

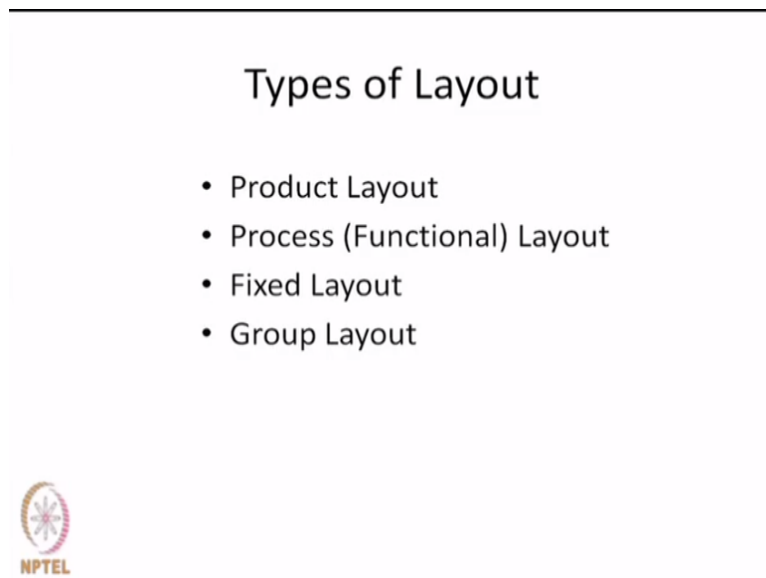
Economics, Management and Entrepreneurship
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Lecture - 40
Plant Layout (Contd.) and Production Planning and Control

Good morning. Welcome to the 40th lecture on economics, management and entrepreneurship. In our last lecture, we had discussed product service strategies and that leads to product focused organizations or process focused organization. We discussed also after that the problem of plant layout. In that we discussed 4 different types of layout. Today, we shall expand on certain specifics of the product and process layout, other layout patterns.

And then we shall start a discussion on production planning and control. So to start with we shall first relook at what we had done in our last class on plant layout.

(Refer Slide Time: 01:34)



We said that there are principally 4 layout patterns, product layout for product focused systems, process or functional layouts for processed focused systems or (()) (01:52) problems, the custom products, fixed layout for very big and heavy products and then a combination of product and process that manifests itself in the form of group layout.

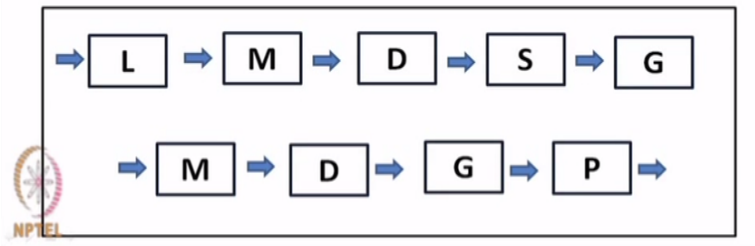
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Product Layout

Machines (facilities) are arranged according to the sequence of operations of a product or a group of related products.

Applicable for mass (or continuous) production.

Product layouts can be production lines or assembly lines.



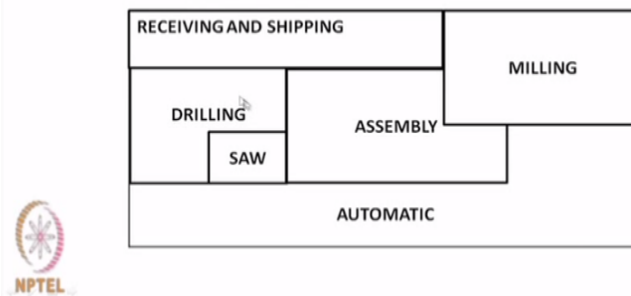
This is an example of product layout, which is in line fashion. The operational sequence and the machine arrangement are the same.

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Process (Functional) Layout

Facilities are grouped according to functions (departments or sections).

Good for products/jobs with differing operation sequences.



In a process layout, different machines or functions are grouped together in the form of a department and they are placed together.

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Fixed Position Layout

The product remains fixed at a location. Workers, materials, and equipment are brought to the location for processing.

Good when the product is heavy and of large size.



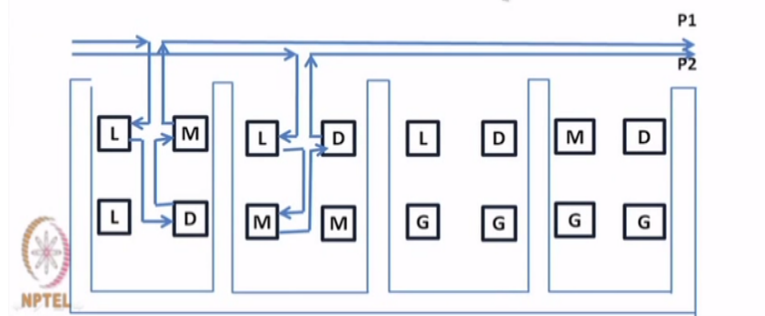
In a fixed position layout, heavy and very long or big products are placed in one place and every other machinery equipment and people they serve in one place. This is a fixed position layout.

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Group Layout

Similar parts and products, perhaps different in sizes or types of the same product are processed in cells (or groups) of machines.

Good for parts/products manufactured in medium-size batches.



And then we had talked of group layout, in this case different products say P1 and P2 they have similar sequences but not exactly the same sequence of operation. Say for product 1 P1, the sequence could be lathe, drilling machine, milling machine and then goes back whereas as in case of product 2 it could be lathe and then milling machine, then drilling machine and then goes back. So these are arranged in groups or sales.

This sort of manufacturing is also known as cellular manufacturing at the layout is in the form of group layout or in the form of cells.

(Refer Slide Time: 03:54)

Group Technology

- Families of parts with similar design characteristics (size, shape, and function) and manufacturing characteristics (type and sequence of operations).
- Group similar machines as in a process-focused system, but arrange the flow of family of products in line fashion.



