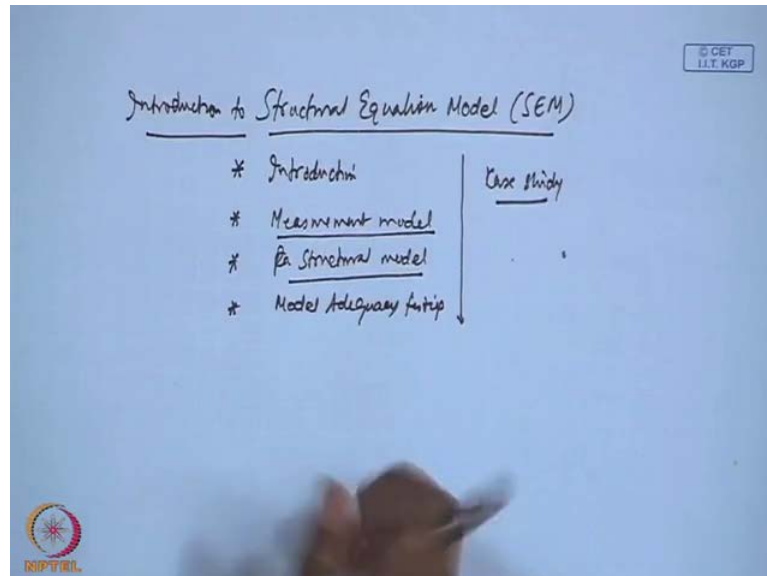


Applied Multivariate Statistical Modeling
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Lecture - 38
Introduction to Structural Equation Modeling

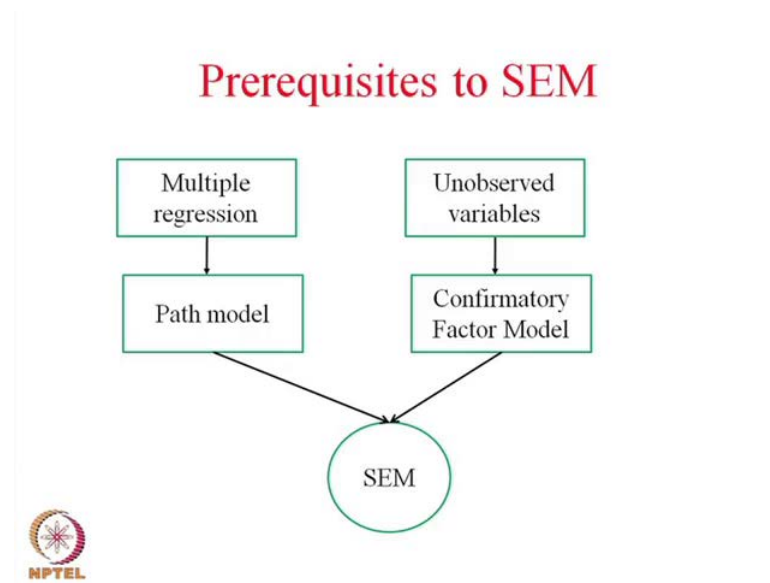
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Good evening. Today, I will discuss structural equation modelling also known as SEM. It is a little bit complex, advanced. It requires knowledge of some other tools and techniques what already covered in this course. I have partitioned this total lecture into three primary headings. One is today's introduction. So, today I will show you that what is structural equation modelling and how it is applicable to real life problem solving.

Then also we will discuss that what are the components of structural equation modelling. For example, it consists of measurement model, it also consists of path model or structural model. So, we will elaborately discuss this measurement model in next class. Then we go for path model discussions, and then there will be the model adequacy testing, adequacy testing and obviously I will discuss case study, one case study all through. Today's lecture is introduction to structural equation modelling.

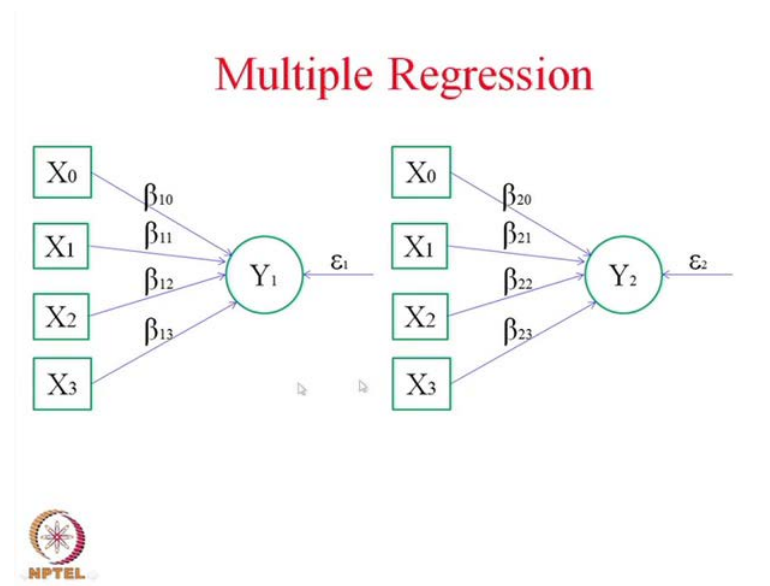
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Let us see that what are the prerequisites to structural equation modelling you must know multiple regression. You must know path model. Although I have not described path model till date, but within the system, we will be describing path model. You must know confirmatory factor model whose prerequisites are exploratory factor model so that unobserved variables can be measured through exploratory factor model.

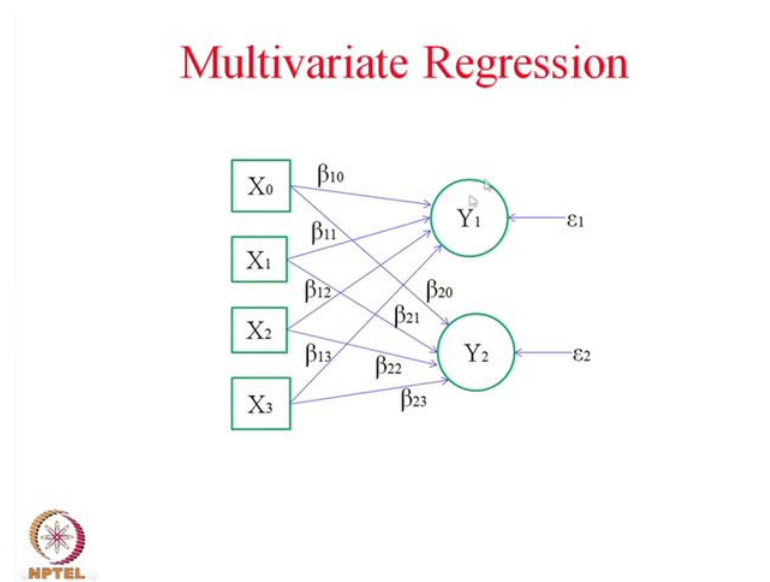
Now, what is the difference between confirmatory and exploratory factor model that we have already discussed? So, that means if you know confirmatory and exploratory factor model, you know path model, which is basically the structural model. Then you will be able to apprehend structural equation modelling.

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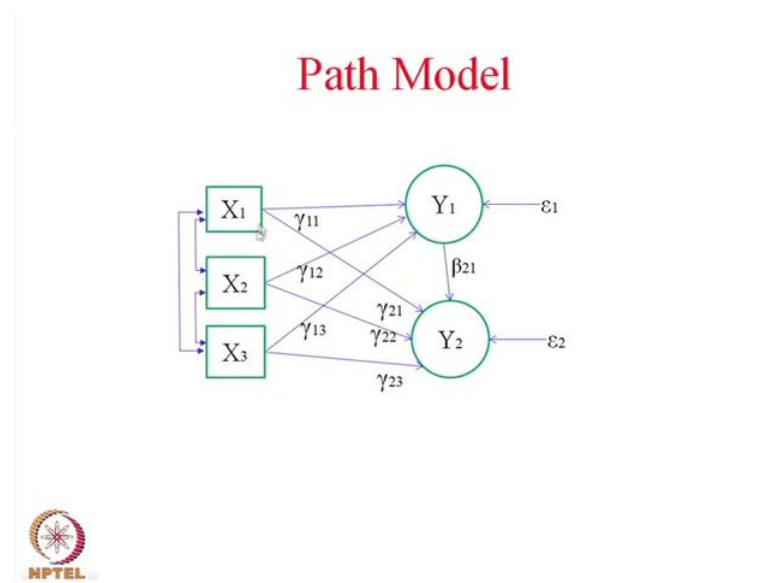
Let me recapitulate little bit of the prerequisites. I know that all of you have seen this type of pictorial representation for multiple regression, where Y_1 , here Y_1 is affected by several variables as well as Y_2 is also affected by the same sets of variables X_1, X_2, X_3 .

