Applied Multivariate Statistical Modeling
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Lecture - 40
SEM Structural Model

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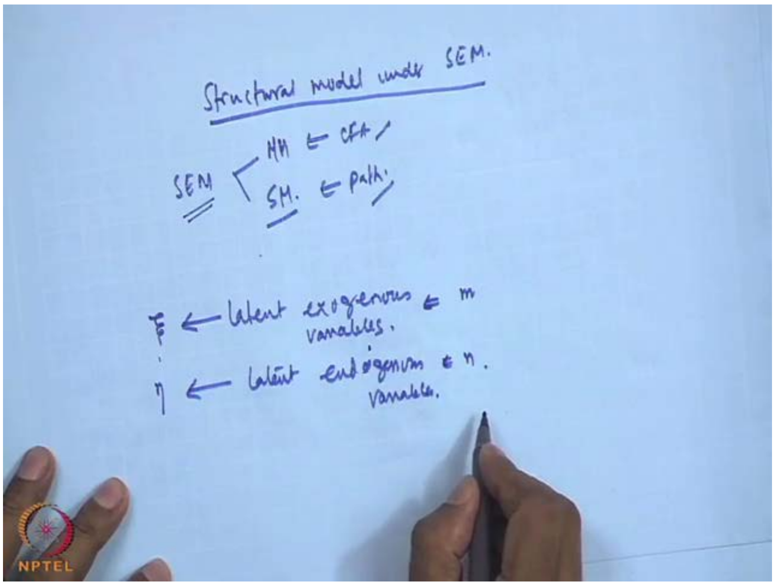

Good morning. Now, we will discuss about structural model under structural equation modeling. Structural model under structural equation modeling, let me see the content today.

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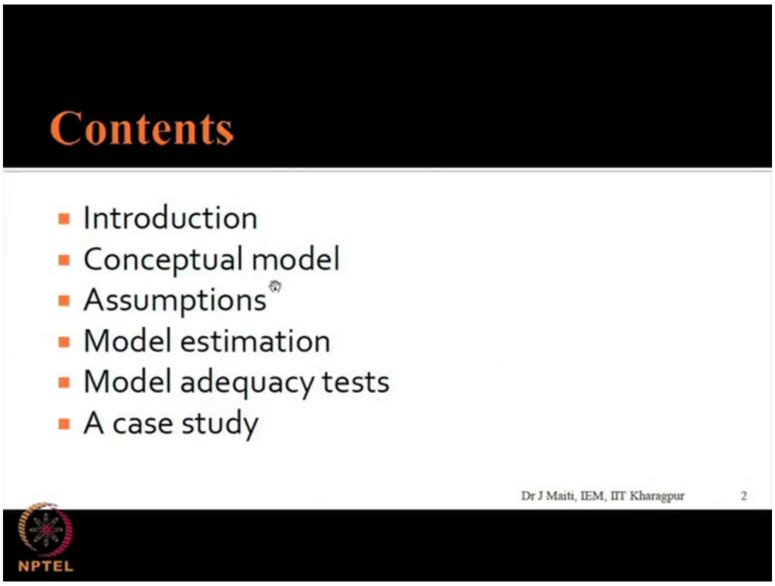

We will start with conceptual model followed by assumptions, followed by model estimation model, adequacy test, and one case study will be shown to you.

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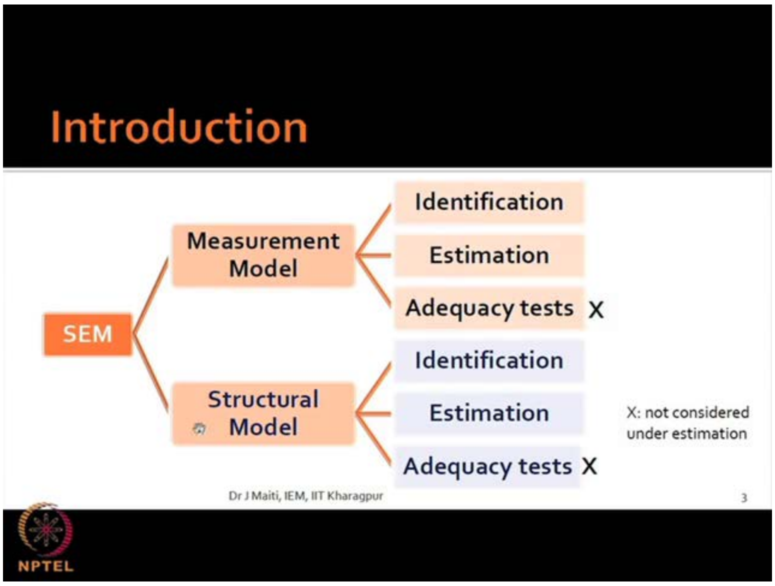

If you recall my last lecture… Where I said that the structural equation modeling has two parts, one is measurement part and another one is structural part. Structural equation modeling entails both measurement as well as structural. What I say that measurement and path or confirmatory factor analysis and path analysis, collectively called structural equation modeling. But, in the last class what is said that you divide it into two parts.

One is structural part, another one is measurement part. Measurement part comes first. You measure the latent constructs and then use the correlation or covariance between those latent constructs as input to the path analysis of the structural part.

So, in measurement part or confirmatory factor analysis, I have assumed the total structure from identification to parameter estimation to model adequacy test and case study. In the same manner, almost in the similar manner, we will be discussing today that what is the structural model there? What do we mean by model identification? How do estimate the parameters? What are the model adequacy tests and how to apply to the same to a case study?

In addition, if time permits, I will tell you the total structure that means need not necessarily be true that you have to divide always the structural model into two separate models and estimate them separately. There is no hard and bound rule. Such rules are there. What is there is that you can go for one time estimation considering both your measurement and structural model together. But, many times, what will happen because of so many parameters to be estimated, so many, as so many variables are involved, then what ultimately will happen? The numerical estimation, it becomes very cumbersome and many offending estimates will result into.

When you relate to the real life situation in about interpreting the parameters to explain the real life situations, you find out that it is difficult in the sense that it is not matching the concept. So, rescue is that you separate them into two parts and then do. If you say no, I want to do altogether and that is my better one because simultaneously measurement and structural part are estimated. That is also good and then your matrix will be bigger and the parameters will be large. So, it is always possible. So, let us start with a conceptual model of structural model that is what is given here.

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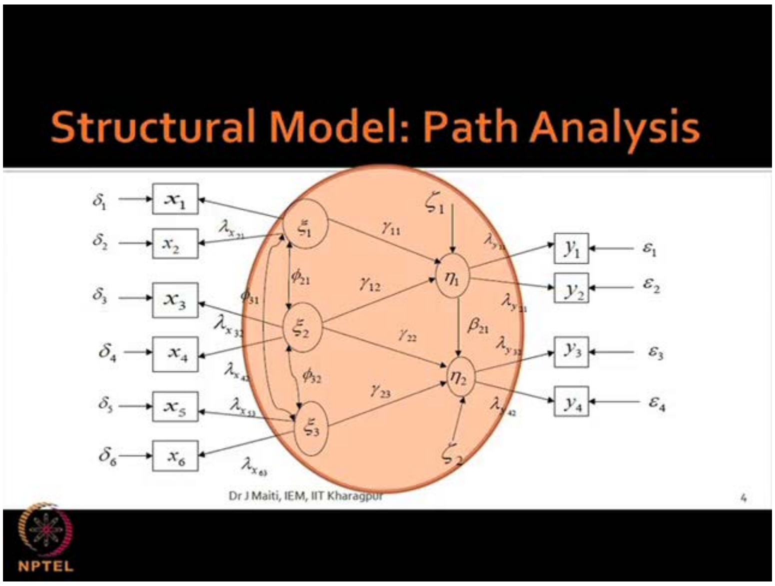

If you go back to my first lecture on structural equation modeling, you will find it that I have given this diagram there. If you see this colored picture oval, this portion where actually two sets of latent constructs are interlinked, link in that sense that there is one set known as psi is and another set known as it is basically eta. So, psi 1, psi 2, psi 3 and eta 1 and eta 2 are two sets of latent constructs. In structural equation modeling, this psi part, the psi 1, psi 2, psi 3, these are all latent exogenous variables and eta is termed as latent endogenous variable.

So, we will be now describing in this manner that there are m latent exogenous variables and n endogenous variables. So, if I go back to this slide, this is that there are two endogenous latent variables and three exogenous latent variables. So, that means n equal to 2 and m equal to 3. Now, see the structure again that these three exogenous variables, there are affecting two latent endogenous variables and their relationship. It is obviously, it is hypothetical relationship. This relationship is formed through literature review or your experience what we are saying that psi 1 is the variable, which is affecting eta 1, not eta 2.

psi is another exogenous variable, which is affecting both eta 1 and eta 2 and psi 1 is another latent exogenous variable, which is affecting eta 1 only. If we consider from eta 1 point of view, eta 1 is affected by psi 1 and psi 1. Eta 2 is affected by psi 2, psi 3 as well as eta 1 and their linear relationships are depicted by gamma. So, any relationship between eta and psi is depicted by gamma, so gamma 11, gamma 12, gamma 23, so like this.

So, gamma 11 means that eta 1 is affected by psi 1. That is why gamma 11 is affected by psi 2 that is 12. So, it is eta 2, 22, 22 and 23, that sense is there. In addition to the relationship between what I can say is the relationship estimate of eta 1 on eta 1 is the effect of eta 1 on eta 2 is beta 21. So, if I consider the relationship here, we can write like this.

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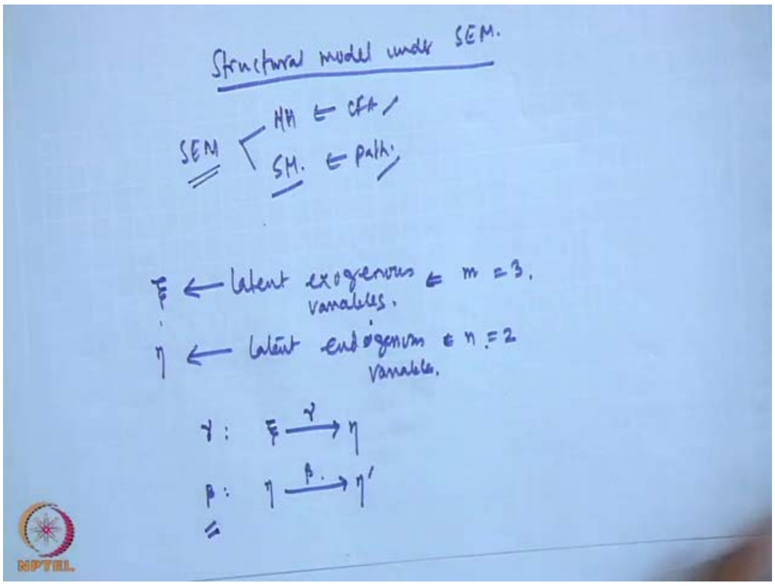

Gamma is something, which denotes the relationship like this; psi is affecting eta through gamma. Now, if beta is something where some eta is affecting another eta that is beta. So, two sets of linear relationship are here. Gamma which is basically exogenous latent variable effect on endogenous latent variable and beta is that one of the endogenous, some endogenous latent variables also affecting endogenous latent variables again. That is your beta.

Now, in addition, although in addition, if you see the totality of this particular picture here, I think you will find out. So, see the left hand side, the exogenous constructs are manifested by x variables and latent endogenous constructs are manifested by y variables. In the last class, you have seen the factor analysis part. There what we have done? We have taken all these psi and eta as they are the factors confirmatory factors and all the x and y; we have taken as x variables.

We have done a confirmatory factor analysis considering all those latent variables, irrespective of whether they are exogenous or endogenous variables. We have taken them simultaneously that they are latent variables without designating, which are endogenous and which are exogenous. Then using confirmatory factor analysis, what you have found out? You have found out the correlation matrix for those latent constructs related factors latent variables. Now, in your path analysis, we are beginning to account another which is something that there is latent exogenous and latent endogenous.

So, if you consider only this portion within this figure, then this is path analysis irrespective of this lambda y 1 and all those things. If I ignore this, the structure like this, gamma and beta are in between and I treat everything as latent and manifest variables, the way we have done in confirmatory factor analysis that would be the measurement model. So, measurement model output is correlation between these two latent variables. Now, subsequently when I am using this path analysis, I am saying that there are two sets. One is exogenous latent, another one is endogenous latent.

The relationship between exogenous latent to endogenous latent is in terms of gamma and within this endogenous to endogenous in terms of beta. Now, in this lecture, in this studios lecture, now what I am saying is that we will be discussing more on this path analysis. Then later on if time permits, I will tell you the totality and what way it will be done. Now, let us see the equations.

