Economics, Management and Entrepreneurship
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Lecture - 03
Elasticity of Demand

Good morning, welcome to the third lecture on Economics, Management and Entrepreneurship. Today, we are going to talk about Elasticity of Demand, before that I would like to go through the slides that I presented in my last lecture on market equilibrium. This was done in the last lecture that was on demand and supply that exists in market equilibrium, I will quickly go through these slides once again, so that we can give the new ideas in the lecture 3 on Elasticity of Demand.

Here, we first gave certain definition of market, demand, supply and market equilibrium. Then we said that there can be different types of competition, perfect competition and imperfect competition, most of our discussion will be assuming that perfect competition exists in market. And then we gave different characteristics of the different types of competition, we talked about demand has been direct demand for consumption or derived demand.

Then we talked about demand function, there we said that demand is a relationship between the quantity demanded in the market, and such other factors like price, price of other goods, income so on and so forth. This is an example of a demand function, then we talked about demand curve where we said that the demand is a function of price alone. Normally, as price increases demand for the product falls, so this is an inverse relationship between demand and price which is called a law of demand.

This is given by a straight line with the negative slope, we can show the demand curve has been shifted from its nominal value when certain other factors other than price is different from the one at which the original demand curve was plotted. Similarly, we talked about supply, supply function and example of supply function. Then we said that supply curve is the relationship between quantity supplied by the firm or by industry.

And the market price alone are not assuming all other factors that affect the quantity supplied to be fixed at certain specific levels. And normally when the price increases, there is a motivation for the firms to produce and supply more to the market, so the slope of the price supply curve or the supply curve is positive. And the effect of other factors can be shown by shifting the supply curve either to the right or to the left.

And then we said that market equilibrium exists when the demand supply demand of the market and the quantity supplied to the market they are equal. This is what we did in the last lecture, today we talk about elasticity of demand. Basically, elasticity of demand will indicate how much the demand will change or what fraction or what proportion demand will change for a unit percentage change in one of the factors.

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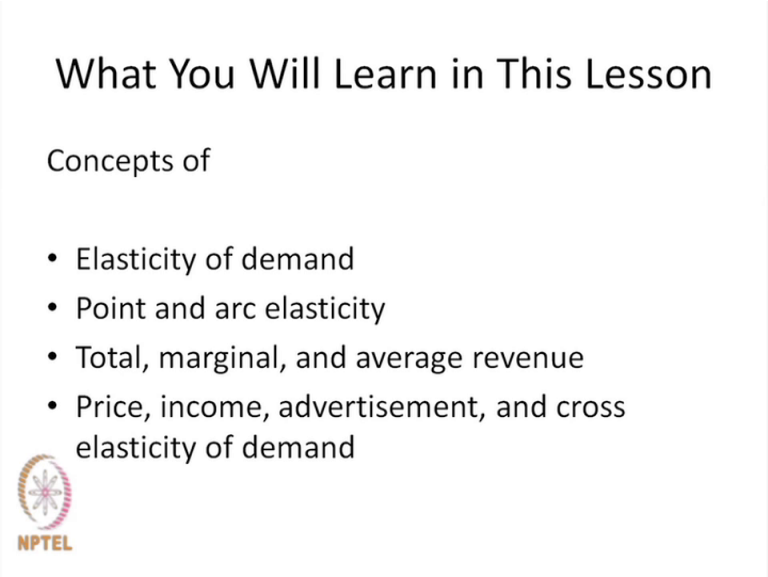

So here we will learn a few things, first the concepts of elasticity of demand. In that context we shall define point elasticity and arc elasticity, we will introduce total revenue, marginal revenue and average revenue. And then we will introduce various types of elasticity of demand such as price elasticity of demand, income elasticity of demand, advertisement elasticity of demand and finally cross elasticity of demand. So let us take up these concepts one by one.

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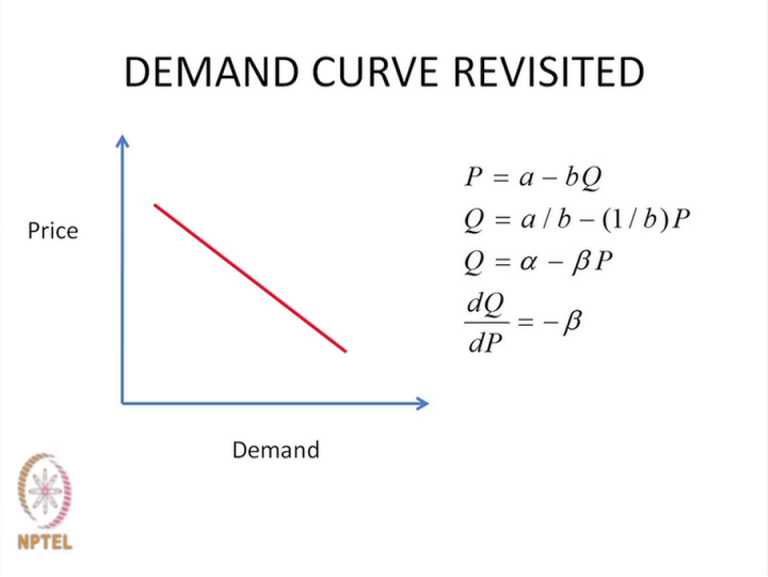

As I so do just now a demand curve is basically it will have a negative slope, we are assuming straight line relationship between price and demand, so the equation of this particular line will be P=a-b Q, and then Q can be expressed as a function of P, so it is a/b-1/b*P the price, Q is the quantity demanded in the market. And assuming a/b as alpha, and 1/b as beta, we get quantity Q=alpha-beta*P.