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When we have more than one dependent variable, we have seen that that multivariate regression is preferable. Now, the question comes if these variables are also correlated and the independent variables are also correlated or if there is even the causal relationship between that Y_1 and Y_2 , so what will happen?

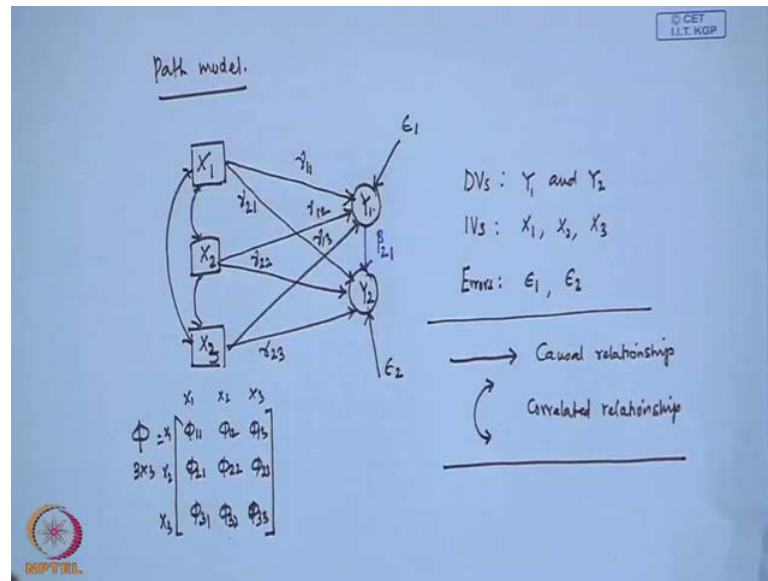
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So, ultimately in order to incorporate the covariance relationship between the X variables, so you create something like this where X_1 , X_2 and X_3 , these three are the independent variable. Y_1 , Y_2 are dependent variables, but the difference of this structure with multivariate regression structure is that in multivariate regression, you are assuming that X_1 , X_2 is equal to X_3 is truly independent.

They are not correlated and there is no causal relationship between Y_1 , Y_2 also. But, in path model, what you are making? You are considering that X_1 , X_2 as well as X_2 , X_3 or X_1 X_3 , they can vary. In addition, what we are seeing that Y_2 can be affected by Y_1 also in addition to the effect of X_1 to X_2 to X_3 . Now, let us see equation for this path model. So, we are deriving a migration from path model.

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Given like this X_1 is an independent variable in this model, but correlated with other independent variables. So, that is basically this correlation can be captured through a matrix, which is phi. If we can say this phi is a 3 cross 3 matrix, so we can write like this, so $X_1, X_2, X_3, X_1, X_2, X_3$, it is phi 11, then we can write phi 21, we can write phi 22, and we can write phi 31, phi 32, and phi 33. Definitely, this is a symmetric matrix and this will be phi 12, which is basically phi 21 equal to phi 21. Then phi 13 is equal to phi 21 and phi 23 is equal to phi 32.

In case of multivariate and multiple regression, we have assumed that this is structure of diagonal elements are 0. Now, we have considered here two independent variables Y_1 and Y_2 and we are assuming that X_1, X_2 and X_3 , they are affecting Y_1 with coefficient like this. Gamma 1 is affected by 1; gamma 1 is affected by 2, gamma one is affected by 3. Similarly, from independent side point of view, this gamma 2 is affected by 1. That means Y_2 affected by Y_1 , Y_2 affected by X_2 and Y_2 affected by X_3 .

Let me repeat here, Y_2 affected by X_1 that is coefficient is gamma 21, Y_2 affected by X_2 , coefficient is gamma 22, Y_2 affected by X_3 , coefficient gamma 23. As these are regression lines like equations, so this is having an error term. Now, this also will have an error term. In addition, here we are assuming that this Y_2 is also affected by Y_1 . If this is the case, you have to give a symbol like beta Y_2 and Y_1 , this beta 21.

So, if you want to write an equation for this, then, there are how many variables are there? How many variables are there? There are different types. One set is DVs that is Y1 and Y2. Another set is IVs that is X1, X2, and X3. Another set is errors that are epsilon 1 and epsilon 2. Then what types of relationships are available here? One relationship is like this. If you see that X1 to Y1, this single arrow, this is known as causal relationship. Another type of relationship with this giving here like this card bidirectional, decision on edge correlated relationship. So, these two types of relationships are available in this. These two types of relationships are estimated in path model.

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Handwritten mathematical equations for a path model:

$$\begin{array}{c} \text{DVs} \\ \underline{DYs} \quad Y_1 \quad Y_2 \quad \text{IVs} \quad X_1 \quad X_2 \quad X_3 \quad \text{Errors} \quad \epsilon \\ \Rightarrow Y_1 = 0 \quad 0 \quad \gamma_{11}X_1 + \gamma_{12}X_2 + \gamma_{13}X_3 + \epsilon_1 \\ \\ Y_2 = \beta_{21}Y_1 \quad 0 \quad + \gamma_{21}X_1 + \gamma_{22}X_2 + \gamma_{23}X_3 + \epsilon_2 \\ \\ \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \beta_{21} & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \\ \\ Y = \beta\gamma + \gamma X + \epsilon \end{array}$$

Now, if you want to write down the equations for these, straightway we can write one is DVs. I can write Y1 and definitely Y2 is there. This side also you can write DVs, you can write IVs, you can write IVs. How many DVs are there? Two DVs, Y1 and Y2, X1, X2, X3, then errors are definitely this. So, the equations will be like this, Y1 equal to that, Y1 is not dependent on that, Y1 again it is not like this. Similarly, if I see that Y1 is dependent on X1, X2 and X3 and this arrow term, not Y2, so nothing will be written here. It will be straightway gamma 11, then X1, then plus gamma 12 X2 plus gamma 13 X3 plus epsilon. This is the regression equation for Y1. Similarly, for Y2, if we see that Y2 is also affected by Y1, so we can write here that beta 2 is affected by 1, Y1 plus gamma 21 X1 gamma 22 X2 gamma 23 X3 plus epsilon 2.

So, in matrix form, you can write Y_1 is equal to Y_2 . Then this side you see here it is nothing means 0, 0, 0 so 2, 1, 0, definitely again you are writing Y_1 , Y_2 , which is the difference from regression model. Then here you have γ_{11} , γ_{12} , γ_{13} , γ_{21} , γ_{22} , γ_{23} into X_1 , X_2 , X_3 plus you are getting epsilon 1 epsilon 2. This is your path model or Y equal to beta Y plus gamma X plus epsilon, beta Y plus gamma X plus epsilon.

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The image shows a hand-drawn derivation of a path model. It starts with the equation $Y_1 = 0 + 0 + \gamma_{11}x_1 + \gamma_{12}x_2 + \gamma_{13}x_3 + \epsilon_1$. Then it shows $Y_2 = \beta_{21}Y_1 + \gamma_{21}x_1 + \gamma_{22}x_2 + \gamma_{23}x_3 + \epsilon_2$. These are then combined into a matrix equation: $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \beta_{21} & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}$. This is simplified to $Y = \beta Y + \Gamma X + \epsilon$, and finally to $(I - \beta)Y = \Gamma X + \epsilon$. A hand is visible at the bottom left of the slide, and a logo for NIPUN is in the bottom left corner.

If you do little more manipulation, then this will be I minus beta Y equal to gamma X plus epsilon.

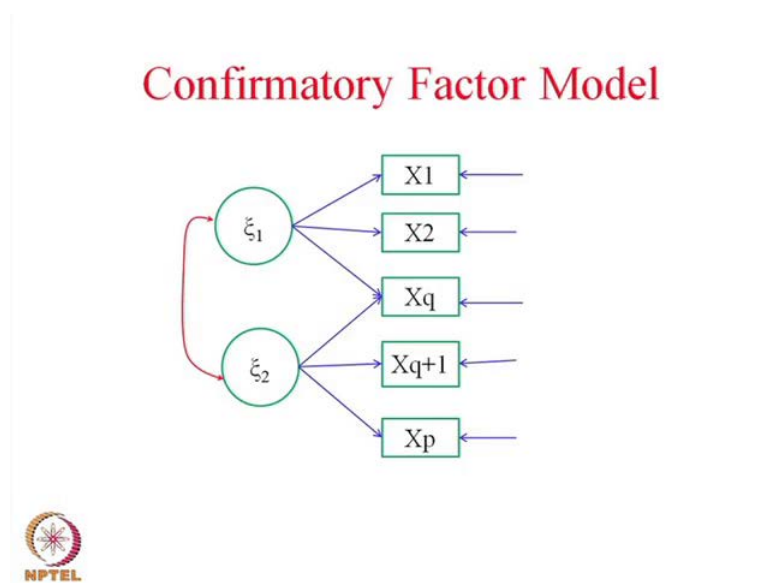
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Structural model of the SEM.