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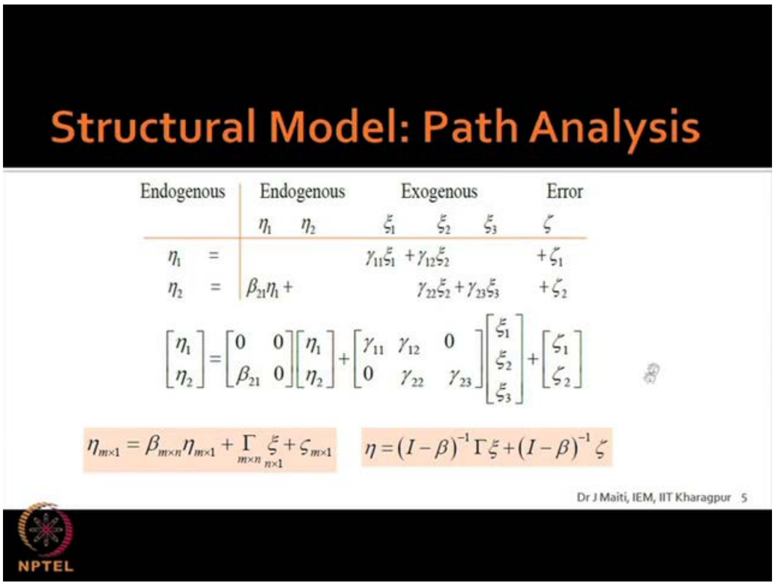

When you write path equations, you please keep in mind that your structure of writing of equations will be like this. It will help you to come to the matrix form. Then you will find that it will be easier for you also to understand how many parameters are related to beta, how many parameters are related to gamma, how many covariant parameters you have to estimate. You see eta 1 is not affected by any of the endogenous constructs; the only affected by exogenous construct is psi 1 and psi 2. This is the question. Similarly, for eta 2, this is the question and if you write down this equation in matrix form, you are getting this equation. Now, this is an example. Now, general equation will be like this.

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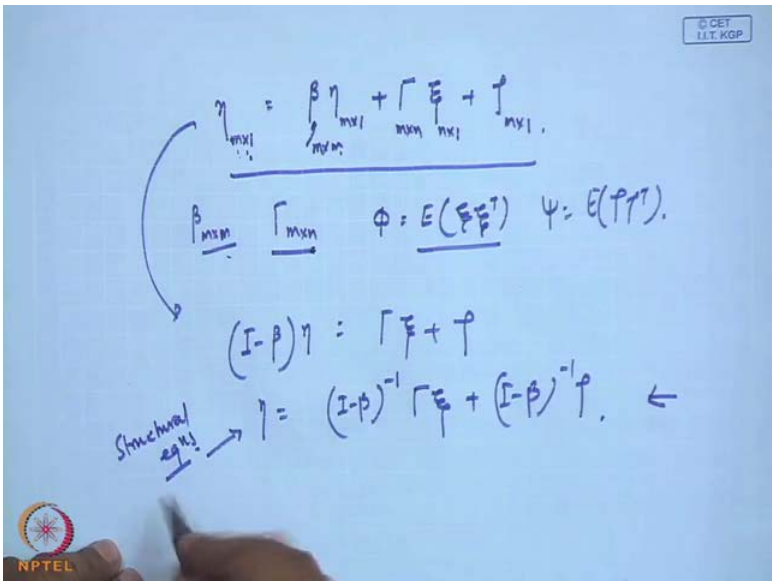

General equation will be like this that eta, how many etas are there, we have considered that eta will be m cross 1. This can be affected by other eta that m cross 1 and this one, beta will be your m cross, it will all depend on, and you want m cross 1. So, m cross 1, if you write, plus it will be affected by psi, psi is n cross 1, then there will be gamma m cross n plus your error term will be there, which is again m cross l. This is the general equation. So, is it matching m cross 1, m cross 1? Yes, it is matching. It is the general equation.

So, beta m cross m gamma m cross n, these are the two parameters what you want to parameters in the sense parameter matrix, what you want to estimate. In addition, there will be other estimates like phi, which is expected value of psi psi transpose. There will be psi, expected value of zeta, zeta transpose, so and so on. Later on we will see if any other parameters are required to be estimated. Now, this equation as if both sides are there, so I will change little bit. So, I will write, I minus beta into beta equal to gamma psi plus zeta. If you multiply both sides by I minus eta inverse that will be I minus I minus beta inverse gamma psi I minus beta inverse zeta. That is our structural equation. This is what is said as structural equations. We can also say these as also path equations.

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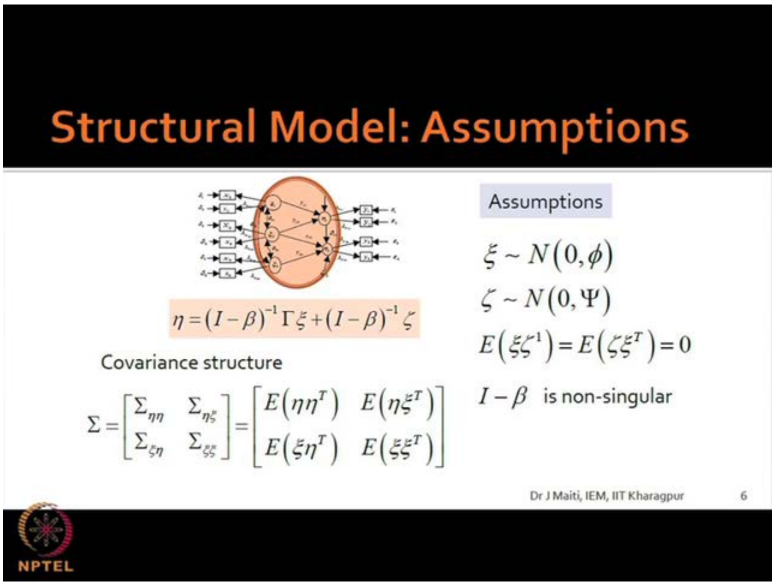

Now, come to the assumption. Like confirmatory factor analysis or this path analysis here or structural model here, there are certain assumptions. First of all assumption is related to the latent exogenous variables zeta, which is definitely multivariate normal and its mean value is 0. The covariance matrix is phi related to zeta, which is again multivariate normal with 0 mean vector and psi is the covariance matrix. What we are saying is that the latent exogenous variable is uncorrelated with the error terms and related to your eta and as well as it is the same thing that we are writing this is 0.

Another important issue is that if you see in this equation eta equal to I minus beta inverse that means this inverse must epsist, otherwise this equation would not be a real one. So, I minus beta is known singular, then only this equation will epsist. So, these are the assumptions of structural model of structural equation modeling in structural equation modeling. So, that means what we have done basically? So, we said that this that these are the zeta and they are the psi and they are uncorrelated.

We cannot make correlation between this. This is a means subtracted observation in the sense that their mean value zero is 0. How many psi is there, how many zeta is there? This zeta, we have considered that zeta will be N, zeta we have considered N zeta. So, that this is in N.

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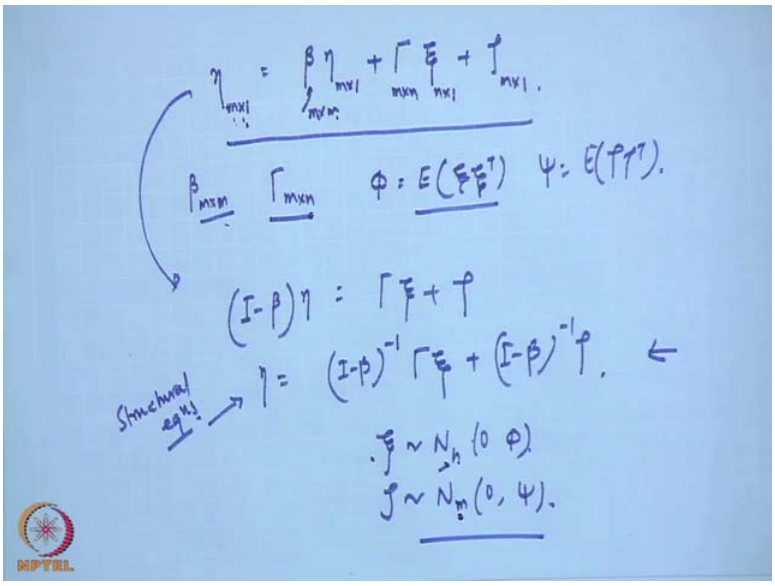

I think that if we write like this that zeta is multivariate normal like this 0 and phi, similarly eta is multivariate normal. How many etas are there? It is N 0 psi this is basically related to the number of psi. This is the number of eta. So, now let us see the covariance structure for this problem.

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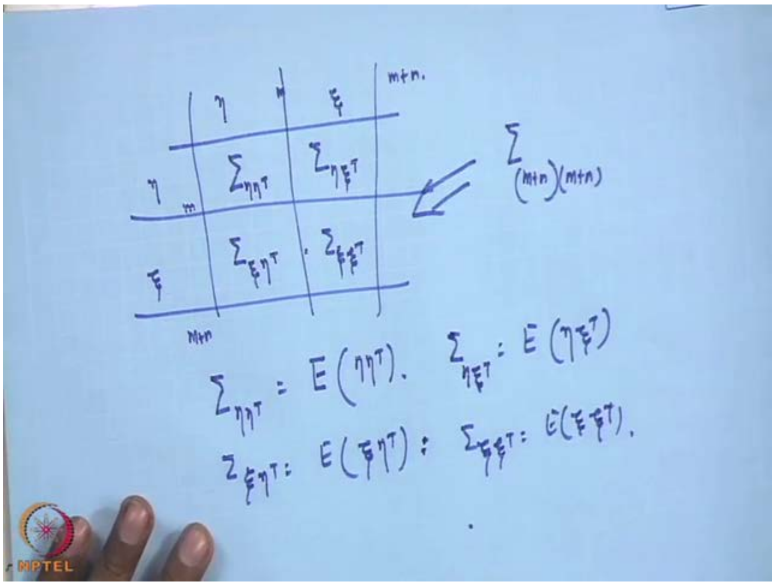

If you see, ultimately the covariant structure is like this eta psi and this side you also write eta and psi. How many etas are there? Eta variable are m variables, psi variables are n. So, the total is m plus n here. So, here also m then m plus n, this is the total. We are talking about the matrix sigma, which is m plus n cross m plus n, this matrix. We are partitioning this matrix into two parts, into four parts. One is this, which is related to eta only, eta eta transpose this one, eta psi transpose this is another matrix, this one psi eta transpose and this one psi psi transpose. So, this eta matrix, this is nothing but the sigma, the total matrix covariance matrix, which is partitioned into four.

So, what does it signify? What is this? This is matrix of eta eta transpose is nothing but expected value of eta eta transpose. Similarly, that matrix of eta psi transpose is expected value of eta psi transpose. Similarly, psi eta, it is expected value of psi eta transpose and last one is psi psi transpose, expected value of psi psi transpose.

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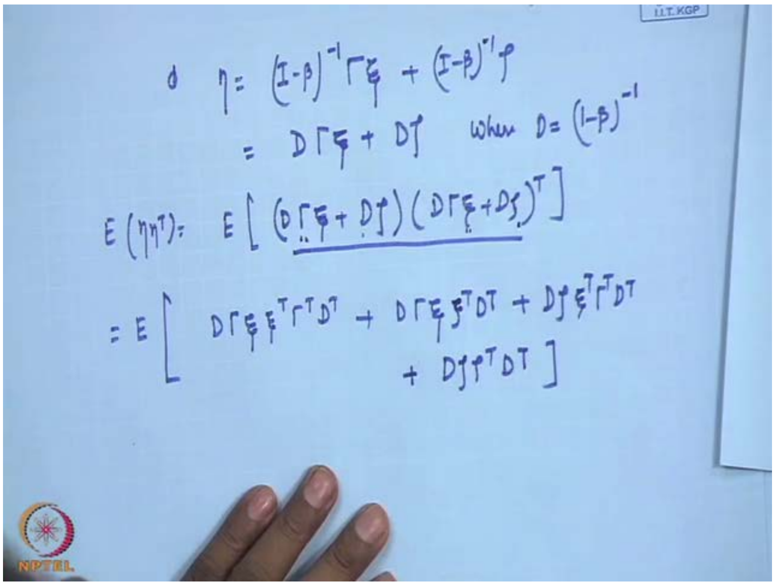

The covariant structure then essentially the covariant structure will be like this that we have eta equal to I minus beta inverse gamma psi plus I minus beta inverse zeta that is our equation structural equation. So, I in order to simplify, I am writing this as D gamma psi plus D zeta where D is equal to I minus beta inverse. So, you require finding out first expected value of eta eta transpose. You can write this is expected value of D gamma psi plus D zeta into D gamma psi plus D zeta transpose.

So, this multiplication you do, you will be getting something like this, so D gamma zeta. So, transpose of this will be just the reverse psi transpose gamma transpose D transpose plus D gamma psi. This one is zeta transpose D transpose, so this into this into this plus this into this we have written. Similarly, I can write D zeta psi transpose gamma transpose D transpose plus D zeta zeta transpose D transpose.