Now this is the demand curve equation expressing quantity demanded as a function of price P, so from here we find out the first differentiation of Q with respect to P which is basically the slope of the supply curve and that =-beta, beta being positive, -beta is negative, and therefore, the supply curve has a negative trend.

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ELASTICITY OF DEMAND

Demand elasticity is a measure of sensitivity of the demand to a change in the factor influencing the demand.

It measures the percentage change in the demand due to one percentage change in a factor.

Other factors are held constant at specified values.

Assume *Q* as the quantity demanded and *X* as the factor.

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We define demand elasticity as a measure of sensitivity of the demand to a change in 1 factor influencing the demand, assuming that other factors remind at certain specific values. So basically we will try to find out is 1% change in the factor brings in how much percent change in the demand Q, so basically it measures the percentage changes in the demand due to 1% change in 1 factor, so other factors are held constant at specific values.

We assume any other factor as X expecting the quantity demanded Q, assuming this let us develop the expression for demand elasticity that is we are trying to find out estimate the percentage change in Q demand when 1% change is made to one of the factors X.

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Point Elasticity:

$$\varepsilon = \frac{\%\Delta Q}{\%\Delta X} = \frac{\Delta Q / Q}{\Delta X / X} = \frac{\Delta Q}{Q} \div \frac{\Delta X}{X} = \frac{\Delta Q}{\Delta X} \times \frac{X}{Q}$$

It is useful in predicting the effect of small change ($\Delta X/X <= 5\%$) in *X*.

Arc Elasticity:

$$E = \frac{Q_2 - Q_1}{(Q_2 + Q_1)/2} \div \frac{X_2 - X_1}{(X_2 + X_1)/2} = \frac{Q_2 - Q_1}{X_2 - X_1} \times \frac{(X_2 + X_1)}{(Q_2 + Q_1)}$$

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It is useful in predicting the effect of "substantial" change ($\Delta X/X > 5\%$) in *X*.

Now in this context, we define 2 types of elasticity, point elasticity and arc elasticity. Point elasticity is basically percentage change in Q/percentage change in X the factor, so it is basically fractional change delta Q/Q/fractional change in X delta X/X, multiplication 100 in the numerator and multiplication 100 in the denominator cancel out leaving only the fractional changes delta Q/Q/delta X/X.

So basically we are trying to find out here, if we give the 1% change in X or 1 fractional change in X, how much fractional change in Q occurs? Now this can be written as delta Q/Q/delta X/X which=delta Q/delta X multiplication X/Q, now we said it is a point elasticity because this requires the value of X and Q to be known at a certain point, at that point the slope is to be calculated. And therefore, this is the elasticity at that point in the demand curve.

And normally when there is a very small change in the value of X, for example the fractional change is about 5% the percentage is only just about 5% in the factor X, this is considered to be a small change occurring at the existing value of X, so there point elasticity is relevant. However, when we bring in a substantial change in the factor X, and conventionally suppose that the fractional change is more than 5%, we consider that changed to be substantial.

When that happens we no longer go for point elasticity competition, we consider that will change its essential. Therefore, we try to find out the change that has occurred from the existing value Q1 to a new value Q2 as a result of the factor X changing its value from its initial value of X1 to its final value of X2. It is defined as the change/the average value of Q/the change in X/the average value of X, so 2 2 cancels out leaving Q2-Q1.

We bring in X2-X1 from here to the denominator and take Q2+Q1 to the numerator and then division therefore, this becomes multiplication this comes towards to the denominator. So the arc elasticity is defined as Q2-Q1/X2-X1 multiplication X2+X1/Q2+Q1, this is normally known as arc elasticity, whenever the change in the factor is more than 5% we consider it substantial, and apply this expression for arc elasticity. And when the change is small < 5%, we apply the concept of for the equation of point elasticity.

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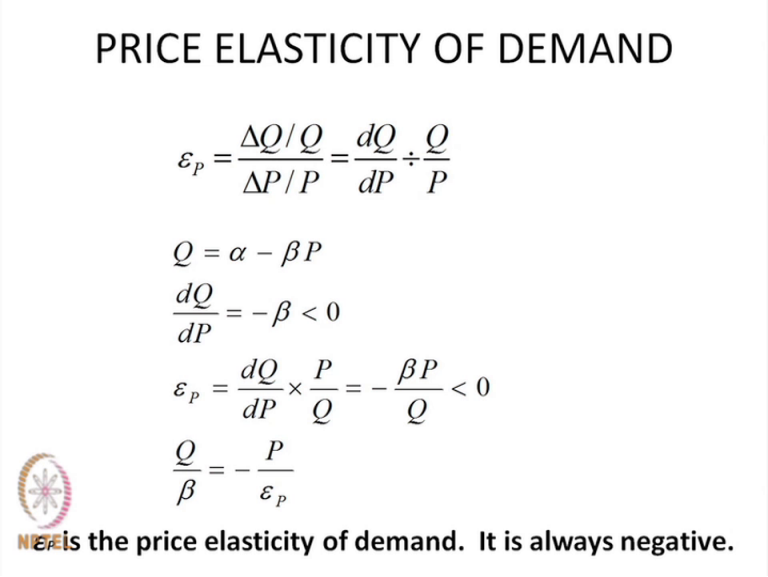

Now in the earliest slide, I just said a factor X without mentioning what that factor is, let us now consider that factor is price, which means that we are now interested to know what is the price elasticity of demand. That means we are interested to know if a change of 1% is given to price then, what is the amount of change? Or what is the percentage of change in the quantity demanded?

