Economics, Management and Entrepreneurship
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Lecture - 21
Time Value of Money

Good morning. Welcome to the 21st lecture on Economics, Management and Entrepreneurship. In the last 20 lectures, we had had elaborate discussion on microeconomics, costing and accounting. This today's lecture and 2, 3 more lectures from today, we shall be discussing on a very important aspect generally known as managerial economics or engineering economics or engineering economy. In particular, we shall now be discussing the time value of money.

As you know, when a company invests a large sum of money in a project, then there are different types of cash flows that occur at different time periods and before an investment is made the company has to make sure that the investment will give proper return and that the project that gives the best return on investment is the one that has been properly selected. Now the main problem that appears here is that cash flows occur at different time periods.

If I give an amount of 100 rupees to somebody today and he returns 100 rupees to me at the end of the day, then the net inflow is 0 for me. There is no problem, but suppose I give 100 rupees to somebody and he returns it after one year the same money 100 rupees, then I am a loser. I am a loser because I could have earned certain interest, if I had that 100 rupees with me and if I invested in some form. If I put it in the savings bank account of the firm, I would have at least got 4-5% interest rate that I have lost. So this is time value of money.

If my friend returns the amount 5 years hence, then I am a bigger loser than if he returns it 1 year hence. In all such cases, time value of money is therefore paramount and this is more so when we consider capital budgeting situations in industries where the industry invests large chunk of money in different projects, in selecting the projects and in deciding the profitability of the projects. So today's topic is time value of money. Time value of money is the topic for today.

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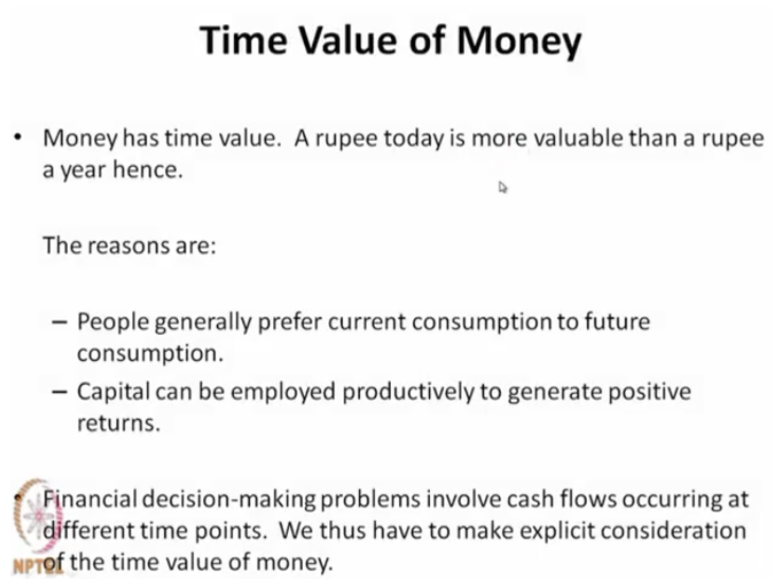

Whatever I told you just now is written here money has time value. A rupee today is more valuable than a rupee a year hence. The reasons are that the people generally prefer current consumption to future consumption. Capital can be employed productively to generate positive returns and for financial decision making problems, we have to understand that cash flows occur at different time periods, thus necessitating explicit consideration of time value of money.

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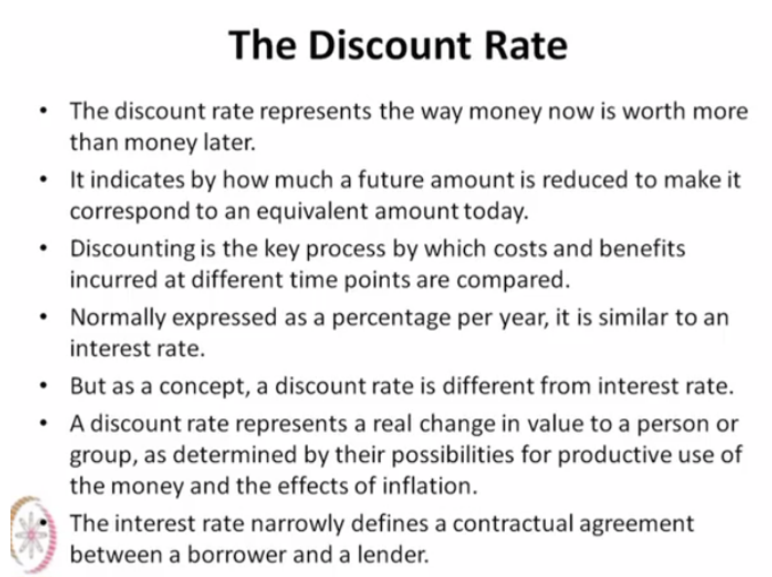

We now introduce something called discount rate. Discount rate represents the way money now is worth more than money later, which is also how discount rate actually represents this phenomenon. It indicates how much a future amount is reduced to make it correspond to an equivalent amount today. Discounting is the key process by which costs and benefits incurred at different time points are compared.

It is normally expressed as a percentage per year and it is similar to interest rate, but as a concept a discount rate is different from interest rate. A discount rate represents a real change in value to a person or a group as determined by their possibilities for productive use of the money and the effects of inflation whereas interest rate narrowly defines a contractual agreement between the borrower and the lender.

Basically interest rate is something like an agreement between who lends money and the person who borrows the money. The interest rate could be 10%, it could be 15%, it could be 20%. It is fixed by an agreement between the borrower and the lender. Discount rate is similar to interest rate, but it is different because it reflects the value of the person who invests the money, a person or the group of persons who invest the money.

What he thinks would be the rate at which he can consider the amount to diminish as time proceeds. We shall give more examples of this later.