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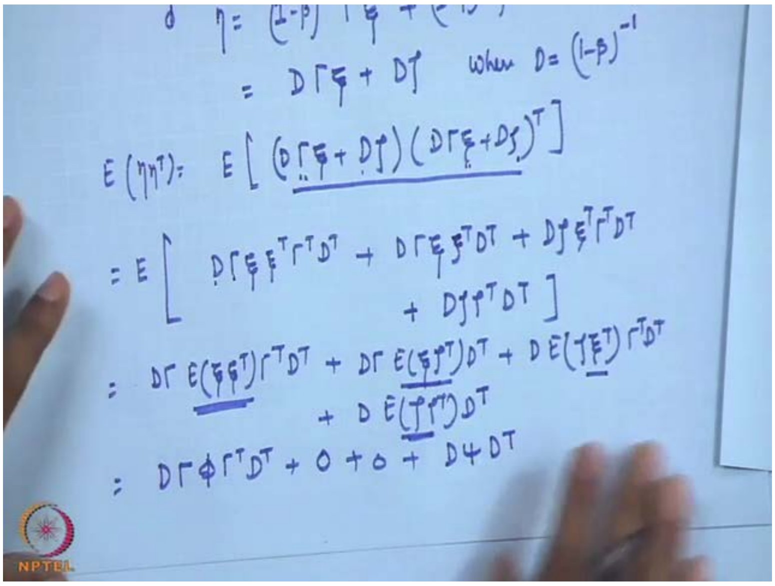

So, you see D and gamma, they are constant. So, you can write in this, D gamma expected value of psi psi transpose gamma transpose D transpose plus D gamma expected value of psi zeta transpose D transpose plus D expected value of zeta psi transpose gamma transpose D transpose plus D expected value of zeta zeta transpose D transpose D gamma.

This one is nothing but phi, expected value of the latent construct, latent exogenous construct is phi gamma transpose D transpose plus, this will be 0 because latent exogenous constructs are uncorrelated with the zeta value, the error term of eta, so this is 0. Similarly, this is 0. So, finally, D, now this one zeta zeta transpose expected value is we say it is psi, then D transpose. So, totally we can write like this.

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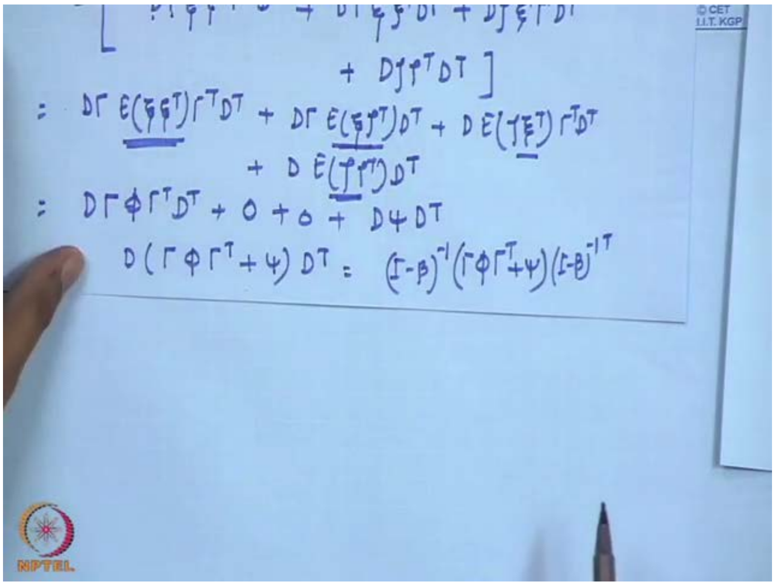

This one is equal to D gamma phi gamma transpose plus phi D transpose. This is nothing but I minus beta inverse gamma phi gamma transpose plus psi I minus beta inverse transpose. So, that is our first part.

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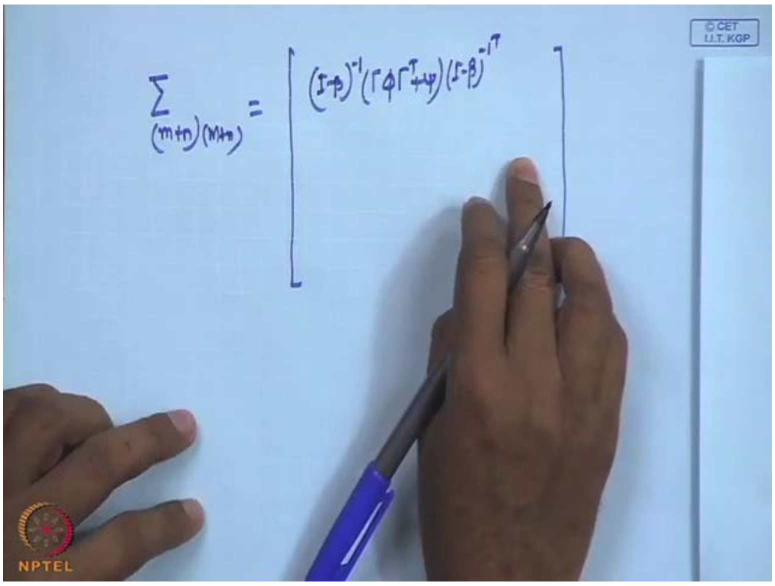

If I say that my covariant structure is this, which is m plus n cross m plus n, my first part is as we have seen earlier, my first part is sigma eta eta transpose, which is covariance of this. This we have already found out. This is I minus beta inverse, then gamma phi gamma transpose plus psi I minus beta inverse transpose, first part you have written. What is our second part? The second part is that eta psi transpose, which is expected value of eta psi transpose. So, let us see what is the expected value of eta psi transpose?

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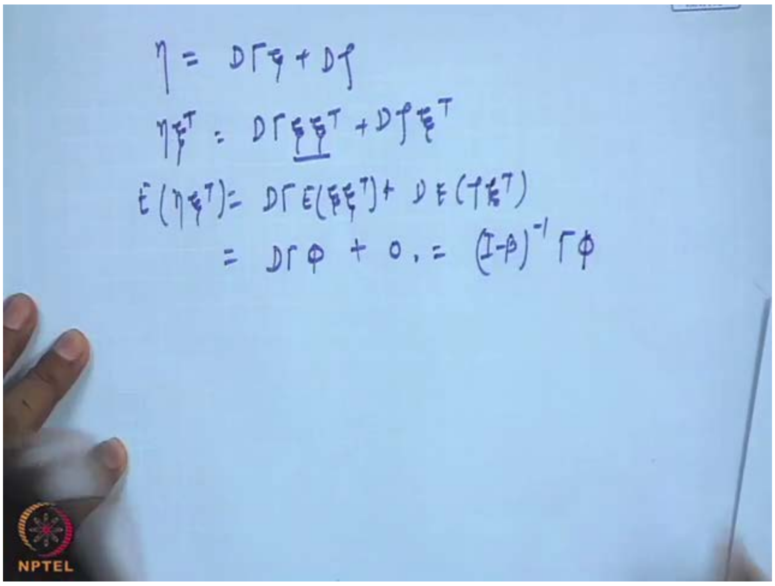

So, we say eta is equal to D gamma psi plus D zeta, we want this into this. So, you can write D gamma psi psi transpose plus D zeta psi transpose. If you take expected value of this, then this is D gamma, expected value of this plus D expected value of zeta psi transpose. This is D gamma phi plus this will be 0. So, our second part is I minus B inverse gamma phi.

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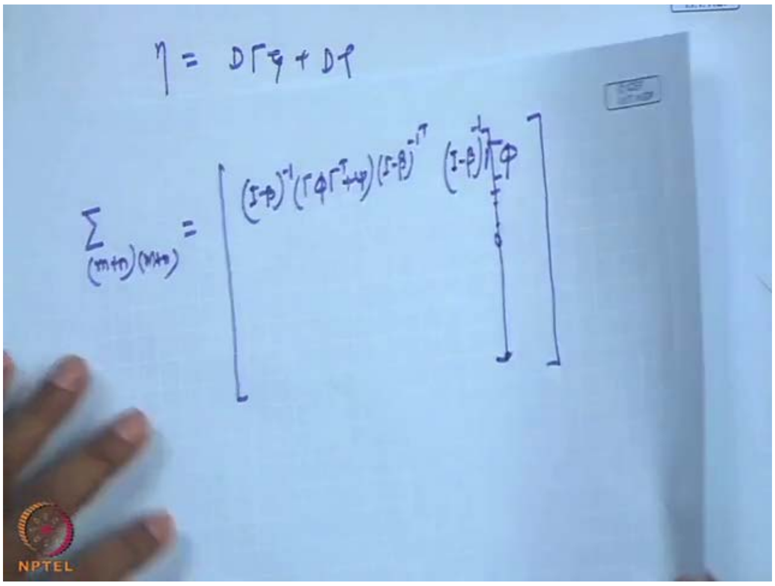

So, you write down here, I minus B inverse phi gamma I minus B inverse phi gamma, sorry just change it to I minus B this. So, this is the case. What is your third portion? Third portion or I can or third element is psi eta transpose.

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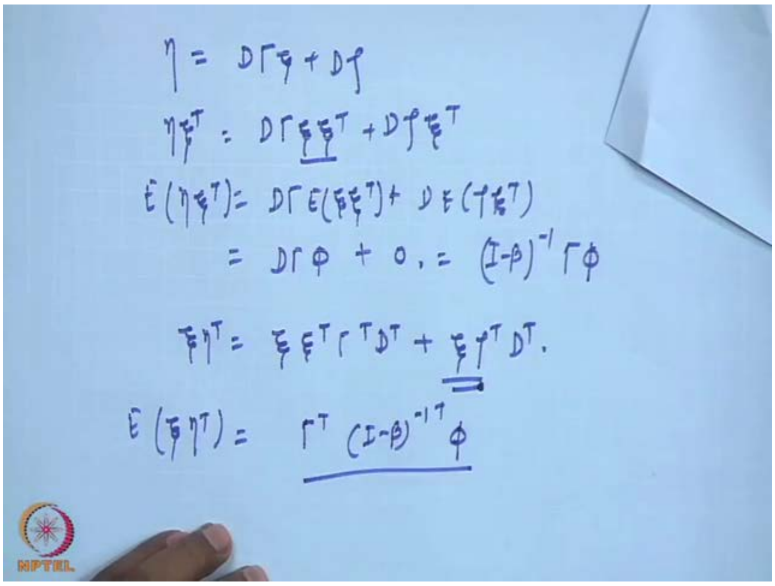

So, you find out psi eta transpose, which will be psi into psi transpose gamma transpose D transpose plus psi zeta transpose D transpose, this will become 0 ultimately. If you take expected value of psi and eta transpose, it can be written like this, gamma transpose I minus beta inverse transpose phi, this term will come. So, I will write here the psi and eta transpose. It can be written like this gamma transpose I minus beta inverse transpose phi, this term will come.

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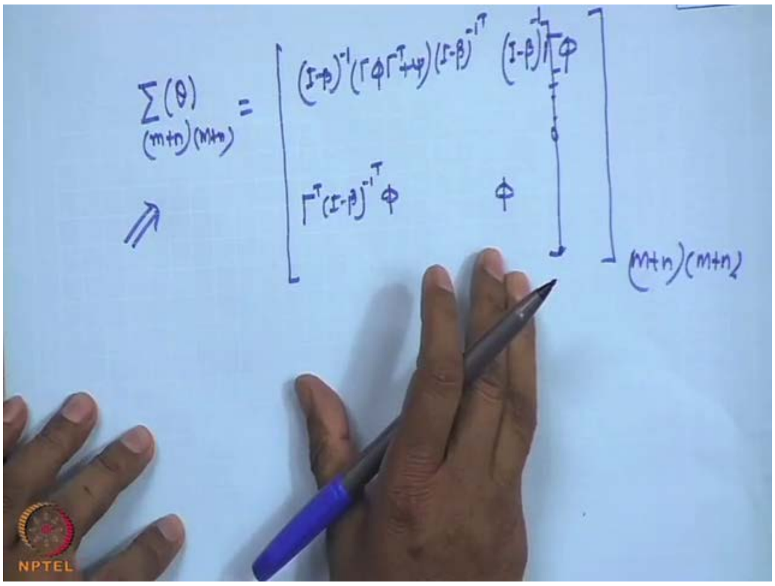

So, I will write here covariant structure here that this is gamma transpose I minus beta inverse transpose phi. Then the final one is this one, psi psi transpose, expected value of psi psi transpose is phi. This is my covariance matrix. So, in terms of parameter estimation, we can say this is sigma theta that these many variables are there, m cross n into m cross n. So, this is the case, the number of parameters. There are many parameters to estimate and we will see later on how this can be done.