We designate this as epsilon P, epsilon for elasticity and P for price elasticity, using the previous equation it is delta Q/Q/delta P/P. So percentage change in P and how much percentage change in Q. Now if we assume the demand curve to be continuous, then we can take this as the first derivative of Q with respect to P, so instead of writing del Q delta Q and delta P, we can write d/dP of Q that is the first derivative of Q with respect to P, and division Q/P or multiplication P/Q same thing.

When we multiply P will come to the numerator Q will go to the denominator. Now assuming that the relationship between Q and P is straight line relationship, we give Q as =alpha-beta P, this is a straight line relationship with a negative trend as we know –beta, so Q=alpha-beta P. And if we take the first derivative of Q with respect to P, we get d/dP of Q=-beta, beta being positive, -beta becomes negative.

Therefore, dQ/dP is always negative meaning that as price increases quantity demanded falls, as we know from our earlier knowledge on the demand curve. Now knowing the value of dQ/dP=-beta, we can find out the price elasticity epsilon P, epsilon P from our first equation is d/dP of Q/Q/P which becomes =this one which is dQ/dP multiplication P/Q, and now that we know that dQ/dP is -beta this then becomes -beta*P/Q.

And since P is positive, Q is positive, beta is positive, but -beta makes it negative. Therefore, epsilon P is negative, this means that the price elasticity of demand epsilon P is always negative. So and from here we can also have an intermediate result which we shall use later which is that Q/beta, we take this to the left hand side Q/beta=-P/epsilon P, so this comes to the denominator this goes this side to the numerator beta comes here.

This is an intermediate result that we shall use in our next slide, so here we learn that price elasticity of demand is always negative, as we know as price increases demand falls. So the price elasticity of demand is expected to be negative, which is so as we have seen in this case.

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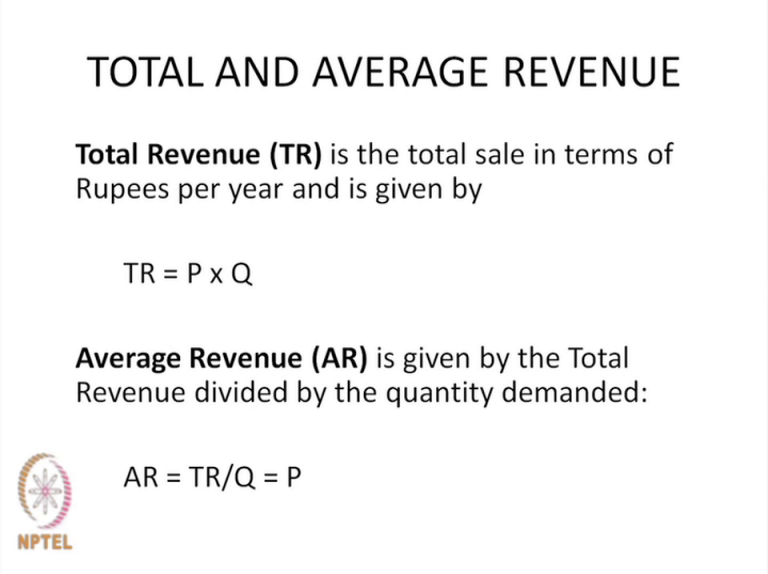

Now before we go further, we would like to now define certain other concepts of total average and marginal revenue, total and average revenue first. Total revenue is the total sale in terms of rupees per year and is given by the quantity sold multiplication unit price P, that is the total sales or just sales or total revenue also called only revenue. So basically we are assuming here that

whatever quantity is demanded it is also sold at a unit price P rupees per unit*Q is rupees per year making it P*Q and that is TR total revenue.

If total revenue is P*Q, the average revenue will be =TR/Q which=P, it is given by the total revenue/the quantity demanded. So average revenue=P when price changes do not occur.

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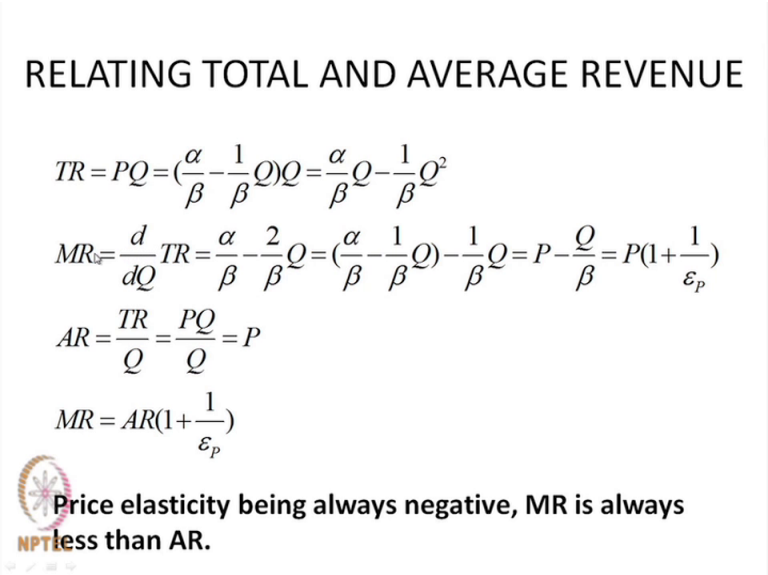

Now here let us relate the total and average revenue and bring in the concept of MR, MR is here and that is marginal revenue which is basically defined as first derivative of total revenue, it says derivative with respect to Q. That means for unit change in the value of Q, what is the rise in the value of total revenue that is d/dQ of TR, unit change in Q gives rise to how much change in total revenue that is called the marginal revenue.

