Economics, Management and Entrepreneurship
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Lecture – 24
Comparison of Alternatives (Contd.)

Good morning. Welcome to the 24th lecture on Economics, Management and Entrepreneurship. In our last lecture we were discussing various methods of comparing economic alternatives. We will continue the discussion on those topics today. To start with let us recall that we had fundamentally 3 methods of comparing among economic alternatives. One was the present worth cost comparison method where we discount all future cash flows to the present.

The second method was equivalent annual cost comparison method where we try to find out the equivalent annuity of these proposals. And in the third method we try to find out the internal rate of return method. Present worth cost comparison method is applicable when the number of interest periods is the same for all the alternatives. Otherwise one has to apply either repeatability condition or coterminous condition to apply present worth cost comparison method.

And when the (()) (02:05) are different the interest period is different for the projects. Then equivalent annual cost comparison method is the best. But if there is only one project and the decision has to be taken as to whether the project is a worthwhile proposition economically or not then we compute the internal rate of return method. And then compare the value of internal rate of return with the minimum attractive rate of return required by the company.

If the internal rate of return is higher than the minimum attractive rate of return, then that project is considered worthwhile. Now, internal rate of return if you recall is basically the rate of return at which the present worth of all cash flows becomes 0. So, we had in the last class taken for example to show how to compute internal rate of return. Now, internal rate of return can be visually seen.

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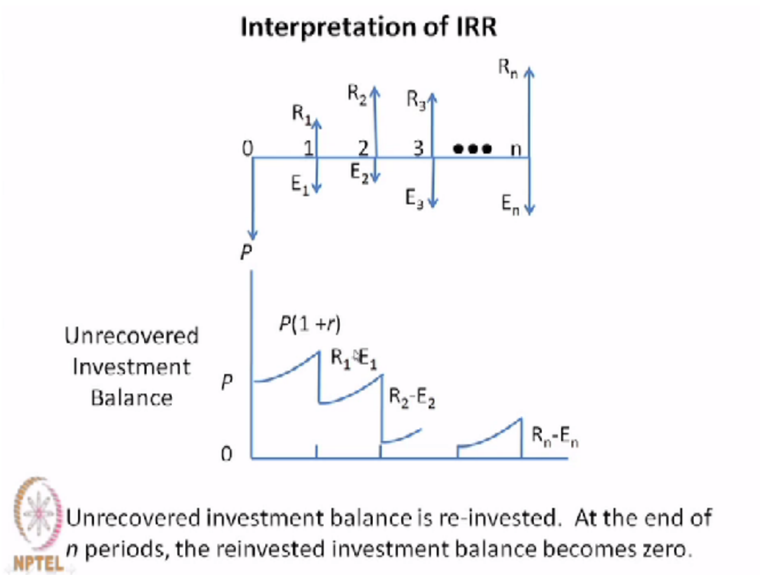

In this interpretation can be made of internal rate of return assume that we have made an investment P initially and that over the life of the project of n years there are different expenses and there are different revenues. So, the meaning of the word internal in the internal rate of return is that this amount is reinvested at the IRR which is R here. So, P is reinvested its equivalent amount at the end of 1 year is P*1+R and then at the end of year 1.

The net revenue is R1 –C1 by that amount the amount is reduced so the unrecovered investment balance is P*1+r –R1-C1 that means it drops to this. Again it is assumed that it is reinvested for 1 years and then again the net revenue is subtracted. Like that this continues till at this point of time it become = 0. The value of R that results in the unrecovered investment balance to be =0 is called the internal rate of return.

Basically it means that the amount that is unrecovered investment balance is reinvested at a rate of internal rate of return. So, this is a difficulty with IRR that actually it is not reinvested. And in particular if let us say the value of IRR is very high of the order of 50% were as the MARR is just about 15% or 20% then it is never imaginable that a particular amount is invested internally at 50% rate of return. So, this is not practical.

This is a problem with internal rate of return. The second problem we of course know that computation of IRR is difficult because it involves a method of trial and error using the values of the factors from the interest table. There is yet a third difficulty and that it is possible to have more than one values of greater return at which the present worth of all cash flows may become = 0. That means there can be multiple values of internal rate of return.

And when there are multiple values interpretation of the multiple values interpretation of the multiple values is difficult as also the calculations may be, may not be correct. We are not discussing these aspects of multiple values of IRR but the fact is that solving for IRR may result in more number of values of IRR and one is not sure which one is the correct value. So, because of these reasons IRR is sometimes not preferred.

But it is quite often cited in the industry for comparison with MARR to find out whether a particular project will be accepted.