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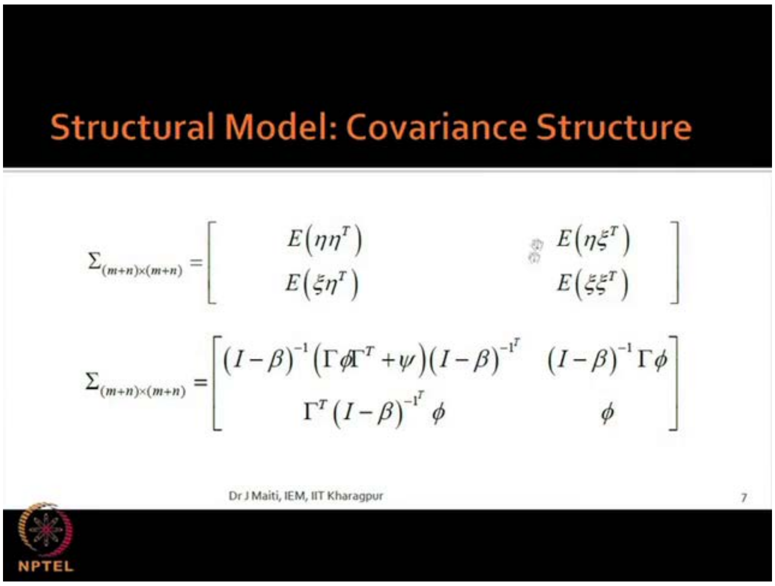

So, let us go to this slide. So, this is ultimately what is the covariant structure what I have already shown that how derivation of this, the structural model in SEM and the covariant structure looks like this in terms of the parameter values.

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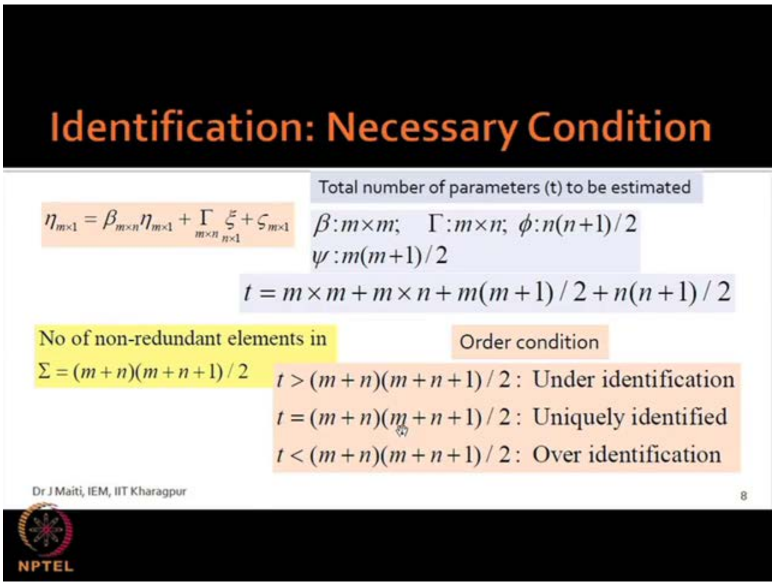

Now, we want to show that what you ultimately what you require to do? You require finding out the number of parameters to be estimated.

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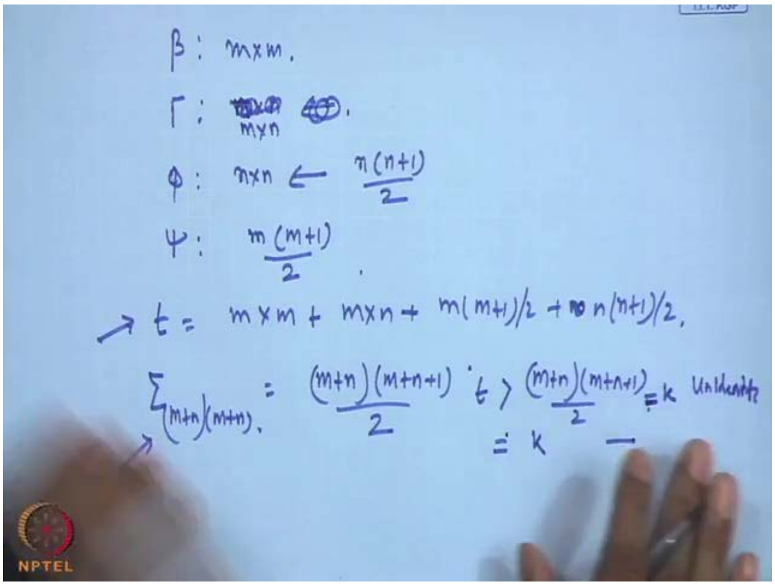

So, I think one is beta that beta in case of beta m cross m parameters to be estimated; in case of gamma that is m cross n, but because of its symmetric nature, gamma is sorry, gamma is m cross n, absolutely no problem. Your phi actually it will be n cross n matrix, but because of its symmetric nature, it is n plus 1. So, similarly, psi m m number of parameters to estimated m plus 1 by 2. The total number of parameters to be estimated is m into m plus m into n plus m into m plus 1 by 2 plus n into n plus 1 by 2.

So, this is the number of parameters to be estimated. So, what is available with you? The thing available with you is this sigma m plus 1 m plus 1. This is the matrix, known matrix in terms of s, it will be known. So, that is why I am saying known matrix. So, then number of your non redundant parameters will be this. This is because of symmetric matrix. So, ultimately what will happen? If your t value is greater than m plus n into m plus n plus 1 by 2; unidentified and if it is equal to this, so if I say this quantity is my k, and if it is equal to k, so this is uniquely identified, so unidentified.

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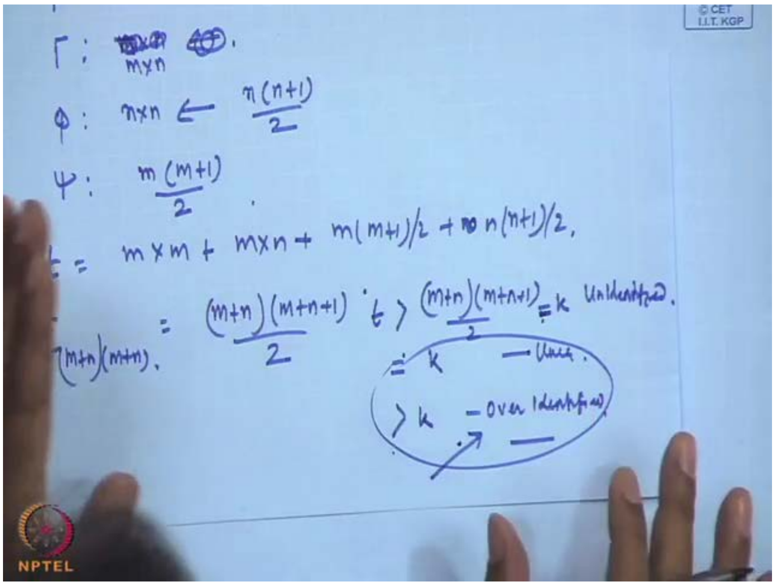

So, unidentified, uniquely identified and if it is greater than k, then it is over identified. We want either of the two, but the last one is desirable. It is the preferable one. So, this condition is satisfied. This model necessary condition is satisfied, which is known as order condition is satisfied and you have to similarly satisfy the rank condition.

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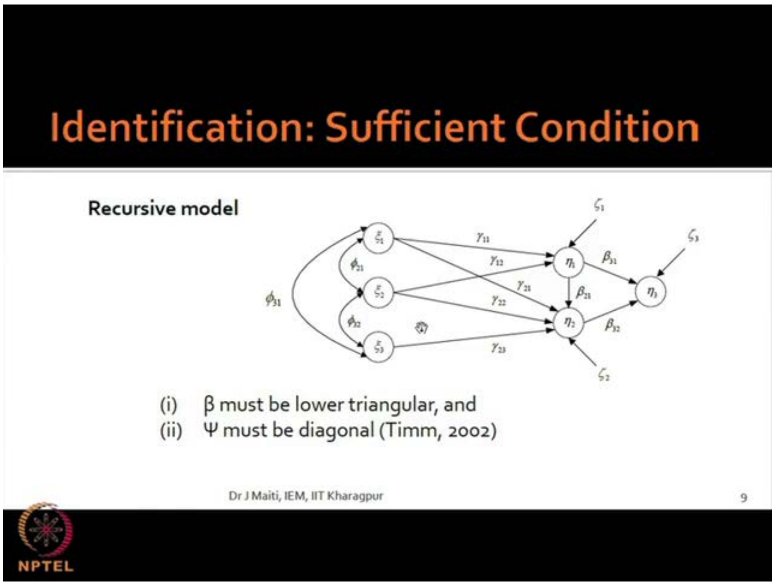

Now, let us see here there are two types of model in path analysis. One is recursive path model, another one is non recursive path model. Now, see this figure. You see that the direction of the error is going in one direction. This is basically psi 1 to this, this one to this. There are no bidirectional arrows. Eta 2 is not affecting eta 1. Similarly, when you find out that to start with will come to the particular point like this, this, then no loop is forming basically.

So, ultimately this type of model is known as if I am able to go from here to here, here to here, and again from here to here that that is some bidirectional loop will be formed. That is for recursive model, this will be the structure. I think for recursive model, the properties, all those things, you can go to path analysis in detail. Now, in recursive model, the sufficient condition, necessary condition what I have described is that the number parameters to be estimated must be less than the non redundant elements in the covariant matrix and the sufficient condition for recursive model, beta must be lower triangular.

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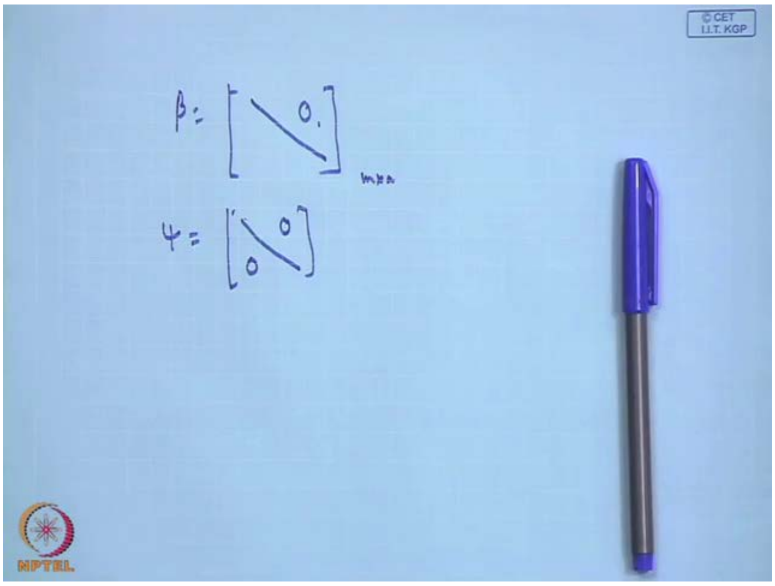

Actually, when you talk about beta matrix m cross n, what we mean to say it will be lower triangular. This side all will be 0 and you see another matrix, which is psi matrix nothing but the relation, the covariance is between the zeta values. They must be diagonal. So, we are saying only these values will be there and often both sides will be 0. If these two conditions are satisfied, then sufficient condition is satisfied or other way we can say that the rank condition is satisfied and in this model is estimated.

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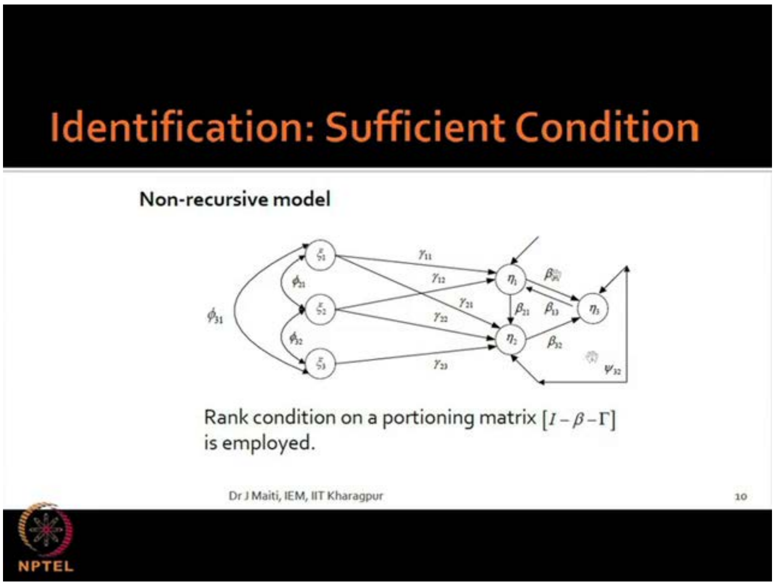

Now, you go to non recursive model. What is happening here is if I take this one, eta 1 affecting eta 2, eta 2 affecting eta 3, eta 3 affecting eta 1, you see I start from eta 1, I am going to eta 2, I am going to eta 3. Again, I am coming back to eta 1. So, a loop is formed and because of these bidirectional arrows, but it is not possible in recursive path model. But, in non recursive path model, it is in this type of model is simple. This type of beta must be triangular phi must be diagonal. This will not be sufficient condition.