$$\Phi \quad (I - \beta)^{-1} (I - \beta) Y = (I - \beta)^{-1} [F X + \epsilon]$$
$$Y = (I - \beta)^{-1} [F X + \epsilon]$$

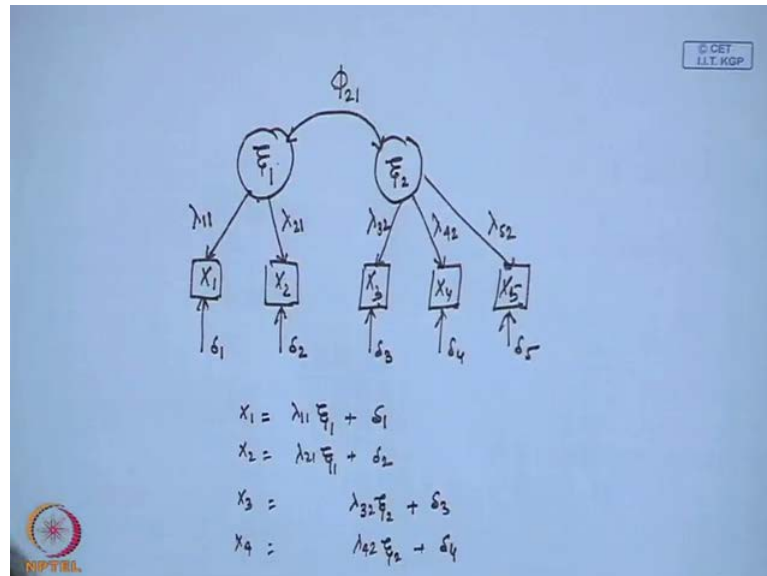
Then, further you may be interested to write this that Y. If I multiply Y minus beta inverse here with I minus beta Y, then I minus beta inverse, you are getting that gamma X plus epsilon. This will be I. So, Y equal to I minus beta inverse F X plus epsilon. The details of this how to estimate this parameter beta, psi and all those things, epsilon, phi, all those things will be discussed under structural equation model, structural model under structural model of the structural equation modelling. So, let us assume that we know this for the time being that this correlation is possible to understand.

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Then, you come to confirmatory factor model. I think you can collect yesterday's lecture will particularly I think last two lectures were on cluster analysis before that.

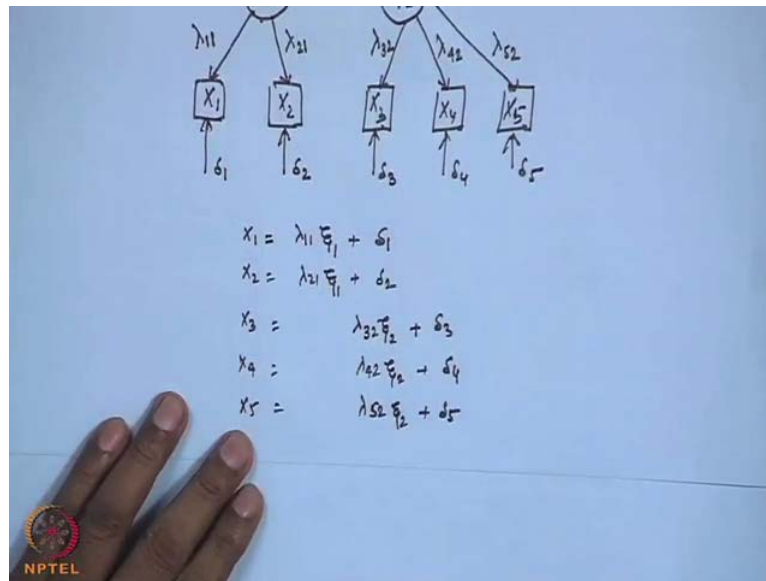
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Then, if I write like this what you are doing, then here in this case, let us concentrate on only two factors and let them be related with X1. First one is related with X1, X2 maybe, second one is related with X3, X4 and X5. There is definitely covariance between the two, that is phi 21 and each of the observed variable X1 are caused by psi 1 here. These are caused by psi 2, psi 1 and psi 2. Now, if I write this one is lambda, then for X1, I am writing 1 for psi 1. I am writing 11 X1 is affected by psi 1, then lambda 2 X2 affected by psi 2, then lambda 32, then lambda 42, then lambda 52.

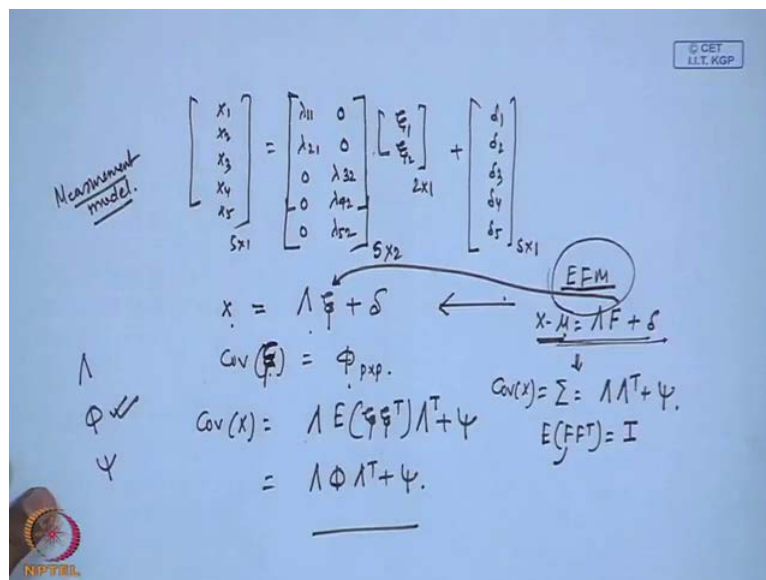
So, this one will be delta 1, this will be delta 2, this will be delta 3, this will be delta 4, this will be delta 5. This is what you have seen in confirmatory factor analysis. We have written this one in this manner that X1 equal to lambda 11 psi 1 plus delta 1, X2 equal to lambda 21 psi 1 plus delta 2, X3 equal to lambda 32 psi 2 plus delta 3, X4 equal to lambda 42 psi 2 plus delta 4.

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X5 is equal to lambda 52 psi 2 plus delta 5. So, the resultant matrix you have written like this.

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So, X1 X2 X3 X4 X5, this is equal to your psi 1 psi 2 is there, so we write down lambda 11, 0, lambda 21, 0, then 0, lambda 32, 0, lambda 42, 0, lambda 52. So, this is 5 cross 1. This is your 5 cross 2 and then psi 1 and psi 2, this is 2 cross 1 and plus delta 1 delta 2 delta 3 delta 4 delta 5, this is five cross 1. The resultant you can write is X equal to, this is you lambda, this one is psi plus we can write this as delta. If you recollect gamma

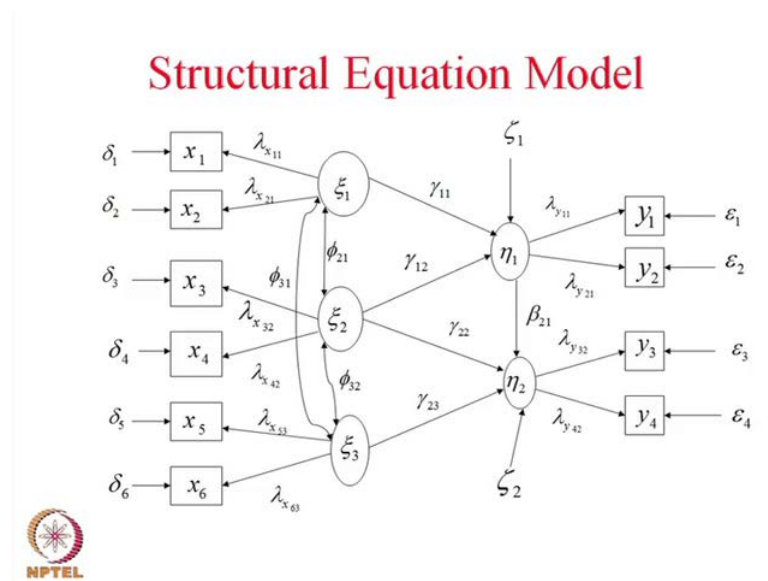
exploratory factor model, what we have written there is $X - \mu = \lambda F + \delta$. It is coming similarly; only that minus μ is not there, but here also we assume that this subtraction is made.