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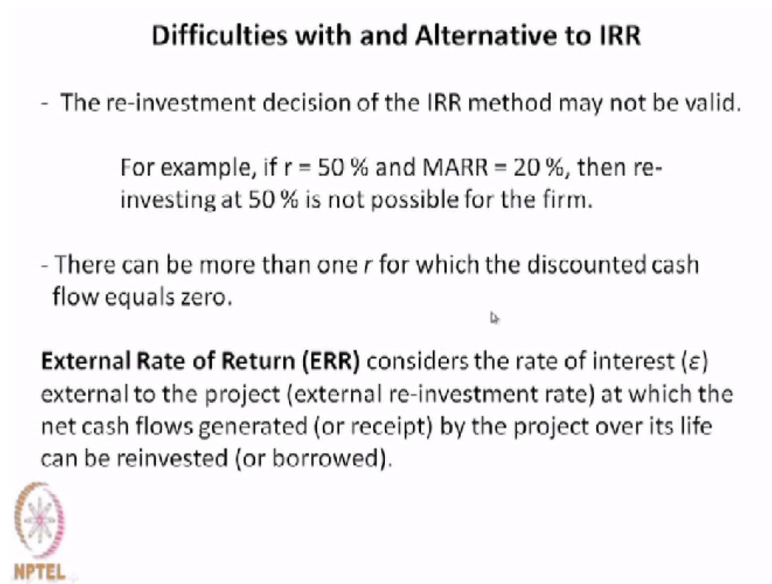

This is what we have written here. The difficulty with reinvestment decision of IRR and the problem of multiple values of R. These are 2 problems for which one goes for what is known as external rate of return. It considers rate of interest epsilon external to the project at which the net cash flow is generated or receipt by the project over its life can be reinvested. And this we are showing in the form of another diagram here.

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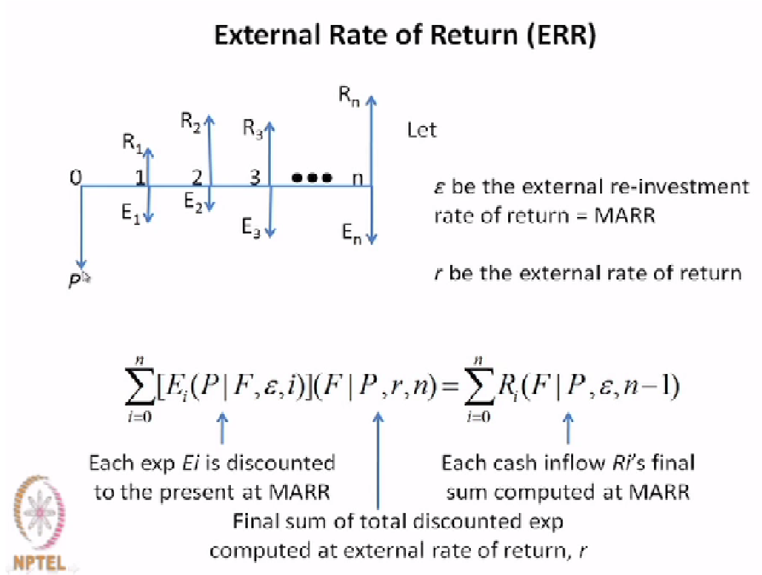

Here as before this is the initial investment P and the expenses over the years E1 through En and revenues over the years are 1 through Rn. What is first of all done is that all expenses are discounted to the present at the external reinvestment rate of return which is normally =MARR. So, if epsilon is the external reinvestment rate of return meaning MARR then all this E1, E2, etcetera are discounted to the present.

So, first of all they are all discounted to the end and then discounted to the present that is what is done here. This is each Ei is discounted to the present so each is considered as an F. So, E1 *PF e, i that is single payment present worth factor is multiplied with E1 + E2 * single payment present worth factor that is multiplied with i=2 here like that and then this whole amount which is considered the present is taken to find out the compound amount here.

So, FP, r,n. So, this is the final sum of total discounted expenses computed at external rate of return r. So, this should be = there is one mistake here this should be P+ because P+ and then this. So, all these are discounted to the present + there was an initial investment P so that is the total investment made by the company and that is then found out the equivalence is found out at the end of the n year. So, multiplication the single payment compound amount factor FP, r, n.

So, r is taken here where as the discounting was made at the rate of MARR we are interested to find out the external rate of return R. So, if this amount was invested externally then we are calling it P, r, n and that is =all the revenues when their final sum is calculated at the end. The final sum is Ri* find F given each one as P. So, P epsilon n-1. So, each one the final sum is found out at this point and all expenses + the initial investment calculated here.

Are the final sum is found at this point? and they are made equal. So, in this case since epsilon is know the value of MARR is known therefore we can find out the value of FP, r, n and therefore r can be determined. So, here we are not making any trial and error considerations. So, basically what we are doing all expenses we are discounting to the present adding to that the initial investment made that is now the present sum.

The final sum is calculated by finding out or multiplying that with the compound amount factor and then equation the whole thing with the compound amount of all the revenues throughout the year. By that process we find out the external rate of return often this external rate of return is used instead of internal rate of return and then the same method of comparison is made ERR is compared with MARR.

If ERR is higher than MARR, the project is considered economically viable and if external rate of return ERR is less then MARR then the project is considered not viable. So, this is an alternative to the very popular internal rate of return method.

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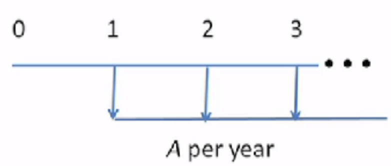
Capitalized Worth Method

In certain cases, same cash flows continue for a very long time.

Cash flows are assumed to continue for an infinite length of time.

