Applied Multivariate Statistical Modeling
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Lecture - 35
Factor Analysis - Model Adequacy, Rotation, Factor Scores and Case Study

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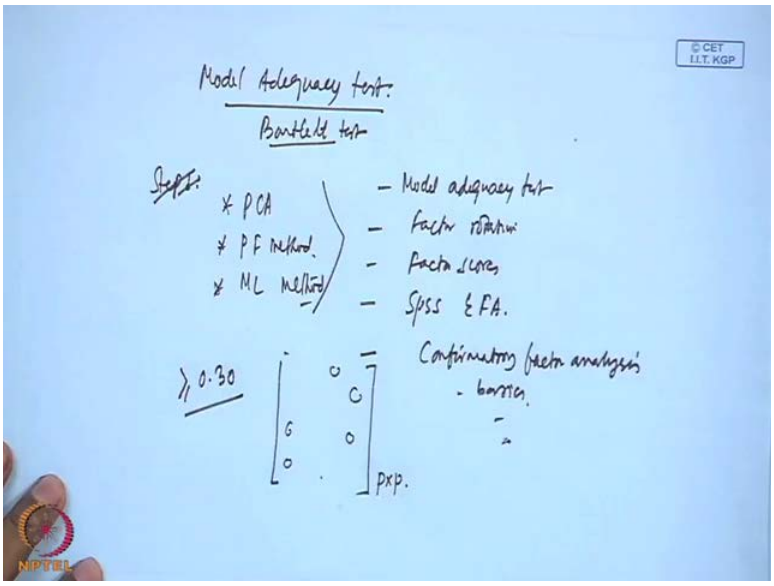

Good morning, we will continue factor analysis, that is the third lecture on factor analysis. In first lecture, we have described the conceptual model and we have given some examples. In second lecture, we discussed about the model estimations, that is from principle component analysis point of view from principle factor method point of view. Also, we have discussed about the maximum likelihood method point of view. This lecture will start with model adequacy test, model adequacy test then followed by your factor rotation, followed by factor scores. Then I will show you Spss exploratory factor analysis and if time permits I will go for confirmatory factor analysis, but only basics. So, under model adequacy test I have already discussed the Correlation matrix.

I said that if there are substantial correlation coefficients greater than equal to 0.30, then you go for factor analysis means suppose you have a large data matrix and correlation coefficient matrix is p cross p. There is large number of values, large number of values having more than 0.3. Second is Bartlett's sphericity test, Bartlett's sphericity test uses this formulation that you find out the correlation matrix.

First take the determinant of it and take logarithm of this, then you multiply with this, in this ultimately follows chi squared distributions with p into p minus 1 by 2 degrees of freedom. Now, if this quantity, if this quantity is more than the tabulated values then the hypothesis is that no factorization possible. That factorization not possible that is not true here, so we will reject null hypothesis in this case.

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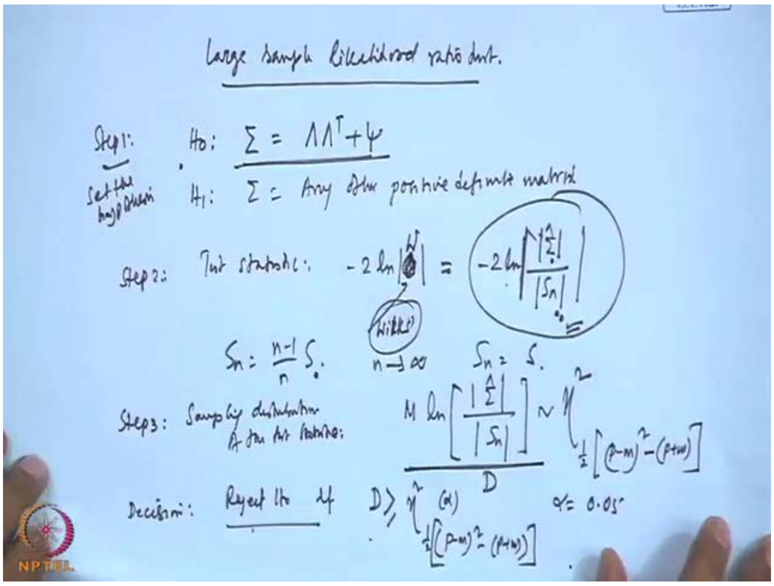

Then, another one is that large sample test which is likelihood ratio test large sample Likelihood ratio test. So, we will again give you the Bartlett procedure here, here the null hypothesis is that the co population covariance matrix is coming from the factor model [FL], we are able to reproduce from the factor model. This population covariance matrix, it is correct and H 1 is that sigma is any other positive definite matrix.

So, this is my step one that means set the hypothesis set the hypothesis, your step two you have to find out the statistics appropriate test statistics, what is your test statistics. As I told you that it is the locality likelihood ratio, so minus 2 log this capital lambda, this lambda is not this capital lambda. So, in this case this is basically, similar to this Wilks lambda what you have seen earlier that similar to Wilks lambda type of things. So, if I write like not lambda here if I write that this is our let it be w.

So, we are creating this and that one is minus 2 log deter log determinant of that sigma cap by determinant of S n where S n is n minus 1 by n into S that is the sample covariance matrix for n greater than m m very large tends to infinity and S n become S.

So, for large sample size you can use S determinant here determinant of S then once I know, so that means what is my test statistics here my test statistics is minus 2 log determinant of sigma by determinant of S n. Then you have to find out that what is the sampling distribution sampling distribution of the test statistics test statistics, now Bartlett says that that m log determinant of sigma by S n.

So, please keep in mind that we are writing the estimate value here sigma this one because this is what is our random variable random component here. So, this follows chi squared distribution with half of p minus m square minus p plus m, that degrees of freedom. So, first this hypothesis related to the population then sample statistics related to the estimates then your sampling distribution related to the estimate.

Now, you have to take the decision, decision will be reject H 0 if this quantity if i write this total quantity as d so greater than equal to chi square half p minus 1 square minus p plus m that alpha for maybe usually alpha will be 0. So, we will reject a 0 for this condition, what does it mean you are saying no it is not that the factor model is not able to reproduce the population covariance matrix. So, if you want to have your factor model acceptable then H 0 should be accepted. Now, this is from the hypothesis distinct point of view correlation matrix Bartlett test, all those things we have discussed.

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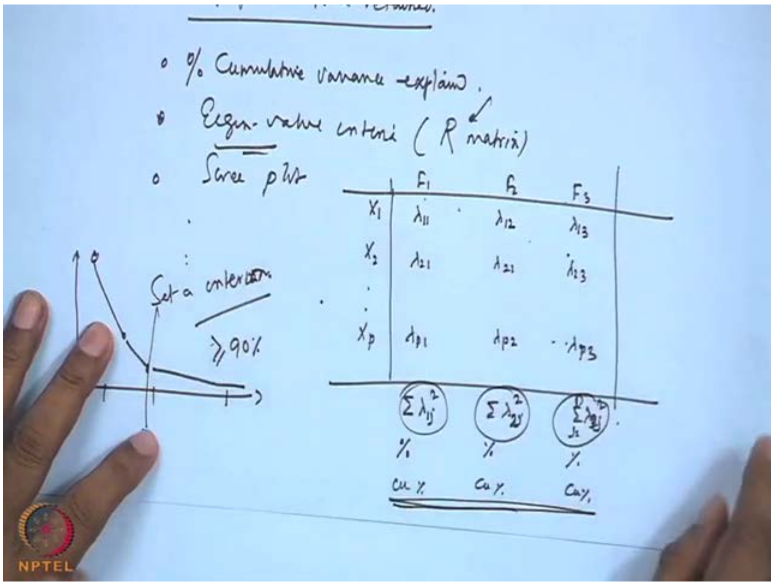

Next issue is that number of factors to be retained, how many factors you will get. So, it is similar to principal component analysis where we have seen that number of component to be kept by several means for example, percentage cumulative variance explained. Then we have also given you Eigen value criteria from much it may be Eigen value criteria, when you use R matrix then screed plot is there. Similar things, similar many things here explained there for example, let we have p variables we have extracted, let it be two factor three factors.