So, actually what happened is that you are getting the same factor model. There is one difference in this case. The difference is in this exploratory factor model, what you found out, when you found out that covariance of X , which is Σ that one is $\lambda \lambda^T + \Psi$ because we assume that expected value of $F F^T$, this is I that covariance between the factors. In this case, we are assuming that covariance of X is Φ depending on, so there are p variables, it will be $p \times p$.

So, as a result, the covariance here we are assuming covariance of Ψ , I am extremely sorry covariance of Ψ , this one correspondence to this; this corresponds to this covariance of Ψ is this. Then covariance of this will be $\lambda E[\Psi] \lambda^T + \Phi$. So, for the time being, we stick to this only.

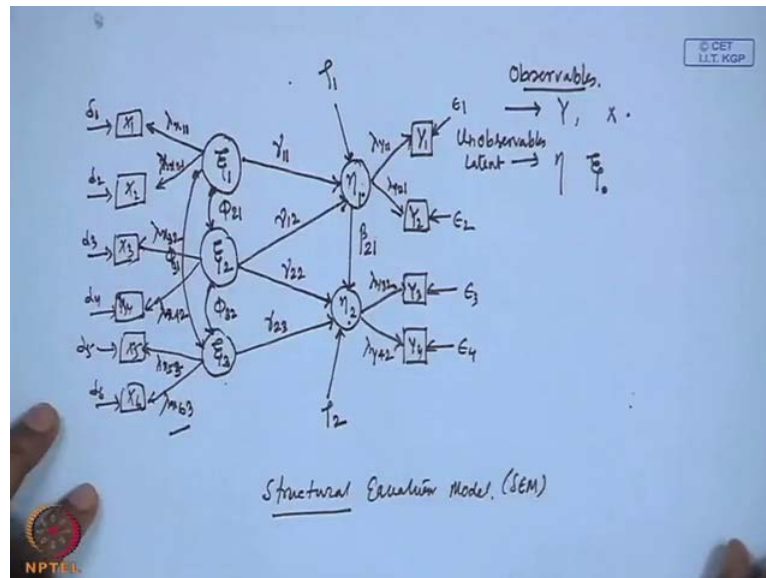
Now, question is how to estimate this λ Φ and Ψ and all those things related to exploratory factor analysis. We have seen that how this can be estimated, but in exploratory initial direction, this Φ matrix was not there. Here, Φ matrix is there. Also, we will be discussing this under measurement model.

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So, these two are very essential for structural equation modelling. Now, see one big picture where we are doing like this. First, let us think of that there is one variable eta 1, another variable eta 2.

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These two variables are affected by some variables called psi 1, psi 2, and psi 3. Let the link is like this eta 1 is affected by psi1 as well as psi2 and eta2 is affected by psi2 as well as psi3 and eta 2 is also affected by eta 1. So, under this condition, if you see the structure path model equation, so in path model equation let me find out this path model.

Yes, this is our path model equation. So, if I keep side-by-side this to side-by-side, so what is happening here is you see, we are writing eta 1 in place of Y1, eta 2 in place of Y2, psi1 in place of X1, psi2 in place of X2, psi 3 in place of X3. Here, everything is linked with all the X variables are linked by Y. Here, we are restricting that there were that eta 1 is affected by psi1 and psi2 and eta 2 is affected by these two. So, there has been this structure is similar to the structure except the some of the links are omitted.

So, I can write this that gamma 1 eta 1 is affected by psi 1 means gamma 11. So, this one is gamma 12. Then this is gamma 22. This is gamma 23 and this one is beta 2 affected by 1. The coefficient as usual we are writing here. Now, the difference between this path model, what I have shown, I have shown in path model in terms of Y and X. When we talk about in terms of Y and X, we say they are observable.

Now, when we are saying in terms eta and psi, they are latent and unobservable or latent that is the difference. So, if this is the case, then what you require to do? You require measuring these unobservable or latent things. We will see this, but before that, some more things that these are cohering, let me write. This is nothing but ϕ_{21} , this is my ϕ_{32} , and this is my ϕ_{31} . These structures are there. So, as I told you that this eta and your psi, they are immeasurable, unobservable, latent. We require measuring this. How do you measure? We have seen earlier that our confirmatory factor analysis or exploratory factor analysis will help us to measure. What it is? These are latent, this is latent variable, this is also latent, and these are manifest variables.

So, these manifest variables are used to measure these latent variables. Now, we will do the same thing here. We may create here like this. Later, we denote it is as Y_1 , this one is Y_2 here, also Y_3 , and Y_4 and definitely there will be error. That error will be suppose ϵ_1 , this error is ϵ_2 , this error is ϵ_2 , and this error is ϵ_4 , error head. Please give it properly. Then what is the difference between this structure and this structure?

If I place these to linking to three indicators, linking to two indicators, variables here eta 1 is linked with two indicator Y variables. Eta 2 is linked with two Y variables. What we are saying these two are latent, these are the manifest variables. This eta 1 and eta 2 can be measured using this. This is nothing but a confirmatory factor analysis. This is what confirmed confirmatory factor model is.

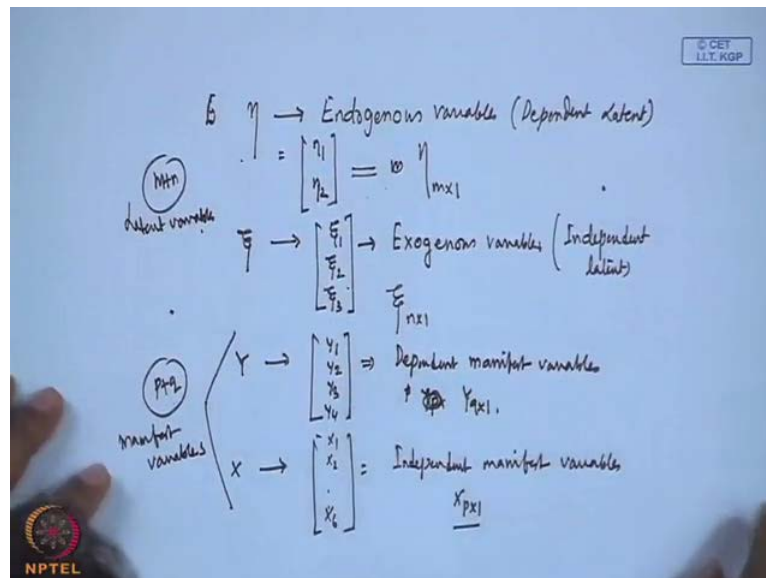
So, you can give a notation here. So, we have given lambda I think. Let us use one another subscript Y and this is 11. Then this one will lambda Y_2 is affected by 1. Then this will be lambda Y_3 is affected by 2. This is lambda Y_4 , is affected by 2. So, we also require measuring this. So, let us assume that this can be measured by indicated manifest variable X_1 . Then your X_2 , then this is your X_3 , this one is X_4 , then your X_5 , X_6 and this side delta 1, this is your delta 2, this is delta 3, delta 4, delta 5 and delta 6.

Now, if you look into this side, what is there? Is there any difference between this, between this and this here ψ_1 , ψ_2 ? You have taken three latent variables and these are the indicator five and indicators two are linked with ψ_1 and three linked with ψ_2 . Here, there are sixteen indicators; two each are linked with ψ_1 and ψ_2 . So, that means this is also a factor analysis part, confirmatory part.

So, then this one, we can write like this lambda, we are taking as X11, this is your lambda X21, this is your lambda X32, lambda X42, lambda X53, lambda X63. What is missing here? These are from confirmatory factor point abilities, these are the dependent, and these are the causes here. These depend on these causes. So, arrow head, arrow terms are given, arrow terms are given, but here from the path model point of view, these are the dependent side and this is independent.

So, you have to give eta. This is what is known as in path diagram, this is known as structural equation model. Why? It is structural equation model because the latent variables, they also have certain structure, some structural relations, which they are. For example, psi1 is affecting eta 1, but eta 1 is affecting eta 2. So, there is a structural path from psi1 to psi7, though here psi1 is not directly affecting eta 2, but psi1 can have effect through eta 1 to eta 2. So, that means that the relationship structure is correctly preserved. So, that is why, this name is structural equation modelling is given here. Now, how to have the estimation of this? This is a big issue. We will be describing later on that all those things, but here, we will want to give you some names in the structure.

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I want to give you some names of the variables. In structural equation modelling, we will use the term like these eta and all eta will be termed as endogenous variable. Endogenous, this is basically dependent latent in the example given, this one each this eta, this is basically eta 1 and eta 2. Now, there is another variable, which is psi, which is

in this example. This is ψ_1, ψ_2, ψ_3 , which is known as exogenous variable, exogenous variables. This is independent, latent.

