Economics, Management and Entrepreneurship
Prof. Pratap K. J. Mohapatra
Department of Industrial Engineering & Management
Indian Institute of Technology – Kharagpur

Lecture - 22
Comparison of Alternatives

Good morning. Welcome to the 22nd lecture on Economics, Management and Entrepreneurship. In the last class, we discussed about time value of money. Basically we said that we cannot compare payments are received if they take place at different points of time. To make a comparison, we must first of all find its equivalence at a single point of time. For that, what is required is to consider the time value of money.

In this case, we had shown that the interest formulae that we had studied in our school days are applicable in this context. However, an investor does not consider the interest rate alone, he considers in addition to the interest rate the minimum profit that he should make by out of his investments in economically feasible projects. So today we shall consider how to compare among alternative project proposals, but before we do that we shall first of all revise certain concepts that we had considered in our last lecture.

In particular, we shall discuss about the background behind minimum attractive rate of return or which is also known as the discount rate. We will also summarize the different factors that are required for equivalence of cash flows. So to start with, we shall first of all consider the principles behind minimum attractive rate of return once again. So topic for today is comparison of alternatives considering, of course the time value of money that goes without saying.

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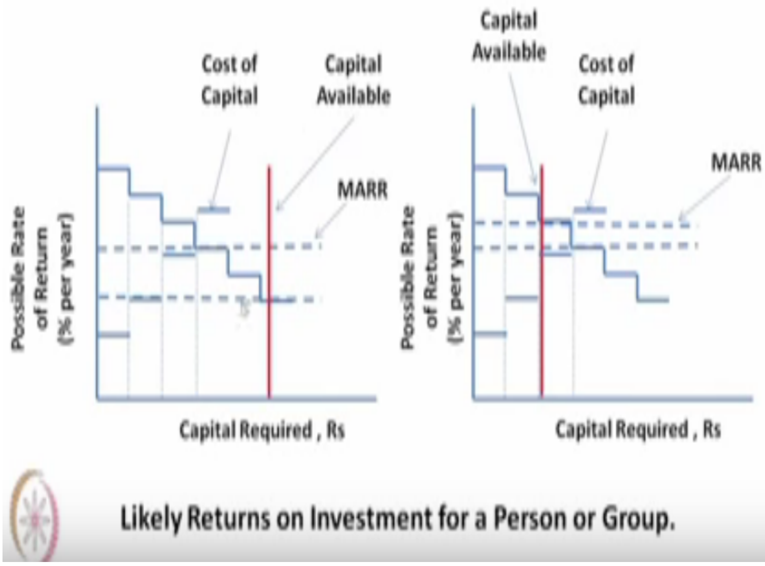

But before we do that we would like to revisit the concepts underlying minimum attractive rate of return, which is also known as MARR. First of all, let us understand that an investor may be having certain amount of money with him. This is indicated by the red line. Suppose that this is the amount of capital the investor is having and he is interested to invest it in certain projects. So naturally he would like to invest in a project, which is likely to give him the highest return.

So suppose that these are different projects, project 1, 2, 3, 4, 5, 6 and so on. Let us say that these projects give different estimated rate of return. So what he will do, he would arrange them in a decreasing order and would like to invest his money maximum in the project, which gives him the highest return. Then if he has some more money, he would invest in the next highest project which is giving the next highest rate of return.

Then he would select they still the next project with the next highest return provided he has sufficient money. Like that he will continue till his money is exhausted. So in this case, a possible minimum attractive rate of return could be this one. However, let us understand that there is a cost of capital involved. It is not that the investor will always have the money with him. He probably would take the money from some financial institution.

So there is a cost of capital involved in it. This is the interest rate prevailing in the market. If he is asking for less amount of money, the interest rate is probably less. More he is borrowing more will be the interest rate. So these ones indicate the cost of capital, higher the money he is borrowing higher would be the cost of capital. So naturally he would not like to go for a higher capital. He would restrict himself to probably only these 3 projects.

First project, second project and third project because the third project gives him profit higher than the cost of capital, whereas the 4th project this one, the cost of capital is higher and the profit the rate of return from this project is lower. So he would not take this particular project. He would be happy with this particular project and therefore his minimum attractive rate of return would be the rate of return associated with the project that he has profited.

So this is similar to the concept of opportunity cost. Remember that the opportunity cost is the cost of the project that is profited. So in this case the project that the company or the investor is not considering because its cost of capital is higher the profit rate that he would have got that would be the MARR. So in this case there are 2 possibilities. It is a comparison between this one the discount rate where his money gets exhausted. So this dotted line is one consideration.

And the other consideration is from the point of view of the cost of capital, whichever is higher would be his minimum attractive rate of return. So this will therefore be the minimum attractive return for the investor in this particular case. Now consider a second situation, in which the capital available is less. So if the capital available is less, in this case the investor would probably invest in project 1 and project 2 and not project 3.

Therefore, the opportunity cost for the investor is this one. The possible rate of return that he is profiting is this one. Now consider the cost of capital consideration that remains as before. From the cost of capital point of view, he could have taken even project 3, but he does not have sufficient capital with him. He does not want to take more than this. So in this case, this would be the MARR. From cost of capital point of view, this would have been the MARR.

The higher of the 2 is the minimum attractive rate of return for this investor. So what we understand from these 2 diagrams is that there are 2 considerations in finding out the minimum attractive rate of return for an investor. One is the cost of capital and the other is the profit or rate of return that the investor get profiting, because he does not have sufficient capital with him considering both the both the 2 cases the higher of the 2 will be the minimum attractive rate of return.

So always MARR is >= the interest rate prevailing or the cost of capital prevailing in the market. So these are the considerations underlying minimum attractive rate of return. Once we do that, we had introduced concepts of cash flows that shows cash flow occurring at different points of time and then we had suggested that anything that takes place any payment or receipt that takes place in the beginning of a year will be called P, the principal sum.