Now total revenue as we know=unit price*quantity demanded or quantity sold P*Q. Now we assume or we have already shown that P=alpha/beta-1/beta*Q, so P*Q becomes alpha/beta Q-1/beta Q square. Now marginal revenue as I said is the first derivative of TR with respect to Q, so taking the first derivative of this expression, we get this as alpha/beta-2/beta Q, this can be written as alpha/beta-1/beta Q put that within the parenthesis and 1/beta Q we bring it outside.

And this is nothing but this which=P therefore, we can write this as P, and we can write this as –Q/beta. So marginal revenue=P the unit price-Q/beta, and by a previous result we have already

seen that Q/beta=-P/epsilon P, epsilon P is the price elasticity. Now this result we now use here Q/beta is-1/epsilon P, and - and - makes it +, so we see we get a relationship between the marginal revenue and the price elasticity of demand.

And it is related to in this fashion it is 1+1/epsilon P, now we already know that price elasticity is always negative, we have already seen that as price increases demand falls and epsilon P is negative, therefore, this quantity is negative and therefore, MR is negative MR<P but P=average revenue, average revenue is total revenue/Q, total revenue is P multiplication Q that/Q=P. So we can write a relationship between MR and AR with the use of epsilon P.

So marginal revenue=average revenue multiplication 1+1/epsilon P, and as I told you epsilon P is always negative and therefore, MR<AR, because it is AR multiplication 1-something, so it is AR-AR into something, so it is always <AR.

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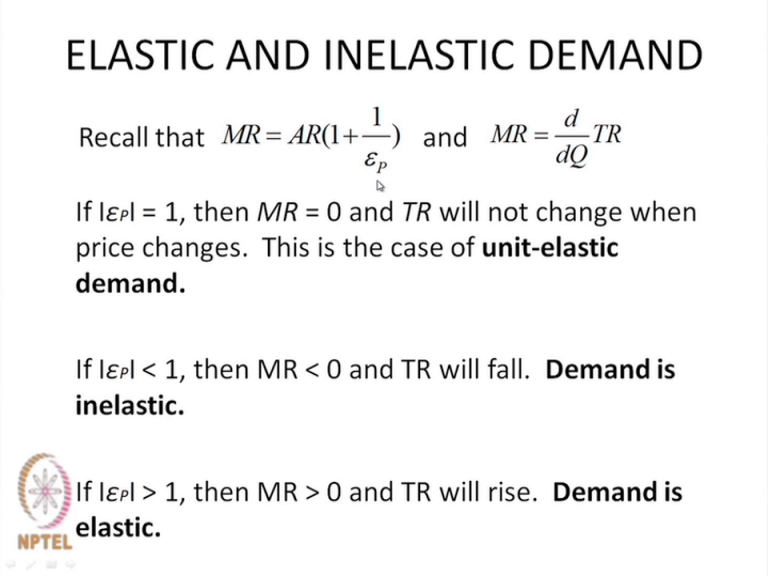

Now use this relationship further, and let us analyze that what happens when epsilon P takes different value. Now we will discuss elastic and inelastic demand cases, here first of all we see that MR is always negative, and <AR and MR=d/dQ of TR that also we know. Now if epsilon P mod value=1, then what happens? Epsilon is negative therefore, epsilon P if it is mod value is 1, then epsilon P is -1, and therefore, 1-1 makes it 0, and therefore, MR=0.

So MR=0 and TR if MR=0, then what is the meaning of TR? TR remains constant because the d/dQ of the TR=MR=0, this means that TR is constant, it means that even when price changes the total revenue does not change. Now this is the case of unit elastic demand that means when price increases their demand falls in such a manner that price in to demand remains constant, price in to demand equals total revenue that does not change, this is the case of unit elastic demand.

Now we take the next case which is epsilon P mod value is<1, and we already know that epsilon P is negative, now it takes the value if mod value of epsilon P<1, then it is MR has to be <0, and TR therefore will fall, which means that as price increases the total revenue will fall, and this is a case of inelastic demand. We will explain these things in more detail little later, now take the case of mod value of epsilon P>1.

Epsilon P>1 but epsilon P is negative the mod value being >1 this quantity MR will rise, and the demand therefore, is elastic.

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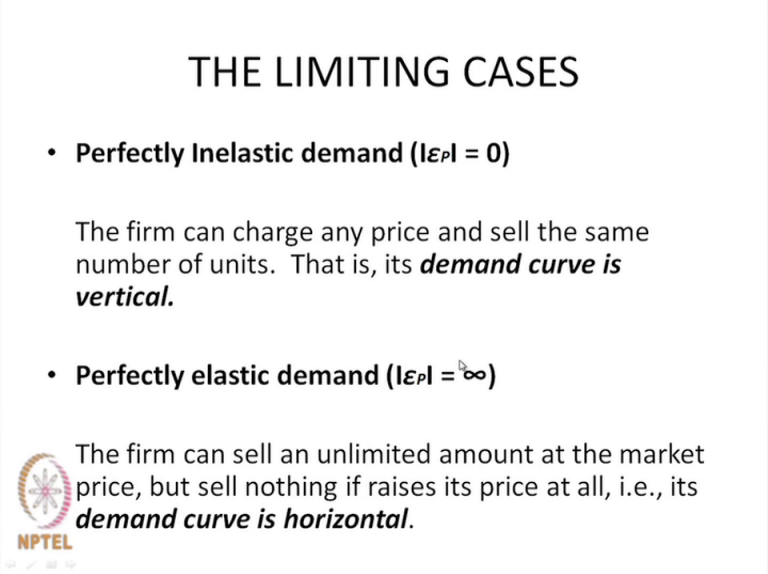

Now let us take the limiting cases, the limiting case 2 cases are there. One is a demand is perfectly inelastic, and the other cases that the demand is perfectly elastic. In a perfectly inelastic demand case, the mod value of epsilon P is taken as =0, whereas in case of perfectly elastic demand the mod value of epsilon P is taken as infinity. Now in this case when we say that the

demand is perfectly inelastic, it means the firm can change any price and sell the same number of units.