So, in general, your η can be $m \times 1$, ψ can be $n \times 1$, those many things will be there or any other means, you can use something else. Then we have Y , which is in our case, in this example Y_1, Y_2, Y_3 and Y_4 . So, this is known as dependent manifest variable. It can be $p, q \times 1$. Similarly, you have X here, we have X_1, X_2 to X_6 . These are independent manifest variables. It can be X that is $p \times 1$. That means in totality, you will be having p plus q manifest variables and m plus n latent variables. In addition, you have other variables like ψ .

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$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}_{m \times 1} \leftarrow \eta_{m \times 1}$$

$$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{bmatrix}_{q \times 1} \leftarrow \epsilon_{q \times 1}$$

$$\delta_{p \times 1} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

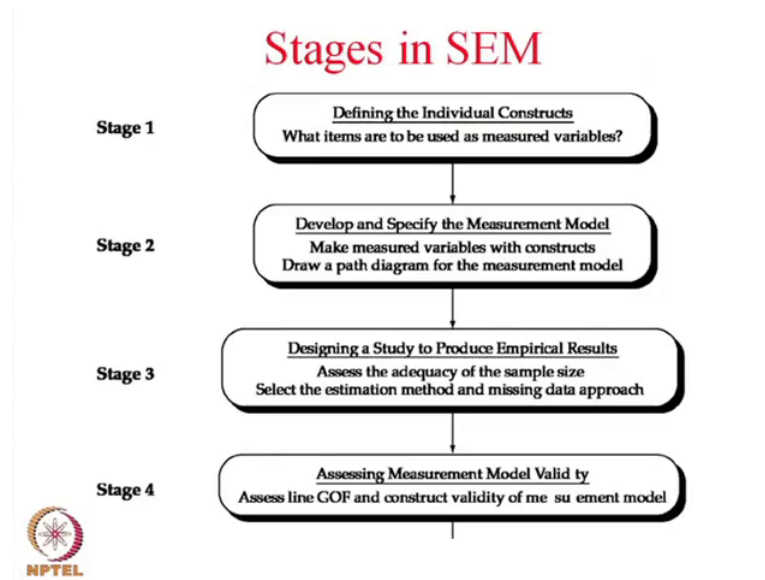
$$\Phi = \text{Cov}(\epsilon)$$

$$\Psi = \text{Cov}(\eta)$$

Other variable in this case, for example is ζ . ζ is nothing but the error terms. In this case, it is ζ_1, ζ_2 . It is related to η . So, depending on the η , if η is $m \times n$, then this also be $m \times 1, m \times 1, m \times 1$. Then another one is your ϵ that will be related to ϵ_1 to ϵ_2 to ϵ_4 . In this case, this is related to your Y . It all depends on what is the value of Y . So, this will be same.

Similarly, related to X , there is δ . The δ is your $p \times 1$ variable vector, these three. You have another parameter called ϕ which is basically covariance of this. You will also have, you will be getting ψ as your covariance of your ζ . So, there are a huge, every large number of, what I can say, parameters to be estimated. Let us assume that yes, everything is possible and we are going for its stages.

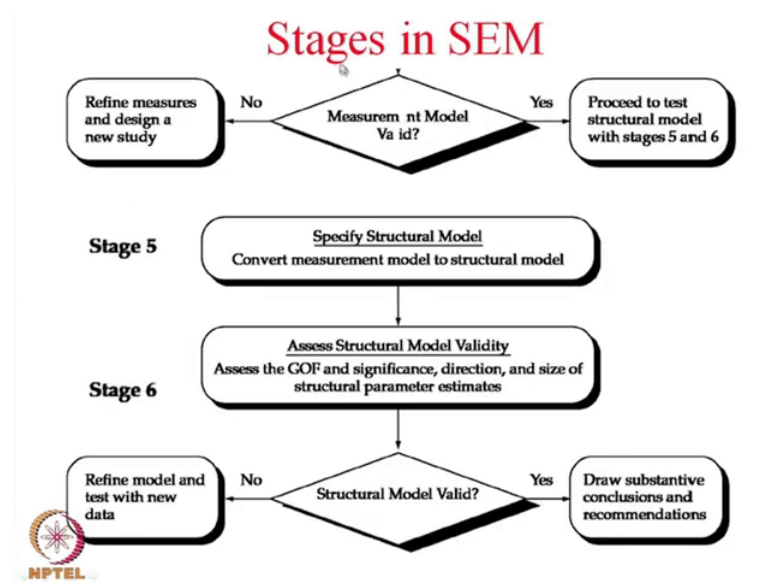
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What is stage one is? It is defining the individual constructs, what items are to be used as measured variables. Then stage two is develop and specify the measurement model, what is the confirmatory factor model, make measured variables construct, draw a path diagram for the measured variables. Then stage three is designing a study to produce empirical results. All multivariate modelling, as such, any model building case, the design, study design is very important. Here, what we are saying, assess the adequacy of sample size, select the estimation method and missing data approach.

This is true for multiple regression, multi variable regression and path model, factor analysis, compound principle, compound analysis, structural model, cluster model, everywhere, it is applicable. Then assessing measurement model validity means these few first four stages; we are talking about the measurement model. Then what is this measurement model? Measurement model is this part, this factor analysis part; this is your measurement effect. Then what is your structural side? Structural side is this one. So, first what we have, it is said that in the stages and find out the manifest variables, then develop the constructs and also measure this, the adequacy of this model.

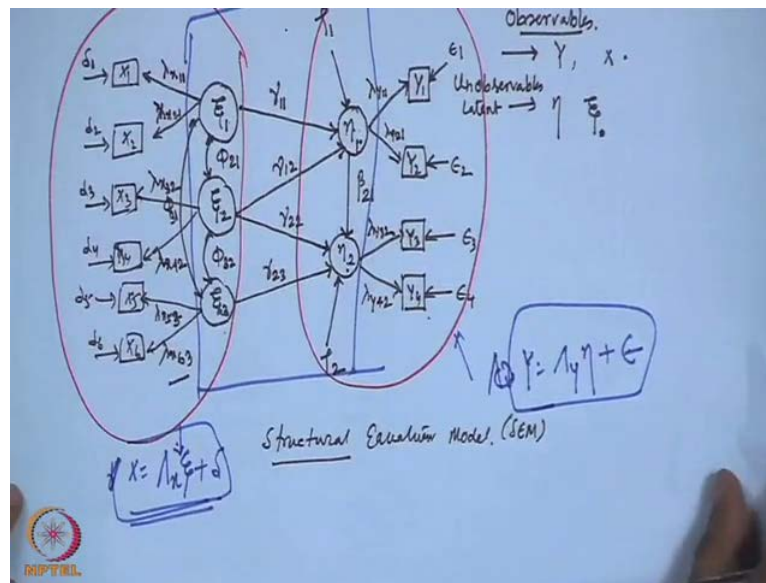
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Then, you have to see if the measurement model is valid, then you have to go through five to six. If the measurement model is invalid, then you have to redefine the measures and other things. Now, specify, then your structural model is coming. Specify the structural model. Convert measurement model to structural model. I think you have seen here ultimately. I will repeat this again. Then stage six is assessing structural model of validity. Then if structural model is valid, then draw substantive conclusion. If it is not valid, refine the model and test with a new data.

In this case, what is it actually given? It is given in a two stage model building. One is first you develop the measurement model. First you develop the measurement model irrespective of your, in this case, forget about the structural part. What you have? You have these many X variables, these many Y variables. Irrespective of eta and psi, you first go for it on measurement model where you will basically factorise, confirmatory you factorise this eta 1, eta 2, psi1, psi2, psi3 from Y1 to Y4. This is the first.

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Once you are happy with this, so that means that if I just follow this one this side, it is X is $\lambda x \psi$ plus δ . This is my measurement model for this site. This side, it will be, Y equal to λy into η plus ϵ . This is my contrary path. But, the process of model building stages is that that you go for measurement model. First consider all these, and then do a factor, confirmatory factor analysis. Find out the correlation or covariance structure of all the latent variables. See here. Then once you have all your latent variables, everything is crystal clear, and then you go for structural model.

The structural model is the building between this path this one you will do and you will estimate and all those things. So, ultimately you will first prepare measurement model. Then from the measurement model, you get the covariance or correlation matrix for the latent variables. Using this correlation or covariance matrix, you will estimate the regression like coefficients, regression coefficients for this structural model. This is one.

Second one is you may be interested to do everything at a time. That means you will feed the model on both structural and measurement part keeping intact at one go. But, because of so many parameters to be estimated and so many computations, permutations, combinations, what will happen ultimately? It is highly likely that you may get several of offending estimates. How to delete offending estimates it is an issue also in structural equation modelling.