You know the loadings, if I say then you know the Eigen values here this is 2 j, 3 j, 3 j equal to 1 to p F square. So, you find out the percentage, what percentage it is, what percentage it is, what percentage it is, and then you find out the cumulative percentage. Set a criteria criterion, let you want the cumulative percentage, it should be greater than equal to 90 percent are these three factors you are considering able to explain this.

If yes, this factor model is good and you can keep three factors, now screen plot also you have seen earlier screen plot what is in that, this Eigen value will satisfy factor one factor two like this Eigen value. Suppose, your values are coming like this, so this is your elbow, so you keep three factors and Eigen value when you use R matrix. So, for those Eigen values which are more than 1that you consider at least 1, [FL] that one variable variability will be explained by this. So, this is similar to usual component analysis, how to keep, how many fact components you want to keep.

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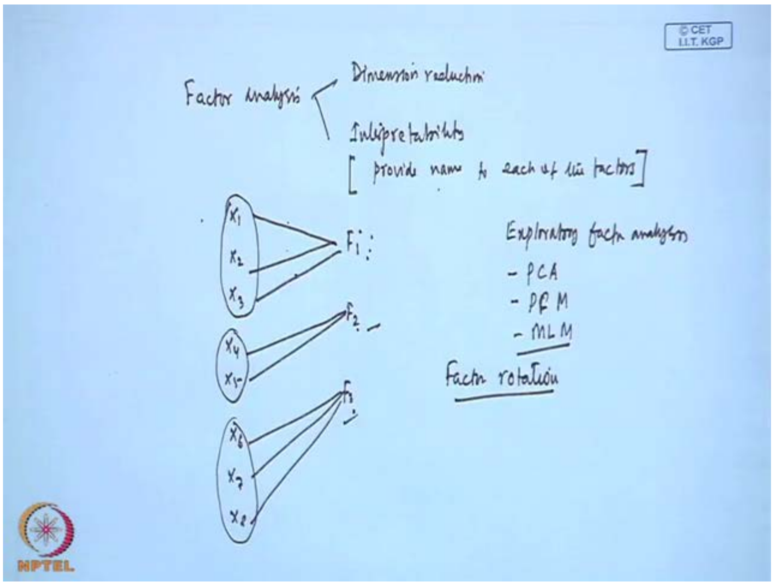

As we have discussed earlier factor analysis has two important purposes, two important purposes, what are those two, one is your definitely dimension reduction, second one is interpretability. By interpretability, we want to mean that we will be able to provide name to each of the factors correct.

So, how to provide name for example, consider that we have three factors here and there are several variables like this and when you have seen the loadings that use lambda values. You may find might have found out a situation similar to this where these loadings are high, where with factor one these loadings are high with factors two. These loadings are higher factor three, but other loadings like x 1, x 2, x 3 loadings on factor two as well as factor three are very small negligible. That means, we can ignore those loadings under such situation what we can say that x 1, x 2, x 3 is creating factor one x 4, x 5 is creating factor two x 6, x 7, x 8 is creating factor three.

Then, we can probably name these factors considering what are these variables what is the nature of these variables the name will be common to this three for F 1. This two for F 2, this three for F 3 can you get this structure immediately when you draw when you basically conduct exploratory factor analysis. The way we have discussed by any of the method maybe your principal component, analysis method, principal factor method, your maximum likelihood method any one of the method you have extracted, sorry estimated.

The factors parameters models are available are you getting this or not it may so happen that you will not get this structure under such situation it is desirable to rotate the factor in such a manner that that the loading of some of the variables. On a particular factor that F 1 that factor that will be maximum whereas, the loadings of other variables will be minimum. Similarly, if you want to find some other set of x excluding the ones we have consider for the F 1 that which will be highly loaded with F 2 and very low loading with other factors. So, this can be possible through factor rotation, so in order to understand this fully I will first show you one example.

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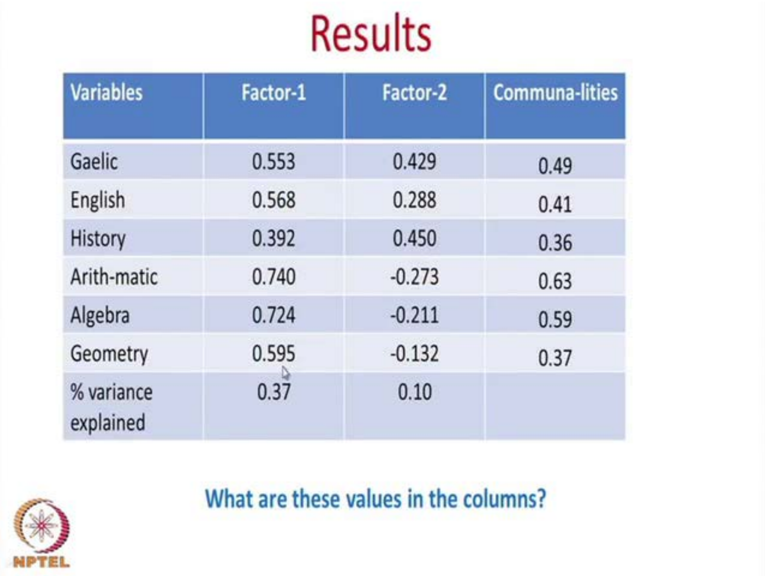

Variables	Factor-1	Factor-2	Communa-lities
Gaelic	0.553	0.429	0.49
English	0.568	0.288	0.41
History	0.392	0.450	0.36
Arith-matic	0.740	-0.273	0.63
Algebra	0.724	-0.211	0.59
Geometry	0.595	-0.132	0.37
% variance explained	0.37	0.10	

See, this is the results obtained by Maxwell and Lawley 1971 paper, they have conducted factor analysis of responses of two twenty students under six variables Gaelic, English History, arithmetic, algebra and geometry can the factor loadings were extracted two factors factor loadings and commonalities, and like this. Now, if you see this loadings Gaelic is equally loaded almost equally loaded with factor 1 and factor 2 because the loading value is 0.553 for factor 1 0.429 for factor 2. Similarly, English also definitely English has certain higher loading on factor1 little lower, but it is not negligible loading for History.

Again, equal pattern almost for arithmetic yes there is higher loading for factor 1 and definitely from compared to 0.74, 0.27 is import a less, but none of the loadings here. If you find that our geo in that sense none of the variables are not loaded with the two factors considered keeping in mind that this negative symbol here, given this is not an issue here. Basically, it may be the negatively loaded or positively loaded, but they are loaded. So, under this condition if we want to see the plot of this factor loading, what you will see?

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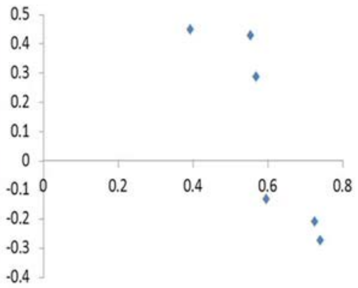
Results (Lawley and Maxwell (1971))

Un-rotated

Variables	Factor-1	Factor-2
Gaelic	0.553	0.429
English	0.568	0.288
History	0.392	0.450
Arith-matic	0.740	-0.273
Algebra	0.724	-0.211
Geometry	0.595	-0.132



What are these values in the columns?

We will see that this basically for different factors for example, factor 1 and factor 2, these are the two axes. Now, Gaelic 0.553 and 0.429, so I think 0.553 is somewhere this and 0.429 somewhere this one. This is the first one second English 0.568and 0.288, 0.568, 0.288this value, then third one is History 0.392 and 0.450 this value then arithmetic algebra geometry coming under this side. So, if we consider now these two axes that first one this one is F 1 and F 2 that 1, 0, 0.6.