Anything that takes place at the end of a series of pavements series of number of years will be called the final sum or F and then if equal payments take place in consecutive years, this will be called A. So for example a cash flow would be in this fashion.

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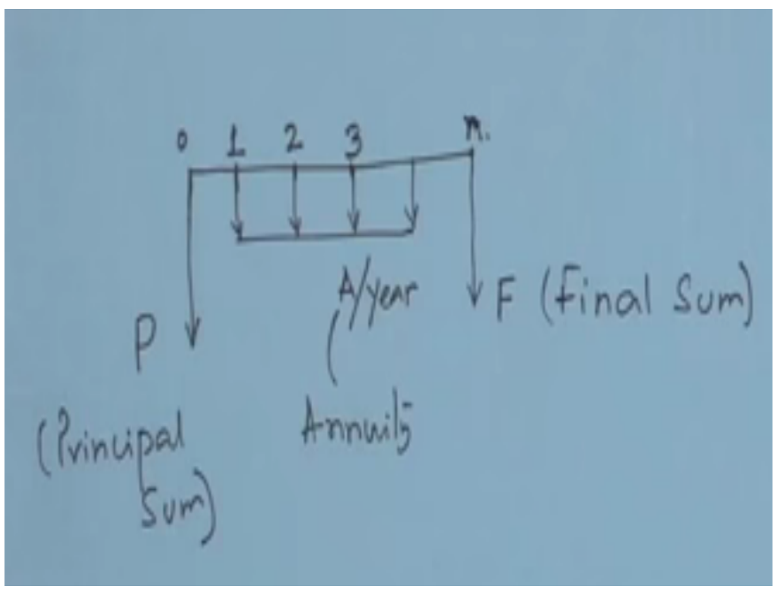

These are different interest periods starting with 0, this is 1, this is 2, 3 and then a payment made here is P and payment made here at the end of the n number of interest periods will be called F and equal payments will be called A per year or annuity for A, F for final sum and P for principle sum. Then we had found 6 formulae to find out equivalence. These ones I have now summarized in this slide.

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$(F/P, r, n) = (1 + r)^n$: *Single- payment compound- amount factor*

$F\ (P/F, r, n) = (1 + r)^{-n}$: *Single-payment present-worth factor*

$[F/A, r, n] = [((1+r)^n - 1)/r)]$:
Equal-payment-series compound-amount factor

$[A/F, r, n] = [(r/(1+r)^n - 1)]$:
Equal-payment-series sinking-fund factor

$[P/A, r, n] = [((1+r)^n - 1)/(r(1+r)^n]$:
Equal-payment-series present-worth factor

$[A/P, r, n] = [(r(1+r)^n)/((1+r)^n - 1)]$:
Equal-payment-series capital-recovery factor.

The first one says single payment compound amount factor.

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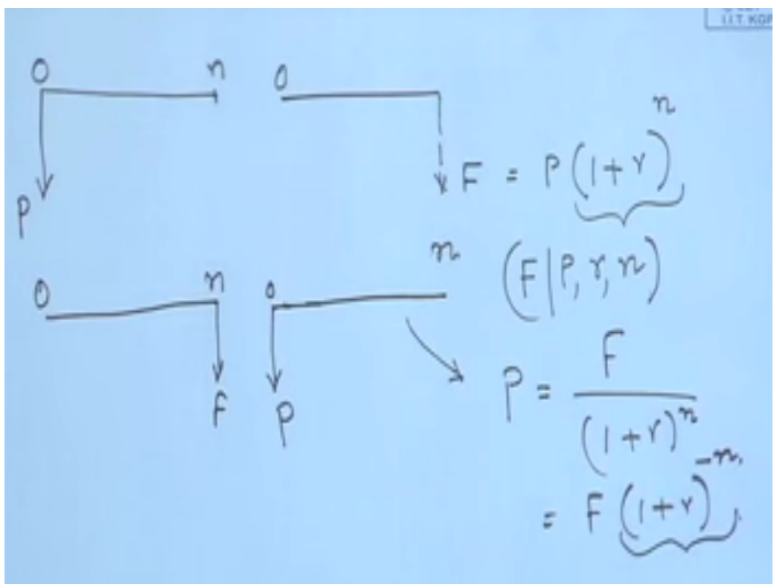

It means suppose that I have single payment taking place at the beginning P, its equivalence at the end of n time periods will be F. F=P* 1+r to the power n. This is said to find F given P, r, n and this is termed as single payment compound amount factor. It is a single payment case and it is compounded, therefore this is called single payment compound amount factor. The second case is that suppose F is given and we are interested to find out its equivalence at the beginning of the year.

Then it is just the reverse P = F/1+r to the power n or it is = F * 1=r to the power –n. This factor is once again a case of single payment, but we are trying to find out the present worth of F. So it is called single payment present worth factor.

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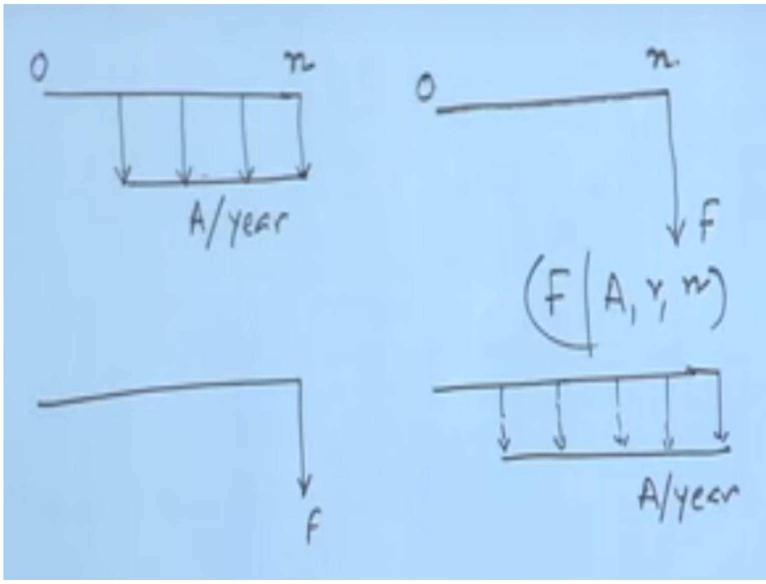