The sufficient condition, you need to test through rank condition on partitioning matrix. You have seen the partitioning matrix. What I mentioned that I minus beta minus gamma, this partitioning matrix, this partition is employed, rank condition of this partition matrix is employed. I told you in earlier class this measurement model for this rank condition, you can follow. In 1973, they have given some clear cut guidelines, how to understand a matrix sufficient enough for estimable estimation for estimation. I hope you are getting.

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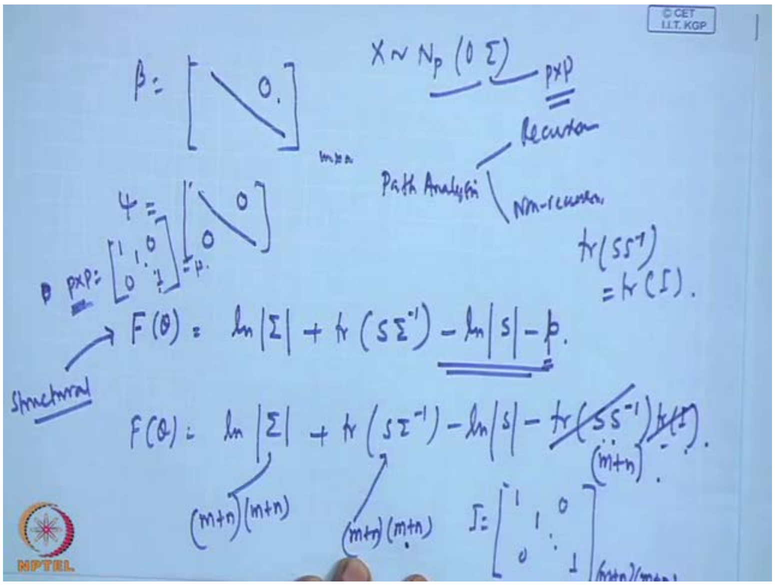

Actually, we started with general structural equation model. Then I showed you the path model inside, then presumptions and covariant structure how to be computed. Finally, what I discuss is a very important one that. In path analysis, there are two types of path analysis. One is your recursive and another one is non recursive. So, sufficient condition, sufficient condition will be different for recursive and non recursive. Please keep in mind. Once the model is estimable identified, it is that is uniquely or over identified. Preferably, it should be over identified.

Once this is done, then you are ready for estimation. You have to estimate. Now, what method you will adopt? Actually, in measurement model, you will develop a function minimization function F theta, which is log of determinant of trace of sample covariance matrix s into inverse of the population covariance matrix sigma minus log of sample determinant of sample covariance matrix minus p. That was the minimization function that they have used. How this p comes into consideration? Actually, this p comes from trace of s, s bar, the second part we found out. When we say that there is a perfect match, then this should be the situation. So, s s inverse is nothing but trace of I.

As in case of measurement model, we have p manifest variables. So, we have p cross p matrix whose all the diagonal elements are one off diagonals are 0. Then the trace of the sum of these diagonals is p and we adopted the procedure that x is multivariate normal with p 0 and sigma. The same procedure can be adopted here. Are you getting me? The same procedure you can adopt here. What will happen ultimately in measurement part? We have taken sigma equal p cross p, but our sigma is n plus 1 is m cross m plus 1. That means in the same manner if you proceed, we will be getting for structural model, you will be getting a fit function, which is known as F theta.

It can be written like this. Then I am writing the order of the matrix is m cross n into s cross n into m cross n plus trace of s sigma inverse. Again, the order of the s will be m cross n into m cross n again minus the same thing will come that s. What will happen? The trace of s s inverse is, now here, s is m plus n cross m plus n. So, that will be I again. If I write this one as trace of I, trace of I, and then I is a matrix, something like this where diagonal elements are 1 and off diagonal elements are 0. It is m plus one cross m plus n. So, trace of this is m plus n. So, again if you write like this m plus n.

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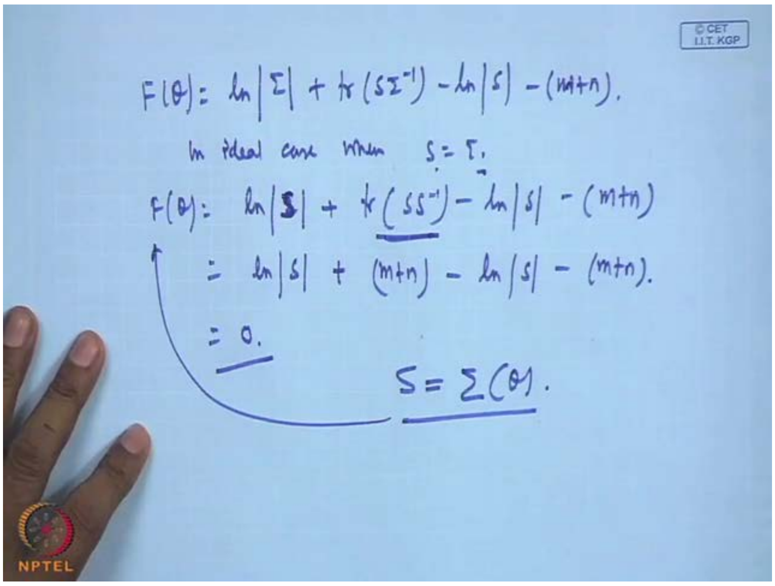

So, the resultant fit function becomes F theta equal to log of sigma plus trace of s sigma inverse minus log of determinant of s minus m plus n. Now, see ideal case what will happen? When s equal to sigma, what will happen to F theta? You see F theta will be log of determinant of this. So, I can write this nothing but s because s equal to sigma plus trace of s s inverse minus log of determinant of s minus m plus n. So, this is log of determinant of s plus this is trace of I, which is nothing but m plus minus log of determinant of s minus m plus n this equal to 0. So, when s is perfectly matching with this, then my fit value is the function becomes 0.

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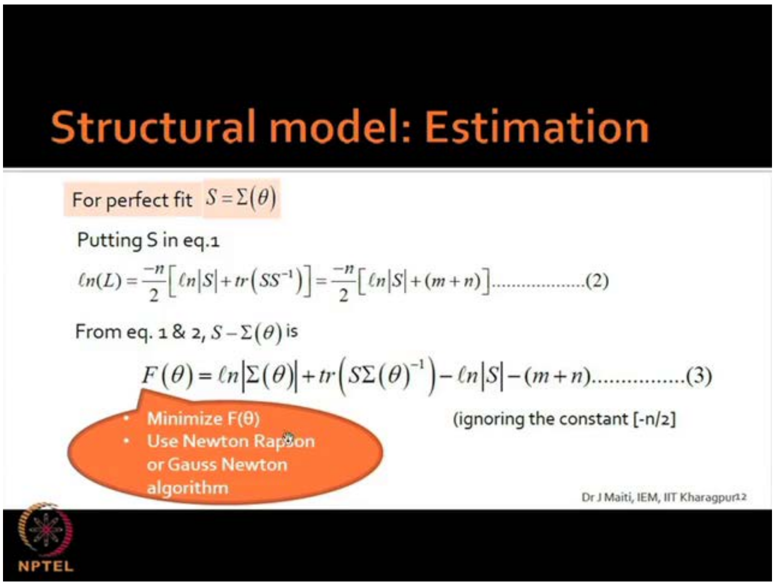

The same principle we have adopted in earlier case also. You see log of this plus this plus this. So, this is my fit function. So, for estimation, you will use this minimization F theta, so this one. So, please remember ultimately, there are many methods of estimation like weighted list square, unweighted list square, two stage list square, maximum likelihood methods, all those things, but it is recommended by many people that you go for maximum likelihood estimation. What is maximum likelihood estimation that I told you long back, I think in estimation when I discussed the estimation statistics, basic statistics, and multivariate statistics.

So, essentially that means the identification and estimation related to structural model is known to you. Now, it is in a very mathematical manner that I have described, but this mathematics is nothing but the manipulation of matrix algebra. It seems to be big, but it is not that complex from just understanding point of view. I am not talking from optimization point of view that F theta you are estimating, but for that you have to go for numerical methods. Now, whatever we have discussed so far in relation to the structural model, let us see with a case study.

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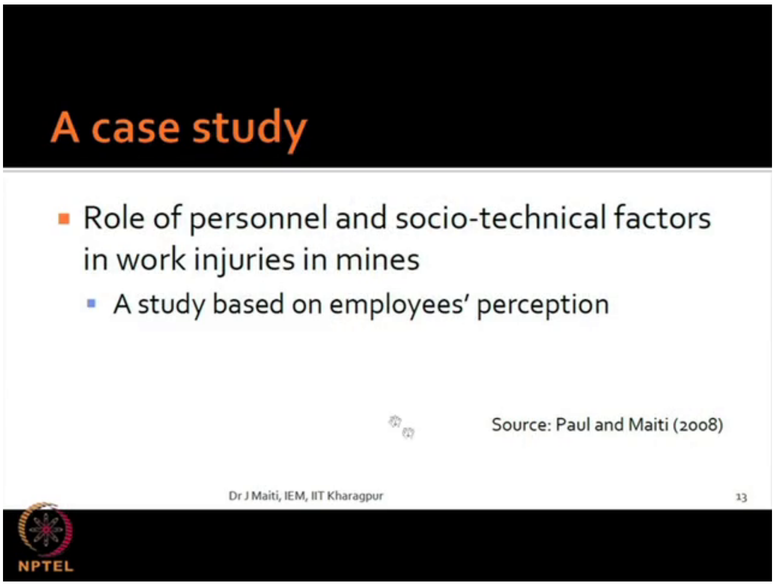

We will proceed with the same case study and the sources are also given here. In the last class, I told you if you are interested, then please go through this case and see that how it is useful.

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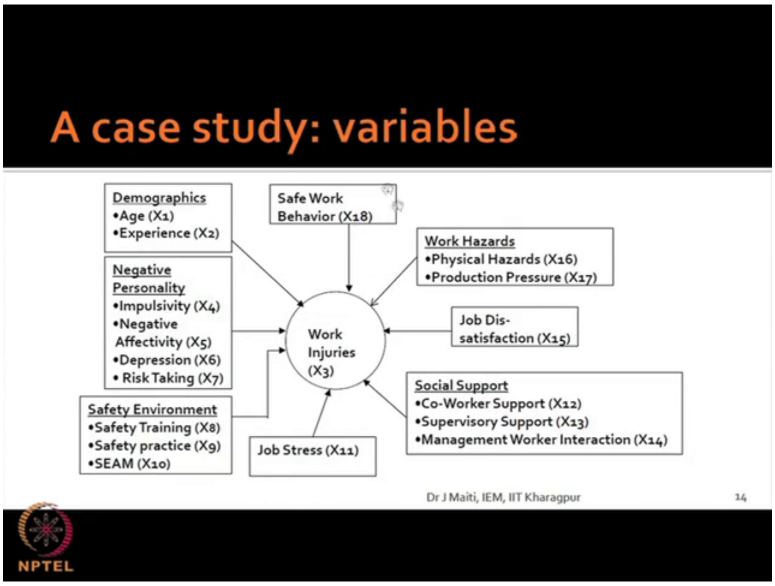

This is fantastic. This is the case study and these are the variables. There are the nine latent variables.

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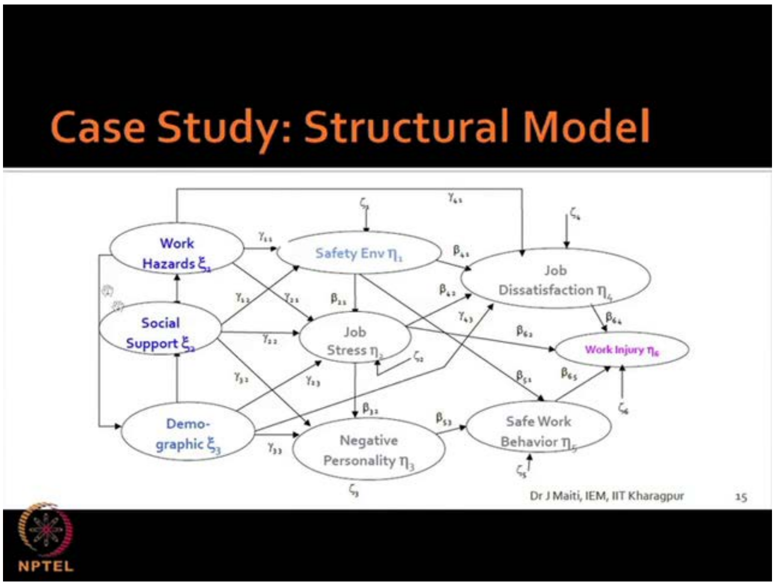

Using these nine latent variables, we are now constructing the structural model. Now, here what we are saying here, now there are two types of latent constructs; latent factors. One is exogenous, another one is endogenous. What we have written here is work hazards, social support, demographic; these three things are not affected by any other variables in the model. So, they are exogenous here. But, if you consider other factors here or construct they are affected by one or more of the exogenous or endogenous factors, so they are endogenous in nature.