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Stages in SEM

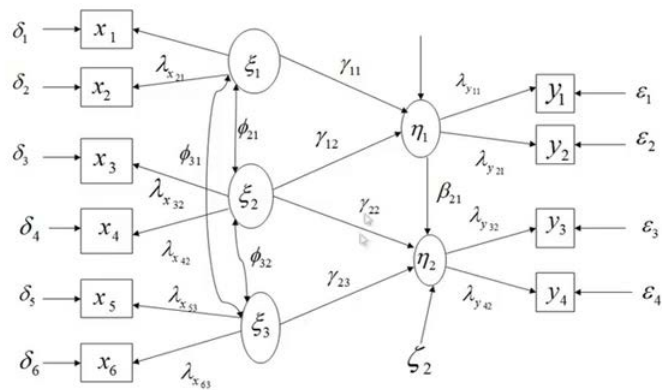
- Stage 1: Developing a Theoretically Based Model
- Stage 2: Constructing a Path Diagram of Causal Relationships
 - Elements of a Path Diagram
 - Examples of Path Diagrams
 - Basic Terminology
 - Assumptions of Path Diagrams



So, now what I will do?

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Conceptual Model




This is the structure I have shown you. Already, I have shown you. These are the stages that what we have discussed.

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SEM Equations

Structural Equations	Measurement Equations
$\eta_1 = \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \zeta_1$	$X = \Lambda_X \xi + \delta$
$\eta_2 = \gamma_{22}\xi_2 + \gamma_{23}\xi_3 + \beta_{21}\eta_1 + \zeta_2$	$Y = \Lambda_Y \eta + \varepsilon$
$\eta = \Gamma \xi + \beta \eta + \zeta$	




These equations also you have seen X and Y, these equations you have seen.

(Refer Slide Time: 42:10)

Stages in SEM (contd.)

- Stage 4: Choosing the Input Matrix Type and Estimating the Proposed Model
 - Inputting Data
 - Assumptions
 - Covariances Versus Correlations
 - Sample Size
 - Model Estimation
 - Estimation Technique
 - Estimation Processes
 - Computer Programs




Then, the stage four is that what matrix you are going to use, you have to choose one input type of matrix that is correlation or covariance matrix. There are different assumptions. Then there is your estimation technique to be used, process, computing programs. These are all the stages one by one you have to do.

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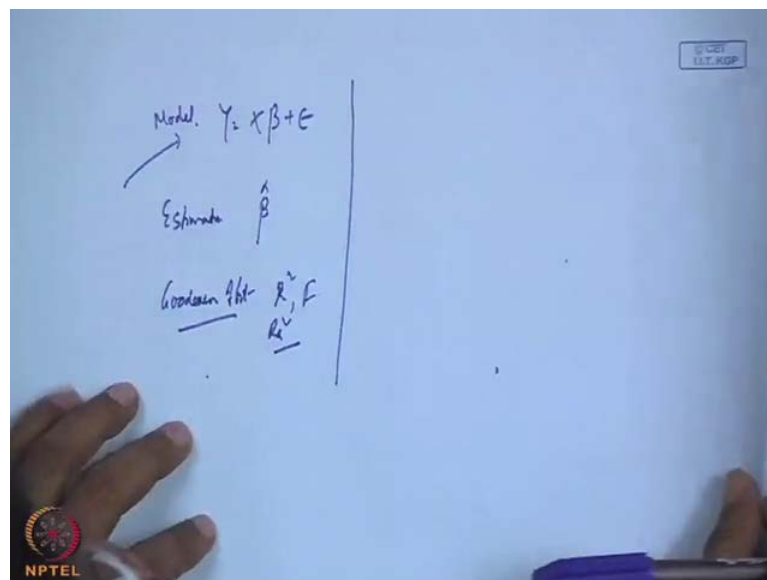
Stages in SEM (contd.)

- Stage 5: Assessing the Identification of the Structural Model
- Stage 6: Evaluating Goodness-of-Fit Criteria
 - Offending Estimates
 - Overall Model Fit
 - Measurement Model Fit
 - Variance Extracted
 - Structural Model Fit
 - Comparison of Competing or Nested Models



Then, suppose this is like your regression model also, you have seen that regression equation is y equal to x beta plus epsilon this is my model.

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Then, suppose this is like your regression model also, you have seen that regression equation is Y equal to X beta plus epsilon. This is my model. Then what we have gone for estimation that is your beta cap. Then what you have done? You have gone for goodness of fit, model adequacy, goodness of fit or model adequacy that is R square F

Re square, all those things that we have used. So, here also, if I see that this is my structural equation modelling, then are you not getting a structural equation like this?

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Handwritten mathematical derivation of a Structural Equation Model (SEM) on a blue background. The derivation shows the relationship between latent variables (η) and observed variables (ξ) through path coefficients (γ) and error terms (ζ).

The variables are defined as:

- Latent variables: η_1, η_2
- Observed variables: ξ_1, ξ_2, ξ_3
- Error terms: ζ_1, ζ_2

The structural equations are:

$$\eta_1 = 0 \cdot \eta_1 + 0 \cdot \eta_2 + \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \gamma_{13}\xi_3 + \zeta_1$$

$$\eta_2 = \beta_{21} \cdot \eta_1 + 0 \cdot \eta_2 + 0 \cdot \xi_1 + \gamma_{22}\xi_2 + \gamma_{23}\xi_3 + \zeta_2$$

The measurement equations are:

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}$$

The final model is summarized in three numbered equations:

$$\eta = \beta \eta + \gamma \xi + \zeta \quad \text{--- (1)}$$

$$X = \Lambda_2 \xi + \delta \quad \text{--- (2)}$$

$$Y = \Lambda_1 \eta + \epsilon \quad \text{--- (3)}$$

The word "SEM" is written in the bottom left corner of the slide.

The equation will be your first one that eta 1 eta 2 in this case and definitely here will be eta 1 and psi, sorry eta and psi and error that path structure. Under this eta 1 and eta 2, if I write eta here, then this is psi 1, psi 2, psi 3 and all those things. So, you have to write the equations. If I write for this particular model the equations, eta 1 is not affected by any of the data 0, 0. So, gamma11 psi 1 plus gamma12 psi 0 plus, plus zeta 1 and this one is beta 21 and this is zero. Then this one is also 0 plus gamma22 psi2 plus gamma 23 psi3 zeta2.

Then, what is your matrix? It is eta 1 eta 2 equal to 0, 0 eta 21 0 eta 1 eta 2 plus, you are getting gamma 11 gamma 12 0, 0 , gamma 22 gamma 23 into psi1, psi2, psi3 plus zeta 1 zeta 2 that is my structural part. So, eta equal to beta eta plus gamma psi plus zeta, this is one equation. Another equation is X equal to lambda x psi plus delta and Y equal to lambda y eta plus epsilon. This is equation number two. This is equation number three. So, your model, this is my structural SEM model and you require to estimate beta gamma, then gamma x, lambda x, lambda y, covariance of this, covariance of this, covariance between these; so many parameters you have to estimate.

Second is parameter estimation. So, we are not discussing here the details of parameter estimation. We will discuss later on separately. So, once you have the model, you require

to know that what will be the goodness of fit measures. So, my model is this and I know that somewhere, I will estimate all those things.


(Refer Slide Time: 46:39)

Goodness of Fit Indices

Absolute fit indices: address the question
Is the residual or unexplained variance remaining after model fitting appreciable? (they are absolute because they impose no baseline for any particular data set).

Relative fit indices: address the question
How well does a particular model do in explaining a set of observed data compared with (a range of) other possible models? (most of these relative fit indices establish as a baseline a “worst fitting” model. The most common worst fitting model is “null model” which models only the variances from the variance/covariance matrix. So, “null model” assumes that all covariances are zero).


Parsimonious fit indices: Adjust number of variables with sample size



Then, I have to go for fit measures. There are three types of fit measures used in structural equation modelling. One is absolute fit indices, relative fit indices and parsimonious fit indices. In absolute fit indices, the question is the residual or unexplained variance remaining after model fitting appreciable. They are absolute because they impose no baseline for any particular data similar to R square.

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$R^2 = 0.90$
 90% \rightarrow $Y \rightarrow z$
 $X - p.$ $(p1q)$
 $\hat{\Sigma} =$ $\begin{bmatrix} s_{11} & a_{11} & \dots & a_{1p} \\ \vdots & \vdots & \ddots & \vdots \\ (p1q) \times (p1q) \end{bmatrix}$
 $S - \hat{\Sigma}$
 $\hat{\Sigma} = f(\theta)$
 $\theta = [\beta, \gamma, \lambda, \lambda\gamma]$



R square, if it is 0.90, it says 90 percent variance of Y is explained by this. Here also, we are basically talking about what will happen to this. Actually, although we will be describing in detail later on, the estimation what is observed with you is this. This is basically if I, how many Y variables, Y is q observed variables and X is p observed variables. You have total p plus q observed variables. So, you have a covariance matrix of p plus q or p plus q. So, this is the observed value and this is nothing but this is what is observed.