The third case is equal payment series compound amount factor. That means we are given A. Remember that A is assumed to take place at the end of the year. So A per year, this is n, this is 0, we are interested to find out its final sum and this is equal payment series they are taking place in series and each is equal payment and compounded to the end. Therefore, it is called equal payment series compound amount factor and the value is given by this.

To calculate F given A, r, and n equal payment series compound amount factor and the reverse is that we are given this F and we are required to find out what could be the equivalent series of payments A per year. It is just the opposite A given F, r, n. This is equal payment series sinking fund factor. This is called sinking fund factor and then the other 2 cases are that we are given A and we are required to find out its present worth.

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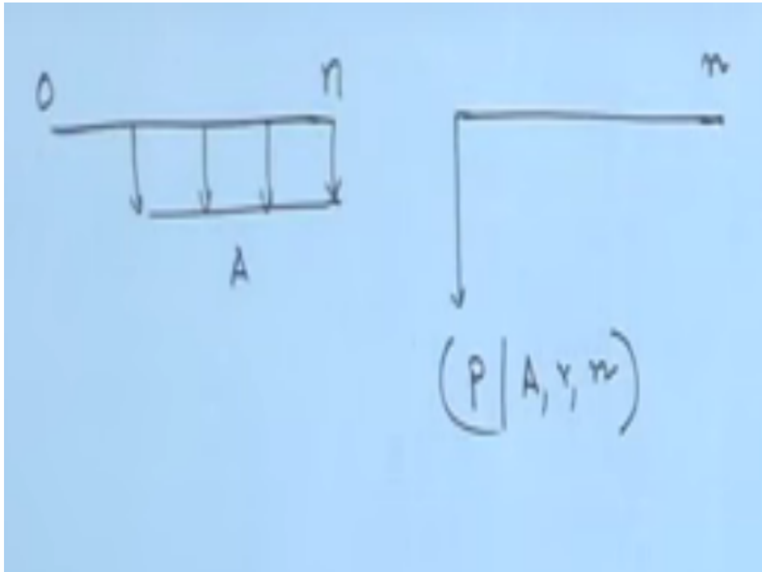

So to find out P given A, r and n so it is present worth, finding out the present worth. So it is equal payment series present worth factor. Lastly suppose that we are given P and we are required to find out what is the equivalent series of payments. Just the opposite, this is to find out A given P, r, n. This is called equal payment series capital recovery factor. So these are the different equivalence factors that we have derived in our last lecture.

So what we are going to do today is to apply these first of all to certain simple problems and then we like to use these concepts to compare among alternatives. Before that let us understand that these expressions 1+r to the power n or 1+r to the power -n and all this they are quite time-taking and sometimes when managers find it difficult to calculate the power functions, therefore like log tables, interest tables are available for different interest rates and for different factors and for different ends the values of the factors are given.

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Interest Tables

Interest tables give values of various factors for specific values of interest rates (*r*) and number of periods (*n*).

Usually, values of factors are given for interest rate values of

1 %, 2 %, 3 %, 4 %, 5 %, 10 %, 12 %, 15 %, and 20 %.

Example:

The value of the capital-recovery factor for *r* = 10 % and *n* =10 years is selected from the 10 % table as:

$(A|P, 10, 10) = 0.16275.$

For example, the interest tables may be given for 1%, 2%, 3%, 4%, 5%, 10%, 12% ,15%, 20% and different interest rates. For every interest rate, there is a table such as this.

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Interest Rate, *r* = 10 %

n	(F|P)	(P|F)	(A|P)	(P|A)	(F|A)	(A|F)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2						
3						
...						

For example, interest rates are 10% for different values of n, the 6 factors could be given, to find F given P, single payment compound amount factor, single payment present worth factor and so on and so forth. The values are calculated and tabulated in the tables for different rates, different tables and for different values of n are available. Therefore, one can straightaway take the values from the interest tables provided they are available.

For example, the capital recovery factor for r=10% and n=10 years, what we have to do we have to go to the 10% interest table look for the row n is equal to 10 and the column capital recovery meaning P is given and we have to find out A, A given P 10 and 10 and the entry would be the value which is in this case is point 0.16275. So if you have a calculator, you can use, otherwise you can use interest tables.

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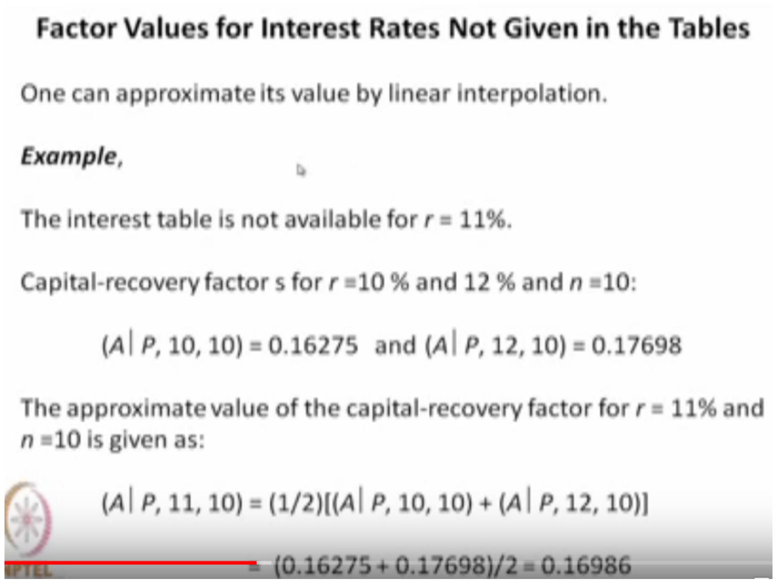

Now sometimes for interest rates tables are not available. Say for example in this particular case we have taken an example of r=11%, for which the table values are not available, r=10% is given and for r=12%, the values are given for a value of n=10 and the values are this is a capital recovery factor, 0.16275 for 10, 10 for 10% and for 12%, it is 0.17698, but we are required to find out for 11%.