Capitalized worth is the present worth of a series of equal payments for an infinite time period:



$$CW = \lim_{n \to \infty} PW = A(P/A, r, \infty) = A \lim_{n\to\infty} \frac{(1+r)^n - 1}{r(1+r)^n} = A/r$$

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Now, we introduce yet another method called capitalized worth method or CW method. Now, in the capitalized worth method we assume that an equal payment or recite is made for a very large number of years particularly let us say the government projects which are very long duration projects, a dam, a bridge, a road, and similar such projects which require large investment and which has a very long life for this purpose we can use a simple formula.

What we can do we can assume that all our receipts or expenses are equal and continue up to infinity that is what we have done here. So, cash flows are assumed to continue for an infinite length of time and so suppose this is this cash flow and this continues up to infinity then we know that this is like A we assume annuity A per year to be constant. And suppose we find out the present worth of this cash flow.

So to find out P given A equal payment series present worth factor given A find P multiply that with A will give us the present worth of this cash flows. And if it was n then the formula is 1+r to the power n-1/ r*1+r to the power n. Now, if n tends to infinity it can be shown that this is = A/r this quantity is 1/r. So, 1/r can be taken out and this is 1-1/this n tending to infinity this quantity becomes 0.

So, this becomes 1/r therefore this is A*1/r =A/r. So, this is a much simpler formula to use. So, suppose for example A is 100 r is .1 then the capitalized worth =100/0.1 which is 1000 this is the meaning.

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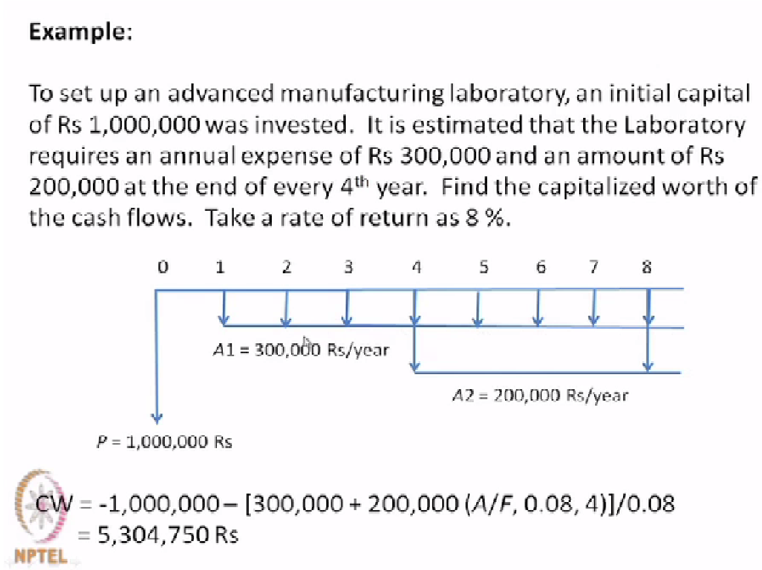

We give here a more realistic example. The company is interested to setup an advance manufacturing laboratory for which it invests 1,000,000 rupees. So, this is the initial investment made. This is a cash flow diagram. It is estimated that the lab requires an annual expenses of rupees 300,000 and an amount of rupees 200,000 at the end of every 4th year. So, 300,000 rupees annual expense for the laboratory.

This is an operating expense and this is at the end of every 4th year there a maintenance expense. So, here there is an expense. Here also there is an expense and 12th year there is another expense of 200,000 whereas 300,0000 rupees per year is continued throughout for every year. Now, this continues for a very long time. We are assuming that long to be infinity and therefore we can straight away use the capitalized worth method.

Meaning we can find out the present worth of this. The present worth of this is first of all. This is taken as – because it is a cash outflow – 1,000,000 rupees and then for this quantity it is 300,000 + if for each of this we find out the annuity. Then that means we consider this as 200,000 and the equivalent amount we can now find out.

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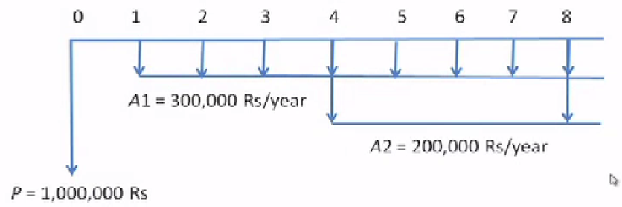
Example:

To set up an advanced manufacturing laboratory, an initial capital of Rs 1,000,000 was invested. It is estimated that the Laboratory requires an annual expense of Rs 300,000 and an amount of Rs 200,000 at the end of every 4^{th} year. Find the capitalized worth of the cash flows. Take a rate of return as 8 %.



$$CW = -1{,}000{,}000 - [300{,}000 + 200{,}000\,(A/F,\,0.08,\,4)]/0.08$$
$$= -5{,}304{,}750 \text{ Rs}$$

This is 4, 3, 2, 1 so the equivalent annuity for this 200,000 will be given by 200,000 multiplication AF, r, 4. This is the sinking fund factor, equal payment series sinking fund factor. So that is A given F, r =0.08, 4. So, this is the equivalent annuity of a payment 200,000. So, similarly for this amount this will also be 200,000 multiplication AF, 0.08, 4. Because of this there is another annuity here and therefore we can actually them that is what we have done here.