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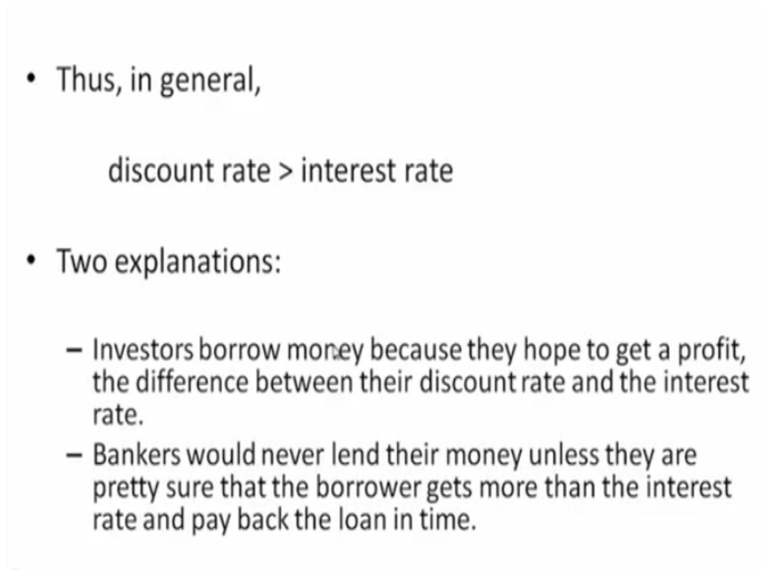

So in general, discount rate is > interest rate. Why because investors borrow money at a particular interest rate, because they hope to get a profit by investing that borrowed money in some business. So the difference between the discount rate and the interest rate is the profit. Bankers would never lend their money unless they are pretty sure that the borrower gets more than the interest rate and pays back the loan in time.

So when a banker a bank a financial institution lends some money to somebody meaning a company let us say, to a company a big amount of money at a particular interest rate usually the interest rate is quite high of the order of 15% let us say, then bank also looks at the possibility at the risk that it is taking of the borrower not paying back the money. The banks should feel that the borrower can invest the money in such a business that it will earn more than 15% to be able to pay back 15% of interest to the bank along with the principal.

So that is discount rate which is higher than therefore the interest rate.

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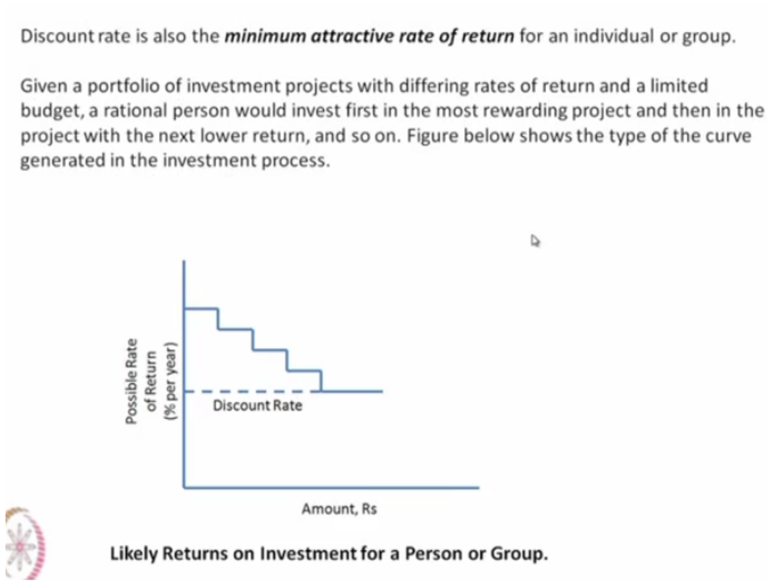

Discount rate is actually the minimum attractive rate of return for an individual or a group. We have given an example here. Suppose that these are projects, each individual project requiring certain amount of money. This is the amount of money required and let us say that this project can give so much rate of return. Suppose that they are arranged in decreasing order, the project that gives the maximum rate of return is plotted here and that requires this amount of money.

The next one gives little less rate of return and this requires so much of money and suppose that I have this amount of money certain amount of money, then naturally I will choose the one that gives me the highest rate of return. If I still have some more money, I will go to the next higher project. If I still have some more money, I will invest in the third project.

I will continue like this till my total money is exhausted and the minimum rate which I think is acceptable to me, which is higher than the bank interest rate and is acceptable to me is the minimum attractive rate of return and that is called the discount rate. So this figure shows pictorially that discount rate is the minimum attractive rate of return, where an investor or at which an investor can invest his money in a project. That is the definition of discount rate. This is always > the interest rate.

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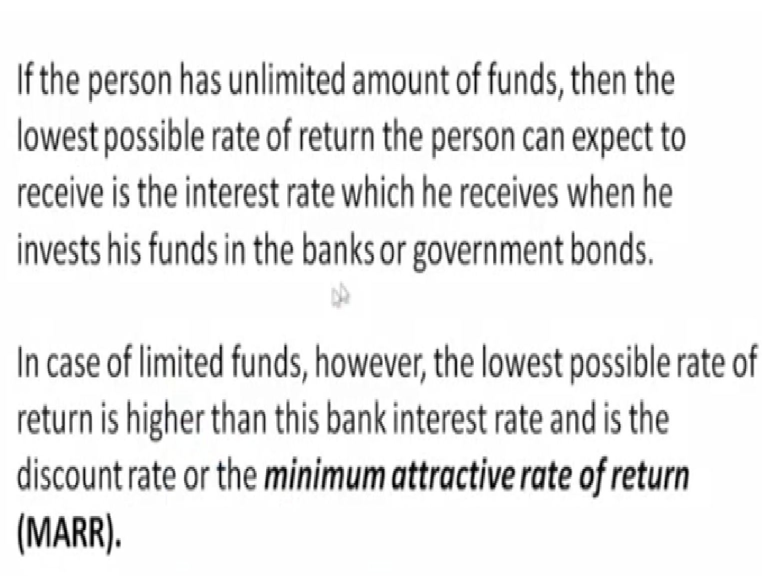

Now although discount rate is conceptually similar but different from the interest rate, they are actually similar as far as its treatment is concerned in a mathematical domain. For example, we are very familiar with compound interest formulae. Suppose I invest on amount of rupees 100 today for 2 years where the interest is compounded every year, let us say the interest rate is 10% and compounded annually, it means that at the end of 1 year, I will have 100 rupees + the interest which is 10% 100 that comes to 10.