If you have a calculator fine, go ahead and do it or else one can approximate the value of A given P 11, 10 by linear interpolation.

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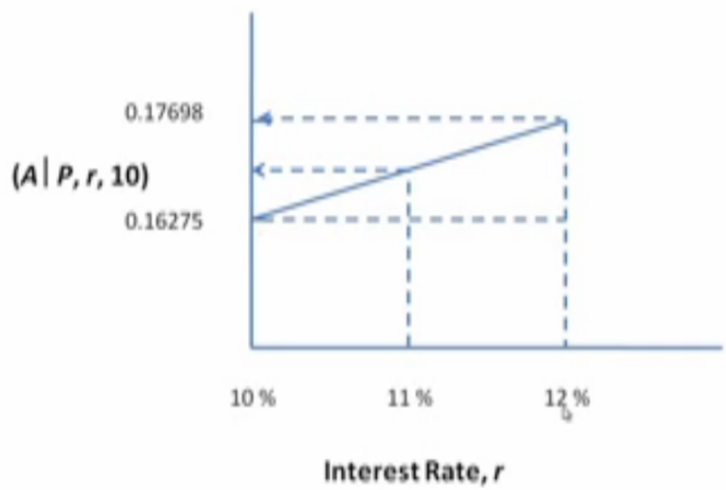

Linear interpolation is very simple. It is something like this what we have in the interest table is if r=10% the value is given as 0.16275. This is the value and when r=12%, the value is given as 0.17698. Therefore 11% is just half of this. So take the half of this, which is this plus this division by 2. So if you do that, it is an approximation it will not be exactly the value, but this is how interpolation we can make and we can get an approximate value.

So in this case, we have taken the average value is which is half of this plus this which is a equal to half of this plus this, which is equal to 0.16986. Therefore, we can determine their interest rates and for values of interest rates for which the interest tables have not available we can interpolate and find approximate values of every factor.

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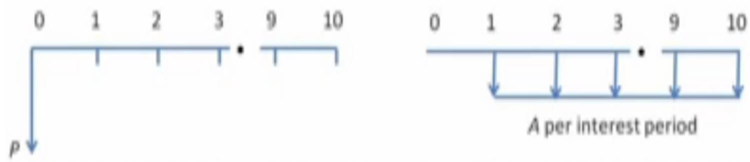
Example

A person wishes to deposit Rs. 10,000/- now for 10 years in a fixed investment that gives him a return of 12 % every year. If he is assured of an equal amount of return every year, how much can he get back every year?



Cash Flow Diagram for a Single Payment   Cash Flow Diagram for Annual Payments

Now let us take some examples to illustrate the use of equivalence. Let us say that we are interested in such a problem a person wishes to deposit rupees 10,000 now for 10 years in a fixed investment to give him a return of 12% every year, which means r=12%, n=10 years and he is depositing the money now. So it is at the beginning of the time. So at zero time the P=10,000 that is what we have to take.

So given P, r=12%, n=10, now if he is assured of an equal amount of return every year starting from the first year, how much can he get back every year. That means in the first year also he is getting back some money, so that money is assumed to take place at the end of the first year, end of the second year, like that for 10 years, he will get back some money giving initially an amount of 10,000. So this is the capital he is investing and this is how he is recovering the capital.

So we have to find out capital recovery factor to find A given P, r, and n and multiply that with P, which is 10,000 that will give us the value of A.

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Given

P = Rs. 10,000/-, r =12 % per year, and n =10 years

To find the equivalent annuity (A) of a series of equal payments.

The value of the capital-recovery factor is read from the 12% interest table:

$[A/P,12,10] = 0.17698.$

Thus the annuity, A, is obtained

$A = P[A|P, 12, 10) = (10,000)(0.17698)$

= Rs. 1,769.80 per year.

So that is what we have done. P is given as 10,000 rupees, r is given as 12% per year, n is given as 10 years. We have to find the equivalent annuity A of this series of equal payments for 10 years. So first of all we read the capital recovery factor from the interest table A given P, r and n that is 0.17698 and multiply that with P, 10,000. So the answer is 1769 rupees 80 paise. So it means that if the person pays 10,000 rupees now for next 10 years including this year, he will get 1769 rupees 80 paisa every year.

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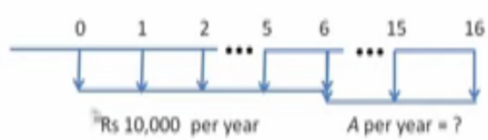
Example–2

A person deposits Rs. 10,000/- every year for a period of 6 years. If he is assured of an equal amount of return every year starting from the beginning of the 7th year for a period of 10 years (i.e., the first payment is made at the end of 6th year), what amount will he get back at a 12 % interest per annum?



Cash Flow Diagram

Take another example, this is a case of a person who deposits 10,000 every year. He deposits every year 10,000 rupees and this is assumed to take place at the end of the year for 6 years. So for 6 years, he continues to pay 10,000 rupees and starting from the beginning of the 7-th year, he gets an equal amount of return for a period of 10 years. That is the first payment is made at the end of the 6-th year. So beginning of 7-th year is the end of the 6-th year.