That is the demand curve is vertical, no matter what the price the firm sets, the number of good that it can sell will always remain constant, a perfectly inelastic case. The other extreme is the perfectly elastic case, where epsilon P mod value is extremely high, it means that the form can sell an unlimited amount at the market price, but if it changes its price anything from the market price anything different, then its quantity comes to 0. This means that it is a demand curve is completely horizontal.

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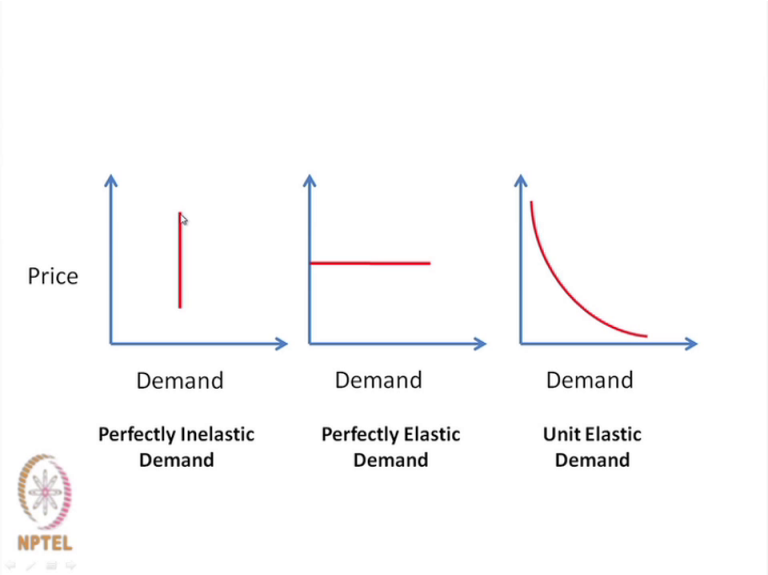

Now let us look at these first 2 are the limiting cases of perfectly inelastic demand and perfectly elastic demand, this says the first diagram says that no matter what price a firm makes for its good for its product, it can always supply and sell the demand the quantity same quantity. This is a case of perfectly inelastic demand, the demand does not change price is changing but the quantity demanded in the market is not changing, this is very unrealistic cases but this is a limiting case.

Here, on the other hand we are saying that at some price P the market the demand and the firm can supply any amount of demand, but if it changes its price then the demand becomes 0 that means it is a case where the demand is extremely sensitive to price change, perfectly elastic

demand case. And this is a case where I said the same percentage change in price leads to same percentage change, 1% change in price leads to 1% change in the demand.

Therefore, the total revenue remains constant, total revenue which is P*Q remains constant, this is P and this is Q, PQ remaining constant is basically a rectangular hyperbola case, for this situation the total revenue will remain constant no matter what the price is set, price may be made more in that case demand will fall so that price*demand is constant, and if price falls demand is more the multiplication of price and quantity remains the same.

So these are these 2 are extreme cases, and this case can occur. But in reality there are many situations where the demand is elastic, the demand is also slightly inelastic in case of items that are considered as necessity items say for example food materials. Even if the price is increased the demand for food materials does not fall to that extent, so to a large extent the food materials are inelastic in nature.

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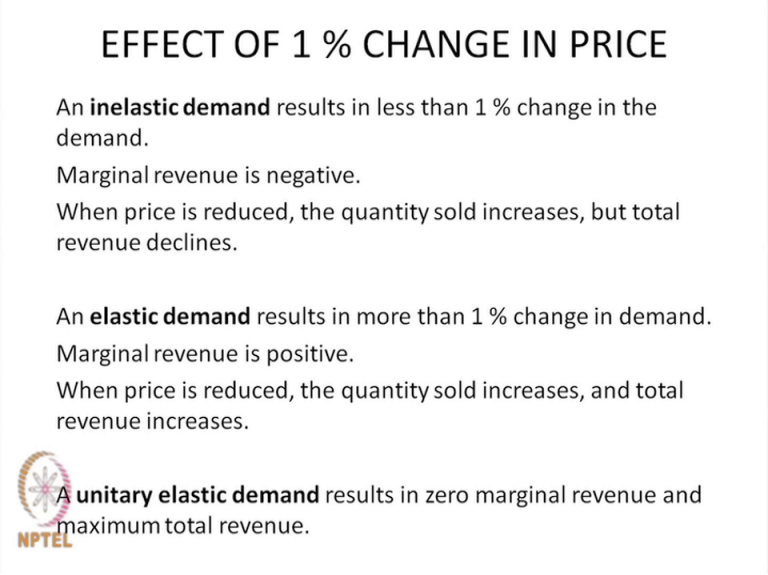

Now this gives the slide gives an explanation of these 3 cases, effect of 1% change in price here we are saying that an inelastic demand results in <1% change in the demand, for a 1% change in the price. That means if we change the price by 1% suppose that we reduce the price by 1% the increase in the demand is not by 1%, but <1%, this is a case of inelastic demand. Here, marginal revenue is negative, and when price is reduced, the quantity sold increases.