So the compound amount is 100 rupees + 10, 110 rupees at the end of the first year. At the end of the second year, 10% of 110 whatever it is 11 rupees, that would be the interest and when we add 110 with 11, it will be 121 and that is the compounded amount at the end of 2 years. This is the usual compound interest formula that we have studied in schools. Now similar treatment is used here when we talk about discount rate, only its interpretation and its values are different.

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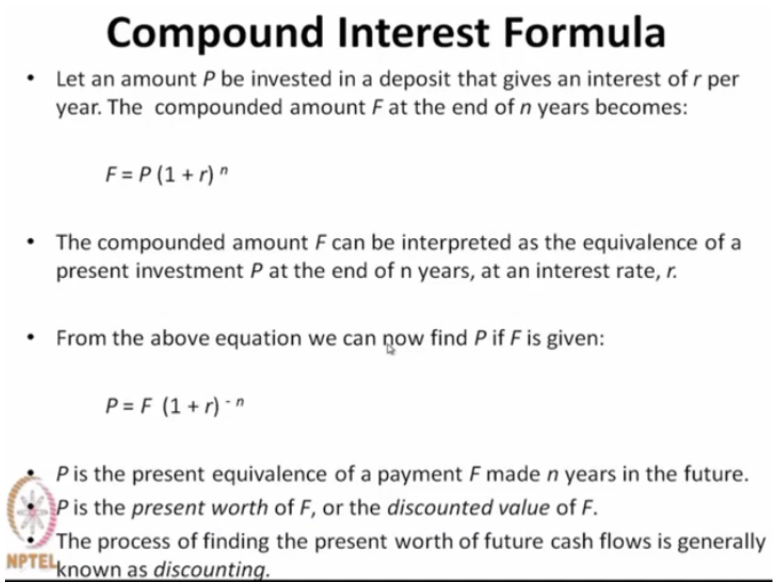

So we call it compound interest formulae and not formula because it is not 1, but there are many, so I will call it formulae. Let an amount P be invested in a deposit that gives an interest of r per year. The compounded amount F at the end of n years is well known to us. The formula is F = P the amount * 1 + r, r is the interest rate per year to the power n number of years that will give me the compounded amount at the end of n years.

We are calling it F. This is a very well known formula. Now we can say from this formula that P which was invested today is equivalent to an amount F with an equivalent F at the end of n years. So this is a concept of equivalence in time. The simple example with the simple formula of F = P * 1 + r to the power n can be interpreted in this manner, that an amount P today is equivalent to F n years hence, if F = P * 1 + r to the power n.

Now since R is the interest rate, it is something like 0.1, you can see that F is quite high, higher than P. So the equivalence of P at n will be much higher, which is F. From the above equation, we can find out what is P related to F. P = F * 1 + r to the power –n. So this says that if somebody is giving me 100 rupees 2 years hence, then its equivalents today are much less, because it is divided by 1+ r to the power -n, n is 2 and if r is 0.1. It is 1.1 to the power -2, which is 100/1.1 to the power 2 by that amount the value of P will reduce.

So P is the present equivalence of a payment F made n years in the future. Normally we say P is the present worth of F or the discounted value of F.

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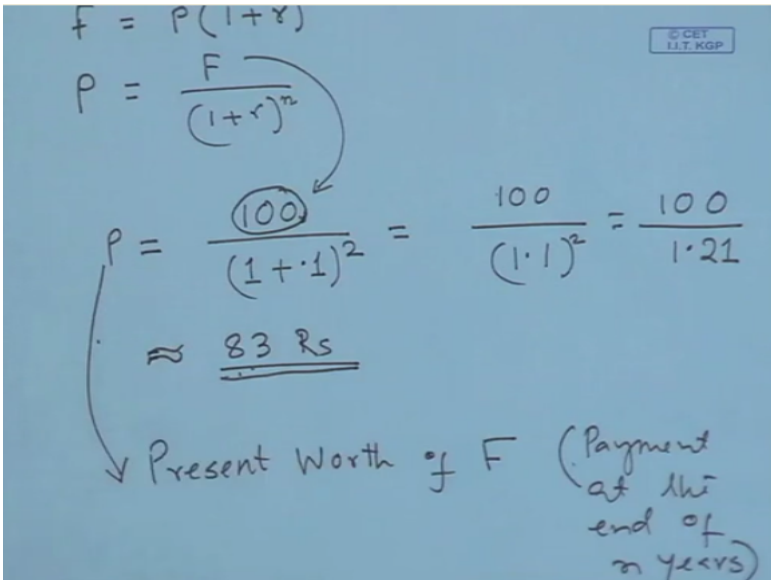

We can see here that P is F = P to the power 1 + r to the power n and P = F/1 +r to the power n. So if F = 100 rupees and r = 0.1, n = 2, then P = 100/1.1 square, which is = 100/1.21, which is something close to 83 rupees something. So 100 rupees payment 2 years hence that is F is equivalent to 83 rupees today. The value has come down. This we say is discounted. 100 rupees is discounted to 83.

The discounted value of today for a payment of 100 rupees 2 years hence is 83 rupees that is why the word discounted and the process of finding the present worth of future cash flows is generally known as discounting. This is called the present worth of F payment at the end of n years. It is present value is less and this is the process of discounting.

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Let

r: the nominal rate of return or the nominal interest rate
n: the number of interest periods
P: the present principal sum
A: Annuity (a single payment, in a series of n equal payments made at the end of each interest period).
F: A future sum, n periods hence, equal to the compound amount of P, or to the compound amount of a series of equal payments A.

Now let us define a few things, r is the nominal rate of return or the nominal interest rate or the minimum attractive rate of return, n the number of interest periods normally interest period means 1 year when the interest is calculated and compounded, but sometimes in many cases the interest is compounded half yearly or even quarterly. In those cases, interest periods may be < 1 year, half a year or one-4th of a year.