Then, you find out that if for this three arithmetic algebra geometry almost highly loaded with factor 1 and lowly loaded with factor 2, but you cannot say that with the other that literature related things like Gaelic, English and History. They are almost at the here this side is that is not highly loaded with the second factor or highly loaded with the first factor. So, under such situation if I rotate the factor then again you get some better picture, so if you rotate then you will get this type of picture. Now, let us see again this one and I will draw that how it is possible.

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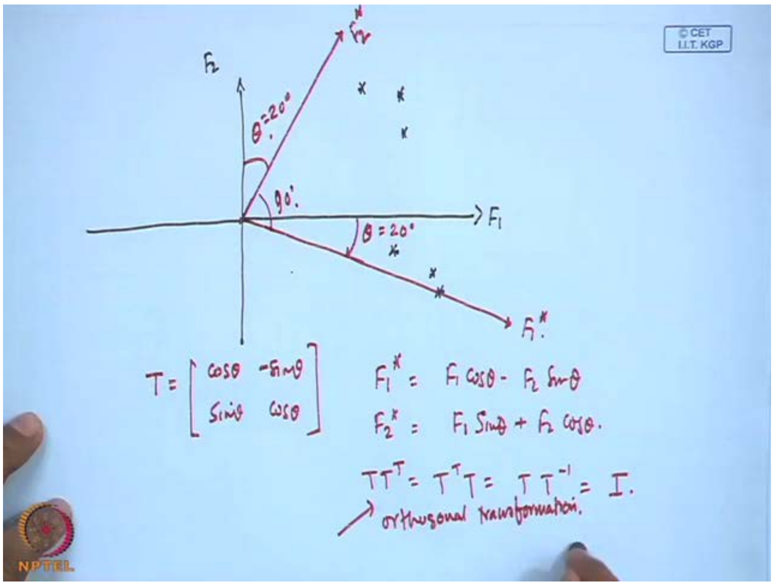

Our case is like this case is like this, this is my F 1 and this one is F 2. Now, let us do some manipulation, what we can do? We will go for result rotation, first let us rotate both the axis like this, this is my new F 1 then if I do perpendicular here, this is your new F 2. I am saying this is star x 1 star and x 2 star F 1 star and F 2 star, so this one is rotated clockwise, so this angle is theta, so this angle is theta. So, if I know theta then we will be able to find out F 1 and F 2 star from F 1 and F 2. That is possible for example, in this curve suppose this is equal to 20 degree.

Then, I think I have given you in principal component analysis one matrix like this T which I say cos theta minus sin theta and sin theta cos theta. That mean what they want to see here F 1 transposes your F 1 cos theta minus F 2 sin theta and your F 2 transpose that new dimension is F 1 sin theta let it be plus F 2 cos theta. Now, this plus minus symbol this one here or here it all depend on the theta value. We also have seen that this one, this T earlier that if you say that T T transpose equal to T transpose T equal to T T n bars equal to i then we say this is orthogonal transformation orthogonal transformation. We have done that because digit rotation keeping 90 degree, in fact we have done this one orthogonal transformation using this transformation.

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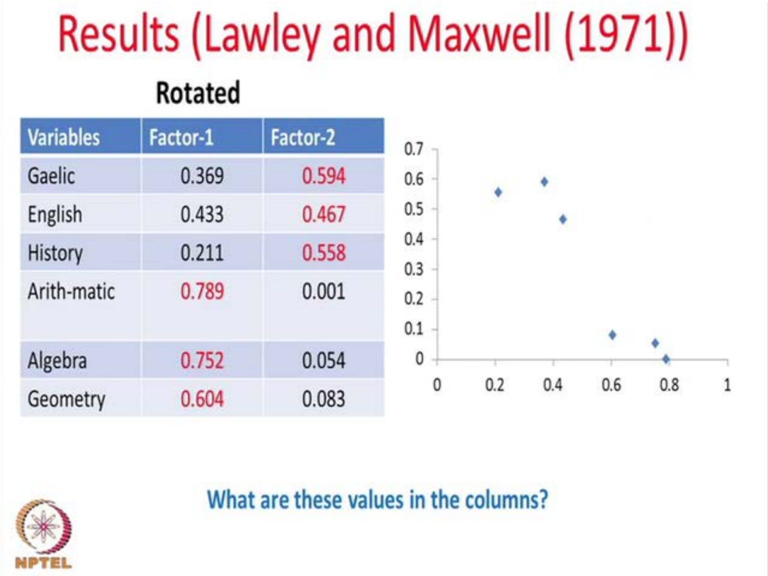

Variables	Factor-1	Factor-2
Gaelic	0.369	0.594
English	0.433	0.467
History	0.211	0.558
Arith-matic	0.789	0.001
Algebra	0.752	0.054
Geometry	0.604	0.083

You got this, you see now these loadings, now if I see the loadings from the arithmetic algebra geometry point of view, and this is the place. So, arithmetic algebra and geometry point of view, we see that an under factor 2 0.001, 0.054, 0.083 almost zero loading, but they are very highly loaded with factor 1 0.789, 0.752, 0.603 this factor 1 and related to factor 2. Now, at least what you are able to see that the first three variables, they are loaded they are higher loading, they have higher loading with factor 2 in comparison to factor 1.

So, this is better picture, so we can say now that these two factors are basically talking about two things one related to arithmetic algebra geometry which are basically mathematical ability. Other one Gaelic English History these are literature based, so this may be your verbal ability. So, Lawley and Maxwell this is that intelligence that have been arithmetic algebra and geometry, they are mathematical ability and verbal ability.

These two components are factors are hidden there and which are manifested in terms of the performance. In the mathematical subjects as well as the literature based subjects, but here one another issue is coming up that issue is, if I go for some other type of rotation which is not orthogonal, and then what you will find out.

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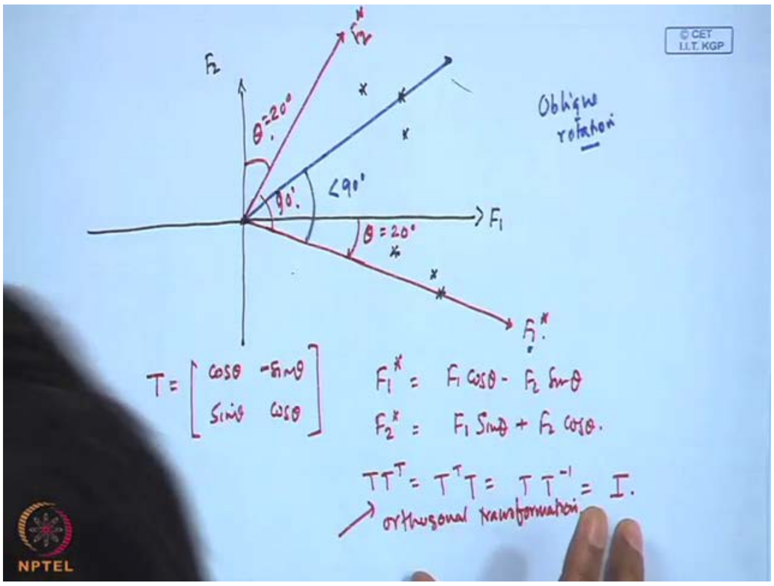

Suppose, if I do there the same this axis, Let it be like this, but the other axis if I rotate like this. I will make this angle less than 90 degree, and then this will be known as oblique rotation oblique rotation. I mean there will be correlation between the two; there will be correlation between the two getting me. Now, what we can do that we want to see under orthogonal transformation or otherwise I can say orthogonal rotations, are we losing anything related to explaining the variability of the original variables.