Now, using these equations, this eta X and Y, this equation, you will also be able to estimate or other way I can say fit this one, you can calculate this. Here, if I use S, then it is s_{11} , s_{12} like s that p q up to p plus q. So, similar values we will be getting. Here, instead of, now I am writing instead of writing s here, we are writing the population sigma. Then that will be in terms of this, beta gamma lambda X lambda Y. So, there will be function of the, sorry, so what I mean to say is we can find out the theta in the nutshell.


I am talking about that beta, gamma lambda X, lambda Y, like this, it is possible to frame this, and then this s and these are compared. There are several ways to compare this. Then here p plus q into p plus q, this is the end. How many unique elements are there those will be compared. When we initialise some value here, some value here, those will be compared.

Then, we finally get the value. Then based on this difference, when we need something like this, this is total. Relative feed index that is the question that how well does a particular model do in explaining a setup of observed data compared with other possible models. You can go for several models and then compare which one is your best. The reference one is null model where we assume that there is no covariance value, relationships in the observed variables. Parsimonious fit index is similar to your adjusted R square where it is the number of parameters estimated, are adjusted. There are different fit indices.

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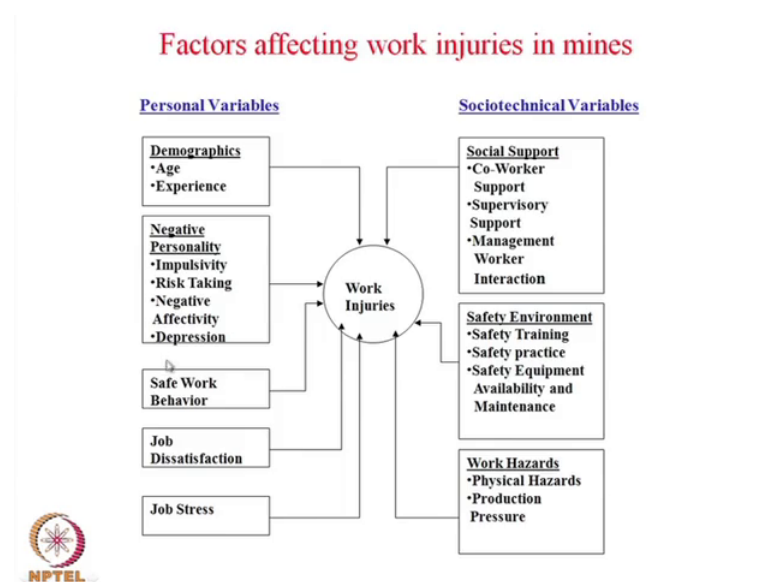
Case Study I

Role of personnel and socio-technical factors in work injuries in mines: A study based on employees' perception



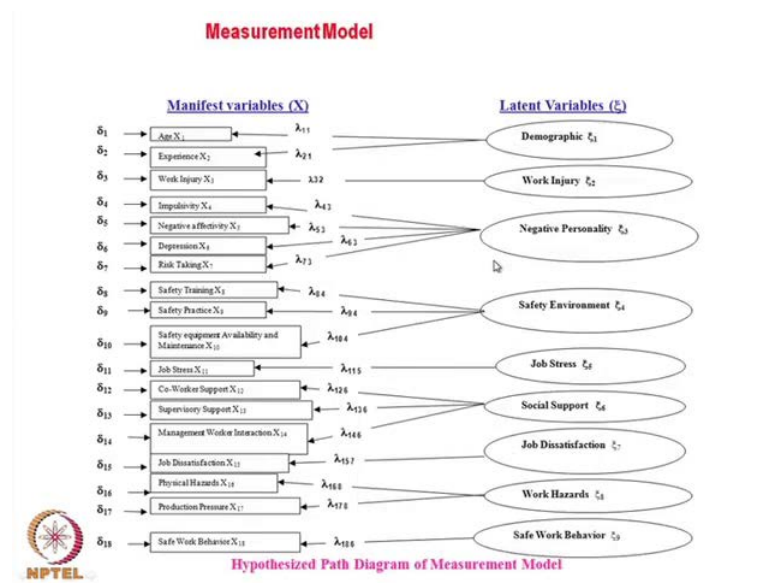
I will show you one case here by five minutes of time. This is role of personnel and social technical factors in work injuries in mines, a study based on employees' perception. This is published in organic general in 2008.

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So, we have captured several variables under personal and social technical variables.

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Then, what we have done? We have first gone for measurement model. All the manifest variables are used simultaneously and then the latent variables, first latent variables are identified and link to them was done. Then then confirmatory factor was done.

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Correlation Matrix

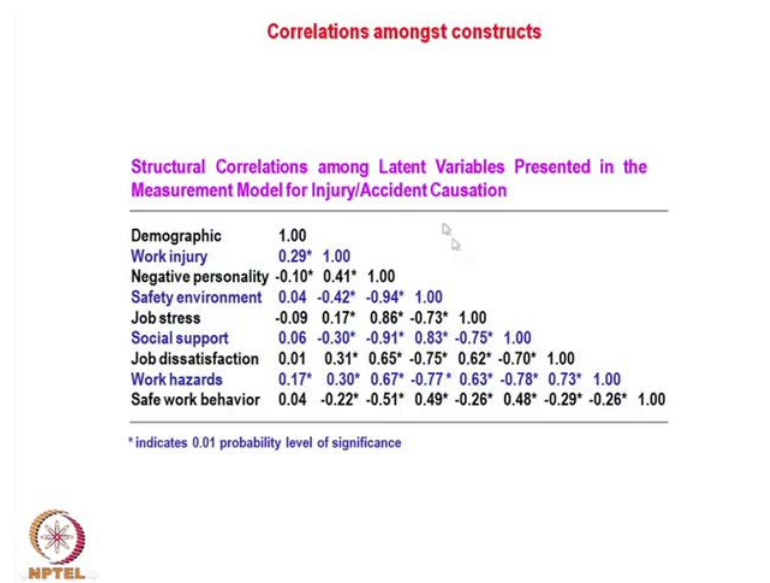
Correlations amongst the Major Variables for Accident Path Model

Age	1.000
Exp	0.816 1.000
Work injury	0.271 0.249 1.000
Impulsivity	-0.143 -0.085 0.214 1.000
Negative affectivity	-0.114 -0.078 0.373 0.600 1.000
Depression	-0.046 -0.059 0.084 0.183 0.409 1.000
Risk taking	-0.001 0.033 0.349 0.467 0.586 0.224 1.000
Safety training	0.074 0.060 -0.251 0.577 -0.444 -0.112 -0.464 1.000
Safety practice	0.052 0.040 -0.358 -0.607 -0.521 -0.251 -0.668 0.523 1.000
SEAM*	-0.024 -0.056 -0.296 -0.350 -0.404 -0.277 -0.557 0.344 0.653 1.000
Job stress	-0.103 -0.054 0.169 0.630 0.672 0.375 0.568 -0.543 -0.600 -0.460 1.000
Co-worker support	0.118 0.087 -0.078 -0.456 -0.315 -0.084 -0.349 0.300 0.501 0.368 -0.375 1.000
Supervisory support	0.042 -0.001 -0.269 -0.600 -0.547 -0.285 -0.660 0.631 0.716 0.560 -0.660 0.357 1.000
M W INT**	0.053 0.012 -0.292 -0.588 -0.515 -0.295 -0.627 0.543 0.799 0.673 -0.659 0.451 0.817 1.000
Job dissatisfaction	0.001 0.026 0.306 0.386 0.475 0.347 0.524 -0.436 -0.573 -0.625 0.617 -0.215 -0.653 -0.665 1.000
Physical hazards	0.094 0.107 0.238 0.301 0.329 0.211 0.424 -0.200 -0.497 -0.514 0.455 -0.251 -0.465 -0.511 0.482 1.000
Production pressure	0.113 0.124 0.169 0.119 0.240 0.413 0.424 -0.293 -0.362 -0.385 0.398 0.099 -0.542 -0.466 0.507 0.459 1.000
Safe work behaviour	0.055 0.010 -0.217 -0.368 -0.303 -0.362 -0.370 0.166 0.539 0.307 -0.258 0.361 0.396 0.387 -0.288 -0.172 -0.181 1.000
Mean	37.34 14.58 0.50 16.02 20.88 8.707 18.58 13.22 39.67 15.73 16.82 12.86 14.91 20.36 23.54 24.10 8.78 20.27
Standard deviation	9.01 9.25 0.50 4.12 5.95 2.55 5.41 3.27 7.53 3.93 3.99 2.18 3.68 5.44 6.16 4.36 2.82 2.99

* Safety Equipment availability and maintenance ** Management workers interaction
Correlation coefficient >0.113 indicates 0.05 probability level of significance