A variation of that is called group technology where part families are made based on similarity of the design characteristics and of manufacturing characteristics. Design characteristics with regard to size, shape and function, manufacturing characteristics with regard to type and sequence of operations. So they are like group layout but there is a difference.

The difference is that the machines are grouped in departments just as they are in a processed focused system but the products flow in a line fashion. This is called the group technology approach.

(Refer Slide Time: 04:44)

Flexible Manufacturing System

- Integrates computer-aided design (CAD), computer-aided manufacturing (CAM), remote-controlled material handling equipment, and robotics into a highly automated manufacturing system
- Computer controls the start of parts from machine to machine as well as the start of work at each machine.




And today we are talking about flexible manufacturing system where computer-aided design, computer-aided manufacturing, remote-controlled material handling equipment and robotics makes the manufacturing system almost fully automated and here the human intervention is much less during manufacturing and very high during the designing and process planning stages.

The computer controls the start of the parts from machine to machine as well as the start of the work at each machine. These are different layout patterns and if you remember we had discussed a specific problem of product layout and in particular about assembly line, we called it assembly line balancing problem.

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Assembly Line Balancing

- Products are assembled in a continuous conveyor line.
- There are several work stations along the line.
- Several operations (tasks) are assigned to each work station subject to the constraints that
 - the operation sequence is maintained and
 - each work station takes (roughly) the same time.
- Other considerations are:
 - Zoning constraint: Certain tasks must (or must not) be grouped together.
 - Positional constraint: Certain task(s) must be done at specific locations.




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Example:

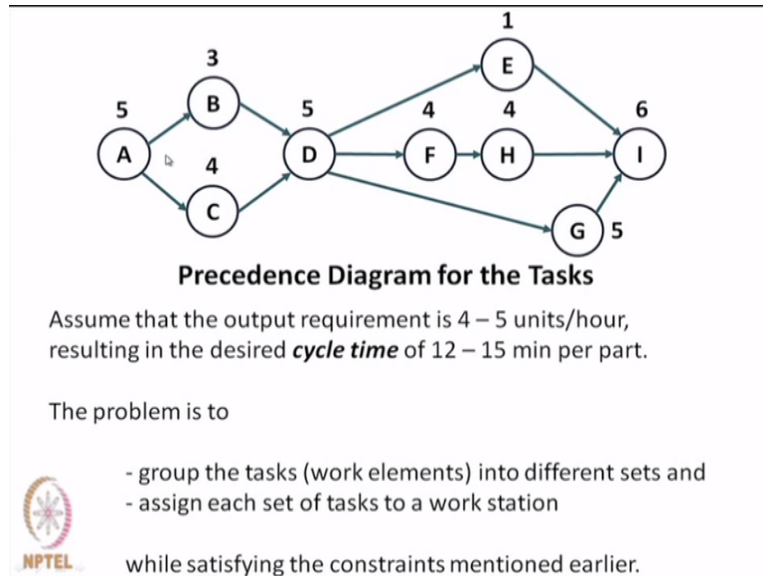
Consider a 9-task (or 9-work element) assembly problem. The *precedence relationships* among the tasks are shown below.

Element (Task) Number	Duration	Immediate Predecessor(s)
A	5	-
B	3	A
C	4	A
D	5	B,C
E	1	D
F	4	D
G	5	D
H	4	F
I	6	E,G,H



Assembly line balancing problem, we had taken an example. There are different tasks or work elements with different durations. We wanted to their immediate predecessors, which are given.

(Refer Slide Time: 06:11)




It can be represented in a network form A, B, C, D etc the work elements and the operation times or element times are given in number. So we have to group these operation times and assign them to various workstations such that precedence relationship is not violated, meaning that D cannot start unless both B and C are complete. So basic constraint must be satisfied.

And there should be a balance of the total time for which various workstations are operating. So we had followed a scheme and with the help of that we had made the arrangements.

(Refer Slide Time: 07:09)


WS 1	WS 2	WS 3
A B C	D G F	H E I
5 + 3 + 4	5 + 5 + 4	4 + 1 + 6
= 12 min	= 14 min	= 11 min
Idle Time: 2 min	0 min	3 min
Cycle time = 14 min		
But we see that the loading of the workstations is uneven.		
We can <i>trade</i> and <i>transfer</i> the stations among stations.		
For example, we can trade task G from station 2 for task H from station 3.		



A, B and C are assigned to workstation 1, D, G, F to 2, H, E, I to 3 and we found that the idle time of various workstations, they vary the maximum is 14 minutes and the minimum is 11 minutes that means workstation 3 will be idling for 3 minutes and workstation 1 idling for 2 minutes and this is very uneven. So what we do? We trade and transfer certain stations.

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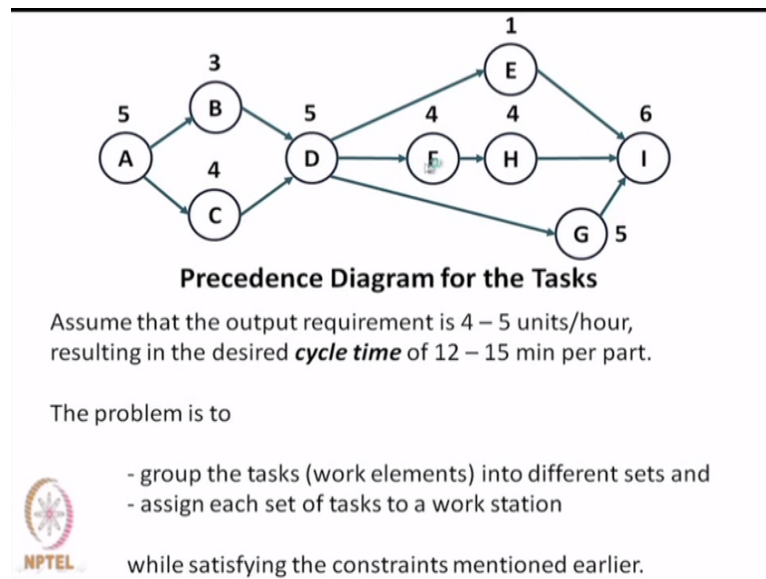
WS 1	WS 2	WS 3
A B C	D F H	G E I
5 + 3 + 4	5 + 4 + 4	5 + 1 + 6
= 12 min	= 13 min	= 12 min
Idle Time: 1 min	0 min	1 min
Cycle time = 13 min		



Now what we do here we have transferred G from workstation 2 and H of workstation 3 that means we have brought H here and G we have taken there. So G has gone here with 5 as the element time and H has come here with 4 as element time. Thus, here it is 13 minutes instead of 14 and here it is 12 minutes instead of 11 that is the idle time is to extend of only 1 minute and the total cycle time is 13 minutes.