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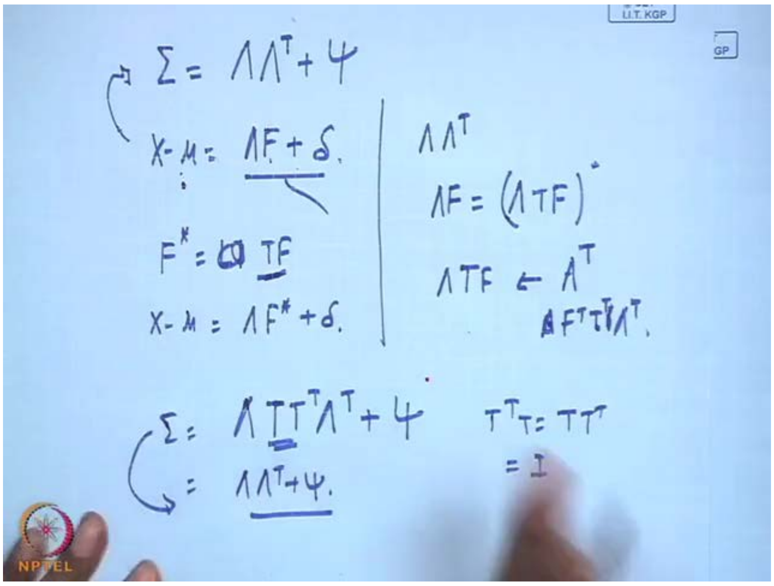

Let us see this one, what is our covariance structure of the original variable that is sigma and we proved that this is nothing but lambda lambda transpose plus sin. Now, if I create something like this, when we have created T his type of transformation rigid transformation that F 2, if F to F star that is F 1 to F 1 star F 2 to F 2 star. Then what is what actually happened, ultimately our model is x minus mu that we say lambda F plus this. This is our original model and from this model you got this structure, now instead of this if I say that F star is nothing but that is T and T F because what you are doing here you are writing F.

Here, with some other matter this T F is T F why T prop has a having this property getting me what I mean my model will change to x minus mu equal to lambda F star plus delta some other directions what will happen to this lambda lambda star, sorry lambda lambda transpose. So, you wish that it is the basically lambda will become lambda F is there this lambda F will become lambda T F, then what I can write, you can give some other symbol.

Then, lambda T f and this side it is coming lambda transpose that will be F transpose again T lamb or lambda if we keep out do not keep out just if I say transpose here F transpose T transpose lambda transpose, the resultant matrix. What will happen, this lambda lambda transpose that will be like this sigma will come lambda lambda transpose case, it will be lambda T T transpose lambda transpose plus sin resultant thing will come like this. Once you manipulate if this one, now what is T T transpose or T transpose T transpose T equal to T T transpose equal to i even the inners also i, so this is lambda lambda transpose plus this.

So, from covariance structure point of view there is no problem, you explain the same structure only thing is that this factor values are changing from F to T F some other dimension. Now, loadings are changing, so that means factor loading will not reduce the explanation power it will keep the same amount of information from the variance explanation point of view, but from the interpretability point of view it improves.

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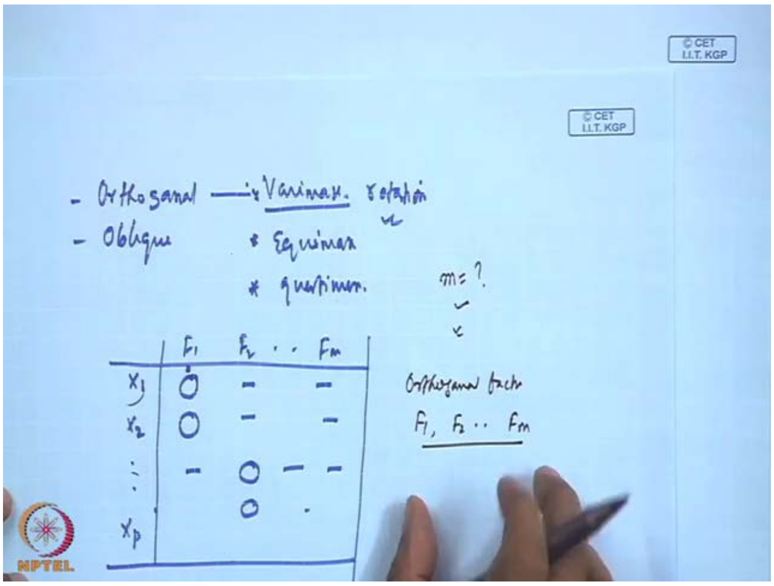

What are the different types of transformation possible? One is orthogonal, other one is oblique under orthogonal, the most commonly used is varimax, varimax rotation. There are other methods like equimax, quartimax, but varimax is more popular and varimax starts basically tries to suppose if row wise I am putting the x variables and column wise. I am keeping F 1, F 2 like even the factors then the purpose of varimax is that it will try that x 1 must be loaded with one of the factors. That means suppose if x 1 is loaded with this factor let it be least, but all other cases it is 0.

Similarly, x 2 together with F 1 be 0 and all other cases will be 0, but the some other can be loaded here all things will be 0, so like this. So, that mean it will allow some of the factor variables to be highly loaded with some of one factor and then rest with other factor. Then rest with other like this a clear structure is visible so that is the advantage and that is what we want also for varimax rotation there are some formulas available. This formulas you can m m go through any standard book, it is possible for finding out those things once you have completed all those things that means, all those things means what I mean to say you know your factor model. You know your number of factors, you have you have satisfied with that number, and you have seen that the adequacy of the model is properly tested.

They are adequate even Bartlett test is also satisfactory all those things, so factor models now can be used for some other purposes also what is the other purpose. Other purpose is suppose if I go for orthogonal factor model factor model, so whatever factor you are basically obtaining they are independent each other.

So, like principal component analysis these factors like principal components can be treated in subsequent regression analysis where this will be used as independent variables. In that case what is required you require knowing what will be the values of all those variables. So, if you want to know the values of the factors obtained these are known for every observations related to x these are known as factor scores.

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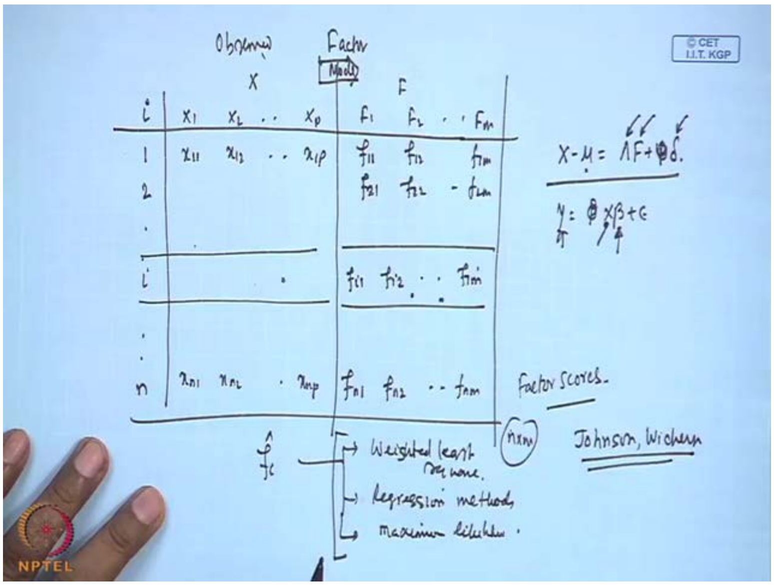

So, pictorially what we can do, we can write like this that you have collected n number of data points. You have x 1, x 2 to x p number of variable manifest variables for every one you collected the data and you have created factors with these data set F m, you are interested to know that for my i th observation what are the values of F 1, F 2 and F m. For example, if I say this f i 1, f i 2 like this f i m, this is I can say multivariated observation i th multivariated observation on all the factors, but this observations are not direct because you are observing directly this x and use you are using factor model.

There is a union this is basically created through the factor model factor model this is observed actually this is unobserved, but you are finding out these values using this. These are known, so for the first one then f 1 1, f 1 2, f 1 m for the second observation f 2 1, f 2 2, f 2 m like this for the last observation f n 1, f n 2, f n m. So, as a result you are having a data set n x cos m, these are known as factor scores.