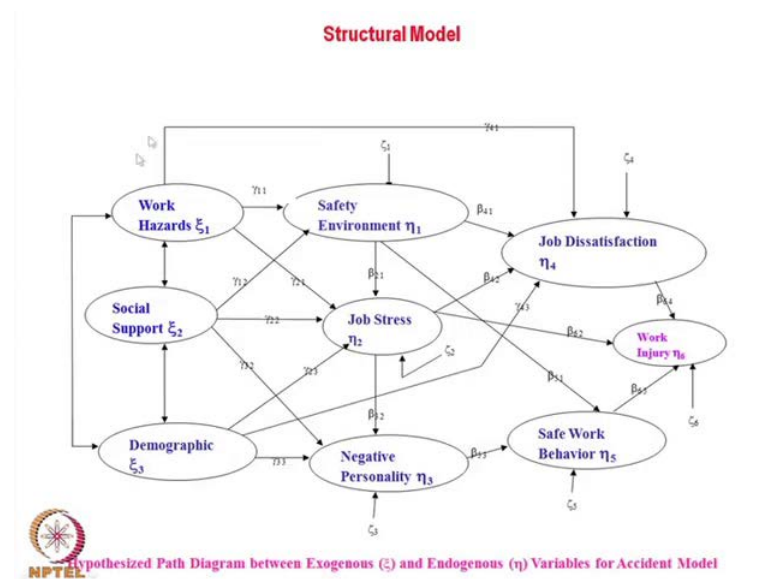
The output of the confirmatory model, this is the correlation matrix.

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This is what the output is.

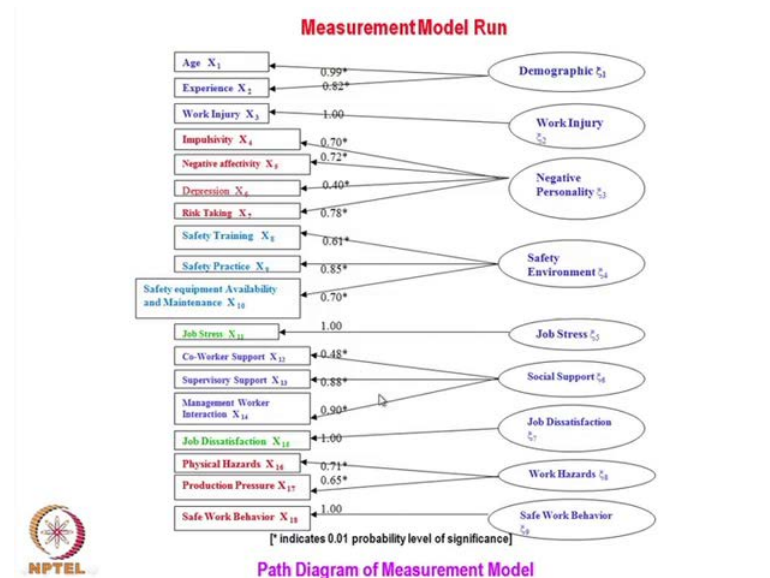
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Once we get this output, this is input to our structural model. This is the first phase where you have to identify the manifest variable and the latent constructs. Then you do confirmatory factor analysis. Find out the correlation or covariance structure of the latent variables because in SEM, your input is either correlation or covariance structure. So, for the measurement model, you give the covariance structure. They will give you the latent correlation or covariance structure.

Now, for structural model again, use the latent correlation or covariance structure, whatever you are getting from the latent variables and then you feed like this. So, you will be getting picture like this. Then definitely these are the some issues what we may be discussing later on if time permits in other classes. Ultimately, this is what is the observed correlation.

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This is the factor and measurement model.

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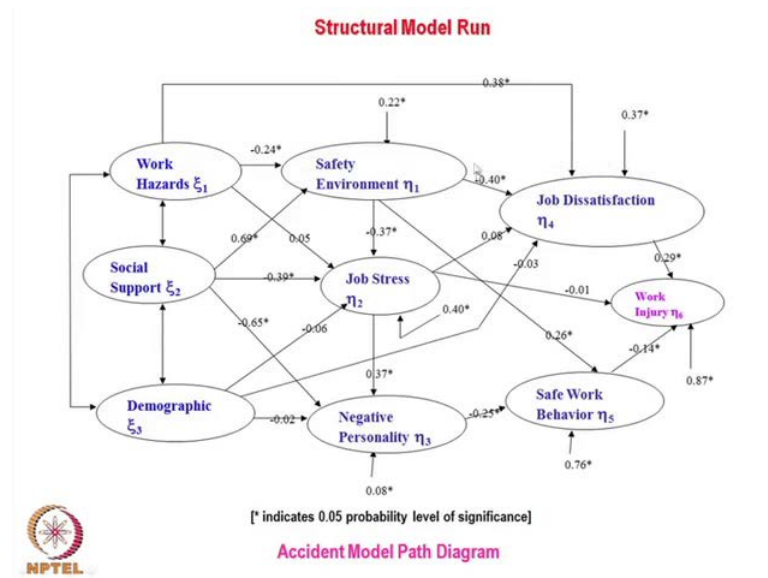
MeasurementModel Fit Indices

Goodness-of-Fit Indices for the Measurement Model of Causal Accident Model

Parameter	Values
Chi-square with 99 degree of freedom	257.24
Root Mean Square Residual (RMR)	0.06
Goodness of Fit Index (GFI)	0.98
Normed Fit Index (NFI)	0.97
Comparative Fit Index (CFI)	0.99
Incremental Fit Index (CFI)	0.99

This is the fit of the measurement model that I told you that I will be discussing fully in the next class. Then this is the correlation among the constructs.

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Then, this is your structural part and these are the structural regression coefficients.

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Structural Model Fit Indices

Goodness-of-Fit Indices for the Structural Model of Accident Path Model

Parameter	Values
Chi-square with 15 degree of freedom	212.23
Root Mean Square Residual (RMR)	0.06
Goodness of Fit Index (GFI)	0.87
Normed Fit Index (NFI)	0.88
Comparative Fit Index (CFI)	0.88
Incremental Fit Index (CFI)	0.88

Then, you see whether the structural model is fit or not. If your structural model is fit, then go for interpreting the things. Here ultimately, there are several structural that fit measures like chi-square with 15 degrees of freedom. This should be as small as possible. Root mean square is 0.06. This should be less than 0.05, less than or equal to

0.05. The goodness of fit index is 0.87, 0.9 or more is better means adequate non fit index, but this is almost 0.9. We can say a structural model is fitting the data. Similarly, we have also seen that the measurement model also fits the data.

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Contributions of constructs

Total Effect of the Significant Variables on Work Injury


Variables	Direct	Indirect	Total	Rank Order
Work hazards	----	0.15*	0.15*	3
Social support	----	-0.14*	-0.14*	4
Safety environment	----	-0.16*	-0.16*	2
Job dissatisfaction	0.29*	----	0.29*	1
Safe work behavior	-0.14*	----	-0.14*	4

* indicates 0.05 probability level of significance

Total Effect of the Significant Variables on Safe Work Behavior

Variables	Direct	Indirect	Total	Rank Order
Work hazards	----	-0.07*	-0.07*	5
Social support	----	0.40*	0.40*	1
Safety environment	0.26*	0.03	0.29*	2
Job stress	----	-0.09*	-0.09*	4
Negative personality	-0.25*	----	-0.25*	3

* indicates 0.05 probability level of significance



So, that means our model or my model fits very much to the data. Then using this structural equation modelling, you will be also able to find out the total effect that is direct effect plus indirect effect. Also, you will be able to, based on the effect; you will be able to rank the variables. Now, what do we see that we found that work hazard vis a vis work injury, work hazard has no direct effects in the model, we have failed, but it is indirect effect. Total effect is this, social support this, indirect affect this, safety environment indirect effect this and total effect this, job dissatisfaction direct effect and safety behaviour, this is the case.

So, ultimately based on this total of effect, we have given the rank one, two, three, four, and five, like this. Similarly, as there are many variables, many dependent variables or dependent constructs, you can find out here. See these are the exogenous, but all are endogenous. So, for all any of the endogenous level variable, you can find out that what are the direct and indirect effects. This will definitely help you in making better policy.


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Application of the developed Methodologies (Contd.)

Total Effect of the Significant Variables on Job Dissatisfaction

Variables	Direct	Indirect	Total	Rank Order
Work hazards	0.38*	0.11*	0.49*	1
Social support	----	-0.33*	-0.33*	3
Safety environment	-0.40*	-0.03	-0.43*	2

* indicates 0.05 probability level of significance



Total effect, so we calculated for this case, work behaviour, for work injury, for job dissatisfaction, what are the variables and what is the rank order. So, thank you very much. Next class, we will discuss confirmatory factor model with respect to structural equation modelling followed by my structural model, which is basically a path model.

Thank you very much.