And the precedence relationships are maintained as you will see D, F and H we can check here.

(Refer Slide Time: 08:39)



D, F, and H all the three after D is complete F can be completed and after F is complete H can be completed. Earlier D, F, and G were put in one workstation, now what we have done, we have put G to workstation 3 and from workstation 3 we have brought H to workstation 2. So 5, 4, and 4 is 13 minutes is spent in workstation 2 and workstation 3 is busy for 1+6+5=12 minutes. This trade and transfer was done purely based on trial and error method.

(Refer Slide Time: 09:22)

Improved Balance

Station	Task	Task time	Station time (S_k)	Idle time ($C-S_k$)
1	A	5	12	1
	C	4		
	B	3		
2	D	5	13	0
	F	4		
	H	4		
3	G	5	12	1
	E	1		
	I	6		

So the final allocation of tasks or assignment of tasks to stations are this. Station 1 A, C, B; station 2 D, F, H; station 3 G, E, I.

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Balance Delay

$$d = \frac{100(nc - \sum t_i)}{nc}$$

where,

- n: number of stations
- c: cycle time
- t_i : time of the i th task

For this case $d = 5.4\%$.

It will give 4.6 parts per hour.



And a measure by which different assignments are compared is by finding out the balance delay, wherever the balance delay is the minimum that is considered as the best line balancing solution. This is what we have done in our last class. We will now take up the process layout design.

(Refer Slide Time: 10:10)

Design of Process Layout

The objective of the layout is to

Minimize the cost of transporting goods between work centers.

Cost of transportation =

$$\sum_{i=1}^n x_i d_i$$

where,

- x_i is a measure of the quantity moving between work centers,
- d_i is the distance travelled during the i th movement
- n is the total number of movements



A good layout minimizes the distances travelled d_i .

Design of the process layout, if you recall process design is done for custom products. If the company or the enterprise is manufacturing custom products, then various machines are grouped together and they are placed in one department. Now the question is how the department should be relatively located relative to each other. Now the basic principle that is followed in the design of the process layout is to minimize the material handling cost or the internal transportation cost.

Now the cost would depend on 2 factors, one is the number of trips that is required to be made between various workstations and the volume or weight of the products. These two will basically determine how many trips are made and what is the cost of material handling. Now we are not going to make a costing of the material handling, it is good if we can make a natural costing.

We shall assume on the other hand that the transportation cost or material handling cost is dependent on these 2 factors, the weight of the product which is transported from one department to another department and the distance between these 2 departments. Now since the weights cannot be controlled in the design of the process layout, we will try to minimize the distance that the parts are moving.

So that is the approach we take and normally once again a judgment based trial and error process or method is used; however, it is backed up by certain objective and graphical methods. Let us see what the methods are. So basically the objective of a process layout is to minimize the cost of transporting goods between work centers. So the cost of transportation is sum of $x_i d_i$ where i varies from 1 to n , n is the total number of movements.

X_i is the measure of quantity whether it is in volume or weight we have not mentioned or number we have not mentioned. It is the measure of the quantity moving between work centers and d_i is the distance travelled during the i th movement and a good layout minimizes the distances travelled d_i because x_i is more or less fixed. Therefore, only d_i we can control. So we would like to therefore minimize the distance of that the parts move.

(Refer Slide Time: 13:33)

Data on Product, Quantity, and Operation Sequence

Input data

Product	Quantity per month	Operation sequence
I	200	A-B-C-B-D
II	100	A-C-D-C-E
III	300	C-B-D-C
IV	200	B-C-E



Now we will take a specific example to illustrate what we are saying or how to solve this problem. We are assuming that there are 4 different products I, II, III, IV, and the quantity produced per month are 200, 100, 300 and 200 and the sequence of operation for various products are given. So you can say that this is an operation and even where that each operation will correspond to a department.

So A-B-C-B-D this is the way the product will move, the material will move. So that after the operation D is complete, the product is ready. Product II passes through these operations or these departments A-C-D-C-E, this is the sequence after A-C then D then C then E. Product III the sequence is C-B-D-C, product IV the sequence is B-C-E. So there are thus 5 departments or 5 operations but the sequence in which each product passes is different.

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From-to Chart

Travel Chart

To From	A	B	C	D	E	A	B	C	D	E
A	-	200	100			A	-	200	100	
B		-	200	200		B		-	900	200
			200							
				300						
C		200	-	100	100	C			-	500
				200	200					300
		300								
D			100			D			-	
			300							
E						E				



Now what we first do is to have a from-to chart. So these are various departments A, B, C, D, E or various operations A, B, C, D, E. So from this to this it is moving. So actually these are 2 tables, 1 is up to E up to this place and the second table is from A, B, C, D, E to this. So let us look at the from-to chart. This is called a from-to chart. So from A it is going to B and the amount is A to B the quantity per month is 200.

So this is 200 A to B, then A to C product II is moving the quantity is 100, so it is 100. Then, B to C and D let us look at that B to C the quantity moving is 200 and then once again here product IV it is going from B to C 200. So 200 and 200 B to C this one is 200. Then there is also a flow from B to D look at that B to D here is this one is 300 product III. So B to D this is 300 and is there any B to D, here is one B to D it is 200.