This originally was 300,000 and because of this it is 200,000 into this. So, this is therefore the equivalent annuity considering both the cash flows. And according to our capitalized worth method this is the A/r is 0.08 that is 8% rate of return is given. So, that makes it this minus this. Once again there is a mistake here. This has to be minus. This is -5,304,750 rupees. This is the capitalized worth if we have similar project of another project with different cash flows.

We can separately find out its capitalized worth compare the 2 and since both are cost the one that gives the lower cost is what is accepted.

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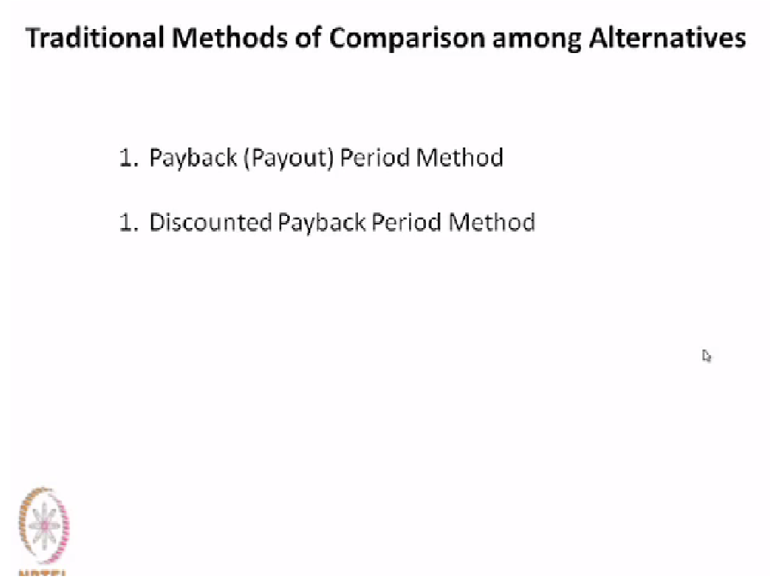

So, up to now we have discussed in fact 5 methods. The present worth comparison method, the equivalent annual cost comparison method, internal rate of return method, external rate of return method and just now we discussed about capitalized worth cost comparison method. Now, all these methods consider time value of money. Now, traditionally however in industries it will use or rather do not use the time value of money so much. What they do?

These are thumb rule that is called the payback period. And of course a variation of payback period is discounted payback period. So, these are the traditional methods particularly the payback period is the most traditional and is very much used as a thumb rule to decide whether a project is worthwhile to make investment. So, pay back or pay out period method and discounted payback or pay out period method.

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Payback (Payout) Period Method

Payback period is the minimum period θ at which the initial investment is fully realized.

$$\sum_{i=1}^{\theta}(R_i - E_i) - I \geq 0$$

- The method does not consider time-value of money.
- A project with a payback period less than 3 years is favoured.
- This method is very popular in industries.
- The method considers liquidity rather than profitability.
- It does not consider the entire cash flow; hence a rejected project may be more profitable in the long run.

Now payback or payout period method is basically finding out a time period theta by which the initial investment if fully realized. That means if I have invested 10,000 rupees in the first year I may be getting back 4,000 the second year I may be getting back 3,000. The 3rd year I might be getting back 4,000. So, totally 4+3 +4 makes it 11,000. That means by the end of 3rd year I have fully realized my initial investment of 10,000 so payback period is little < 3 years.

This is how the payback period is calculated and in the symbolic form it is given in this fashion Ri-Ei revenue –expenses for every year some sum over are = 1 to theta – the initial investment I should be >= 0. As you can see this method does not consider the time value of money. A project with a payback period of < 3 years is usually favoured.

Because the company has investment an amount of I is getting back his money in cash form or in some form in 3-year time. The net revenue if 3 years if it is so then that project is considered good. This is very popular in industries however this method considers only liquidity rather than profitability. Because it does not consider the entire cash flow hence a rejected project may be more profitable in the long run this I can explain in this manner.

Let us say that a project is paying back its money in 3-year time. Another project paid back its or can pay back its money in 5-year time. So, naturally, this first project is preferred because it takes less time to get back the initial investment. But we have really not considered the future cash flows. It can happen that the second project gives very high profits in the 5 years, 6 years, 7 years, 8 years. And the first one does not give so much profit.

That for if the entire cash flow is taken may be the first project is not economically better that the second project. However, since all the future cash flows beyond 3 or 4 years have not been considered. Project 1 or the first project has been considered better. Therefor this is a problem with the payback period or pay out period method.

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Discounted Payback (Payout) Period Method

Discounted payback period is the minimum period θ at which the discounted cash flow is greater or equal to the initial investment.

$$\sum_{i=1}^{\theta}(R_i - E_i)(P|F,r,i) - I \geq 0$$

Year	Net Cash Flow	Cum Cash Flow (Payback Prd)	PW of Cash Flow (r = 10 %)	Cum PW (Discounted Payback Prd)
0	-25000	-25,000	-25,000	-25,000
1	8000	-17,000	6,667	-18,333
2	8000	-9,000	5,556	-12,777
3	8000	-1,000	4,630	-8,147
4	8000	+7,000	3,858	-4,289
5	13,000	+25,000	5,223	+934
		θ = 4		θ = 5

To take care of the time value of money sometimes discounted payback period is calculated. So, here what is done this is a concreate example. So, what is done that the net revenue, revenue – expense the net income, the present value is found out. So, this is considered as a final sum. The present value is calculated that means that the single payment, present worth factor is multiplied with the income and that is subtracted from I and over a period of theta.