Let P be the present sum that means a payment or a receipt today A is the annuity. It is a single payment in a series of n equal payments made at the end of each interest period that means suppose that there are n interest periods then at the end of each interest period, suppose that there is an equal amount payment and that amount every year is called A or every interest period is called a and let F be the future sum n periods hence, which is an equivalent compound amount of P or an equivalent compound amount of the series of equal payments a.

Now these things just now would not be very clear to you, but as we proceed you will see that there very simple notations and they will be very clear as we go on.

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We assume the following:

- The end of one interest period is the beginning of the next interest period.
- *P* occurs at the beginning of an interest period at a time regarded as being the present.
- *F* occurs at the end of the n^{th} year from a time regarded as being present.
- *A* occurs at the end of the period.
- Any receipt or payment made during an interest period is assumed to take place at the end of the period.

First we assume that the end of 1 interest period is the beginning of the next interest period that p occurs at the beginning of an interest period at a time regarded as being present. PA will be considered as being present that is time T = 0. F occurs at the end of the n-th year from a time regarded as being present that is time T = 0. So F occurs at time = n, P occurs at time = 0, A occurs at the end of each period. That is 1, 2, 3, 4, 5, up to N.

One important thing here is that any receipt or payment made during an interest period is assumed to take place at the end of the period. Now once again they will be very clear once we go here.

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Single-Payment Compound-Amount Factor

Cash Flow Diagram for a Single Payment

- The beginning of the first period is marked as zero. The vertical arrow indicates that a payment, P, is made at the beginning of the first period.
- Equation below gives the equivalence, *F*, of the payment, *P*, at the end of *n* periods. The equation is written as

$$F = P\,(F/p, r, n)$$

$(F/P, r, n) = (1 + r)^n$: *Single- payment compound- amount factor.*

Now this is an example of a cash flow diagram. In a cash flow diagram, we saw the x axis as the time and we have in this case n time periods at an interest periods, we are assuming that every interest period the amount is compounded and the beginning of a year is 0. Beginning of this axis considered as being present end of the first year is written 1, end of the second year is written 2, end of the third year is written 3, end of n-first year is written as -1 and end of n years is written n.

Here we are drawing a cash flow diagram for a payment P considered to be made at the beginning of the first time period that is 0, that is what I was trying to tell you the principal sum P made at the present time. This is considered present. All these are considered future. The beginning of the first period is marked as 0. The vertical arrow indicates that the payment P is made at the beginning of the first period and the equation below gives the equivalence of F of the payment P at the end of n periods.

So if a payment P is made today, what is its equivalence at the end of n periods. We have already seen. It is = P * 1 + r to the power n and that is called F. So we say the equivalence of P at the end of n time periods is capital F and that is = P multiplied by a factor that factor is 1 + r to the power n and that is given a symbol F given P, r, n. Look this factor is multiplied with P to give a value F. So given r and n and P this factor will be multiplied with P to give a value F.

So a symbol F given P, r and n can also be written in place of 1 + r to the power n. This is a convention to write in this fashion F = P multiplied by a factor, which is given a symbol F given P, r, n and the name of this factor is single payment because it is a single payment case compound amount factor when this is compounded up to n periods, then what is its value. The factor value is 1 + r to the power n. This is called single payment compound amount factor.

So when one knows single payment compound amount factor multiplies that with P, one can find out capital F.

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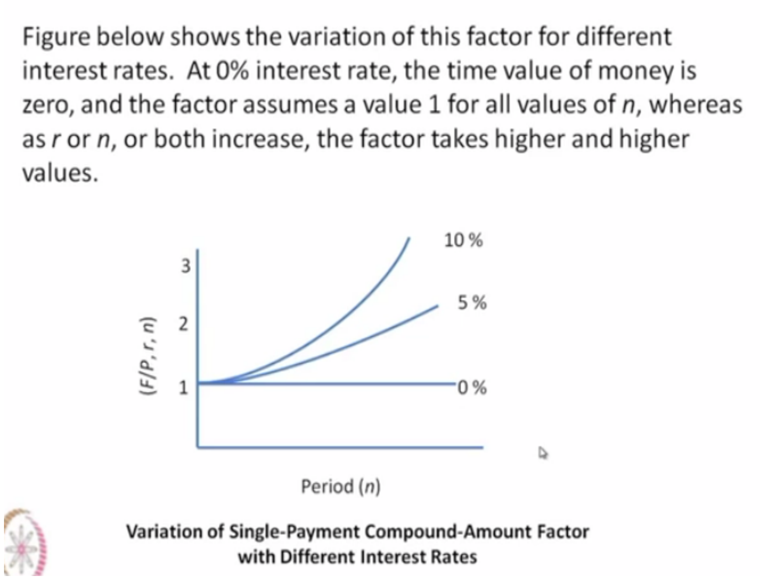

Now in this diagram, we are saying how r changes at different time period n, how this factor value changes. Now if r = 0, look at this 1 + r to the power n, the value of this is 1 + r to the power n. Now if r = 0 then 1 to the power n becomes 1. It means this value remains at 1. So if there is no interest rate, then F value will remain as equal to P, but as interest rate increases from 0 to 5% to 10% to 15%, then the compound takes place in an increasing fashion that means a value P today is equivalent to a large amount of money 2 years, 3 years, 4 years, 5 years hence.

The higher the value of n, the higher is the value of F and the higher the value of r, higher also is the value of F. This is what is shown in this diagram. It shows how single payment compound amount factor changes its value with different interest rates as n increases.