So B to D is 200 here, so like this we have written down the quantity moving from one department to another department. Now you see here A to B is we are now not considering whether the movement is taking place in a reverse direction. For example, let us consider this B to C $200+200=400$ quantities are flowing and then C to B there is once again 200. C to B here this one C to B is 300.

This is here and C to B probably there is another one somewhere C to B is here, it is flowing 200 so this is C to B 200 and once again C to B 200. So these are 500 and earlier we have seen B to C 200 and 200 400. Therefore, the quantity moving between B and C in either direction whether from B to C or C to B it does not matter. In either direction, the total movement is $200+200+200+300$ that comes to 900.

So the travel chart is basically a consolidation of the from-to chart where the direction of movement is disregarded. Thus, this 900 source that the amount traveling between B and C in either direction is 900. So from here we can derive the travel chart. So A to B is 200 because there is nothing from B to A. A to C is 100 because there is nothing from C to A. B to C is 900 as I have calculated.

B to D is 200 because there is nothing from D to B. Now come to C, C to A there is nothing because we have already taken care of C to A. Therefore, this is an upper triangular matrix there is nothing in the lower below the diagonal, is all the points all the values are coming

above the diagonal. For example, C to B we have already taken care of when we considered B to C, D to A, B, C we have already considered when we considered A.

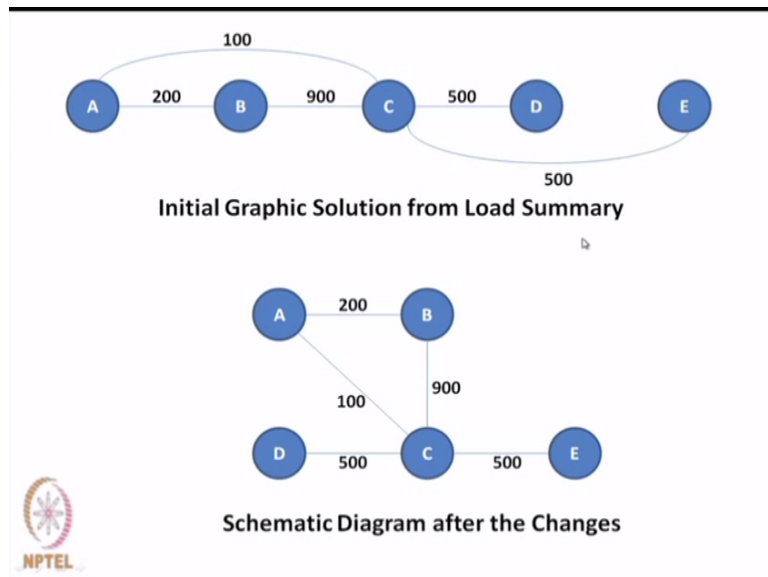
Thus these are all 0s, so here we have C to D, C to D is 100 and D to C is 100 and 300. So total 500, $100+100+100$ so this is 500 here C to D and then finally between E and C. C to E is 200 here and C to E is 100 here, thus C to E is 100, C to E is 200 so C to E is 300 and between C and B let us see how many C and B 200 and C and B 300. So this entry is okay 500.

This is okay but between C to E, it is 100. Now we are considering D to E okay, now D to E there is nothing, C to E is 100 here and C to E is 200 here or total 300. So that is what we have written C to E 300 but we should have shown it here. C to E is 100 and C to E is again 200 that is missing, so there should have been another here. So that makes it alright now C to E now is 300 and that is what we have done here.

So now we have the quantity traveling between different departments are now calculated here. So what our strategy would be that if between A and B the amount traveling is 200 and between C and D the amount traveling is 500 maximum is B to C is 900 then B and C should be as close as possible to each other. Followed by C and D, the amount traveling is 500, therefore C and D should be as close to each other as possible.

Now D is not connected with A, B and C. D is not connected with A is here, not connected with A. Therefore, D can be away from A but cannot be away from B or C. So this is the way we do.

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So what is now done is that from these values we construct an initial graphic solution where we show almost in a linear fashion first of all, all the departments or operations. So these are A, B, C, D, E and the quantity traveling between various departments are written down here along the arc that joints these notes. So between A and B the amount traveling is 200, between B and C 900, C and D 500, A and C 100 and C and D 500.

Now let us put them as close to each other as possible. Now this is one way in which we can put it. A and B they are close to each other and A and C is also close to one another. So instead of putting them in a linear fashion, we put it here because this distance is 100 it can afford to be a little longer and C can be close to 900, D can be somewhere here because E has to be close to 100 C can be here.

So this is purely heuristically we can put the relative location of the departments. So the ones that carry highest amount of material they should be as close to each other as possible. The one that carries least thus they can be little away; the diagonal distance is higher than this distance. So this is a purely based on judgment.

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Space Requirement	
<u>Machine-related</u> Machine Machine travel Machine maintenance Plant services	<u>Materials-related</u> Receiving and storing In-process Storing and shipping Storing and shipping of wastes and scraps Tools, jigs, fixtures, dies, and maintenance materials
<u>Personnel-related</u> Operator Material handling	<u>Aisle</u> Between objects for movement of persons For movement of material handling equipment



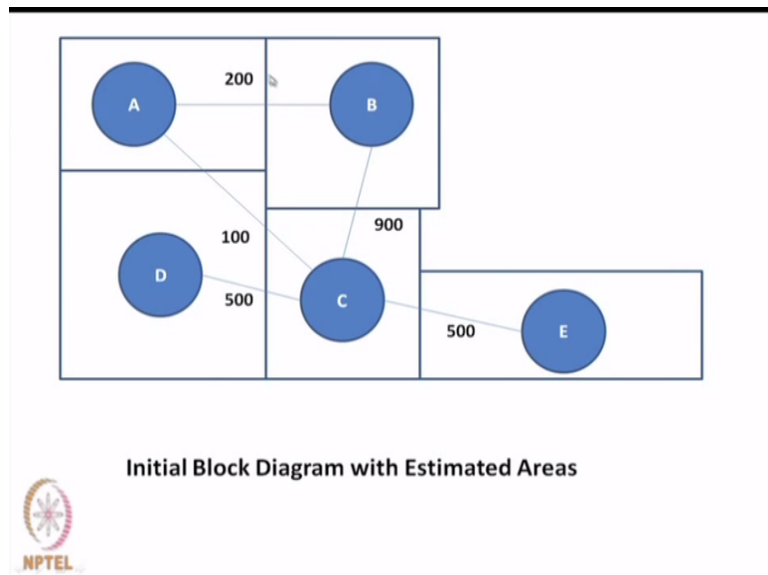
Now we consider the amount of space that is required for every department. Now there are different considerations. Firstly, the machine consideration, material consideration, people consideration and aisle that must be left between machines for movement of material handling equipment and for persons. So every machine has a volume and a certain machine also needs to travel a little.