This would be >= 0 to find out the value of theta. Now, this is shown here. Let us say that initial investment is 25,000 and the net cash flow for the project for 5 years is let us say 8,000, 8,000, 8,000, 8,000 and 13,000. Now considering only, the payback period initial investment was 25,000 and we get back 8,000 therefor 17,000 remains unrealized after the end of the first year.

After the end of the second year 17,000 -8,000 its 9,000 after 3rd year it is -1,000. Only in the

4th year it becomes positive. Therefore, theta is =4 so this is the payback period. Now, when we calculate the discounted payback period what is done that 8,000 its present worth is calculated. So, at a MARR of 10% the present worth is 8,000/1+r. So, that comes to 6.667 so what remains unrealized is 25,000 which was initial investment –the present worth of the income.

After the first year. S, unrealized amount is 25,000 – this which is -18,000. After the second year similarly this 8,000 is discounted to the present meaning 2 years, year is taken as 2 that means 8,000/1+r which is 10% square that amount is lower than which comes to 5,556 and therefor the remaining amount is this amount is this minus this. Which is 12,777 and similar calculations are now made after the 3rd year.

The present worth is 4,630 if you subtract this amount from here we get the unrealized amount of 8,147 and after the end of 4th year it is still negative. Only in the 5th year it becomes 934 it becomes positive. Therefore, the whole amount of initial investment is realized only after the 5th year or only in the 5th year and therefore theta =5. So, you can see that in the payback period where we do not consider time value of money.

The pay way period is 4 and the discounted payback period it is 5. However, the fact remains that the future cash flows are not considered in either of this methods. So, these are traditional methods and therefore they are quite popular however. But they do not have the fundamental bases of consideration of time value of money and therefore one should not depend on or one should not take conclusions or base conclusions on the bases of the payback period or even the discounted payback period.

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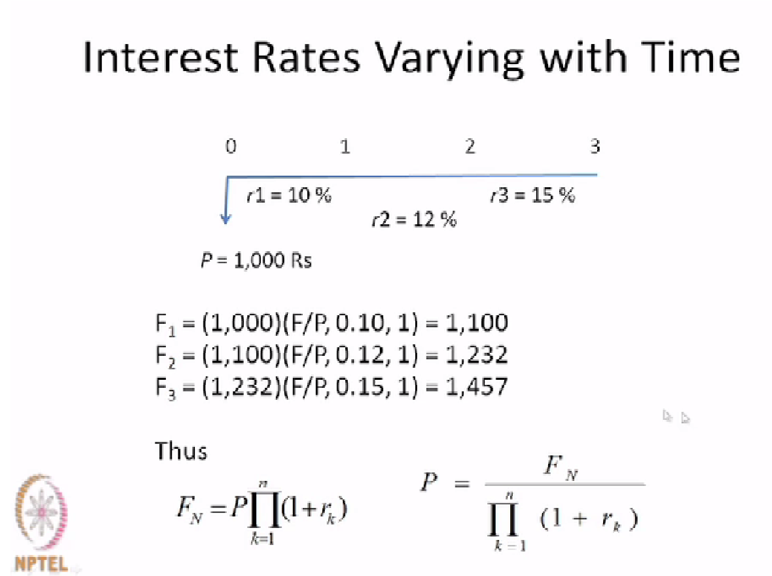

Now in practice there are some issues concerned with varying interest rates. This is one example of interest rates that rates that vary over time. Here suppose that an investment is made or a payment is made for 1,000 rupees it is quite possible that in the first year the interest rate is 10% in the 2nd year the interest rate is 12%, in the 3rd year the interest rate is 15%. So, this is an example of interest rates varying with time.

So, if I calculate the final sum here the compounded amount of this payment at the end of the first year then it will be a F1 =1,000 *1+r1 which is basically this. The final sum F this factor is single payment compound amount factor. To find FP at the value of r1 as = there is no need to write here r because already we have given the value of r, similarly here. So r1 is 10% and this value can be obtained from the interest table.

Otherwise we know that it is nothing but 1/1+.1 which is 1.1. Not divided by multiplication. So, this multiplied by 1.1 is nothing but 1,100. Then suppose we are interested to find out F2 then taking this as the principle we can find out its compound amount which is 11,000 * for 1 year it is 1.2 that comes to 1,232 and here it is compounded at the interest rate of r3=15% therefore it is 1,232 * 1.15 and that comes 1,457.

So, in general if we are interested to find out the equivalent final sum FN it is P*product of 1+r1 * 1+r2 * 1+r3 written in this manner. Or equivalently if we are interested to find out P even the final value then its discounted value can be found out. So, when interest rates vary then we can use this formula.

