Applied Multivariate Statistical Modeling

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Lecture - 28
Multivariate Linear Regression- Estimation

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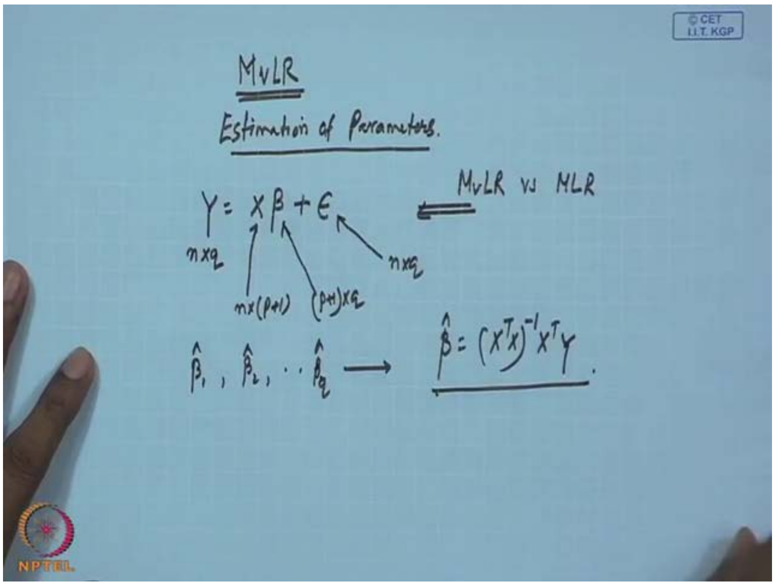

Good afternoon. Now, we will continue Multivariate Linear Regression; first we will discuss about estimation of parameters. So, last class what we have seen, we seen that Y is function of X in terms of regression coefficient beta, as well as error term. So, here the matrix looks like this, Y is n cross Q matrix, our X is n cross P plus 1 matrix and beta definitely should be P plus 1 cross Q matrix. Our error term is also n plus Q matrix. So, we also have discussed the relationship with MvLR, that is multivariate linear regression versus multiple linear regression.

We have also seen that if we go for multiple linear regression of individual Y variables and then estimate the parameters. The estimates like beta 1 cap, beta 2 cap beta your Q cap, they will be similar same, exactly same. If I estimate jointly using the multivariate linear regression formulation, the formulation remains same, beta cap is X transpose X inverse X transpose Y. So, now we will discuss that with an example that how we can estimate this beta cap.

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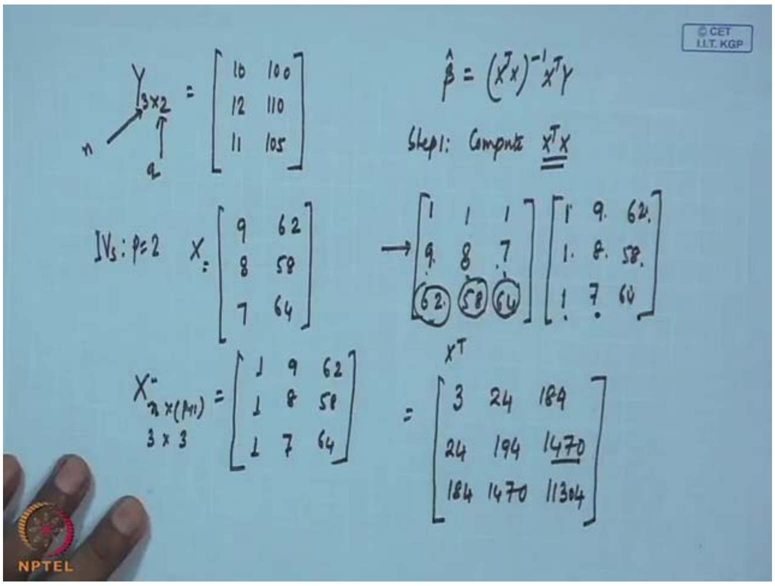