But because the marginal revenue is negative, the total revenue declines. Similarly, or rather unlike this in an elastic demand case, the demand is elastic it means that when 1% change is made in the price, this leads to more than 1% change in the demand. This is the case of elastic demand, here the marginal revenue is positive, and when prices is reduced. This is an example when price is reduced the quantity sold increases as it should following the demand curve.

And also unlike in the case of inelastic demand, the total revenue increases. Now in the case of unitary or unit elastic demand case, the marginal revenue is 0, the total revenue is maximum.

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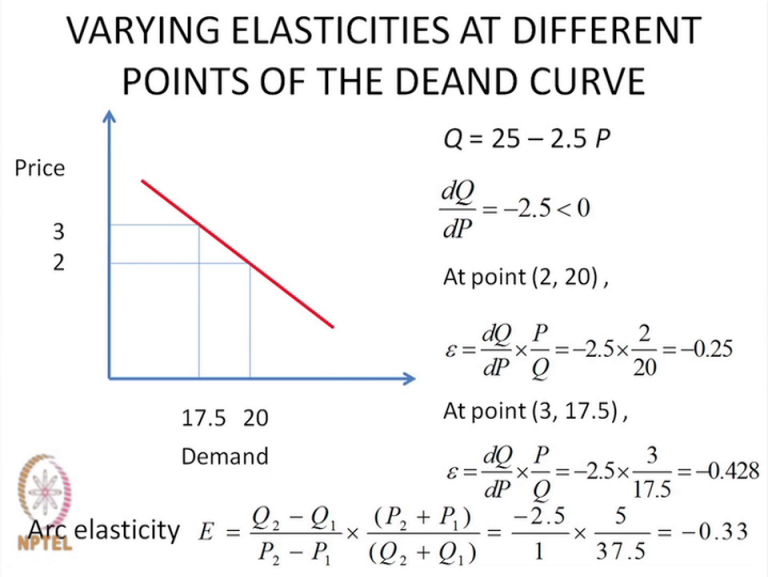

Now here in this slide we shall illustrate that along the different points of the demand curve, the price elasticity is would differ with the concrete example we are illustrating this case. We are first of all assuming that the relationship between quantity demanded in the market or quantity sold in the market and the unit price is a linear function with the negative slope 25-2.5 P. Therefore, taking the first derivative dQ/dP =-2.5 which <0.

Now I have taken 2 points let us take this is the first point, if price is 2 then putting this value here 2.5*2=5, 25-5 is 20, therefore, the demand is 20. So this is the point of price 2 and quantity 20, now at this point if I calculate my point elasticity, the equation is dQ/dP multiplication P/Q, now already we have found out dQ/dP that =-2.5, so this value I put here as -2.5 multiplication

the values of P and Q at this point P is 2 this one, and Q is 20, so I put -2.5*2/20 which is -2.5*1/10 which is -0.25.

So the point elasticity at the point 2, 10 price 2 and demand 20 2 and 20 is -0.25. Now consider another points 3, this point second points, but second point let us say that the price is more and that=3, so if price is 3 then the corresponding value of the quantity will be 2.5*3-7.5 that brings it down to 17.5. So as price increases demand falls, if the demand falls we once again now use the same equation for finding the elasticity at that point 3 and 17.5, price=3 and demand=17.5.

As before dQ/dP=-2.5 this one, this one, this one multiplication the value of P is 3 and the value of Q is 17.5 therefore 3/17.5 approximately =-0.428. So we see that at different points in the demand curve the elasticity of demand changes, in fact at this point it is more negative than at this point. Now suppose that we consider a substantial change from 2 to 3 as substantial is actually 50% change from 2 to 3, so the change is delta X 3-2 which is 1/2 which is 50% change.

Now if there is a 50% change in price this is considered a substantial, and for such a substantial change we should not apply the concept of the point elasticity, instead we should use our arc elasticity concept. Arc elasticity if you recall the corresponding equation is the change in Q/the change in the price multiplication average price of course 2 has eliminated 2/2, divided by 2 here, divided by 2 here they cancel out leaving P2+P1 and division Q2+Q1.

Now in this case Q2-Q1 is 17.5-20 which is -2.5, P2-P1 is 1 from 2 to 3, so this is P2 3-2 is 1 multiplication addition of the 2 is 5, and addition of the 2 is 37.5, this value comes as -0.33 lying somewhere between these 2 values. So this slide tells you or gives you a complete example of how to calculate the point elasticity and the arc elasticity.