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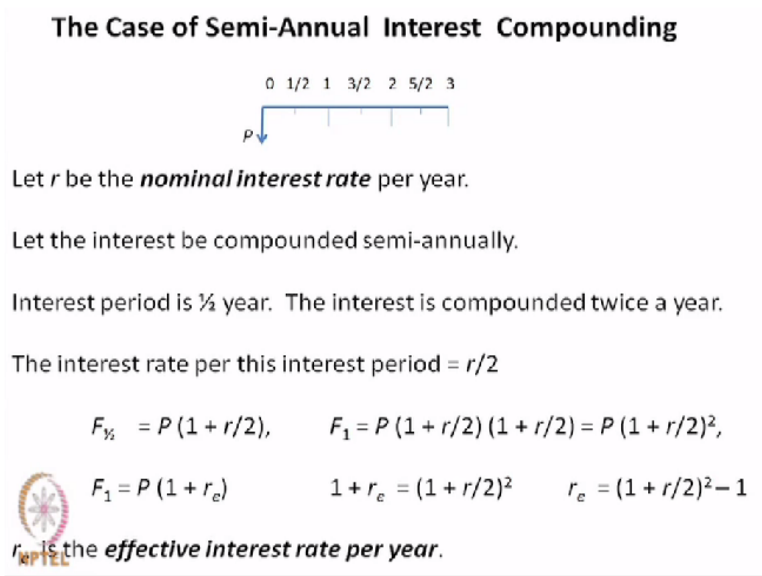

Now, another interesting case is the case of semiannual interest compound. We have so long considered interest rates that are defined for a particular year. Although we said that its particular period but we almost implicitly assumed that 1 period is equivalent to 1 year. Quite often interest is compounded more frequently than once in a year. In this particular example we are considering interest rates that are compounded twice in a year.

That means the time period for which the interest is compounded is 1 / 2 year. So, it is a case of semiannual compounding. So, this is the case flow diagram for such a situation. This is 1 year, end of 1 st year, end of 2 nd year, end of 3rd year. And this is 1 / 2 the year, this is 1 and 1/ 2 year. This is 2 and 1 / 2 year. So, time period if you write it will be 0, 1, 2, 3, 4, 5 and 6 so in terms of years these are what is written here in terms of years.

But if write in terms of time period then it will be 0, 1, 2, 3, 4, 5, 6 etcetera. For example, this should be (()) (35:39) in terms of years this is 1, 2, 3. But in terms of periods I will write this is 0, 1, 2, 3, 4, 5 and 6 interest periods. This is what I am trying to say that if it is semiannual and if we are dealing with a problem of 3 years then it is a matter of 6 interest periods. We now define suppose that the interest rate is defined for full year.

We call that as the nominal interest rate that is not the actual interest rate. The nominal interest rate is defined for the full year that is r but if the interest is compounded semiannually that means for after every 6 months it is compounded. In this case the interest period is therefore 1/ 2 the year. And the interest is compounded twice a year. The interest rate for this half year is nothing but r/2.

So, if r is the nominal interest rate for the full year for that one interest period that is r/2 therefor the compounded amount F at the end of first interest period meaning at the end of 6 months of the year is =P*1+r/2 because r/2 is the interest rate given for this 1/ 2 the year. And then for the full year it will be this amount F 1/ 2 that is F calculated at this point will be compounded here. So, this is F 1/ 2 which is nothing but P* 1+ r/2 multiplication for this amount it is compounded.

It is 1+r/2. So, this quantity becomes P*1+r/2 whole square. So, if it is compounded semiannually at the end of the year it is not P* 1+r but it is P*1+r/2 whole square that is the difference. So, what is the equivalent interest rate which we say effective interest rate. Let that we re so in this way where we are calculating compounding twice a year the amount comes to P*1+r/2 whole square where as the effective interest rate suppose it is re.

Then it would be =P*1+re therefore this 1+re will be = 1+r/2 whole square. That is what we have written here. 1+re =1+r/2 whole square that now defines re as =1+r/2 whole square -1. So, you can see that this re is greater than r. The effective interest rate per year is higher than r and we can show it with an example such as this here.

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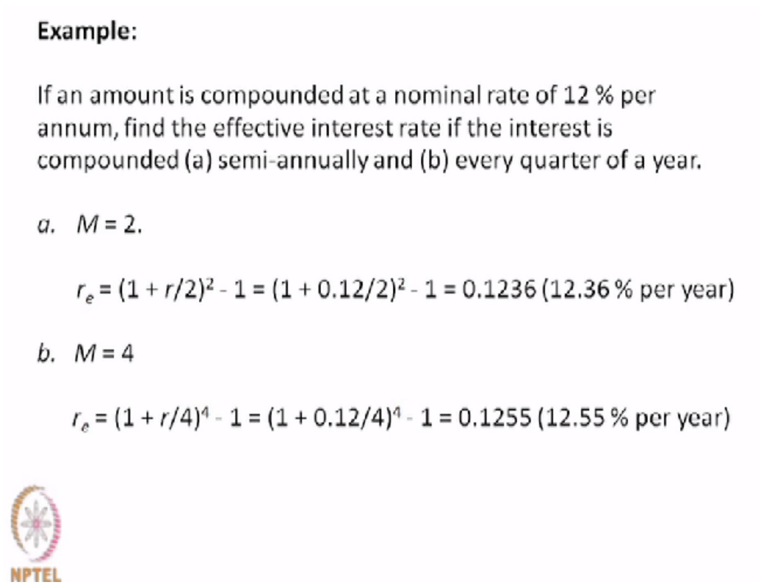

Let us say that an amount is compounded at a nominal interest rate of 12% per annum so that is the r. So, it can be compounded semiannually and it can be compounded every quarter meaning 4 times a year. So, we are required to find out the effective interest rate re in each case. So, I am defining a quantity M. Actually basically what I am trying to say is that if an amount is compounded M times in a particular year and suppose that r is the nominal interest rate.