Now, how do you get this factor scores, what are the methods available? If you see the factor model x minus f equal to x minus mu equal to lambda f plus delta what are the things unknown, unknown thing. This is also unknown, this is also unknown, and this is also unknown only you know this side. Again, this one may be x bar, so though it looks like a regression equation, but in regression equation, you know y equal to beta l x beta plus epsilon.

So, there you are interested to find this one, this one no x was given y was given, but it is not like this here, so this structure is complicated. Now, finding out this factor scores what I can say that f i cap, it is a difficult one, but there are certain methods like weighted least square method is there you can go by as it is regression like equation. So, regression, regression methods you can use and maximum likelihood methods are there also several methods are there.

If you go through good books like Johnson and Wichen book applied multiply statistical analysis, this three discussions are given and how to go for this factor scores. Then our as I said that what you have computed that means the adequacy part is over factor rotation and factor scores and users factor scores is like this. Then what I want, I want to just add little more about the confirmatory factor analysis here.

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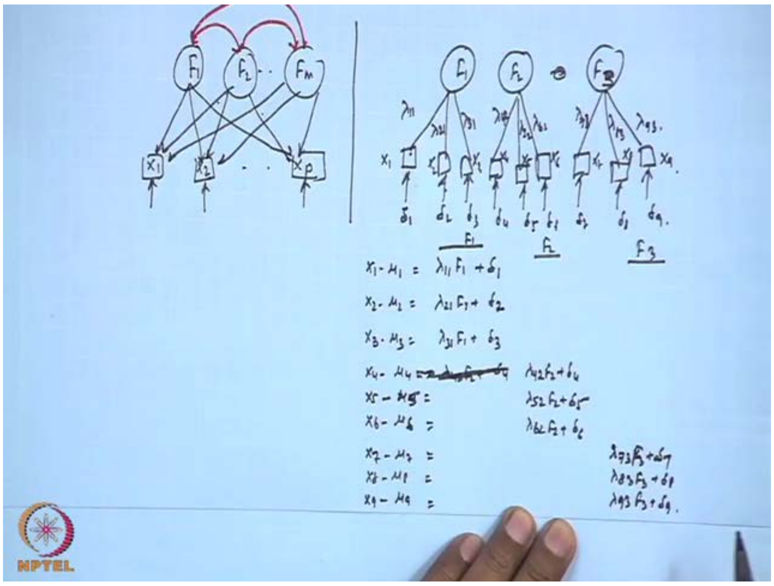

In explanatory factor analysis, you have m factors and there are three observe variables and you are linking each like this, and when it is orthogonal then there is nothing like nothing but when it is oblique there will be relationship between the factors. So, orthogonal and oblique in confirmatory factor analysis will be similar, but with a very important difference. This difference is I know, what the factors linked with which variable are getting.

So, since if I take only two factors with three variables suppose that three factors suppose with x 1, x 2, x 3, x 4, x 5, x 6, x 7, x 8, x 9 then this is my lambda 1 1 lambda 2 1, this is my lambda 3 1 then this one is lambda 4 2 lambda 5 2 lambda 6 2. This will be lambda 7 3, then lambda 8 3 lambda 9 3, this is delta 1, delta 2, delta 3, delta 4, delta 5, delta 6, delta 7, delta 8 and delta 9. Now, if you want to write in the equation, you can write like this x 1 minus mu 1 equal to what will be lambda 1 1 F 1 plus delta 1 x 2 minus mu 2 this will be lambda 2 1 F 1 plus delta 2 x 3 minus mu three lambda 3 1 F 1 plus delta 3.

Now, x 4 minus mu 4 will be lambda 4 2 F 2 plus delta 4. So, let me do little manipulation here this equal to this related to F 1, now F 2 let it be related to f 3. So, what is happening here then this x 4 minus mu 4 is lambda 4 2, F 2 plus delta 4. Similarly, x 5 minus mu 5 will be lambda 5 2, F 2 plus delta 5, F 6 minus mu 6, x 6 minus mu 6 equal to lambda 6 2, F 2 plus delta 6 then your x 7 minus mu 7 x 8 minus mu 8 and x 9 minus mu 9. If you write this all will come here that x 7 minus mu 7 is lambda 7 3 f 3 plus delta 7 then lambda 8 three f 3 plus delta 8 lambda 9 3, f 3 plus delta 9. This is we are talking about under confirmatory factor analysis.

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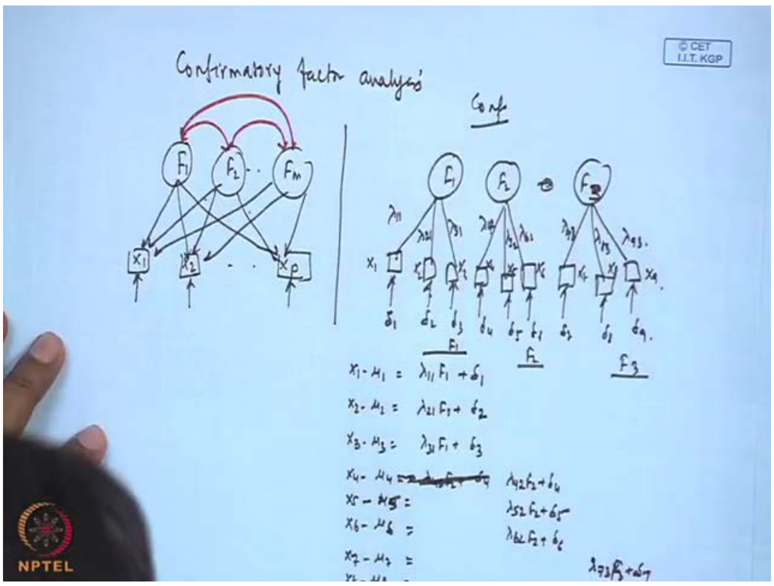

So, as a result if I see this in terms of matrix what you will get you will get x 1 minus mu 1 x 2 minus mu 2 to x 9 minus mu 9, this side the factor is very interesting.

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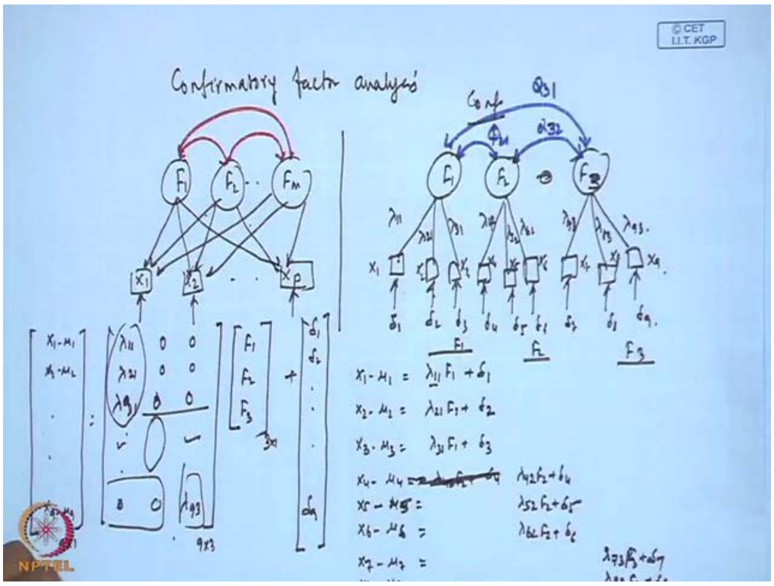

So, for see x 1 minus mu 1 for F 1 you are getting so that means this is lambda F 1, F 2, F 3 will be there. So, this one is your 9 cross 1, this will be 9 cross 3 and 3 cross 1 plus your delta will be delta 1 delta 2, so delta 9 this is the case, so what I want x 1 minus mu 1 equal lambda 1. So, lambda 1 F 1 is there F 1 is there here one what will happen 0 then lambda 2 1 0, 0. Similarly, 0 letting how many things are there three probably 3 lambda 3, 1, 0, 0. Similarly, I know last one also 0, 0 lambda what we are taking 9 3, you will be getting first three here three variable then 0 second, three variable you will be getting here 0, third this will be 0 here, you will get some values.