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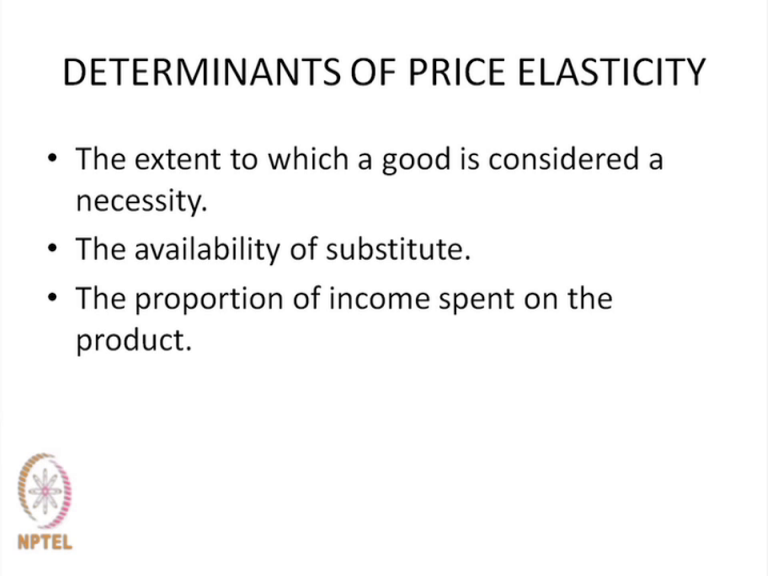

Now we go further, we are assuming that the demand is a function of price, and that we are trying to find out how much what fraction or what percentage the demand changes for a unit percent change in price, that is called the price elasticity of demand. Now the price elasticity of demand values will differ from one good to another good or one product to another product.

Say for example, we have a product which we consider highly necessary for our daily life such as food materials, compared to a product let us say a car or television or an air conditioner which is probably a luxury to certain society. Now if the product is a necessity then a change in the price of the product a small change in the price in the product will not change the quantity so much, because we need it as a necessity of life.

So this is the case of inelastic demand, demand does not change much as price changes. But if the car price rises, then probably the demand in the market may fall, so these are situations or cases where the price elasticity figures can change from type of good from one type of product to another type of product. Price elasticity will also depend on the availability of a substitute, if for example the substitute there is a substitute for a product that you are manufacturing, and its price reduces or falls then it is likely that your product demand will be affected.

Because many demands in the market will be met by your competitors who sell the substitute product, so your demand for your product will fall. Therefore, if in the presence of substitutes

elasticity value will differ from product to product. The is the proportion of income spent on the product, if families spend a large percentage of their funds for the product under consideration that will have one type of elasticity value compared to when the family spends less.

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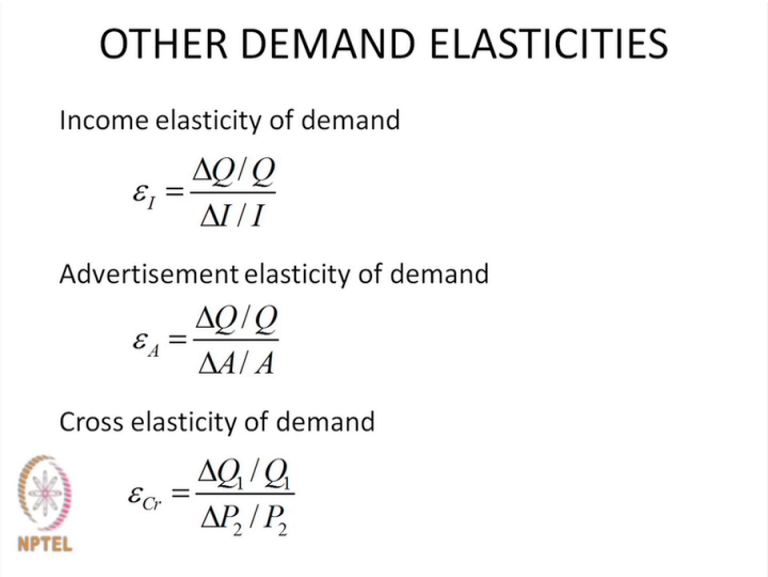

Now there are different other types of demand elasticities, let us spend some time we have been talking about only price elasticity of the demand, we will now consider different other types of elasticity. In particular we would like to talk about these 3 types of elasticities, 1 income elasticity of demand, 2 advertisement elasticity of demand, 3 cross elasticity of demand.

As you know in general whenever a factor value changes by certain fraction a percentage, the elasticity of demand can be calculated to find out how much percent change occurs in the demand that is the elasticity of demand. Now when that factor is income we call it income elasticity, when that factor is advertisement we call it advertisement elasticity and when that factor is a substitute or a compliment then we call it cross elasticity.

Just as we had defined price elasticity as delta Q/Q/del P/P, here we say it is delta Q/Q/delta I/I where I is the income disposable income. In case of advertisement it is delta Q/Q/delta A/A, where A is the amount of advertisement expenses. And this is as I said this is the another products price, particularly as I have told you a substitute product let us say a compliment complimented product, its prices change.

And to what extent the quantity demanded for your product changes for a unit or a 1% change in the price of another product which is a substitute or a compliment, or another product in general is called the cross elasticity of demand. Let us elaborate these cases in some detail.

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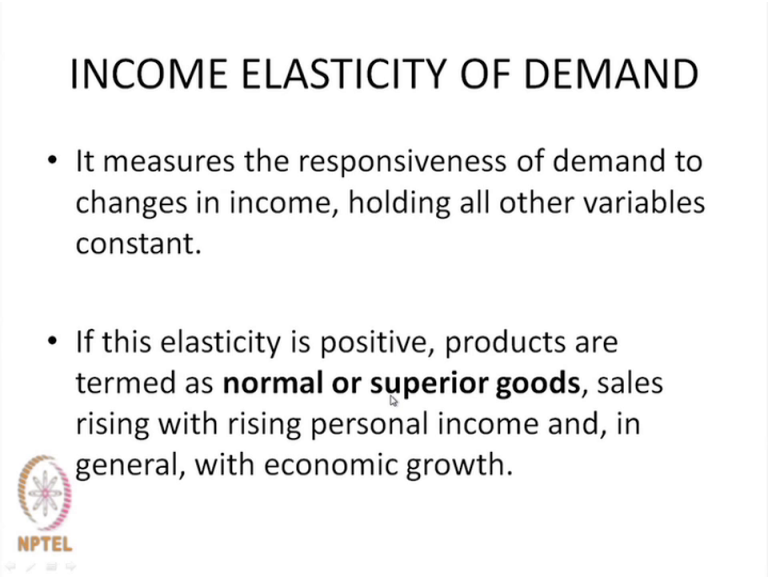

Income elasticity of demand, it measures the responsiveness of demand to changes in income holding all other variables constant, and if this elasticity is positive that means it is expected that as income increases, the demands should rise. And if this elasticity is positive this is called a normal or a superior goods sales rising with rising personal income, and that normally occurs when economic growth occurs, this is income elasticity.