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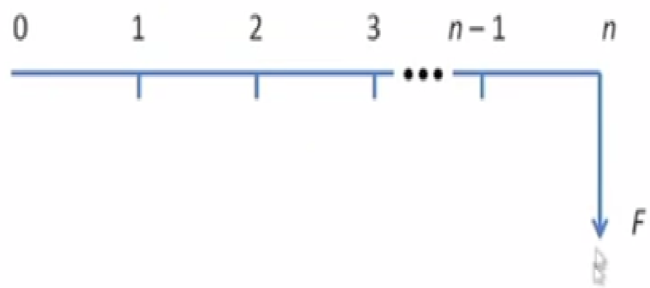

Cash Flow Diagram for a Single Payment *n* periods hence

Now we consider the reverse case. This case we considered the reverse case. The reverse case is that this is the cash flow diagram for single payment n periods hence, meaning we are here n periods in the future, suppose a payment is made of F at a value F that will be called F. We are calling such a value such a payment made at the end of n time period as F and not P. P is when we are making a payment at the present time, at the future time we are saying a future sum F.

Then what is its equivalence today. We have seen P = 1 + r to the power n, P * 1 + r to the power n is F, therefore F given the equivalent P is 1 + r to the power -n * F, that is what is written here. The present worth P of a future payment F is another equation given by this, which is 1 + r to the power n. Now this is called to calculate P here we are calculating P given F, r, n. The earlier one was given F calculated P, r, n. This is given F, r, n calculate P and the value is 1 + r to the power -1.

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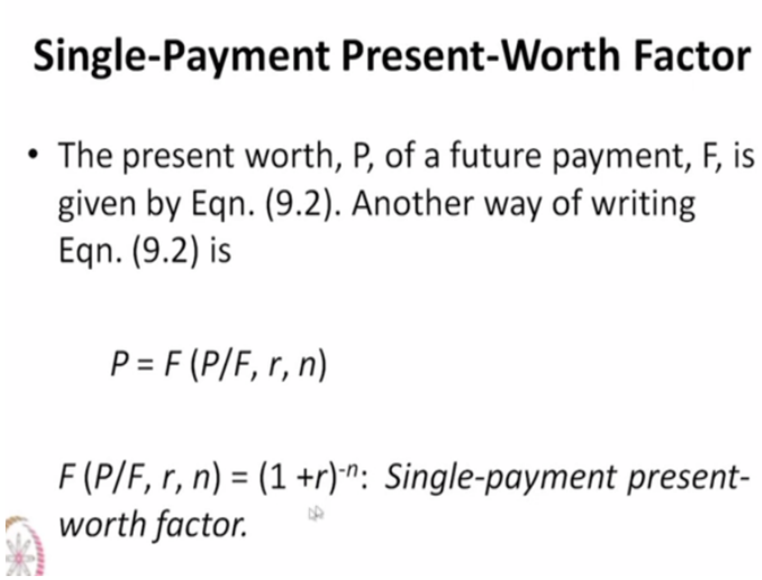

This is called single payment present worth factor. So the earlier one was called single payment compound amount factor and in this case it is a single payment present worth factor, because this was a single payment and we are trying to find out its present worth. So this factor 1 + r to the power -1 is called single payment present worth factor and how the value changes.

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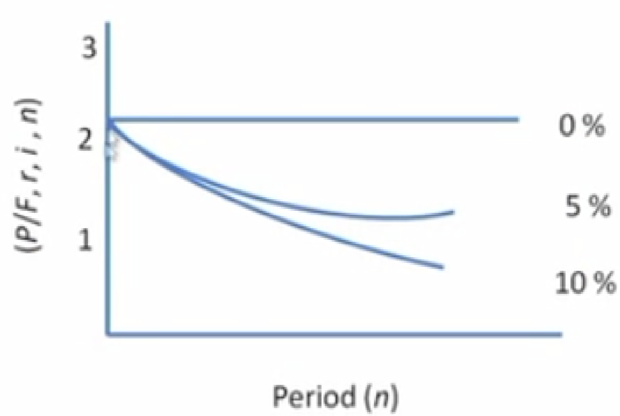

Variation of Single-Payment Present-Worth Factor with Different Interest Rates

There is a mistake here. This value will be 1 and this value will be much less, something like 0.5 or so. So here we are showing how the value of this factor the single payment present worth factor is less and less as interest rate increases, because the expression for this factor is 1/1 + r to the power n. So as in r increases the value reduces further. This is called discounting. As r increases or n increases, the value of P further reduces. This is evident from this equation

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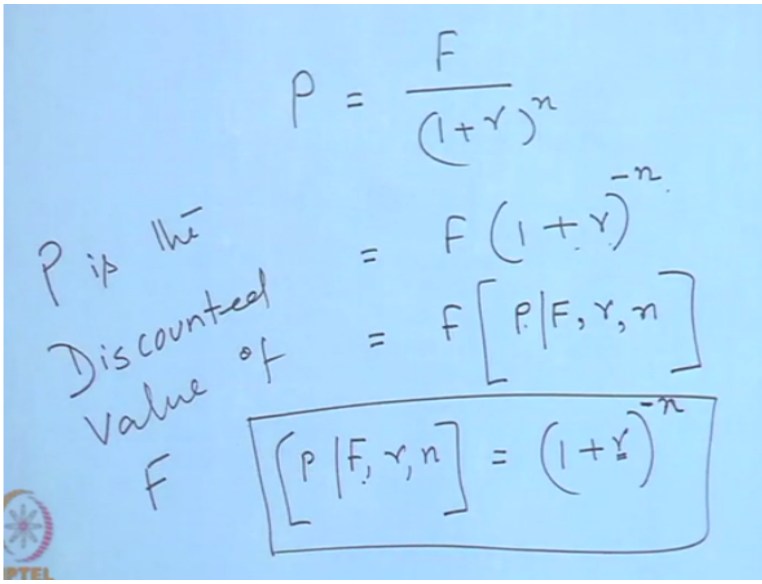

P = F/1 + r to the power n. This we are saying F * 1 + r to the power –n. This we are saying F multiplication to find P given F, r, n where P given F, r, n is nothing but 1 + r to the power –n. So P given F, r, n = 1 +r to the power –n. This is the single payment present worth factor and you Can see because this is –n, as n increases or r increases, the value falls. Therefore, this is called discounted value. Discounted value P is the discounted value of F.

Now this is what is shown here. This we have already discussed.