Then for machine maintenance and for other plant services, we need some space. In fact, they should be calculated. For materials, materials have to be received and between different machines in-process inventories built up a space must be available there. After the work is completed, space must be there for storing and shipping them and if there are wastes and scraps, they must be stored and shipped.

Additionally, we also need space for tools, jigs, fixtures, dies, and maintenance materials. Then, there is a space required for movement of the operator for operator himself and even for material handling equipment and there must be some path or aisle left for a person to move and for material handling equipment to move. All these are required to calculate the space requirements.

And there are standards if it is a fork-lift and how much should be the aisle space, aisle width. If it is some other thing, then what should be the aisle. There are standards that are available for various machines and for various aisles.

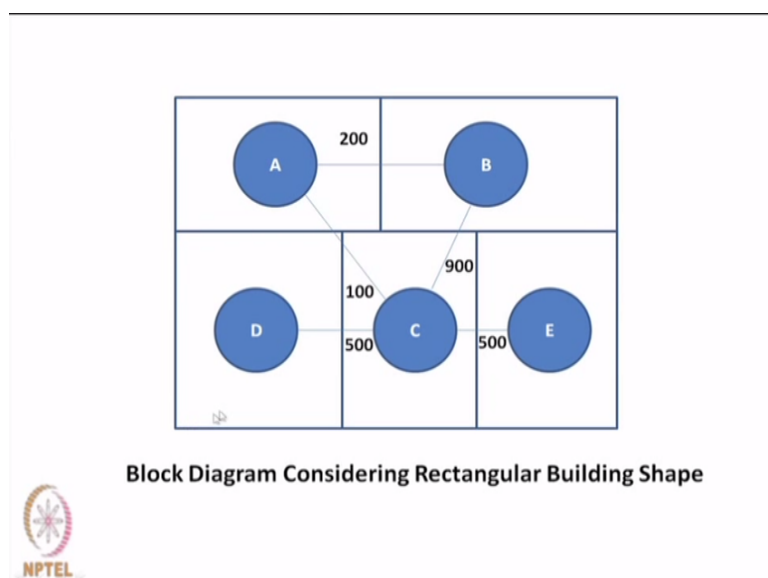
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Once this space is estimated for every department, we already know the relative location of A, B, C, D, E. Now this space requirement for A could be this, for B it could be this, for D it could be this, for C it could be this, E it could be this. So first of all we make an initial block diagram with estimated areas. Now you can see that this is very irregularly set but the actual set could be a rectangle or could be a square.

Hence, one has to now reconfigure this diagram so that this total area remaining same it is something like a rectangle or a square.

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That is what we have done here. So we are keeping the relative location similar. We have tried to accommodate it within a rectangle and this is the block diagram considering rectangular buildings set. Now this is a high level design, one can of course decide what

should be the aisle, how the exact machines in the department D should be put, whether it should be linear fashion or it should be a separate thing.

These details time does not permit me to discuss the details of how the machines in a particular department should be laid out, where the entry and where the exits should be, where the material handling equipment should move, where should be the space for vests, for finish products and for in-process material, where is the place for tool room etc we are not discussing these details.

But we have now at least discussed in the context of process layout, how relatively the departments should be laid out. Now the basic consideration is the load that is transported from one department to another department and forever large amount of load is traveling between 2 departments, those 2 departments should be as close to each other as possible. So this is the principle that is applied.

And there are various software packages such as craft and other such packages that can be used to help a designer of a process layout problem. So friends we have now discussed different layout problems and 2 specific assembly line balancing and process layout design problems we have discussed in some detail. Now we shall take up the production planning and control aspects of an enterprise.

So the topic we are now starting is production planning and control. Now before an enterprise starts manufacturing a job, first thing to do is to plan for production. Naturally, planning would require annual plan, monthly plan, weekly plan, how much to produce. Then each plan must be broken down into schedules that says that this machine should be loaded at this point of time for this particular job and the job must be completed, at this time on this day the next job should start.


These details are who will do the job, which operator will do the job, how the job should move from one department to another department who is going to authorize the operator to start the work, how much material should be procured from the stores, which tool should be used, all these are planned in advance and this is usually done by the production planning and control department.

This is like the nerve center of a manufacturing organization. So let us study what the different functions of a production planning and control department fondly called PPC department, let us see how it functions.

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Production Planning - Definition

Production planning is concerned with translating overall sales orders and plans to specific schedules and meeting them by efficiently coordinating and integrating the factors of production.



Firstly, a definition production planning is concerned with translating overall sales orders and plans to specific schedules and meeting them by efficiently coordinating and integrating the factors of production. So the input to production planning, basic input is sales; however, to be able to meet the sales that the company has the demand or the order that the company has received.

It has to know what its present machine capacities are, how much of raw materials it is having, whether it is already having certain finish product, inventory of that product, how many people are available, whether certain machine has broken down, whether the tools that are required are available for the product I am saying for manufacturing of the product. So these details the production planning control department must have in order to commit how much it can produce and deliver to the stores for final shipment to the customers.