This 6 means this is the end of the 6-th year. So he gets back some money for the next 10 years. So up to the end of 16-th year, he gets back. So it will be 16 or 15 if he gets here 6, 7, 8, 9, 10, 11, 12, 13, 14, 15. So this should be, there is a mistake here this would be 14 and that should be 15 because the first payment is made at the end of the 6-th year. The 10-th payment will be made at the end of the 15-th year that will make it 10 years.

The question is if he has been making a payment for 6 years at the rate of 10,000 rupees per year and if he starts getting his money from 6-th year till 15-th year, that is for a total period of 10 years, what amount he is likely to get every year. If the interest rate prevailing is 12%. This is the question.

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The present worth of the deposits

= Rs.10,000 [P | A, 12, 6) = Rs.10,000 x 4.111= Rs. 41,110/-

The present worth of the receipts

= A* [P | A, 12, 10] x [P | F, 12, 6] = A x (5.65) x (0.5066)

[P| A, 12, 10]: To find the equivalence of the series of receipts (7^{th} through the 16^{th} year) at the beginning of the 7^{th} year (i.e. at the end of the 6^{th} year).

[P| F, 12, 6]: To discount the A * [P| A, 12, 10] to the beginning of the 1^{st} year.

Equating the present worth of deposits and receipts, one gets:

$$A = \frac{41{,}110}{(5.65)(0.5066)} = 14{,}410.74 \text{ Rs./year.}$$

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So what you supposed to do, first of all we can find out the present worth of the deposits. So he has been making a payment of 10,000 rupees for 6 years. So let us find out the present worth of this amount of money, that means find out the equivalence of this amount at this point of time. That means we find out the present worth, equal payment series present worth factor, given A find P. That is what we are now.

Let us now take another example. This is the case of a person who deposits rupees 10,000 every year for a period of 6 years, if he is assured of an equal amount of return every year starting from the 7-th year for a period of 10 years, what amount will he get back at a 12% interest per annum. For this problem we have first of all drawn a cash flow diagram. Now here we are showing that for 6 years 10,000 rupees have been paid.

Payments are assumed to take place at the end of every year and he gets back money 7-th year through 16-th year that is for a period of 10 years. All these payments are assumed to take place at the end of the year. If he makes a payment of 10,000 rupees per year, what is the value of A. That means how much he would get back in equal payment for 10 years starting from the 7-th year through the 16-th. This is the question, so what we can do?

We can consider there are different ways of finding out the values. One method or one way that I have followed here is, find out the present worth. This is the amount that I am now paying and that is 10,000 rupees per year. We now take up a second example.

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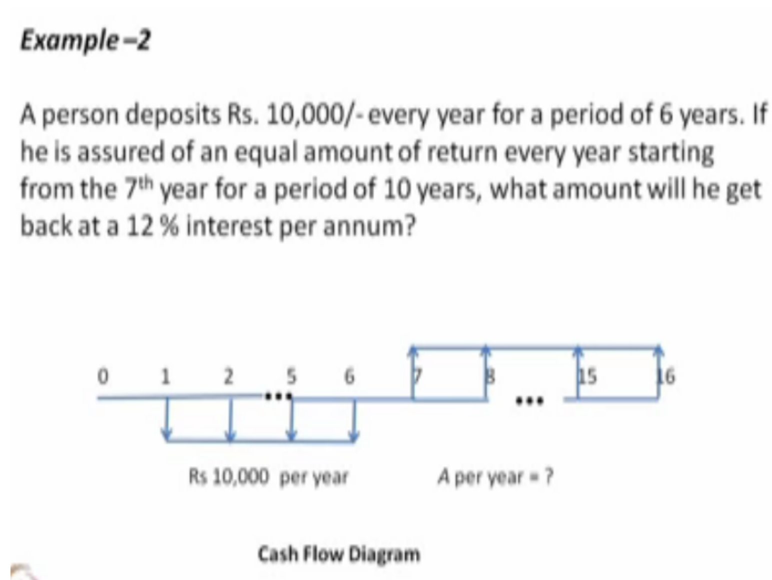

The example reads like this. A person deposits rupees 10,000 every year for a period of 6 years. If he is assured of an equal payment of return every year starting from the 7-th year for a period of 10 years, what amount will he get back at a 12% interest per annum. So here we are showing the cash flow diagram. For 6 years he is making the payment 10,000 rupees every year and then from 7-th year onwards, he is getting payment. He is getting receipts. Basically it is receipt, these are all receipts pointing upward and these are payment pointing downwards.

Payments are equal payment in series, so therefore this is A = 10,000 rupees and this is for 10 years, 7-th year through 16-th year, what is the value of A? This is the question. So what we can do, we can treat this as one cash flow and we can treat this as another cash flow. So what we first do is treat the first payments.