So, you are not getting all the random value because which is not required that is the difference and another important difference will be here that we want particularly for confirmatory factor analysis. We want this also the covariance structure will be there, so that means this will be my 5 2 1 2 1, this is 5 3 2; this is 5, 3, 1, 5, 2, 1, 5 the third one this structure also we want to keep this.

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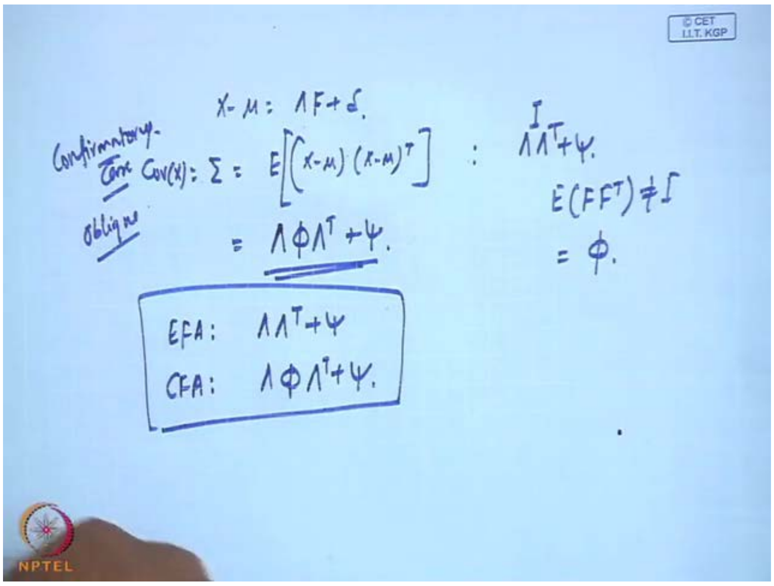

Now, then if I go by the factor analysis that x minus mu equal to lambda F plus delta then what will happen you found out. Finally, that sigma is the covariance of x which is ultimately expected value of x minus mu x minus mu transpose and then we ultimately what we found out for orthogonal case lambda lambda transpose plus sin. We found out because in between there was an i we have when you created there is an i, so that i is nothing but expected value of F F transpose.

Now, in confirmatory factor analysis not only we have done this type of that so many coefficient lambda loadings are 0 apart from that there is covariability among the F. So, this will not become i this is not i rather this will be your phi, so then what will happen, the covariance structure will be this phi this plus sin. So, confirmatory case why it is because we are going for oblique factor, so keep in mind that that there is definite purpose here.

Why we are going for oblique, because it is basically we are we want to major this factors actually there inter relationship this inter relationship is of importance is very important for followers. Otherwise, there is no difference supposing, if I go for exploratory factor analysis orthogonal then this plus sin and confirmatory factor analysis which are terminally, this phi this plus sin phi is the correlation matrix for the factors.

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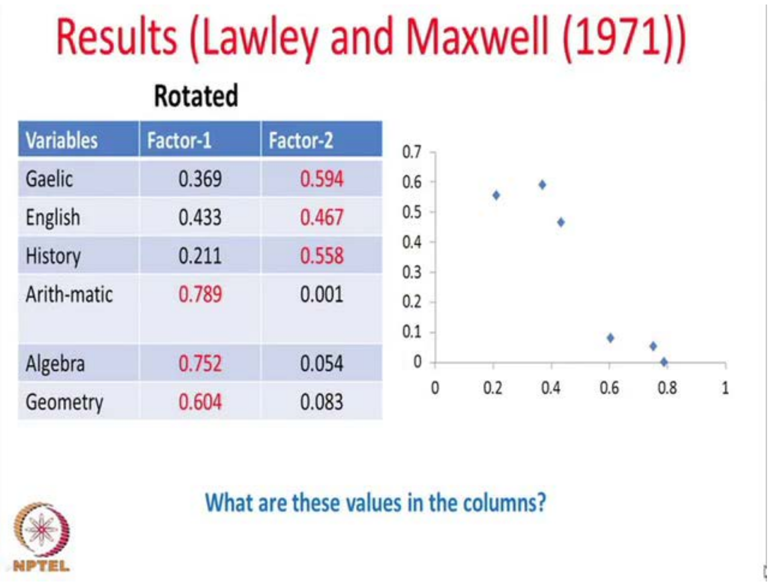

Now, with this I say that we can now prepare to see subtract whether see Spss last Class I have shown you Spss which is close also let us see Spss that how Spss can be used for factor analysis any question if you have any queries, we can discuss.

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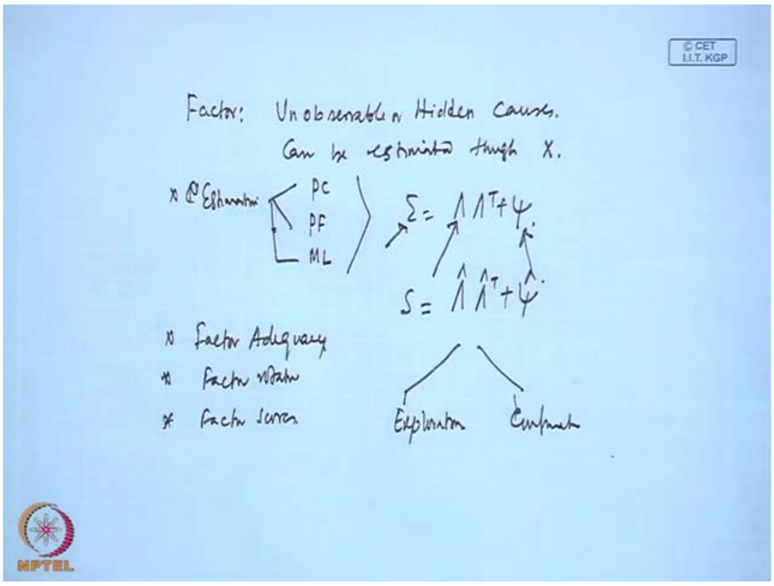

Please keep in mind few things for factor analysis, what is factor? Factor is unobservable or hidden we say this are the causes can be can be estimated through manifest variable x. There are estimation methods, principal component method, principal factor method, maximum likelihood method that covariance structure is this, where this is loading matrix. This is the pacific variance this is your original variance you may go for S you may go for S that also then you will estimate using one of the method there are several test. So, factor adequacy keep in mind keep, in mind your factor rotation factor scores then types of model exploratory confirmatory.

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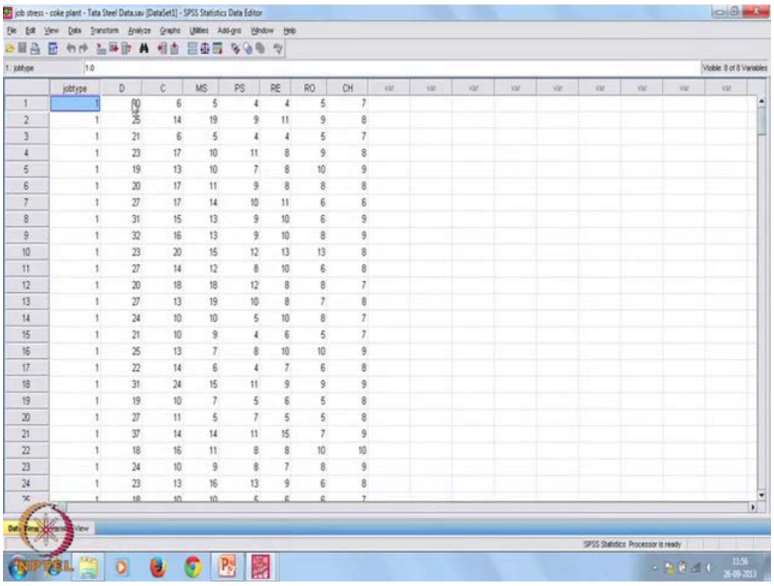

Let us see this one, I will take one data set state from here and then let this data set is like this, so there are suppose there are one two that seven variables I want to do factor analysis. So, we use Spss go to analyze then I think under dimension reduction, so go to analyze then go to dimension reduction then click factor and here it will be asking what are the variables you want to consider let you want to consider d to C H all variables.

