ. Here several assumptions are made, first of course that the demand is deterministic that it is continuous and that it is constant. Also assumed to start with is that as soon as we order we get the supply, there is no delay.

The supply lead time is equal to 0. Later, we shall relax this situation and we shall consider the constant lead time. Also, we are considering one single item and we are considering that it is a multi-period infinite horizon case that means we order and the material is received instantaneously and then it is depleted as on when it is taken off items or taken off for use and then again we order etc.

So this continues at infinitum, thrice of the item is constant. The set of cost K, K is the constant procurement cost, K is constant and that no demand is lost or backordered. There is no stock-out cost involved.

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This shows the situation in this case the x axis is the time and the y axis is on-hand inventory and the demand rate is constant. Therefore, if initially we are holding an inventory of Q and if the demand rate per day demand let us say is lambda then at that rate it falls, the on-hand inventory position falls because it is getting depleted at a rate of lambda per unit time and at some point it goes to 0.

The on-hand inventory becomes 0. Let this length we call capital T, the cycle length because this continues. When it comes to 0, we place a fresh order and because of the assumption of instantaneous supply Q is immediately obtained. The on-hand inventory position immediately goes from 0 to Q and then starts falling again because of the demand lambda. Now this continues.

So we can see that Q is nothing but $\lambda \cdot T$ that is Q that is what we have written here. $Q = \lambda \cdot T$. Now our interest is to find out what is the value of Q which minimizes the inventory cost? Now there are 3 types of inventory cost as we have seen. The procurement cost, the inventory holding cost and the stock-out cost. Now in this case there is no stock-out, so we shall deal with only 2 types of cost.

Procurement cost and inventory holding cost, now procurement cost if you recall is the fixed cost independent of the demand $K + \text{the cost of the item} \cdot Q$, $K + CQ$. This is the procurement cost. K is often called the set up cost + the inventory holding cost. How do we calculate the inventory holding cost? We shall find out the average inventory level and the average inventory level is nothing but the area under this T .

The area of this is $\frac{1}{2} QT$, Q is λT , so from here we can find out that the average inventory level is nothing but $Q/2$. So $Q/2$ is the average inventory held and it is held for how long? It is held for T periods. So $Q/2 \cdot T$ or the average inventory held is nothing but the area under this curve which is $\frac{1}{2} QT$. So $\frac{1}{2} QT$ is the average inventory level \cdot the time for which it is held.

This \cdot this C is the capital invested in the inventory during that cycle, $C \cdot Q/2 \cdot T$ is the amount of capital that is invested and blocked in the inventory during that cycle and I if you recall is inventory charge which is rupees per unit time per rupees invested in the inventory, which is basically percentage per year. So it is something like 12% per year or up to 35% per year it can go.

So this amount of capital invested into I is the inventory holding cost. So total cost within a cycle or per cycle is $K + CQ$, which is the procurement cost + the inventory holding cost $I \cdot C \cdot Q/2 \cdot T$. If I want to calculate total cost per year, then I should divide by capital T , T is the cycle length that will give me $K/T + CQ/T$, Q/T is nothing but λ , so $C \lambda + \frac{1}{2} I C Q/T$ is $\lambda \cdot T$.

And this can be written as $K/T + C \lambda + \frac{1}{2} I C \lambda T$ and C multiplied can be written as h defined as $h \cdot \lambda \cdot T$. So TC total cost per year contains T in the denominator and T here.

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Taking derivative with respect to T,

$$\frac{\partial (TC)}{\partial T} = -\frac{K}{T^2} + \frac{h}{2}\lambda = 0$$

$$T^* = \sqrt{\frac{2K}{h\lambda}}$$

$$Q^* = \lambda T^* = \sqrt{\frac{2\lambda K}{h}}$$

When to Order?

When the on-hand inventory is zero.

How much to order?