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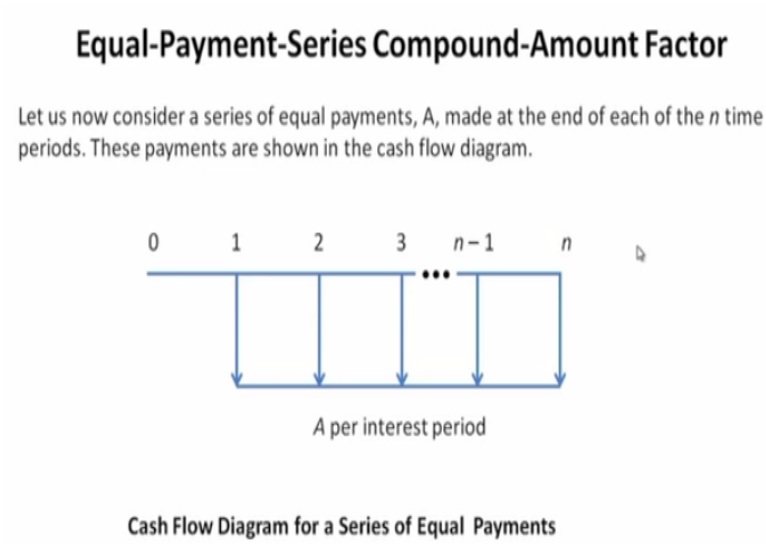

Now we are considering, look at this A. We had already introduced a symbol A. This is called equal payment series compound amount factor. Equal payment meaning the same amount is paid equal amount of A per interest period is paid for a number of years. We are assuming here that A amount is paid in all the periods each of the n periods and as per our assumption all payments are assumed to be made at the end of the year.

So even though there are payments taking place within a year or within an interest period, we are assuming that as if they are all made at the end of the interest period. So there are how many payments here n number of payments, each = A. Our interest is to find out if this is there what is its equivalent amount at the end of the n periods and equivalent amount at the present period. When we will talk about the equivalent amount at the end of the n periods, the factor will be called compound amount factor, equal payment series compound amount factor.

When we shall consider its present worth, we shall call it little differently. So just now we are considering equal payment series compound amount factor meaning its equivalents of a series of equal payments at the end of the n periods.

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We wish to find the equivalence of these payments at the end of n time periods.

Obviously, the equivalence of the last payment A is A itself, the equivalence of the last but one payment is A (1+r), and so on.

Thus, the equivalence at the end of n time periods is given as

$$F = A + (1 + r) + A\,(1 + r)^2 + \ldots + A\,(1 + r)^{n-1}$$

$$= A[((1+r)^n - 1)/((1+r)-1)]$$

This equation is often written as

$$F = A\left[\frac{(1+r)^n - 1}{r}\right]$$

$$F = A\,[F|\,A, r, n]$$

where, $[F|\,A, r, n] = [((1+r)^n - 1)/r)]$

and is called the ***equal-payment-series compound-amount factor***.

Now you see we can consider this. Suppose that we are considering only this particular payment, so its equivalents are this place is A itself whereas the equivalents of this particular payment is A*1+r. The one previous to that will be A*1+r square and like that it will continue and that is what I have written down here the equivalent of this series of payments is the last payment was A, the one previous to that, there is a mistake here.

There will be an A here as well, A*1+r. The one before that it's equivalence will be A*1+r square and like that it's A*1+r to the power n-1. This is a geometric series with progression ratio 1+r. A is common. So A comes out so it's value is 1+r to the power n-1/1+r-1, that 1 and 1 cancels out denominator is r and that is written here. The denominator is and here we have 1+r to the power n-1.

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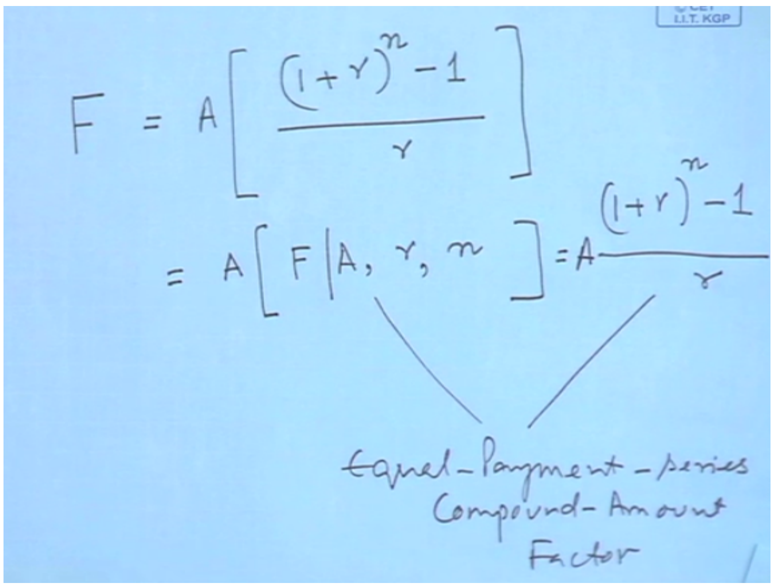

So basically we have here in this case F = A*1+r to the power n-1/r. As before we can give it a symbol to calculate F given A, r, n and this value is nothing but this value which is 1+r to the power n-1/r and this into A and this is called equal payment series compound amount factor because it is trying to find out the compounded amount at the end of n periods. So that is simple.