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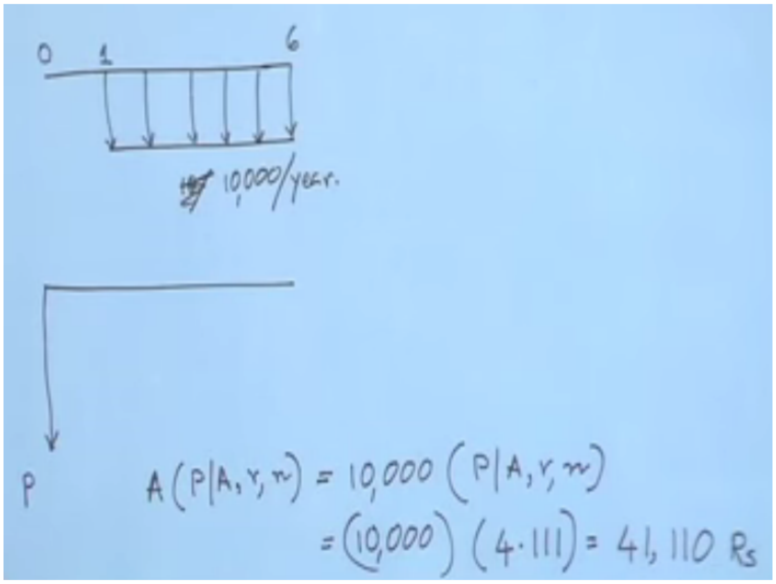

We take this as, what is its equivalence at the present time, what is the P. So we find out first of all the present worth of this equal payment series. For this we have to multiply A with P to calculate P given A, r, n. P = 10,000 and we have to find out P given A, r, n, n=6, r=12%. So we go to the 12% interest table and find out the value. The value of this is 4.111. Therefore, the present worth of this payment 41,110 rupees. Now what we do, we take the remaining portion of the cash flow.

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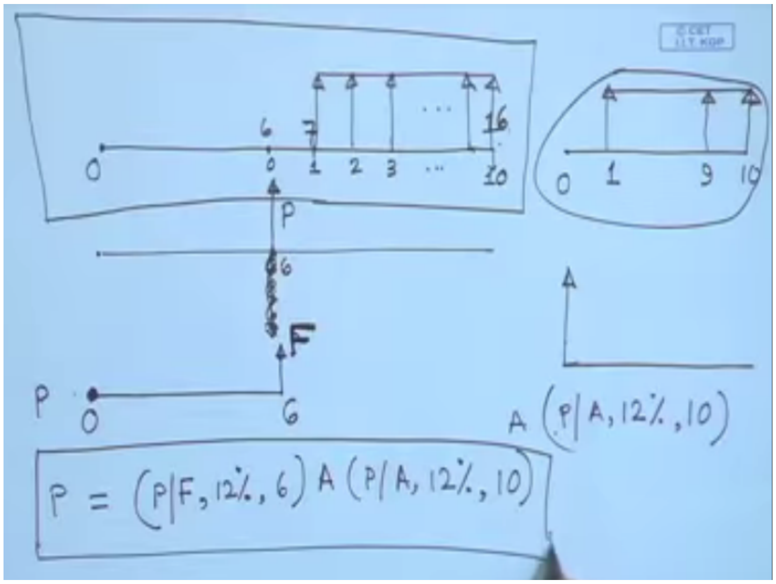

And that is starting from 7-th year through the 16-th year, there is a receipt, equal payment in series 7th through 16th. So we also can find out the present worth of this. For that what we are first of all doing we are assuming, we are trying to find out the present worth at this point of time. At this point of time I am sorry because this is a receipt it will be pointing upwards. So we have this is equivalent to writing 0 here, 1 here and 10 here. So instead of 7 through 16, we can write 0, 1, 2, 3 and like this 10.

Therefore, this particular portion would be something equivalent to like 10 here, 9 here, 1 here and 0 here and we find out first of all the equivalence of this at this point. So this is like calculating P given A, r is 12% and n is 10, multiply that with A that will give the present worth of this value here and that is what we are now putting here as equal to P. This is 6, so now we have a situation where we have P here and we can find out.

Whatever was P here now becomes an F here, the final sum and we try to find out the present worth at this point of time. Thus the present worth of this particular series of receipts will be given by first of all discounting this to this point treating this as a final sum and discounting once again to the present. So this P will be equal to, to calculate P given F 12% 6 * F and F is nothing but equal to this. So this is the value of P and that is what we have shown in this slide.

The present worth of the receipts are equal to first of all find P given A, r, 10 and then multiply that with, find P given F, 12, 6 and A is the amount of receipt that we are interested to find out and the values are read from the interest tables. They are this and this. Now we equate the 2. So what we have deposited its present worth is 41,110 and what we will be receiving in 10 years starting from 7th through 16th year, its present worth is equal to this. We now equate them.

So A is equal to this divided by this into this. That is what we have written here and that comes to 14,410 rupees 74 paise. It means that if the person deposits for six years consecutively 10,000 rupees, then he will get back 14,410 rupees and some paise for the next 10 years. So the idea behind this is, first of all find out the present worth of all the future receipts and the present worth of all the cash outflows in the form of payments and equate them to find out the value of A.

So these 2 examples illustrated the use of the factors and the interest tables and how they should be used to find equivalence of series of payments and receipts made at different points of time. Now we shall use these concepts in actually comparing among different project alternatives or economic alternatives.

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Now there are different methods of comparing among alternatives, but the most elementary ones are these 3. The present worth comparison, the equivalent annual cost comparison and internal rate of return or also called IRR comparison. We shall see what each one is all about. We already know present worth.

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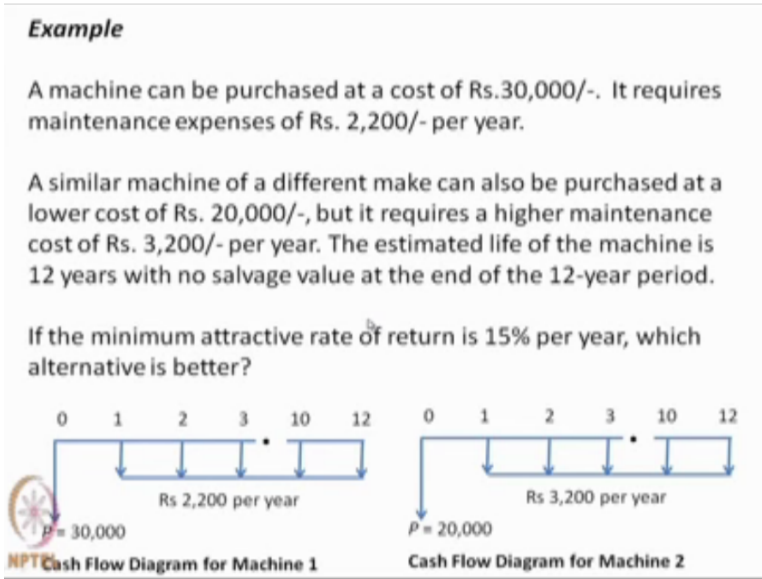

Let us take this particular example and use it and illustrate the 3 methods of comparing among the alternatives. It is a case of a small entrepreneur who would like to purchase a machine at a cost of 30,000, but he estimates that the machine would require a maintenance expense of rupees 2,200 every year. A similar machine of a different make can also be purchased at a lower cost, which is 20,000 rupees.