Q*



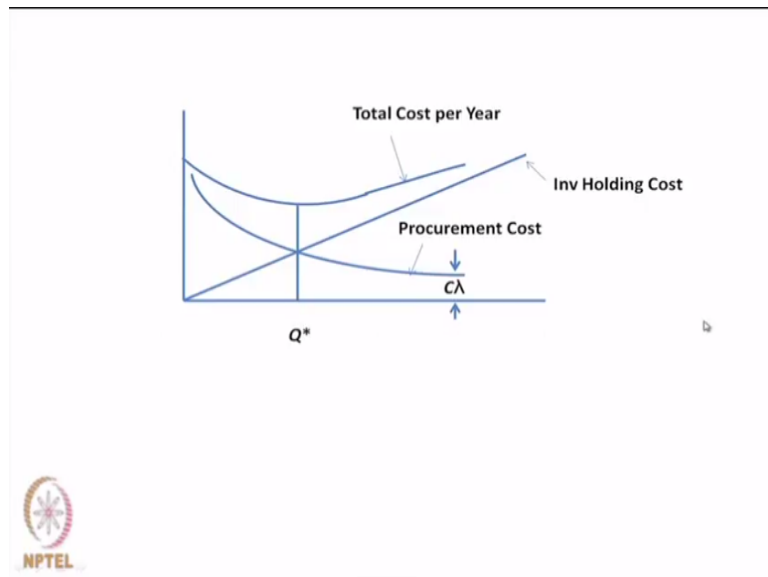
If I take the first differentiation of total cost with respect to T, I get $-K/T^2$ because there was a T in the denominator $+h/2\lambda$ and if I put that=0, I get a value of T that minimizes the total cost and that becomes equal to $\sqrt{2K/h\lambda}$ and Q star then can be derived as λT^* . Now it is T star, so $Q^* = \lambda T^*$, which is equal to $\sqrt{2\lambda K/h}$.

This is the lot size formula due to Wilson who first developed this and it is known as Wilson's lot size formula, $\sqrt{2\lambda K/h}$. Now this is the order quantity, how much to order is basically Q star given by the square root formula of the economic order quantity formula or Wilson's lot size formula, $\sqrt{2\lambda K/h}$ and when to order is very simple when the on-hand inventory is 0.

Now look at the diagram, when the on-hand inventory is 0, place the order, on-hand inventory is 0, place the order that means that one has to always look at the inventory position as soon as a transaction occurs the inventory position must be recorded or updated and a decision made when it is 0 place the order. Now the next thing is that we have made an assumption that the supply lead time is 0 or that the supplier is instantaneous.

Now this is not a very good assumption. We would like to now take a constant lead time and before that we are plotting these costs against Q.

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Now you see the procurement cost and the inventory holding cost. The more amount of inventory we are holding the more is the cost here and the procurement cost is total cost per year now. Let us look at the picture. This is the total cost per year equation, $K/T + C\lambda + \frac{1}{2} h \lambda T$. Now λT is Q , so it is $\frac{1}{2} hQ$. So this portion is a straight line when arising straight line.

And K/T is basically K/T so $1/T$ is λ/Q . So T comes here, now it is something like this.

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$$Q = \lambda T \Rightarrow \frac{1}{T} = \frac{\lambda}{Q}$$

$$TC/year = \frac{K}{T} + C\lambda + \frac{1}{2} h \lambda T$$

$$= \left\{ \frac{K\lambda}{Q} + C\lambda \right\} + \left\{ \frac{1}{2} hQ \right\}$$

If $Q = \lambda T$ then $1/T = \lambda/Q$, therefore TC per year $= K/T + C\lambda + \frac{1}{2} h \lambda T$ and this can be written as $1/T$ is this $K\lambda/Q + C\lambda$. This is procurement cost $+ \frac{1}{2} hQ$, this is inventory holding cost. You can now see here, this part is $C\lambda$ and that is

shown here, this is $C\lambda$ which is constant and then this one $K\lambda/Q$ is basically gives the shape such as this.

And this is the inventory holding cost because it is $1/2 hQ$ directly proportional to Q therefore it rises from here to here. So when these 2 costs are added, we get the total cost per year. You can see that at this point the total cost is the minimum and that is the value of Q^* . The value of Q at which the total cost takes a minimum is the economic order quantity and normally this curve is quite flat around this point.

That means if the Q is changing if because it is quite possible that some of our assumptions may not be correct in the sense that λ may not be correct or that the various things that we have taken is not correct then the actual Q may vary a little from the Q^* that we have calculated. This curve says that even if Q changes a little bit to the left or to the right, the total cost does not change much.

That is that it is insensitive, the total cost per year is not very sensitive to the variation in the value of Q^* because of any change in the value of C or I or whatever we may have estimated. So friends today we took up inventory control and we defined inventory, we said different types of inventories could exist. We gave motives why inventory is kept and then we consider a multi-item situation and we say that it is possible to selectively control the items inventory by classifying them as A, B and C.

We said that A class items have to be controlled with a lot of focus whereas C class items such controls cannot be necessary, by doing so we are reducing our scope meaning that we are reducing our focus, we are concentrating more on certain a few items rather than a large number of items and then we said that there are different types of costs. The 3 classifications of costs are the procurement cost, inventory holding cost and the stock-out cost.

The simplest situation is modeled at the end of the lecture and we said that it is possible to find out the order quantity and that deterministic and known situations and that is given by the square root formula known as Wilson's lot size formula and there we have seen that when to order is when the on-hand inventory is 0 and how much to order is given by $2\lambda K/h$ root over.

So we stop here today and we will continue to discuss inventory management in the next lecture also. Thank you.