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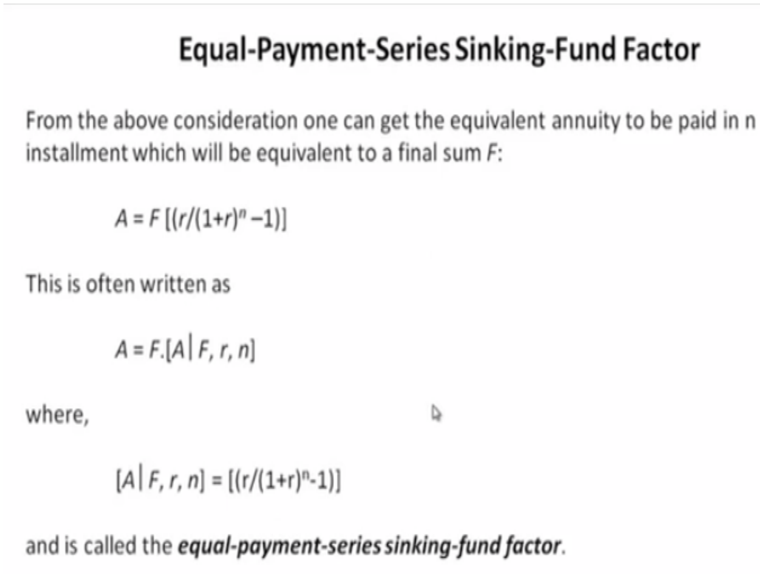

Now I told you that we may be interested also to find out the present worth of all this payment. This is usually called equal payment series sinking fund factor. Now I am sorry, this calculates the value of A given F that means that.

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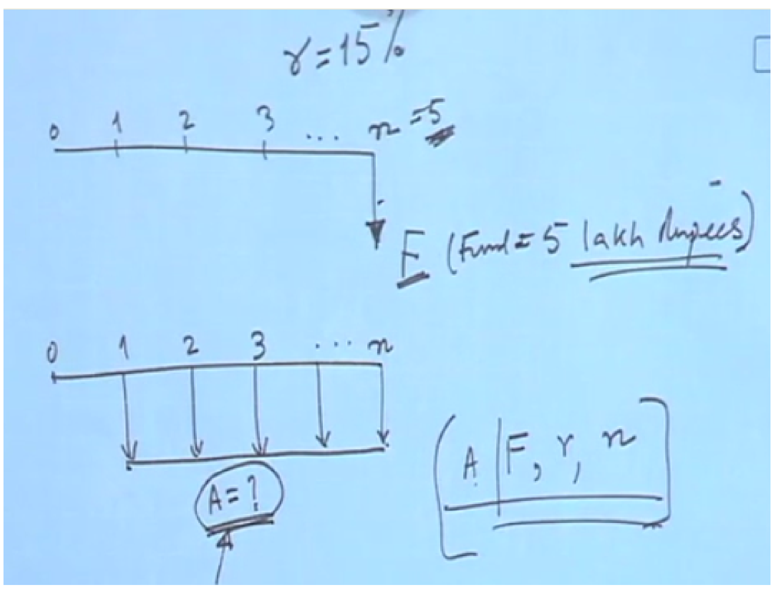

Suppose we have a payment F made at the end of n years. We are trying to find out what would be a series of payments, so suppose that we have to create a fund F at the end of 5 years and that amount should be 5 lakh rupees, how much fund we should deposit every year. What should be the value of A that should that can create 5 lakh rupees at the end of 5 years. If this is the situation, in this case F is given, n is given, the minimum attractive rate of return r could also be given as 15 percent and we are required to find out what amount we should deposit every year.

So given F to calculate A given F, r, and n. Earlier we were talking about this case where A was given and we were to find out its equivalent F. So find out F given A, r, n. This is just the opposite given F find A. Now the equation previously was F is equal to this. So from here I can write just the opposite equation A = F*r/1+r-1. So from this equation, you can derive this equation and from here we can find out this expression that is what we have done here.

A = F*r/1+r to the power n-1. This is what I had written here A= F*r/1+r to the power n-1. That is what in the slide also I have shown here and this can be written as A multiplied by this factor and this factor is then = r/1+r to the power n-1 and this is called equal payment series sinking fund factor that means you create a fund by putting in A amount of money to create F, the final sum at the end of n interest periods. So what is that value A.

If you have to know that find the value of this equal payments series sinking fund factor, multiply that with F and that will give you the amount that you should deposit every interest period.

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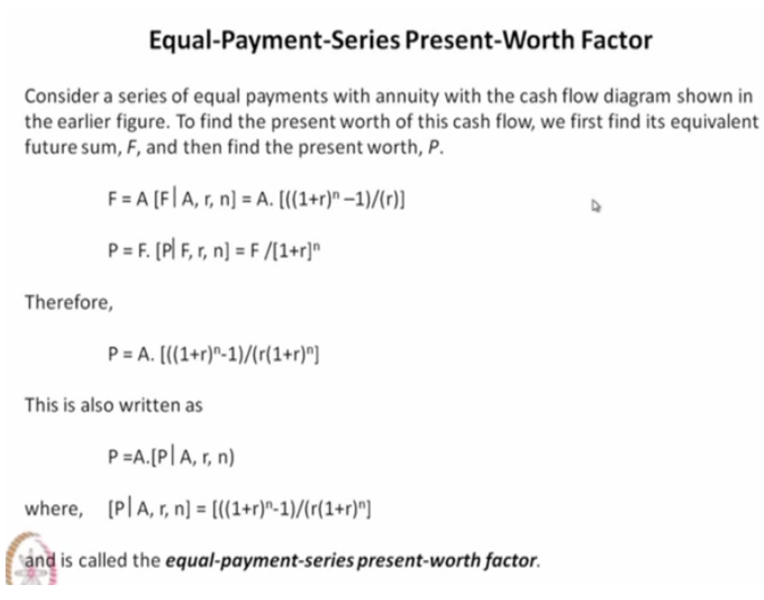

Now we talk about equal payment series present worth factor.