This machine is 30,000 and this machine is 20,000, but it requires a higher maintenance cost of 3,200 rupees per year. Maintenance cost for this machine was 2,200. Maintenance cost for this machine is 3,200. If the estimated life of the machine is 12 years with no salvage value at the end of the 12-year period, it means that the machine either machine 1 or machine 2, it will work for 12 years and after that it has to be thrown away.

It cannot be used and if the MARR that is a discount rate is 15% per year then which of the 2 alternatives is better. So this is a case of comparing between 2 alternatives. The first cost of the machines is different and the annual maintenance expenses are also different. However, the annual expenses are distributed over time. The first cost of course take place at the beginning of the times. Now how to compare these 2 alternatives.

We will use all the 3 methods, but first let us make or draw the cash flow diagram. Cash flow diagram is like this. These are all cases of payments, no receipts. The first payment is the purchase price of 30,000 rupees for machine 1 and the annual payments or the annuity is 2,200 rupees per year. This is for machine 2, the values are 20,000 and 3,200 per year. This is very simple.

So the idea for the present worth cost comparison method is find out discount all flows cash flows to the present, for machine 1 and for machine 2 and then find out which is lower, go for that particular alternative.

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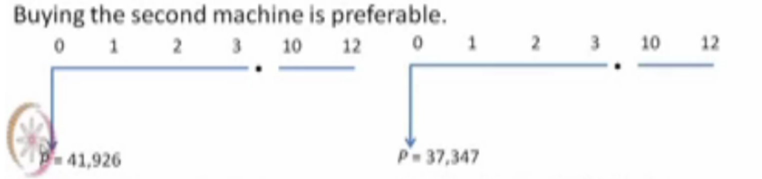
Present-Worth Cost-Comparison Method

The present worth of each of the two sets of cash flows is:

$PW_1 = 30{,}000 + (2{,}200)(P|A, 15, 12)$
$= 30{,}000 + (2{,}200)(5.421) = \text{Rs. } 41{,}926/-$

$PW_2 = 20{,}000 + (3{,}200)(P|A, 15, 12) = 20{,}000 + (3{,}200)(5.421)$
$= \text{Rs. } 37{,}347/-$

Buying the second machine is preferable.



Cash Flow Diagram for Machine 1

Cash Flow Diagram for Machine 2

That is what we have done here. In this diagram, all the costs associated with machine 1 are discounted here. All the costs associated with machine 2 are discounted to the present. Now the initial cost was 20,000 or 30,000 that remains. So present worth of the payments are 30,000 remains as it is and these were equal payment series we have to find out the present worth of equal payment series.

So this equal payment series present worth factor to find P given 15%, 12 number of years. So from the interest table, we found out its value as 5.421 and the value of the annuity is 2,200 multiply that add that to 30,000, we get 41,926 rupees. A similar procedure we followed for machine 2, initial price 20,000 and these are the annuity 3200 maintenance expense multiplied by equal payment series, present worth factor read from the table as 5.421 that came to 37,347.

So we found because it is a payment, this payment is less than this. It means the second machine is less costly and therefore the second machine is preferred. It is a very simple application of present worth cost comparison method.

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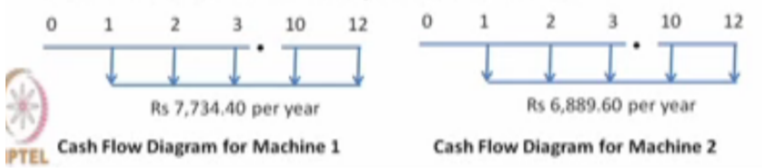
Equivalent Annual Cost Comparison Method

The equivalent annual costs of the expenses associated with machine 1 and machine 2 are as follows:

EAC_1 = 30,000 [A|P, 15,12] +2,200
= 30,000 [0.18448] +2,200 = Rs. 7,734.40 per year

EAC_2 = 20,000 [A|P, 15, 12] +3,200
= Rs. 6,889.60 per year.

Machine 2 is an economically better alternative.



Now let us take off the case of equivalent annual cost comparison method. In this case, we convert all the cash flows into annuities. If you remember annuities are equal payment series. The first will be taking place at the end of the first year up to the end of the final layer. This is the equal payment series. These are called equivalent annual cost. So this is what we are supposed to find out. Already we know 2,200 rupees was the maintenance expense every year.

So that was like an annuity. What we have to do now is that the first payment that was made which was 30,000 that should be converted into equal payment series. So that is what we have done. The first payment was 30,000 and we have to find out what is A. To find out A given P, r, n. So we are trying to recover the capital. This is capital recovery factor at 15% and 12 years. So 30,000 is the initial P present some P to be multiplied with the capital recovery factor read from the table as 0.18448.

Add to that the already existing annuity of 2,200 that gave a value of 7,734 rupees 40 paise. This is the equivalent annual cost for machine 1 and similarly we calculate equivalent annual cost for machine 2 which is this. Here is 20,000 was the initial price and multiplied that by the capital recovery factor, then add to it the maintenance expenses, which is already in an annuity form. So that gave a value of 6,889 rupees 60 paise and this is obviously less than this.