Then there are many other thing like descriptive that variable, you may interested variable description initial solution. You may be interested to know coefficient standard determinant k m o and Bartlett test k m o test is there which is known as that Kaiser Meyer o stands for Olkin probably that Oklin Kaiser Meyer Oklin test. So, that is fine then extraction which method you want to use we have discussed principal component maximum likelihood principal axis factor this two three. We have discussed there are un weighted generalized many things are there. Let it be principal component then scree plot you may be interested in covariance matrix.

Now, what do you want to what will be your extraction criteria based on Eigen value Eigen value greater than 1 or fixed number of factors. So, let it be Eigen value greater than 1, so what will happen. Let us see then rotation you want to know I guess let there be varimax rotation then you go for scores, you want score yes you save as variables. If you want to know the vector score coefficient matrix click here and then there are several options how to deal the missing values, what is the display that is possible.

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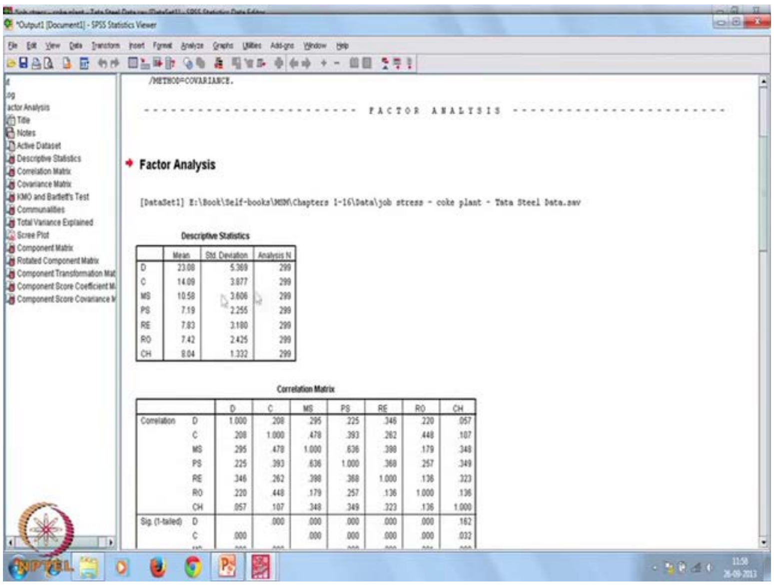

So, click now descriptive statistics mean standard deviation then correlation matrix is given here.

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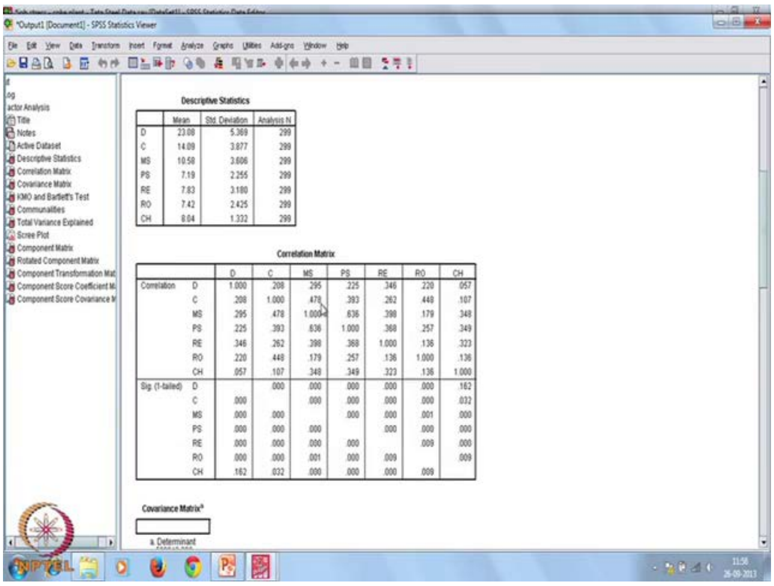

Now, see the correlation matrix first there are many vary many correlation coefficients which are greater than 0.3. Now, that Kaiser Meyer Oklin sample adequacy test based on that approximate chi square that degree Bartlett test of sphericity significance is 0.

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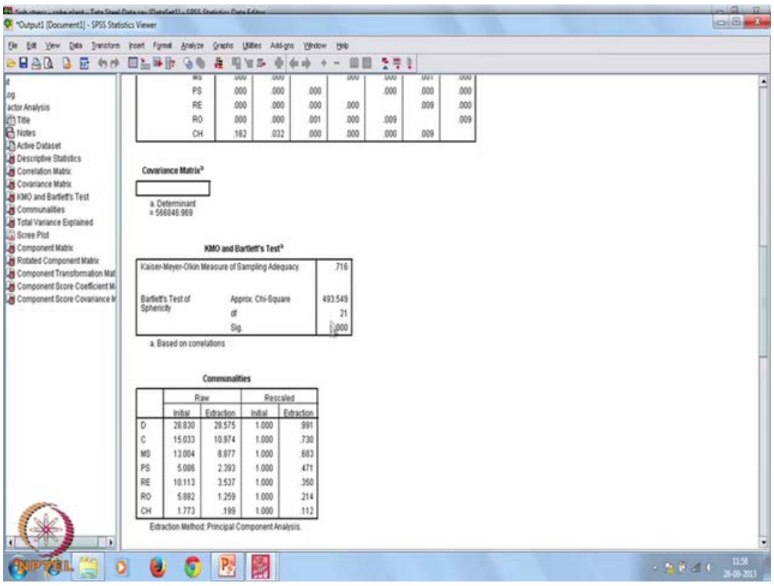

It simply Says that these are this data set is factorable and Adequation sampling. Adequation test value is not rejecting null hypothesis, that the data means it can be captured then the commonalities. I think at this point in time, you are very much in able to explain what commonalities are. So, extraction and initial raw, and rescaled means with respect to standardized variables.

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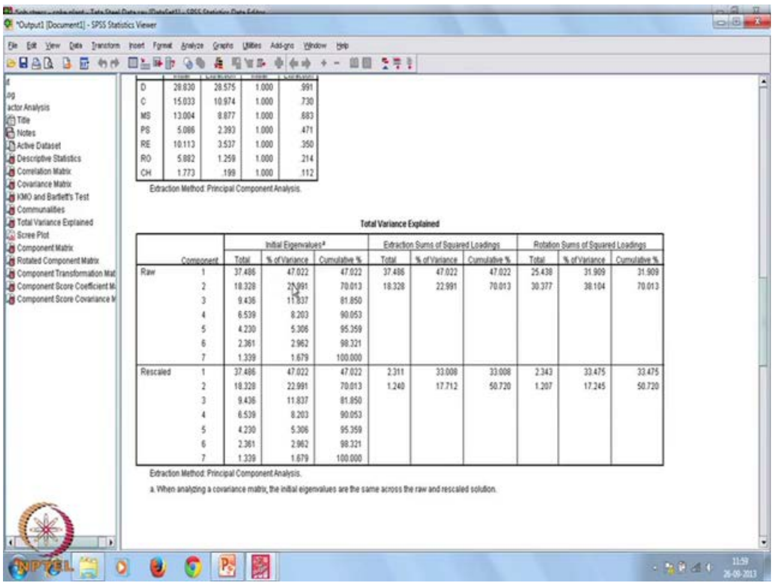

Now, total variance explained if you see here ultimately these are the can go up to maximum seven components, but first component percentage variability explained is 47 percent. Second component explaining 22 percent, that too in total they are able to explain that 70 percent of total variability. As we have seen, we have basically selected that we want the Eigen value criteria of one or more using correlation matrix.