So, accordingly whatever the same nomenclature we are using, we have used here that the relationship or the effect of exogenous construct to endogenous construct is determined by gamma. So, all gammas is coming here. So, this is essentially exogenous construct and these are all endogenous construct. Now, each exogenous construct, wherever it has an effect on endogenous construct that is denoted by gamma and within endogenous construct, it is denoted by eta. The ultimate aim of this model was whether we were able to explain the structural relationship of the variables leading to a work injury; not necessarily here we are interested to know that what are the latent constructs that are affecting the work injury?

We are interested to know the structure of relationship means work hazards to safety environment, may be safety environment to be job dissatisfaction, these types of things. So, this type of relationship, later on I will explain that the direct and indirect effects are there. Then it will be much clearer from the relationship point of view, but from the structural model point of view, path analysis point of view, I do not find you are facing any problem. Are you facing any problem?

If you are facing any problem, you take your own example, your own case, this is one example case and here we are demonstrating things. You take your own case and then and develop similar like this. You use software like this. There you can use your software is there, but the software is available where the structural model and path model can be analyzed. Now, let us see next what happened.

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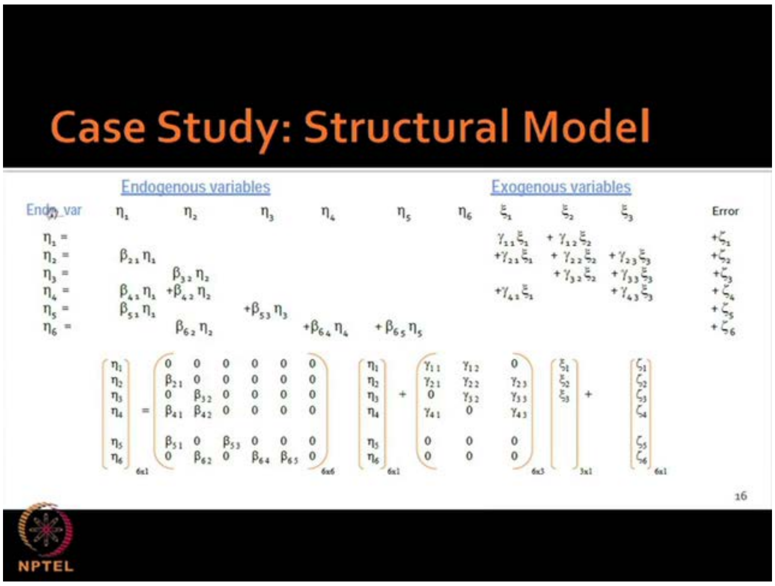

I have shown this diagram. This is known as is path diagram. This is converted into path equation or structural equation. This structure is very much known to you now. This is because you started with a diagram and this type of equation. If you write in matrix form, so this is the case. Now, see although we say beta will be m cross m here there are six eta, so beta can be written as six cross six here, but see many of the betas are 0 here. Many of the betas are 0. If it is not, six thirty six parameters have to be estimated since this reduced to one, two, three, four, five, six, seven, eight, nine beta parameters to be estimated here. Similarly, the gamma case is also one, two, three, four, five, six, seven, eight, and nine. Then the rest are the covariance matrix between psi between zeta. That is what you have to estimate.

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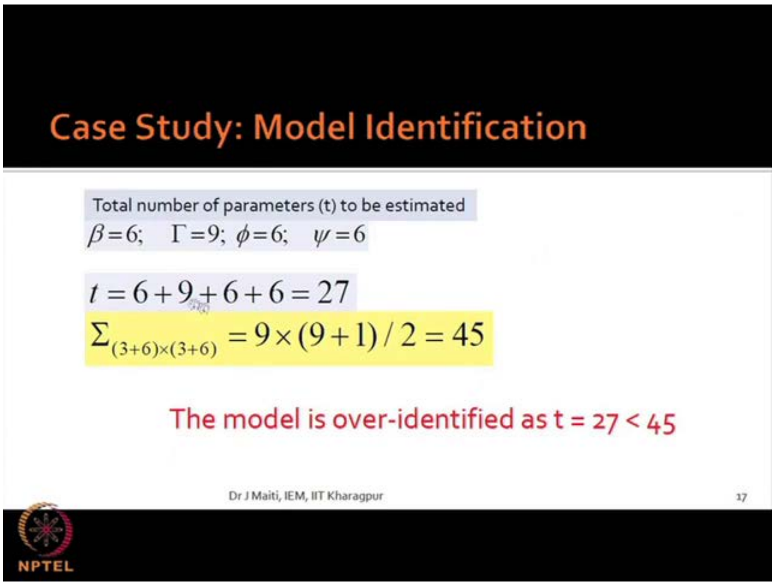

So, if I see in terms of model estimation, you see what is happening, we are basically, we had to estimate 6 beta parameters, 9 gamma parameters and 6 psi parameters. It is 6, 1, 2, 3, 4, 5, and 6, no it is basically 9. So, this will be 9, 9, 6 and phi is 6 because there are 3 C 2, 3 by 1 into 2, 6 will be there, n plus n into n plus 1 by 2 that is 6. That is fantastic. So, 6 is there. So, ultimately you are estimating 30 parameters.

t is number of parameters to be estimated is 30. Now, what are the independent elements, known elements from sigma point of view? It is 9 into 9 plus 1 by 2 that is 45. So, the model is over identified as t is number of parameters to be estimated much less than the number of non redundant elements in the covariance matrix, which is known from sample data. So, it is not 27, I think. So, let me check. Let me check beta 2 1 2. It is not 27, it is 30. So, this minimum correction, this minor correction you do.

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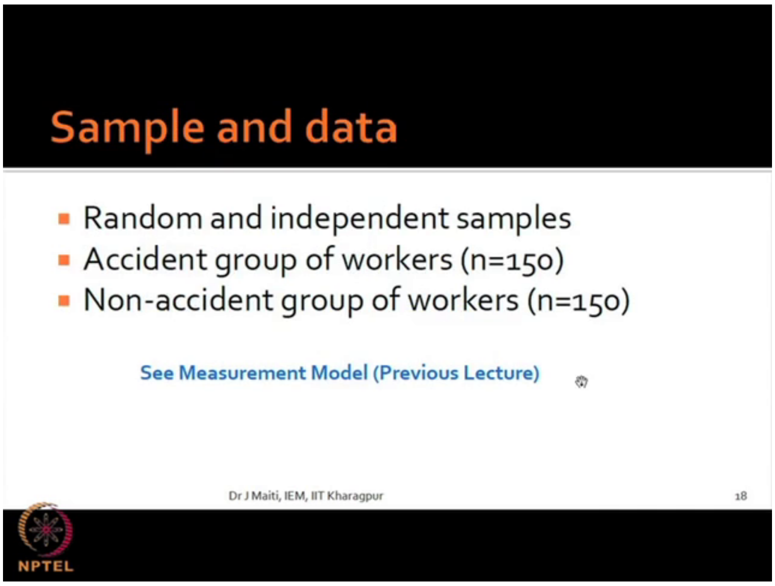

So, see that sample and data case here. I do not want to explain further because we have explained much on this in the measurement model.

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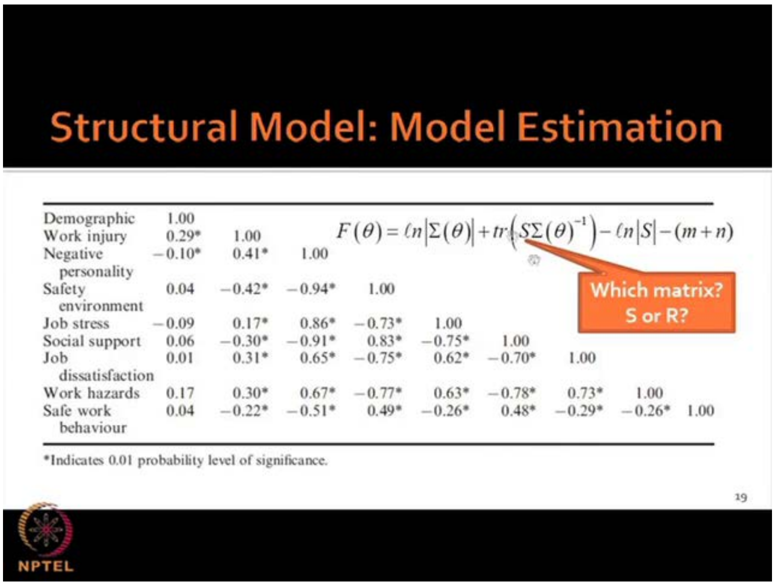

Demographic	1.00								
Work injury	0.29*	1.00							
Negative personality	−0.10*	0.41*	1.00						
Safety environment	0.04	−0.42*	−0.94*	1.00					
Job stress	−0.09	0.17*	0.86*	−0.73*	1.00				
Social support	0.06	−0.30*	−0.91*	0.83*	−0.75*	1.00			
Job dissatisfaction	0.01	0.31*	0.65*	−0.75*	0.62*	−0.70*	1.00		
Work hazards	0.17	0.30*	0.67*	−0.77*	0.63*	−0.78*	0.73*	1.00	
Safe work behaviour	0.04	−0.22*	−0.51*	0.49*	−0.26*	0.48*	−0.29*	−0.26*	1.00

*Indicates 0.01 probability level of significance.

Now, measurement model has given you this correlation matrix. Again, here we are using correlation matrix and we are interested to see the pattern, and trace pattern and the relationships. So, this is the input to your software.

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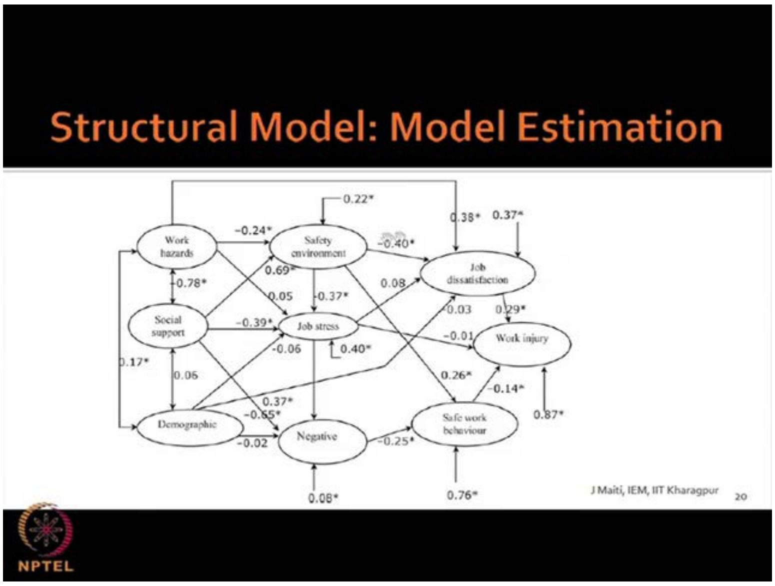

So, the model result is like this. These values gamma, beta values come from the model. Now, the star is given everywhere. Somewhere star is there that is not there, so wherever it is not; there the relationship is not significant, which we are talking here in terms of test of parameters.