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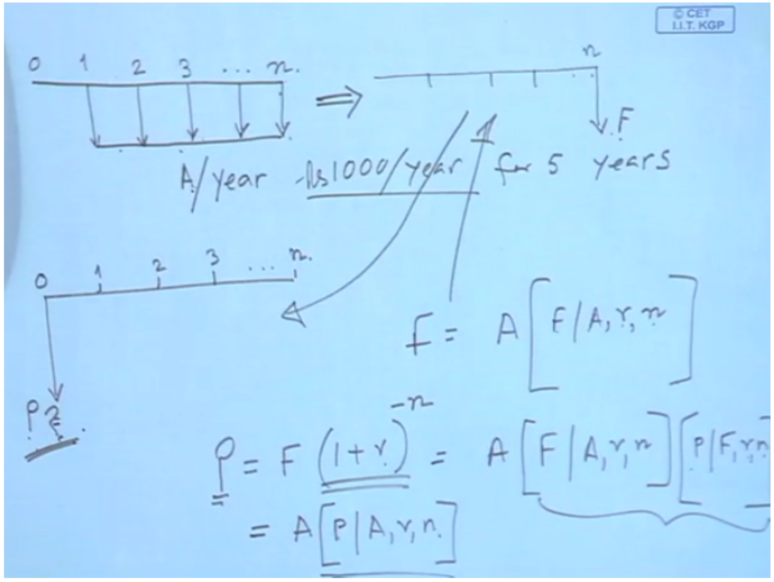

Now in this case, basically we are saying that if there is a equal payment series of this sort A per interest period, let us say interest period is here so A amount rupees per year and these are the years. What is this equivalence today? The problem is similar to this. What amount we have to deposit today in a bank in a fixed deposit such that we are ensured of getting A amount every year for n years. This is the interpretation of this problem.

That if I am interested to get 1,000 rupees per year for 5 years, then how much I should deposit today. What should be P. This is the present condition that I am now talking the equal payment series present worth factor. To calculate this, it is very simple, we have already seen its equivalents at the end of n periods. Already we know this that from here if I multiply A with equal payment series compound amount factor, I get F.

So I know for this, I can write F = A* calculate F given A, r, n, calculate F. This factor is known as equal payment series compound amount factor, that gives me the value F and I know that from here, I can find this by discounting F. Therefore, P I can write as F*1+ up to the power –n. This is single payment present worth factor. So this I can find out. Therefore, I can write for F, I write this AF given A, r, n, multiply that with this to find out P given F, r, n.

To find P given F this whole thing is F and r, n that gives me in the value P. So this can be written as A multiplied by this into this. This is to find P given A, r, n. I can multiply this and give another symbol call it P given A, r, n given A, r, and n find P. So this is nothing but equal payment series present worth factor and that is what I have written down in all these places and that comes as 1+r to the power n-1/r*1+r to the power n.

This thing can be derived from the previous expressions and this is called equal payment series present worth factor.

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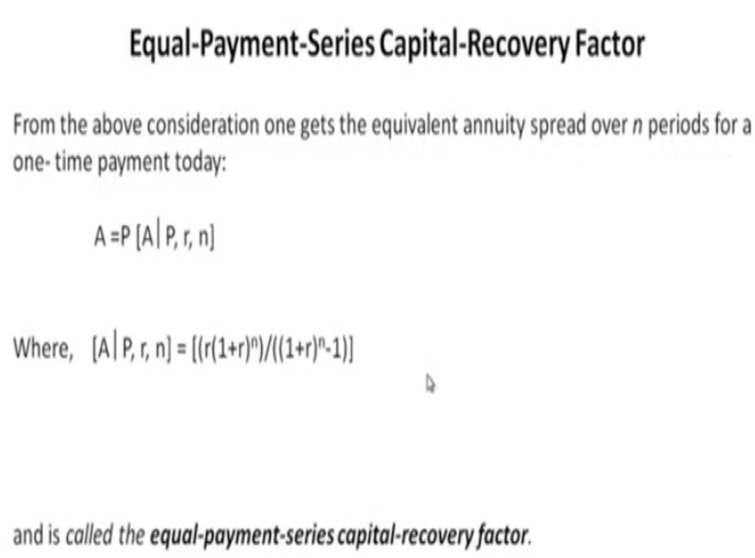

Now we talk about the last factor. It is equal payment series capital recovery factor. Here we are trying to find out given P what is the amount A that we can derive. From the above consideration one gets the equivalent annuity spread over n periods for a one-time payment today. So we had been looking at the case when a series of payment is made and what is its equivalence P or equivalent F.

Now we are trying to consider a case where P is given and we are trying to find out what is its equivalent annuity for n years. A is the annuity for n years. So it is just the opposite. Earlier we had found out P = A into this. Now we are interested to find out A equal to what. So it is very simple. We have already derived P as equal to A, P given A, r, n. So A will be equal to P/P given A, r, n and that we say as A given P, r, n.

This factor is called equal payment series capital recovery factor. Why it is called capital recovery factor, because it is like saying that we have invested this amount of money in the beginning of the time period. Today this is the capital invested, then how much every year we should get back, so that we can recover our capital. That is why it is called capital invested. So friends today, we have discussed a very important topic and that is time value of money.

This is important because to select projects for the purpose of investing large amount of money, the company has to be very careful. It has to consider the present investment, we survey the future cash inflow and outflow and it has to see that it is making adequate returns. The future is uncertain but estimates have to be made regarding cash flows. The main thing here is the consideration of equivalents of cash flows that take place at different time points.

For that the time value of money is important. The basic philosophy comes our basic mathematical derivations come from the knowledge of compounding that we have read in schools the case of interests and how interests are compounded. We are not using the term interest rate; we are calling minimum attractive rate of return. A rate of return that an investor considers as the minimum attractive to him that is taken as the rate at which future cash inflows and outflows will be discounted to the present.

In this connection 3 types of payments are receipt we considered. One is payment made at the present that is considered today at time T=0 that is given a symbol P. Anything a single payment at the end of a future time period n is considered capital F and if equal payments are made for a number of years in series in that year, the payment is considered as A. Then we found out their equivalences. 6 formulae were developed given P what is A and what is F, given A what is P and what is F, given F what is P and what is A.

We defined 6 factors and they involve power functions 1+r to the power n, etc. It is difficult to calculate. They are normally given in different tables called interest tables just as we have log tables, there are interest tables. One does not have to really make calculations. One needs to only know the name of the factor whether it is a single payment present worth factor, single payment compound amount factor or equal payment series compound amount factor or equal payment series present worth factor or sinking fund factor, etc., etc.

So we will continue to discuss this in our next lectures, then things will be more clear. As for today, thank you very much.