Therefore, machine 2 is an economically better alternative. Similar conclusion we are getting as we got for by applying the present worth cost comparison method.

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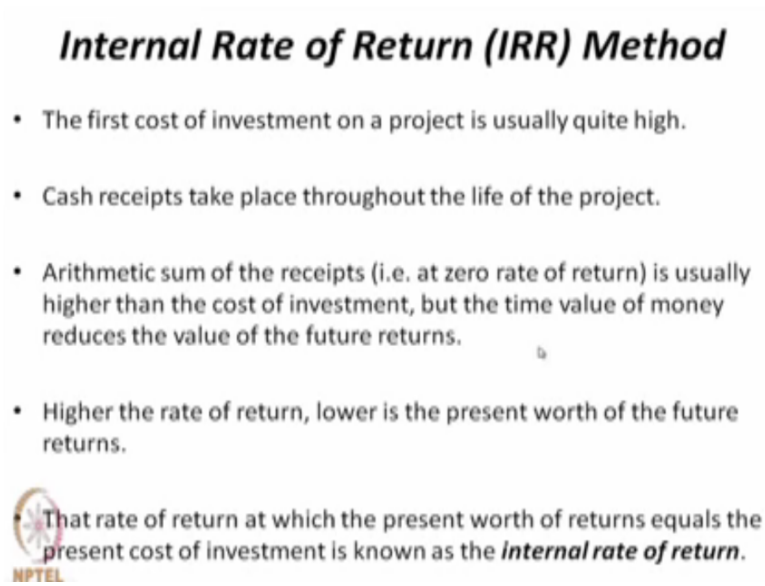

Now we introduced the third method, which is called the internal rate of return method. Internal rate of return method is little more difficult than the other 2. So first of all what we mean by internal rate of return. We know that the first cost of investment on a project is usually high and receipts take place throughout the life of a project, the cash inflows. The first cost is already cash outflow.

Now the arithmetic sum of the receipts that means if it is a zero rate of return, should be always higher than the cost of investment for the project to be acceptable. But the time value of the money reduces the value of future returns. The sum of the receipts is usually arithmetically higher than the initial payment. If it is not, then it is not at all viable, but even if the arithmetic sum of the receipts future receipts is higher because of the time value of money the discounted value is always less and it can go below the initial cost of the project.

The higher the rate of the return, the lower is the present worth of the future returns. That rate of return at which the present worth of returns equals the present cost of investment is known as the internal rate of return. So although the arithmetic sum that means at 0 interest rate of future returns could be higher than the present investment, because of some positive rate of return, a positive discount rate, the discounted value of the future returns becomes less and less as the interest rate or the rate of return or the discount rate increases.

A time comes when it is equal to the initial cost of investment and at that interest rate, at which the cost of investment and the discounted returns equals is called the rate of return. Now this is shown.

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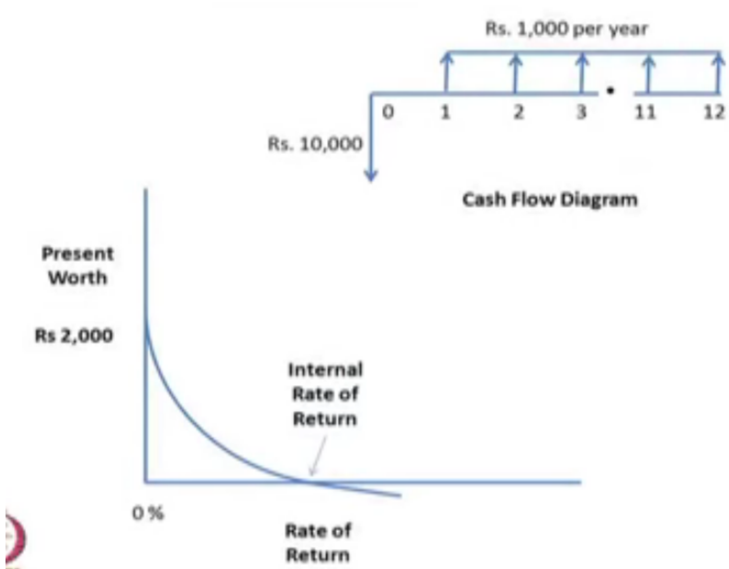

So friends what we have seen in the last lecture and today's lecture is that there are different methods of making comparison economically among alternatives and in the last example, we have taken how to use internal rate of return method and how to use it to compare between 2 alternatives. We have already seen in the past that the most preferred method is present worth cost comparison method.

WE can also use equivalent annual cost comparison method and the third method that we discussed just now is internal rate of return method and we have seen that in the internal rate of return method, basically we have to consider the differential project and we will have to see whether the internal rate of return is higher than the minimum attractive rate of return. If the IRR is > MARR, then the differential project is preferred, otherwise it is not.

So in this particular example, we have seen that the IRR is somewhere here and the value of the rate of return is this value and if this value is higher than MARR, then the differential project is definitely better. Now we shall also consider where these 3 methods are actually applicable to different projects. First thing is that if the life of the projects are equal, then present worth cost comparison method is applicable.

But if the lives of the projects are different, then the equivalent annual cost comparison method is preferred. At that time the present worth cost comparison method is not applicable. Internal rate of return method when applied to comparing between 2 or more alternatives is quite difficult. Assets calculation of IRR itself is time taking and therefore difficult to apply, but when we have only a single project, in that case IRR is the best and is the only method applicable.

Because there, we are finding out the IRR and then comparing it with the minimum attractive rate of return. So these are the relative advantages of the 3 methods of comparing among alternatives. We will take up further in our next class, other issues concerning time value of money. Thank you very much.