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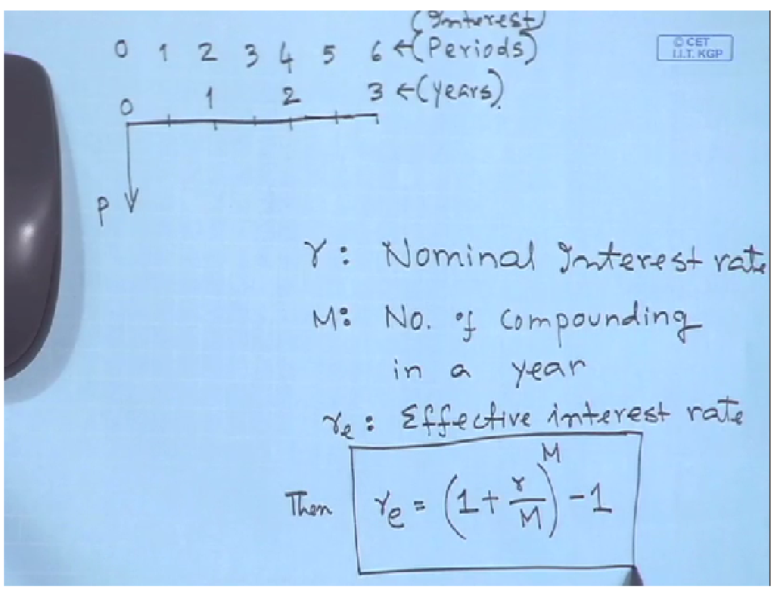

And M is the number of compounding, number of times interest is compounded in a year. And re is the effective interest rate then re will be =1+r/M to the power M -1 compare that this formula with this it is re =1+r/2 square -1 and when the number of times the interest is compounded is M then this 2 will be replaced by (()) (42:37) and the power, the exponent will be also replaced by

M. This becomes 1+r/m to the power M -1.

So, when suppose that it I semiannual compounding M=2. r is defined at 12% therefore it is 1+0.12/2 whole square -1 and this quantity is obtained as .1236 which is 12.36% per year. So, although the nominal rate is 12% when it is compounded semiannually the effective interest rate is higher at 12.36% per year. When instead the interest rate is compounded 4 times a year then M =4 so this becomes effective interest rate.

Then becomes 1+r/4 whole to the power 4 -1 and that is = 1+0.12/4 to the power 4 -1 this quantity is 0.1255 which is 12.55% per year. Now, you can compare this the higher the frequency of compounding of interest the higher is the effective interest rate. This is even higher than this therefore the idea given here is that the interest rates are compounded more than once in a year the effective interest rate also arises.

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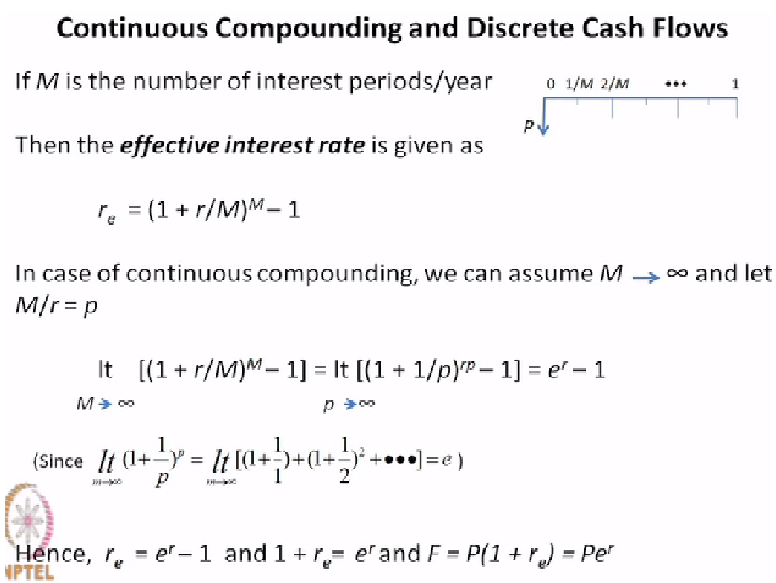

Now, if we carry this idea to the extreme and say that compounding take place continuously. Then what is the effective interest rate? Now, this is quite interesting this is also quite practical in practice it is not that the rates are given only at the end of the year it actually it gets reinvested and the interest rates are compounded almost instantaneously. So, in that case how to calculate the effective interest rates? How to calculate the equivalent present worth? final sum values.

This we can now extrapolate our previous idea in this fashion. Here we are considering that there (()) (45:31) are M number of interest periods per year. So, if this is 1 year and every interest period is 1/M, 2/M. M/M becomes 1. M numbers of compounding of interest. So, the effective interest rate is re=1+r/M to the power M -1. Now, if we assume continuous compounding, it means that M the number of times the interest is compounded can go to infinity.