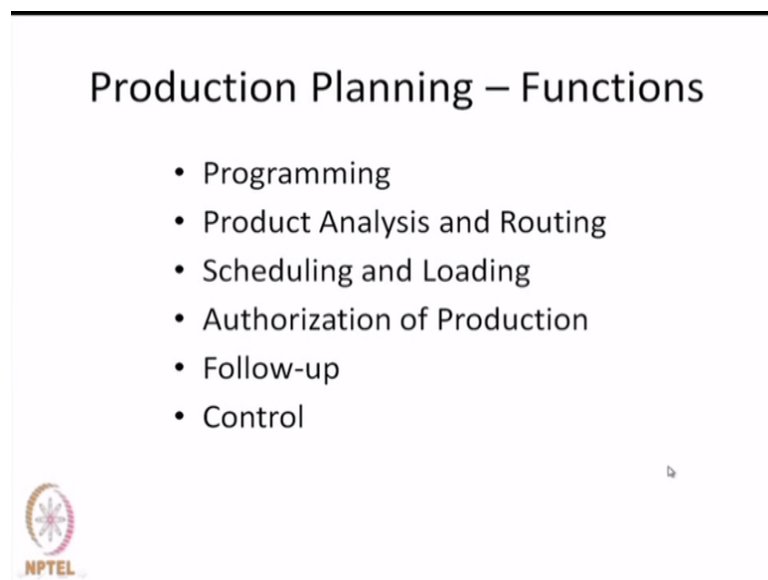
So a lot of information must be available. Once this information is available the next question is how best the information can be utilized to prepare a plan. Normally, as we shall see lot of mathematical tools are available to help formulate a planning problem and solve it. Some of them take the problem as static problem given a particular order how to solve it, some of them take it as dynamic problem.

Things are changing, the sales are changing with time, the inventory are changing with time, machine availability is changing with time. Some consider only the production planning as a problem how much to produce and some consider not only production but also production and distribution.

Some consider the planning as not to just produce but produce how much of overtime to be used, how much of subcontracting should be used, whether one should go for additional number of workers or it can lay off certain contract workers. Now these details which were normally done or which for a long time are still being done in some companies completely subjectively and now being done mathematically using mathematical models.

Now time does not permit us to go into those mathematical models but we shall definitely give you an idea as to how such models are made. Now let us consider these details now.

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The production planning functions can be grouped in 5 headings, production planning proper which is I am calling programming, product analysis and routing, scheduling and loading, authorization of production, follow-up and control. I think I should add here another point that is control.

(Refer Slide Time: 37:46)

Programming

- Preparing production forecast from sales forecast
- Making master production schedule
- Planning material procurement
- Preparing a schedule of personnel
- Making a schedule of operating capital
- Planning alternative strategies



Now first programming, prepare production forecast from sales forecast. Make master production schedule. Plan material procurement, how much raw material to procure. Prepare a schedule of personnel, how many people are required every month, make a schedule of operating capital, something like a budget, how much money we require and plan alternative strategies.

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Master Production Schedule

Dept	Jan	Feb	...	Dec
A	10	15		
B	8	10		
...		
K	20	10		




Now this is a programming function, these 5 or 6. Master production schedule almost looks like this. It is made for the whole year. It is something like an annual plan, month wise January, February etc up to December. These are the departments. In terms of the output of the department, it specifies how many units are expected as output from every department at the end of every month.

So the department head knows that at the end of the month how much to make, this is an ultimate output prepared in the programming function of PPC as a master production schedule MPS.

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**Coping with
Widely Fluctuating Demand**

- Influence demand by price cutting, etc.
- Expand product line by including products with less demand variability
- Build up inventory in lean time
- Organizational adaptation – overtime, using part-time workers
- Subcontracting
- Aggregate planning



Now we shall take off a situation when the demand fluctuates almost seasonally. Now when demand fluctuates then it influences demand by price cutting. The company can cut the prices if the demand is less. It can cut its price in order to induce the customer to buy more so that the fluctuation is less. Expand product line by including products with less demand variability.

Already we know this product line diversification to meet seasonality in the demand, a company can have more number of products, product type such that the variability in the total aggregate demand is less or it can meet the high demand by producing more and keep it as inventory in the lean time or it can go for overtime or using part-time workers or can subcontract.

It can also go for aggregate planning. So almost these things we have discussed earlier. This is a new term aggregate planning, which I have introduced in the context of production planning and control. Let us study that in some detail.

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Aggregate Planning

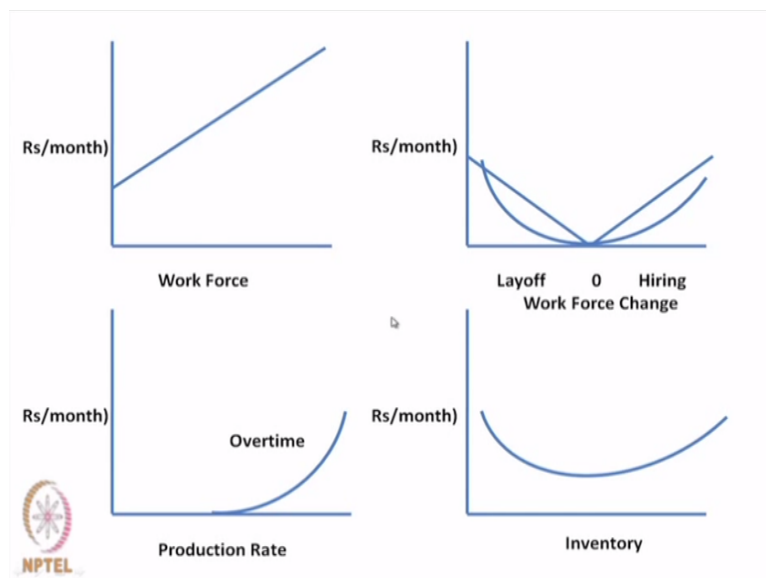
- Plans for medium term (usually for one year),
- Aggregates all types of products (tons or units of output, all forms of workers (labour-hours), and all forms of facilities (machine-hours),
- Forecasts in these aggregate terms,
- Estimates variation of costs (of inventory, payroll, layoff/hiring, production change, etc.) with volume change.
- Necessitates a second-stage planning



Aggregate planning is usually done for one year. It aggregates all types of products. It does not make specific plans for individual products instead it aggregates all types of products and expresses it in terms of tons or units of output. It aggregates all types of workers in terms of labour-hours and all forms of facilities in terms of machine-hours. Thus, it forecasts only in aggregates terms not specific product type, all products put together was the forecast.