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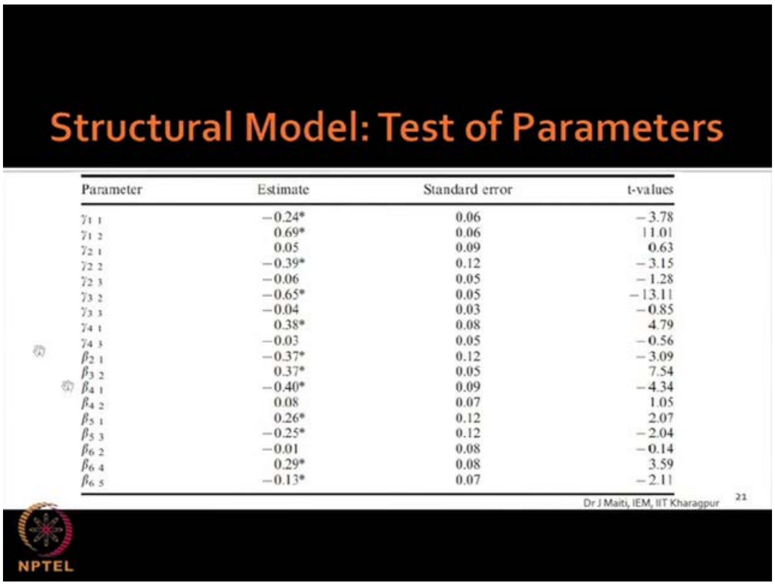

Structural Model: Test of Parameters

Parameter	Estimate	Standard error	t-values
$\gamma_{1\,1}$	−0.24*	0.06	−3.78
$\gamma_{1\,2}$	0.69*	0.06	11.01
$\gamma_{2\,1}$	0.05	0.09	0.63
$\gamma_{2\,2}$	−0.39*	0.12	−3.15
$\gamma_{2\,3}$	−0.06	0.05	−1.28
$\gamma_{3\,2}$	−0.65*	0.05	−13.11
$\gamma_{3\,3}$	−0.04	0.03	−0.85
$\gamma_{4\,1}$	0.38*	0.08	4.79
$\gamma_{4\,3}$	−0.03	0.05	−0.56
$\beta_{2\,1}$	−0.37*	0.12	−3.09
$\beta_{3\,2}$	0.37*	0.05	7.54
$\beta_{4\,1}$	−0.40*	0.09	−4.34
$\beta_{4\,2}$	0.08	0.07	1.05
$\beta_{5\,1}$	0.26*	0.12	2.07
$\beta_{5\,3}$	−0.25*	0.12	−2.04
$\beta_{6\,2}$	−0.01	0.08	−0.14
$\beta_{6\,4}$	0.29*	0.08	3.59
$\beta_{6\,5}$	−0.13*	0.07	−2.11

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We have so much of parameters and their estimated values, t values. Using the t values, you will be able to find out which are the parameters. Another issue what I want to see here is that these tests are similar to regression parameter test. I hope you are getting it.

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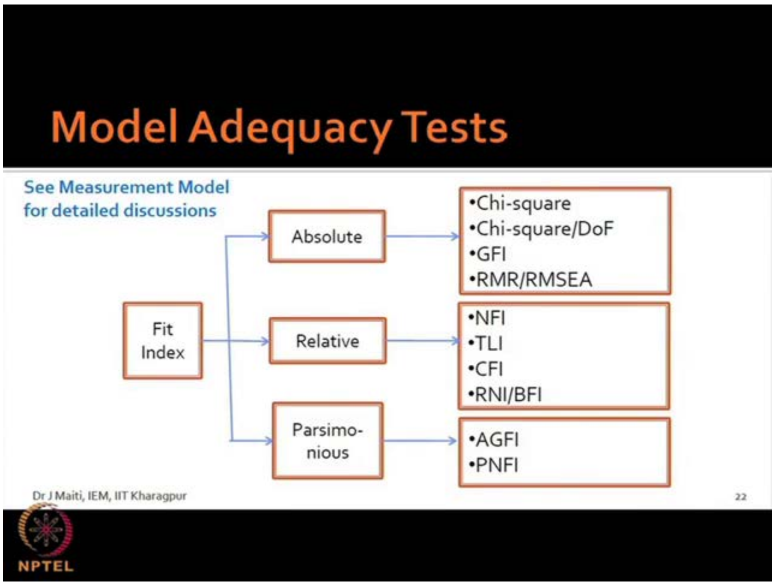

Adequacy test, all the adequacy in this case what we have used in the measurement model, I have discussed in detail what are all those things. The same thing is applicable here for the structural model also.

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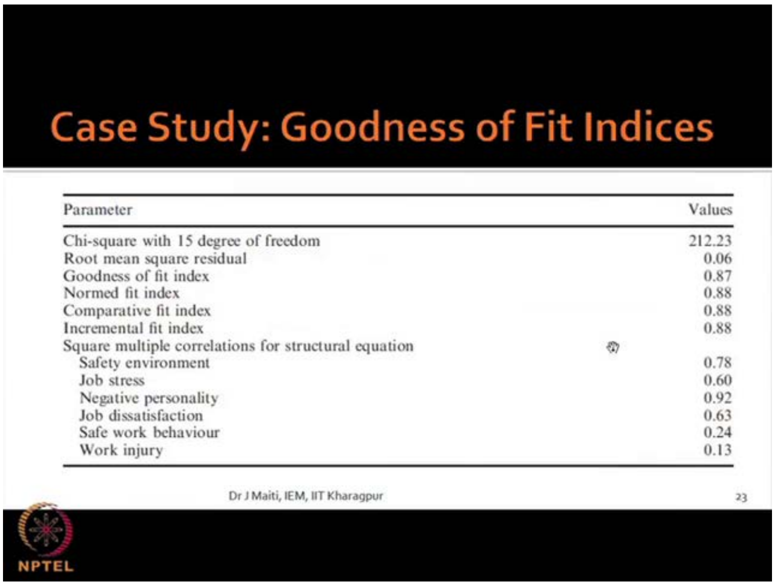
Case Study: Goodness of Fit Indices

Parameter	Values
Chi-square with 15 degree of freedom	212.23
Root mean square residual	0.06
Goodness of fit index	0.87
Normed fit index	0.88
Comparative fit index	0.88
Incremental fit index	0.88
Square multiple correlations for structural equation	
Safety environment	0.78
Job stress	0.60
Negative personality	0.92
Job dissatisfaction	0.63
Safe work behaviour	0.24
Work injury	0.13

For our case, this is the scenario for case, mean case study. You see that the case study results are convincing in the adequacy point of view, it is convincing. Most of the majors are very much related to almost to near 0.9 and in addition to goodness fit indices, there is square multiple correlation is similar to R square values, how much variability is explained for all the endogenous latent constructs. You will find out that for safety environment, it is high; for personality, it is high. For work injury, it is less, but the path is traced here and with a high degree of relationship, the adequacy it is good enough.

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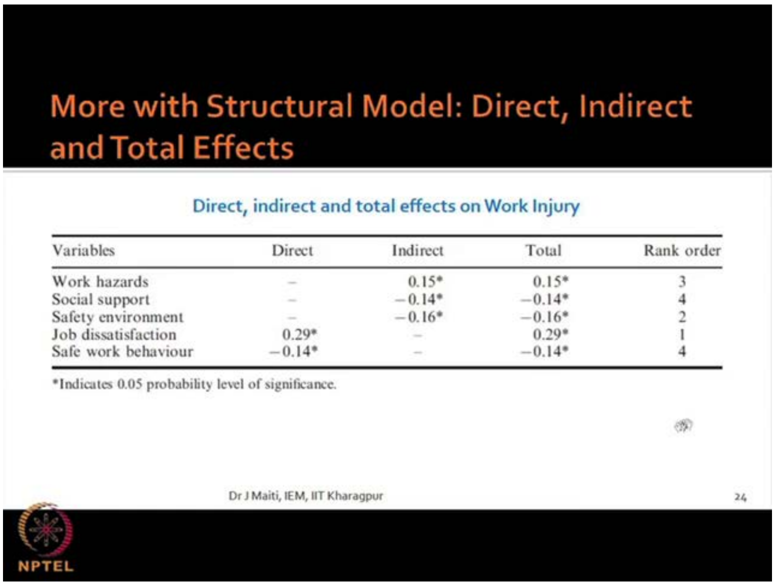

Another important issue is in structural model, the direct effect and the indirect effect. What is direct effect and what is indirect effect?

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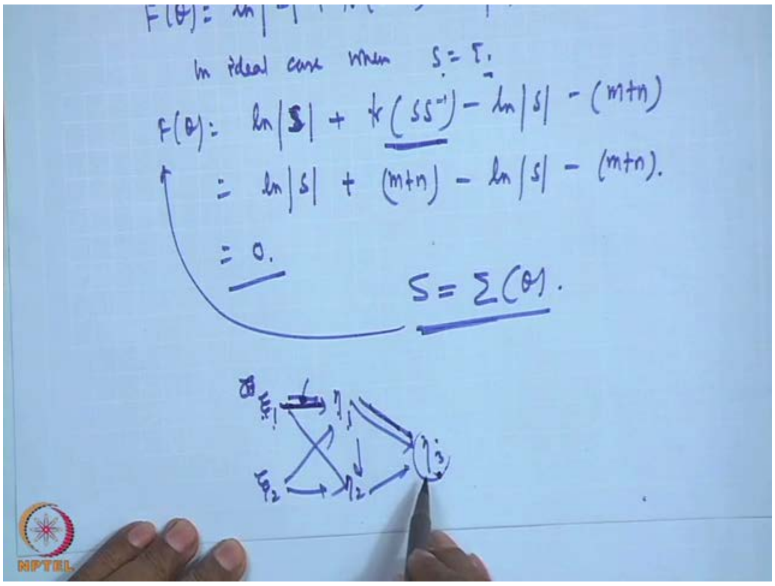

For example, I am saying this is eta 1 and this one is suppose psi 1 and this is psi 2 and this is eta 2 and this is eta 3. Suppose the linkage is like this. I want to know what is the effect is of psi 1 on eta 1. This effect is direct, but if you want to go for these, I cannot get direct. I have to go through this part. So, this multiplied by this gives me the effective. So, this is indirect effect. So, that means every endogenous construct might have direct effect as well as indirect effect. So, within this context here, if I want to know what is the direct effect on work injury, then you will be getting only here this, this and these three variables, job dissatisfaction, safe work of behavior and job trace. It has direct effect or the other has indirect effect.

Now, see that result here. This work has no direct effect, indirect effect, indirect effect plus the indirect effect is the total. So, in that manner, we have calculated and found out that job dissatisfaction is the main reason for work injury followed by this one, safety environment. Safe work behavior is coming under fourth position.

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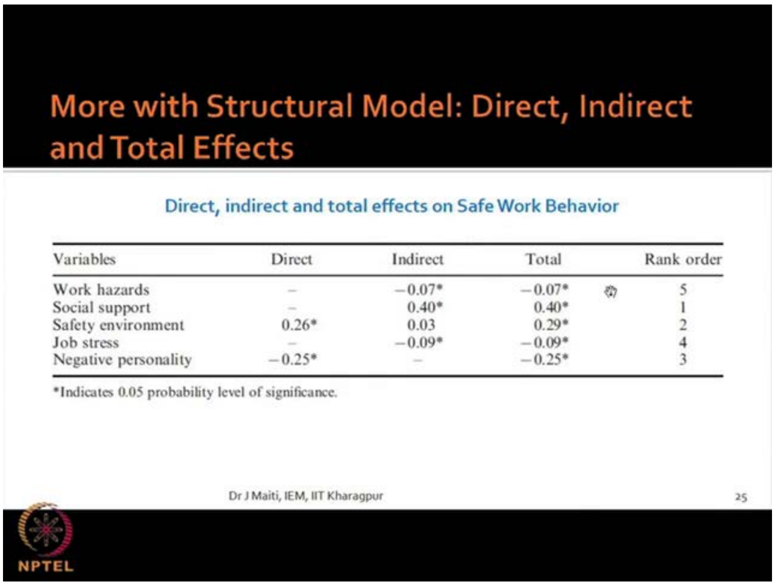

Variables	Direct	Indirect	Total	Rank order
Work hazards	–	−0.07*	−0.07*	5
Social support	–	0.40*	0.40*	1
Safety environment	0.26*	0.03	0.29*	2
Job stress	–	−0.09*	−0.09*	4
Negative personality	−0.25*	–	−0.25*	3

*Indicates 0.05 probability level of significance.

The same thing we have done for safe work behavior. Suppose we want to know what are the factors that are contributing to the safe work behavior, what is the rank and the direct indirect effect? Finally, we found that social support is primarily responsible for safe work behavior. The more social support the better the safe behavior. It is not job safety environment or job trace in that sense. But, social safety environment is coming very close together. From the total effect point of view, this is large compared to others.

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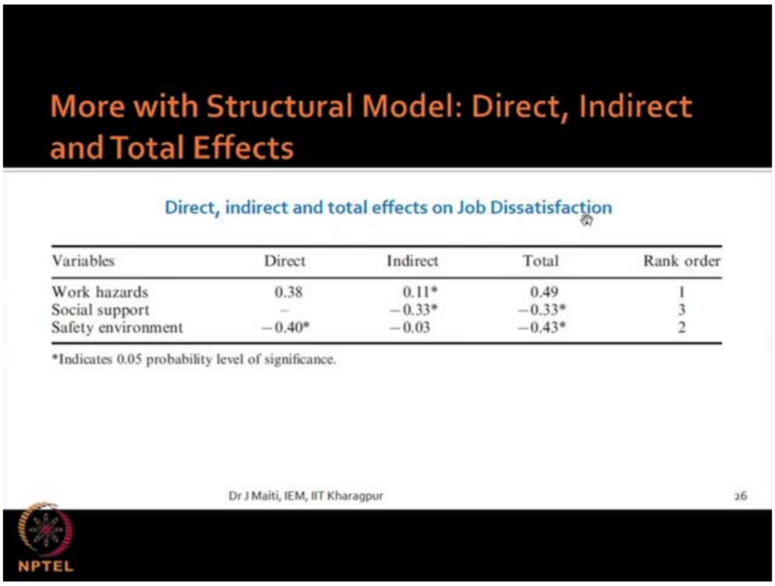

Similarly, for job dissatisfaction, interestingly work hazard is coming. So, the contrast in this situation, we usually understand that work injury is very much affected by work hazard. It is true, but it is not direct. So, because if you see work injury, where is the work hazard? Work hazard is coming in third position where we are talking about your job dissatisfaction; it is coming in the first. When I talk about work injury versus job dissatisfaction, it will be coming in the first position.