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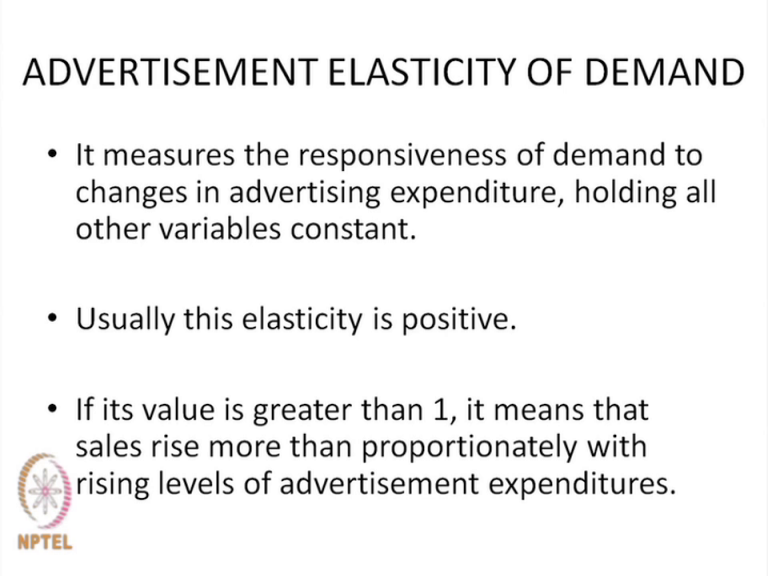

Advertisement elasticity of demand, it measures the responsiveness of demand to changes in advertisement expenditure holding all other variables constant, usually this elasticity is positive as we know it is expected that as we increase our advertisement expenses. The awareness in the market about your product rises, and therefore, the quantity sold in the market are demanded in the market rises. So we expect this elasticity to be positive.

Now if its value >1, it means that the sales rise more than proportionately with rising levels of advertisement expenditures.

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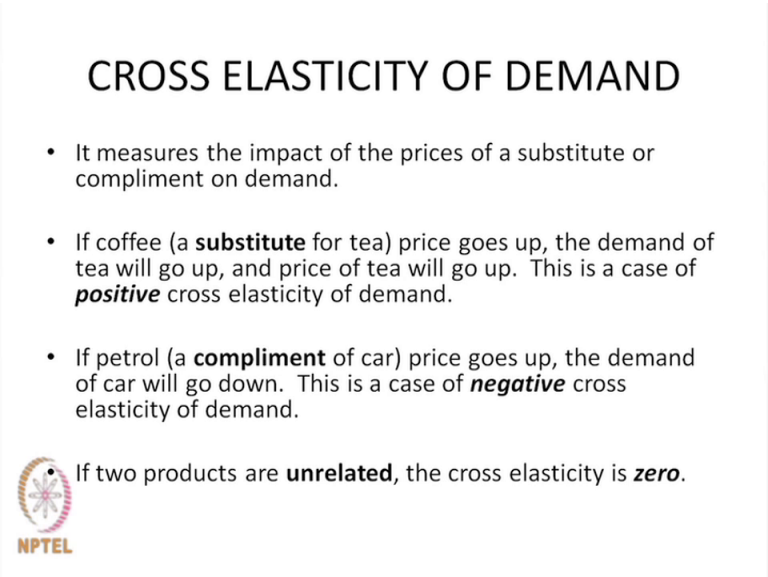

Now we come to the cross elasticity of demand, it measure the impact of the prices of a substitute or a compliment on demand. Now this is an example of what we you mean by substitute, and what we mean by compliment. Let us say that the product in under consideration is tea that a company is manufacturing, the substitute for tea in the market is coffee. Now if coffee price goes up, then the demand of tea is likely to go up.

Because as the price of coffee rises, the demand for coffee reduces demand for tea rises, and therefore, price of tea is expected to also go up, this is the case of positive cross elasticity of demand. Now consider a case of petrol and car, car is the product under our consideration, to run a car we need petrol which is prepared or which is sold by another company, and there is a compliment. Petrol is a compliment of car, now if petrol prices go up then it is likely that the demand of car may go down, this is the case of negative cross elasticity of demand.

Whereas if 2 products are unrelated, the cross elasticity is 0. Gentleman and ladies, basically we introduced to you the concept of elasticity of demand where we said or where we wanted to quantify or measure give a measure of how much fraction or how much percentage change is brought about in the quantity demanded for 1% change in one of the factors, when all other factors are assumed to be held constant.

We gave certain examples, we gave certain illustration, particularly saying how point elasticities are calculated, and how arc elasticities are calculated. We also gave examples of different factor elasticities price elasticity, income elasticity, advertisement elasticity and finally cross elasticity of demand. In this context, we also defined substitutes and compliments, thank you very much.