That is why the name aggregate planning. Estimates variation of costs such as inventory, payroll, layoff hiring, production change, etc and thus for actual planning of production of individual products one has to go for a second-stage planning. Now let us see how it is done.

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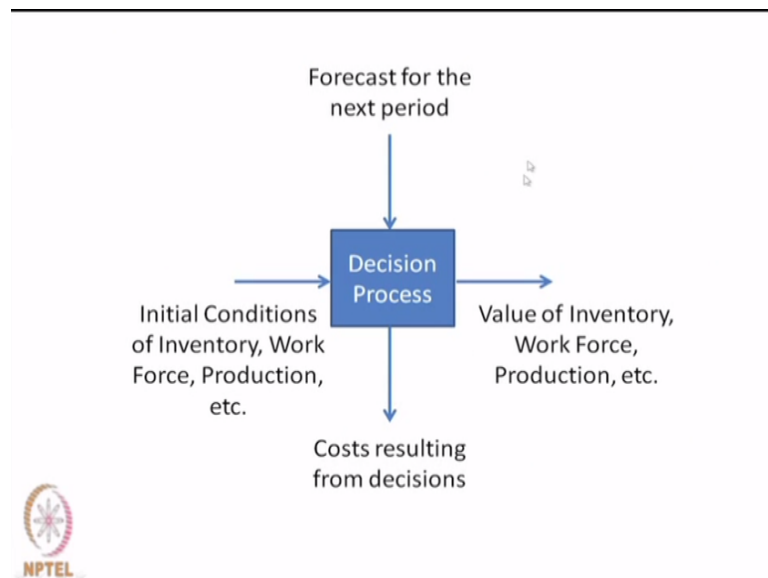
Normally, it assumes that the costs vary either in a linear fashion or in a quadratic fashion. For example, work force increases, as work force increases their wages increase is linear.

Now suppose there is a possibility for an enterprise to either lay off workers or hire more workers, then the workforce changes as an effect on the (()) (43:19) payment and it usually goes up in a linear fashion in this fashion and in this fashion if it is laid off you have to also give them more amount of money to lay them off just as if you hire you get pay more.

So this sort of structure can be approximated by a quadratic cost function. When one pays overtime to increase production rate then also it goes up and that also can be approximated as quadratic and inventory normally at the particular value of inventory the cost is the minimum, we shall consider this in more detail when we discuss about inventory control.

But at this moment take it for granted that if inventory increases the value associated with it increases and if inventory is less then also the value increases because of opportunity cost associated with it. So all the cost at least these 4 costs the production rate change, the inventory change, the workforce change can be approximated as quadratic and this cost remains more or less linear.

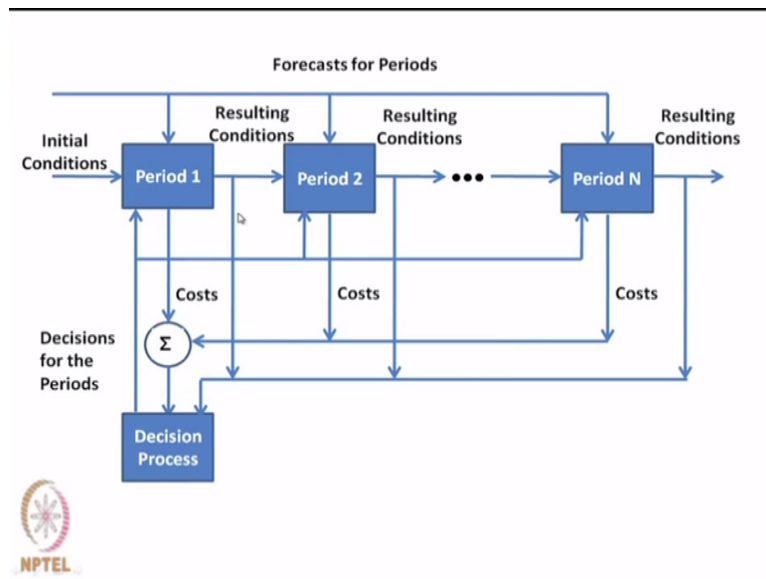
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So aggregate planning what it does it gets the forecast for the next period, already we have discussed how different forecasting methods can be used and it uses its cost functions and it gets information regarding inventory, workforce production etc and uses a decision model to calculate the cost resulting from the decision and it tells how much inventory should be built up, how many work force to deploy and what should be the production.

So there is a mathematical model usually it is a quadratic problem and one can find out this value.

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It is given in much more elaborate form in this fashion for every period, period 1, 2 to period N forecasts are met and initial condition is known and all the costs that result they are added up here. Now the decision is made and the fed back. This decision is fed back for period 1 and then fed back to period 2 and then fed back to period N. So a model that is made for N period it is basically it takes the complete model and then solves it.

This is the structure of an aggregate planning problem. I did not discuss this in more detail because the model is quite complex and it is difficult to discuss here at this stage; however, we shall consider a small problem, problem of product mix problem.

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Optimal Product Mix Problem

Example:

A manufacturer produces four types of household products fabricated from sheet metal. The production rates in hours per unit and available production hours are given known for different departments. Also known are their selling prices, variable costs, minimum and maximum sales. Furthermore, only 2,000 sq. m of sheet metal used for products 2 and 4 are available. Product 2 requires 2 sq. m and Product 4 1.2 sq. m per unit.

Find out the units of the four products that the manufacturer should plan for.



A product mix problem says that we have different products and how much of these products should be manufactured, so that it maximizes the contribution to profit and satisfies the constraints. This we will take up right away. Let us take this example. A manufacturer produces 4 types of household products fabricated from sheet metal. So there are 4 products. Finally, we are asked how many units of each of these products have to be manufactured of course to maximize the profit.