So, that is why these two factors we are keeping here and please remember we are using basically the covariance matrix in the raw data, skilled data that is what the correlation matrix is. See, the values are like this, but here the values this values is 2.31 and 1.24. So, two factor model is possible because 70 percent of variability of job stress component if I can explain using two factor instead of three factors, it is not bad we can go for that.

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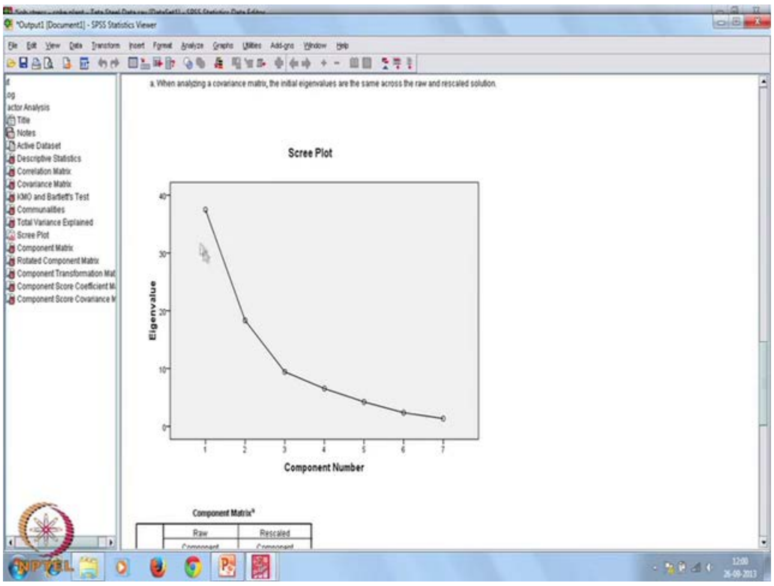

Now, scree plot will tell you that where you want to I think the three factors should be better because elbow is coming here. So, that means this s i not 1, but it may be more than 0.7 because the Juliet book I have found that it says that if we go for Eigen value of 1 or more criteria, then what will happen ultimately you will find out that many factors to be excluded. So, he suggested that go for 0.7, 0.7 is better, so go for point seven so that mean three factors can be extracted and that will give you little better structure. Now, this is the component matrix your raw component versus scale component that loading factors is giving here and your rotated component also given first one is this.

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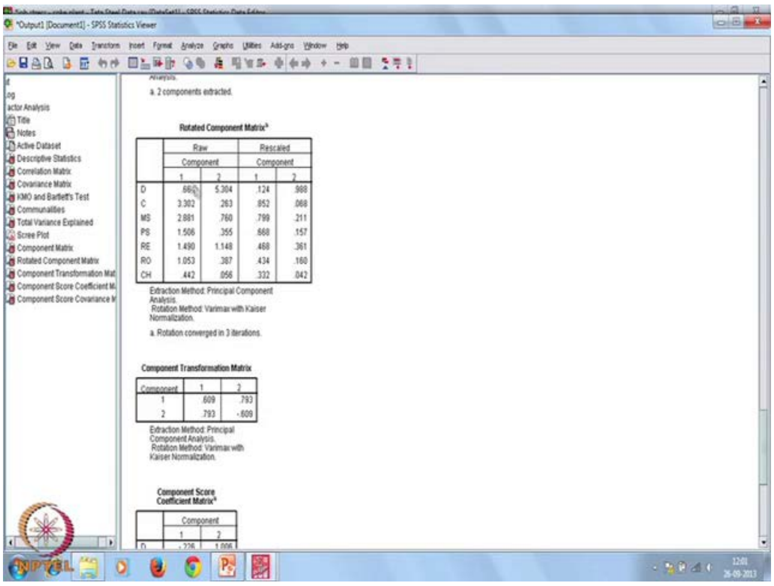

This one is your original case, this is your rotated case and component transformation matrix using rotated factor. So, under rotated case came Bartlett test, all those things again it is repeated and what is more important here. I think we have not taken this one that the plot factors that component plot, we have not considered here, but you can probably do now. I am going for three factors because we found that there factor the better one.

So, let us go to this extraction, I will write number of factors that is three then principal component, let it be then rotation let it be loading plots. Now, I am giving loading plots rest I am keeping as it is you come to this what is happening here your 81 percent of variability can be explained if you go for three factors. It is it is a better one and now after that I will show you the component lading plot.

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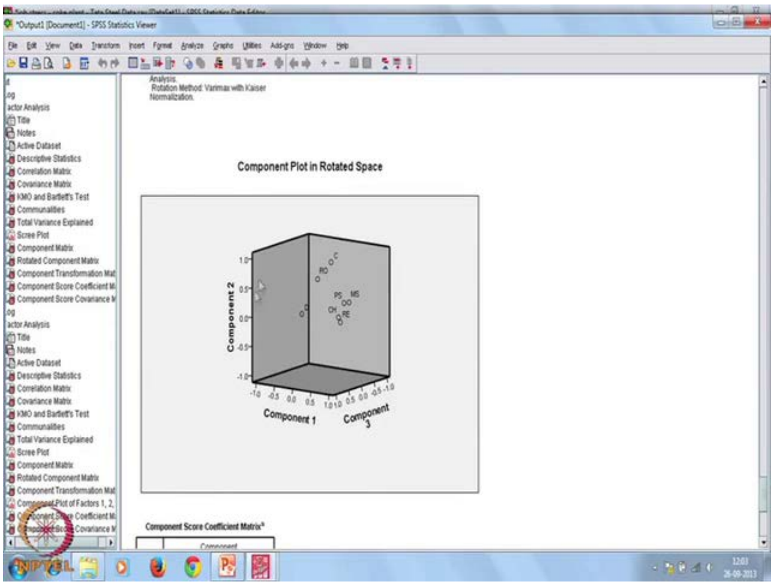

Three components at a time plots are given, but it will better if we go for two components at a time that will be much better visibility will be much better.

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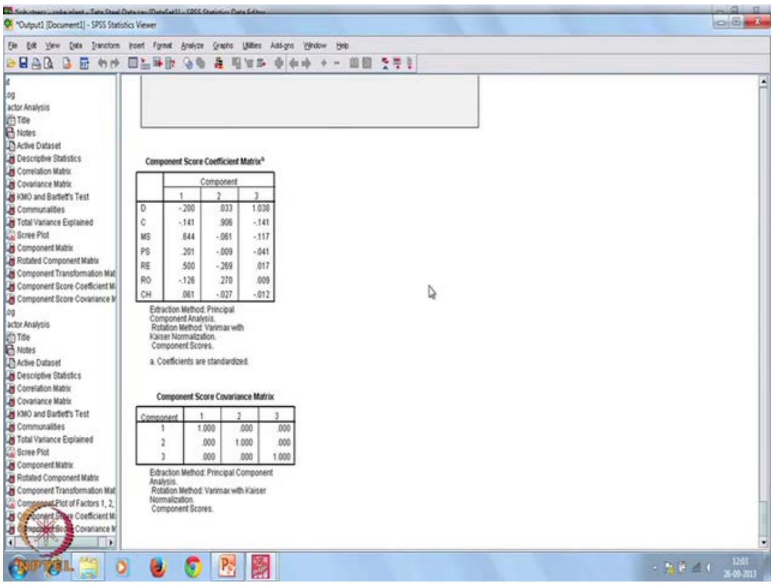

So, I think in Spss it is very easy to conduct factor analysis provided, you know the theory behind it. Otherwise, what you will get, you will not be able to interpret properly confirmatory factor analysis under there for confirmatory factor analysis. There are many other methods of estimating the model fit for what I can say that model adequacy, what is the model adequacy that you want to estimate. That is possible through different methods different index are developed, but that those things I will be discussed in structural equation modeling again and which are equally equal to confirmatory factor analysis.

Thank you very much.