Now, let M/r be given as p be defined as p r/M we are saying M/r. So, what happens to this quantity 1+r/M to power M to the power M -1 that is re. Limit M tends to infinity is therefore = if M tends to infinity then p also tends to infinity if p is defined in this fashion. So, I can write p tends to infinity and this is one remain to 1. r/M I now write as 1/p just the inverse of M/r is r/M therefore it is 1/p and the exponent M is nothing but rp so it is rp.

Now, this quantity is nothing but p to the power r. This is a series if we expand this for different values of p then we will see that it is nothing but e to the power r the first term is 1+1/1 to the power r. The second term is 1+1/2 to the power 2r. 1+1/3 to the power 3r and this will come as e to the power r. So, this quantity becomes = e to the power r because of this reason. Hence r e the effective interest rate is therefore =e to the power r -1 or 1+re is =e to the power r.

Hence if we are considering a present investment of p and if the interest is compounded continuously at the end of the year the value then becomes p*e to the power r. And if there are n time periods it becomes p*e to the power rn. So, this is the expression for continuous compounding and discrete cash flow case.

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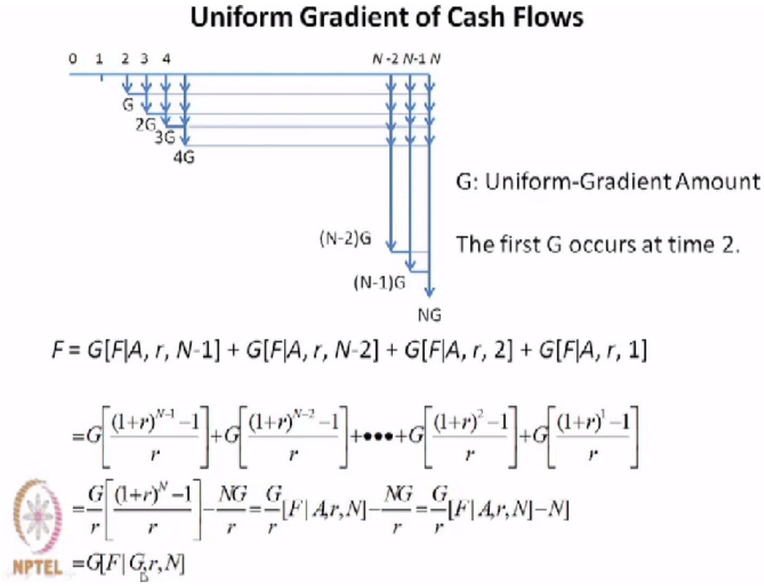

Now, we will discuss a very interesting case of cash flows that increase in a uniform gradient manner or with a uniform gradient. Look at this we are now considering a cash flow at the end of the second year I am assuming that there is a cash outflow of the amount G. At the end of the third year I am assuming it is 2G at the end of the 4th year it is 3G and like this it continues up to N here. Now, I can find the equivalent amount at the end of the Nth here. This amount I can find out in this manner. I can assume this 2G as =G + an extra G.

This 3G as = G +G+ still another G and like this I can continue up to this. Hence I can consider all the G this is an equal payment series of value G. This is another amount of another (()) (50:52) equal payment series of G. This is yet another and lastly this. So, I can find out the equivalent amount at the end here. For this it is G multiplied by 1+r to the power N -1/r for this it is N-2 for the third one it is etcetera, etcetera it continues.

So, this can be summed and the value can be found to be a simpler one. And that can be given a notation G given F G, r, N instead of F given A, r, N.

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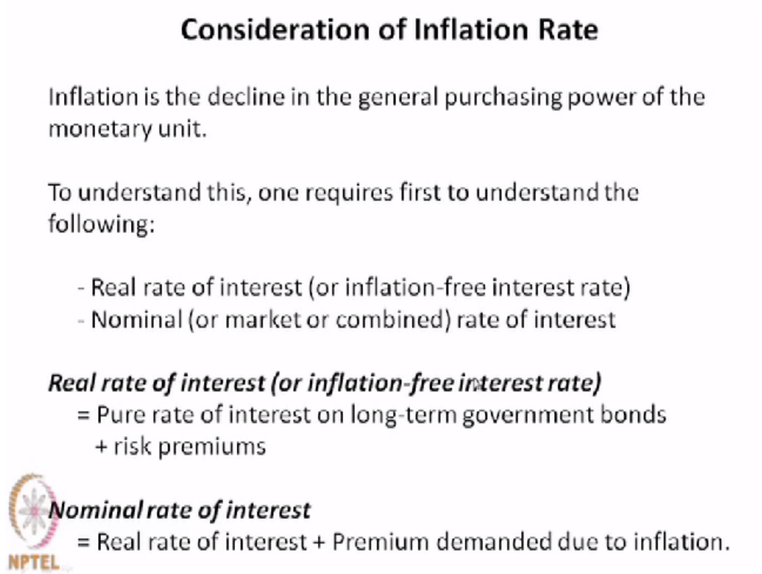

We stop here at this moment and we will take up in our next lecture the consideration of inflation rate and also we shall start a new topic on depreciation. Thank you very much.