The production rates in hours per unit and available production hours are given and are known for different departments and given on the next slide but are also known are their selling prices, variable costs, minimum and maximum sales that are given in the next slide. Furthermore, the sheet metal available is 2000 square meter, which are used for products 2 and 4 and product 2 and 4 require 2 square meter and 1.2 square meter per unit each.

So there is a constraint as far as the sheet metal is concerned.

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Production Rates in Hours per Unit					Production Hours Available
Dept	Product 1	Product 2	Product 3	Product 4	
Stamping	t_{s1}	t_{s2}	t_{s3}	t_{s4}	T_s
Drilling	t_{d1}	t_{d2}	t_{d3}	t_{d4}	T_d
Assembly	t_{a1}	t_{a2}	t_{a3}	t_{a4}	T_a
Finishing	t_{f1}	t_{f2}	t_{f3}	t_{f4}	T_f
Packaging	t_{p1}	t_{p2}	t_{p3}	t_{p4}	T_p

	Product 1	Product 2	Product 3	Product 4
Net Selling Price	P_1	P_2	P_3	P_4
Variable Cost/unit	v_1	v_2	v_3	v_4
Min Sales	S_{1min}	S_{2min}	S_{3min}	S_{4min}
Max Sales	S_{1max}	S_{2max}	S_{3max}	S_{4max}

Now look at this table. It says that each product passes through 5 departments. Stamping, drilling, assembly, finishing and packaging and the time it takes in different departments are given, t_{s1} stamping product 1, t_{d1} drilling time for product 1, t_{a1} assembly time for product 1 etc. So accordingly of course the numerical values are not given symbolically the values are given here.

But the actual values are given are have to be known and in a year or in a month whatever is the time for which we are making the plan stamping time available is T_s total time available in that let say month. Drilling time in that month is T_d , assembly time t_a , finishing time T_f , packaging time T_p . So you can very well understand that suppose we produce product 1 X_1 in number, product 2 X_2 in number, product 3 X_3 in number, product 4 X_4 in number.

Then the total time consumed for product 1 in stamping is $X_1 * t_{s1}$, product 2 takes $X_2 * t_{s2}$ time, $X_3 * t_{s3}$, $X_4 * t_{s4}$ and all these time in stamping must not exceed the total time available T_s . So this is a constraint $X_1 t_{s1} + X_2 t_{s2} + X_3 t_{s3} + X_4 t_{s4}$ should be $\leq T_s$. Similarly, for drilling, assembly, finishing and packaging.

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Let X_i be the number of products of product type i , $i = 1, 2, 3$, and 4 .

Machine Constraints:

$$\sum_{i=1}^4 X_i t_{si} \leq T_{si}, \sum_{i=1}^4 X_i t_{di} \leq T_{di}, \sum_{i=1}^4 X_i t_{ai} \leq T_{ai}, \sum_{i=1}^4 X_i t_{fi} \leq T_{fi}, \sum_{i=1}^4 X_i t_{pi} \leq T_{pi}$$

Material Constraint:

$$2X_2 + 1.2X_4 \leq 2000$$

Sales Constraints:

$$S_{\min} \leq X_i \leq S_{\max}, i=1,2,3,4$$

Objective Function: Maximize $\text{Max} \sum_{i=1}^4 (P_i - v_i) X_i$

Non-negativity Restrictions: $X_i \geq 0, i=1,2,3,4$

This is what I have written down here, machine constraints okay. This should be $\leq T_{si}$ etc, 4 products X_i is the number of products or product type * T_{si} $X_i T_{di}$ etc. Now there are other constraints when we come to this table. The minimum sales and the maximum sales for each product type is given that means the constraint is that X_1 should be between S_1 minimum and S_1 maximum.

X_2 should be between S_2 minimum and S_2 maximum, X_3 between S_3 minimum and S_3 maximum, X_4 between S_4 minimum and S_4 maximum. That we have written down here sales constraint that is excised between S_i min and S_i max. Now there is a material constraint if you recall product 2 and product 4, 2 and 4 they require a particular sheet metal, available area is 2000 square meter.

And each unit of product 2 requires 2 units, product 4 1.2 square meter, so $2X_2 + 1.2X_4$ should be ≤ 2000 . I have written $2000=2000$ it should be \leq , please make a correction here. It should be \leq . This is a material constraint and there is another restriction called non-negativity restriction that each X_i must be either 0 that is do not produce or produce something, X_i should be ≥ 0 .

So we have machine constraints, material constraints, sales constraints and non-negativity restrictions and then what we are supposed to do we maximize an objective function, which is maximize the contribution. Contribution is the price-the variable cost that is the contribution from the i th product, X_i we are producing, sum over $i=1$ to 4 that should be maximized. I think we do not have to write twice.

We can take it out. So friends basically what I did so far in the beginning I completed whatever I wanted to discuss in plant layout and then I started a discussion on production planning control. Two things I said under programming aspects of production planning and control one is aggregate planning in which I said that irrespective of the type of product that a company is producing, type of machine it is having, type of work force it is having it makes an aggregate plan.

And it tells how much to produce of all demand all products put together or how much workforce it can hire or it can deploy no matter where and how and how much to produce. We only gave the model structure without going into the details but next we gave introduced a product mix problem. How many products of each product type should be produced under certain constraint and this is called a linear programming formulation.

The objective function which was to maximize the contribution to profit was linear and the constraints were also linear. Standard linear programming packages are available to solve linear programming problems. If one solves this problem, it will give the optimal values of X_1 , X_2 , X_3 , X_4 meaning it will say how many products to manufacture of each product type.

Production planning control and control literature is rampant with various mathematical models, static models, dynamic models that considers every period as one stays, it can assume deterministic demand pattern or it can assume stochastic demand pattern that means demand changes with time with the probability distribution. There can be different other types of constraints that can be taken.

A systemic outlook can be taken of production and distribution whereas one can only discuss production. So production planning and control literature is reached with different mathematical models. We do not have time or scope to discuss these mathematical models but we have introduced only the most basic ones. In our next class, we shall discuss on scheduling and other functions of production planning and control. Thank you.