So, this structural relation is very important for taking decisions because for organization, issues are very important. You know, not necessarily the technical issues or the engineering issues will always contribute particularly in a complex situation of work injury. So, apart from this, I think that I want to tell you one more issue here. Suppose I will not go for measurements in structural models separately. I will go for only one model, all measurements and structures simultaneously.

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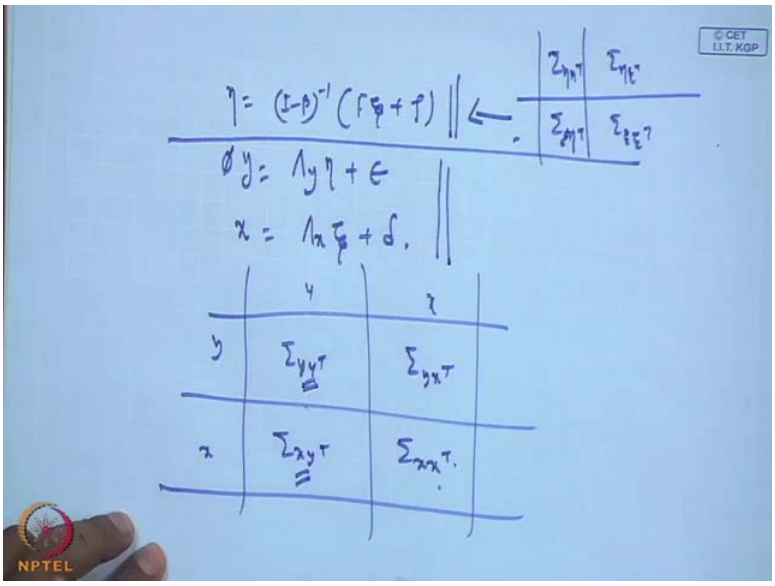

Then, you will be having an equation like this. You will be having I minus beta inverse gamma psi plus zeta, this will be your one equation. Then another equation will be your y is lambda y eta plus epsilon. Another equation will be x lambda x psi plus delta. So, these three questions when you split into two parts, first you established this through measurement model and then this one through structural model. If you want to estimate all those things simultaneously, then you are doing same, taking everything in one go.

So, in this case, now your covariant structure will be like this what matrix you will be analyzing. Finally, so this will be y y transpose. This will be covariance y x transpose, this will be covariance x y transpose and this will be covariance of x x transpose. Earlier here, you have seen that we have done like this. Covariances of eta eta transpose covariance of eta psi transpose covariance of psi eta transpose covariance of eta eta transpose that you can calculate. Now, here this is a different case. You are getting me? Now, if this is the case, then what you require to do?

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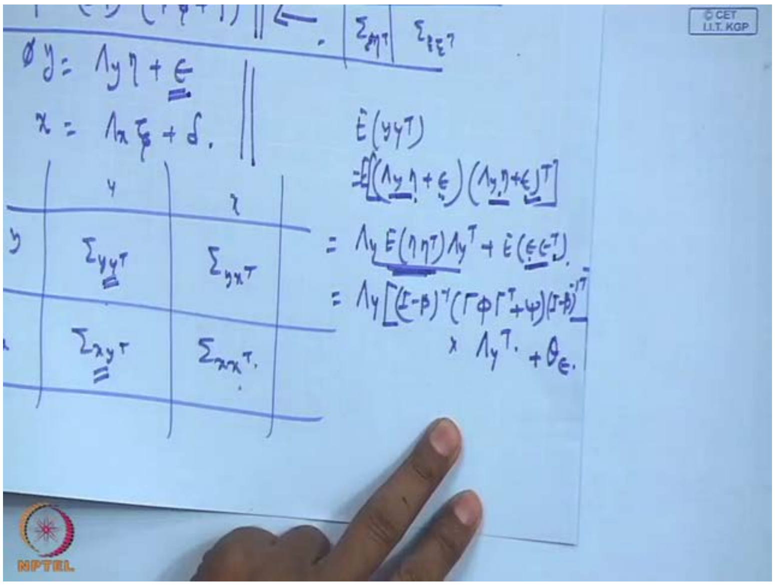

You require to find out is expected value of y y transpose. Then this is nothing but you just see lambda y eta plus epsilon and lambda y eta plus epsilon transpose and expected value of this. So, you see what will happen ultimately? Lambda y expected value of eta eta transpose lambda y transpose will come from here. Now, from assumption point of view, eta and the epsilon are uncorrelated, epsilon is uncorrelated plus, so those two expected values will be 0 is what you will get expected value of epsilon epsilon transpose. Then essentially it will be this.

You are seeing that we have found out this one in measurement model. What is this value, we have found out. I will show you this value. We have found out earlier. Earlier, I think you just see expected value of this is I minus beta, this is a bigger quantity. So, if I write here, then this part will become like this, I minus beta inverse, then gamma phi gamma transpose plus psi I minus beta inverse transpose, this into y transpose plus what will happen is epsilon epsilon transpose. You will see that they are talking about this equation. This equation is the error term, which we can write delta epsilon. This is the covariance between epsilon and epsilon.

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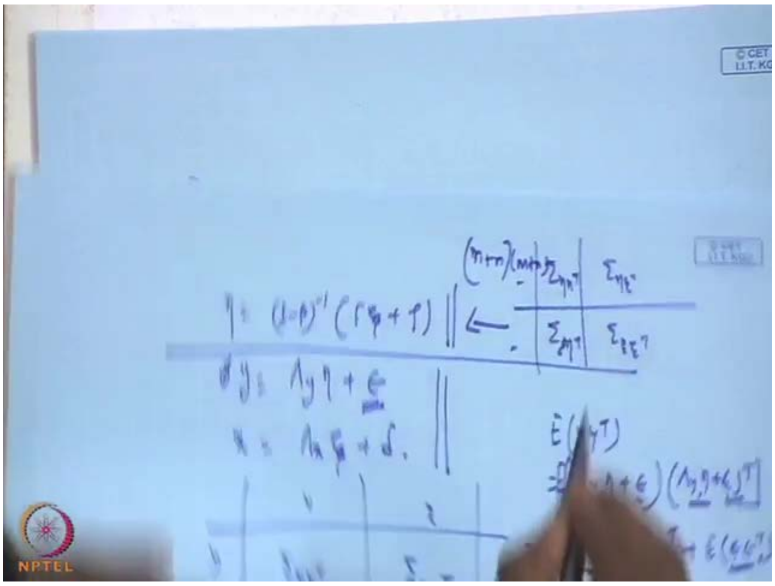

So, essentially that means this one matrix become not m cross, here it is m plus n cross m plus n. here, it is suppose we have p y variable in or p x variable and q y variable. Then the total variable is p plus q.

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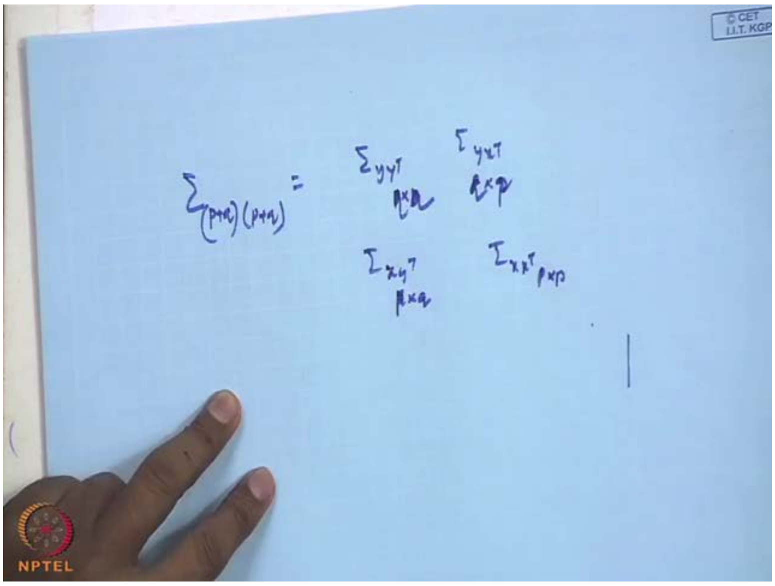

So, our matrix is p plus q into p plus q. This one is partitioned into, first one is sigma y y transpose that is p cross p. Then this one is sigma y x transpose, this is p into q. So, like this sigma x y transpose that is q, this is q cross q, this is q cross p and this one is p cross

q. Like this, this is x x transpose. So, that means what you can do you to find out this matrix for everywhere like this. So, your matrix will be there.

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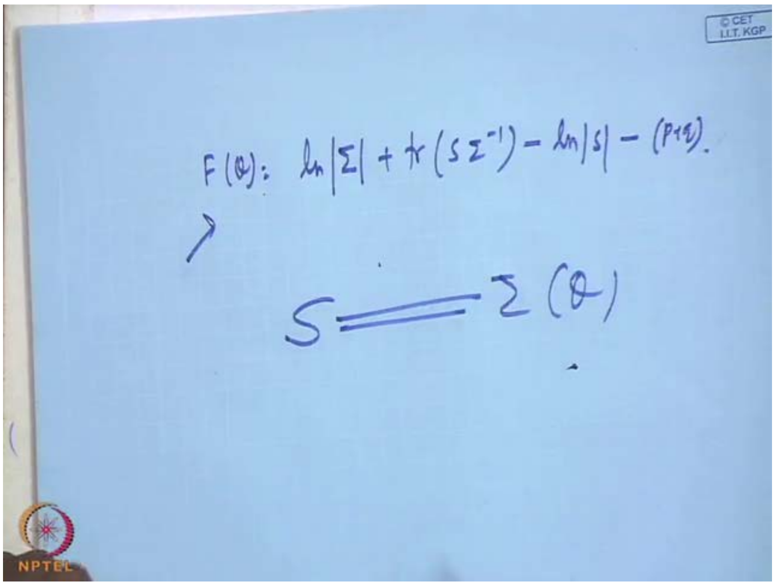

Now, your F theta again will become in the same manner, log of this plus trace of s epsilon inverse minus log of s minus now p plus q. This is the function you are going to minimize. So, you have one end s, another end sigma theta, now minimising this function, finding out this value. Then the number of parameters to estimate, it is definitely much more because the measurement model parameter and structural model parameter, all are coming into consideration. All will be taken into consideration here. From the model identification point of view, it will be difficult because so many parameters are involved.

Anyhow, this is also possible and many times we do this. The advantage of doing this in one go that means the interaction between everything , every variable, that interaction that is controlled of the parameters estimate is controlled by the effect of other parameters in a logical manner. This is in a nutshell. In true sense, this last one, when you combine everything together, and estimate everything at a go that is the complete structural modeling. So, structural equation modeling is a big issue. It is not that as simple all together.

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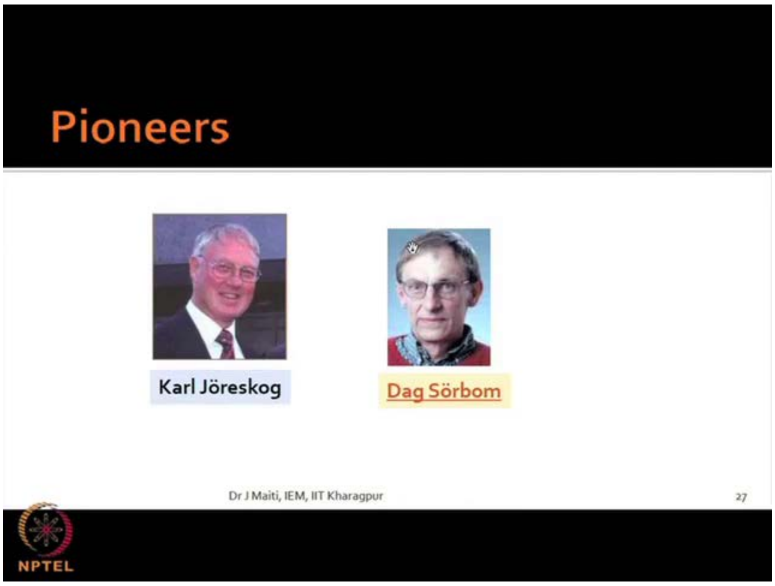

Again, I must show my gratitude and respect to the persons who have developed these useful things and for all of you, you have to practice it. You use the software, find out case where you have expertise, not arbitrarily hypothetical case, you have to get inside. There are many nutty gritty. Now, the structural equation in total what I have described, this is the legally linear scale. It is non linear also. So, a lot of advancement has taken place over the years and some advance structural equation modeling if you find out, please go through the fundamentals.

Thank you very much.