So, let us take a very simple small example, where Y is 3 cross 2, that means n is 3 observation Q is 2. So, Q is 2, so on two dependent variables the values are 10, 12, 11, 100, 110, 105. So, let us consider one, two independent variables also, two independent variables. So, I can say that P equal to 2, in that case the data matrix maybe like this, where 9, 8, 7, 62, 58 and 64. These are nothing but the first three observations for the city can example, we have given earlier. So, what we require to do know? We require to find out the design matrix X, which will be n cross P plus 1. Here n is 3 cross P plus 1, that means 3. So, this one is 1, 1, 1 that is for the constant term, then 9, 8, 7, 62, 58 and 64

So, what is our beta, estimate beta? Estimate is beta cap equal to X transpose X inverse X transpose Y. Our step one compute X transpose X, so if we write like this, then X transpose X is 1, 1, 1. That is basically the transpose of this matrix 9, 8, 7 then 62 58 and 64 this is your X transpose. Now, what is our X matrix? X matrix is 1, 1, 1, 9, 8, 7, 62, 58 and 64. If you multiply this what you will get what will be your multiplication 1 into 1 plus 1 into 1 plus 1 into 1. So, this will be 3 similarly 1, 1, 1, into 9, 8, 7. So, 9 plus 8 plus 7 this will be 24. Similarly, third one is 62 plus 58 plus 64, this will be 184.

Now, for you consider the next row 9 into 1 plus 8 into one plus 7 into 1, which is nothing but 24, then 9 into 9 plus 8 into 8 plus 7 into 7 the square term. So, it will become 194. Then 9 into 62 into 58 plus 7 into 64, this will become 1470 then it will be a symmetric matrix. So, symmetric and square, so this 184 will come here 1470 will come here and the remaining term will be 62 square plus 58 square plus 64 square. So, this total quantity will be 11304.

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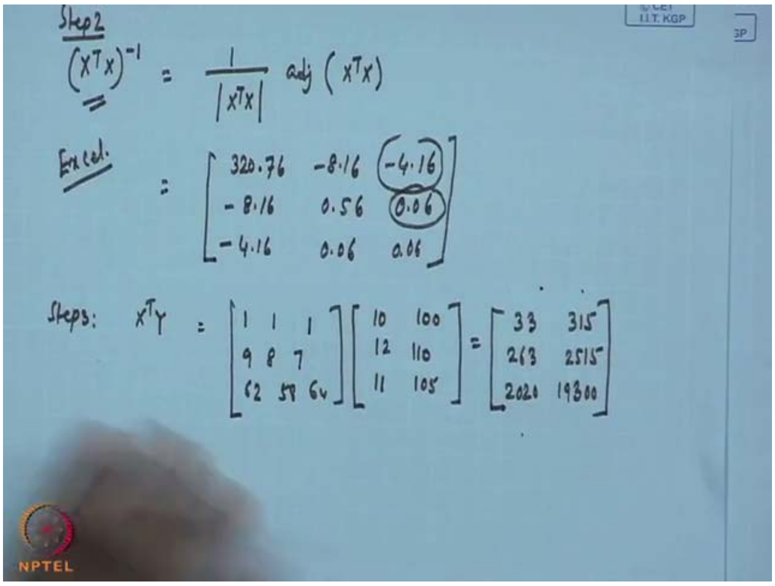

So, this is what is our X transpose X, then you have to compute X transpose X inverse. This is our step two, step two says X transpose X inverse, this will be one by determinant of X transpose X adjoint of definitely, X transpose X. So, ultimately the resultant quantity you will find out it will be if you use any suppose you use excel I have used excel. Then the value what I got this value is for this data set that is 327 minus 8.16 minus 4.16. As symmetric matrix this will be 1 minus 8.16. Then this one is 0.56, this one is 0.06, so this value will repeat here. So, minus 4.16 this value will repeat here, 0.06 then the last term is your 0.06 also, this is what is inverse X transpose X inverse.

Your step three you compute X transpose Y. So, if you write down X transpose Y 1, 1, 1 then 9, 8, 7, 62, 58, 64, Y values are 10, 12, 11, 100, 110, 105. Again, if you go for matrix multiplication of this two, if you use excel what you will be getting? You will be getting, X transpose Y equal to 33, 315, 263, 2515, 2020, 19300. So, with respect to this problem that means, what we are discussing? Now, we are discussing that we are given this problem that Y is 3 cross 2 matrix X is again 3 cross 2 matrix from the data point of view.

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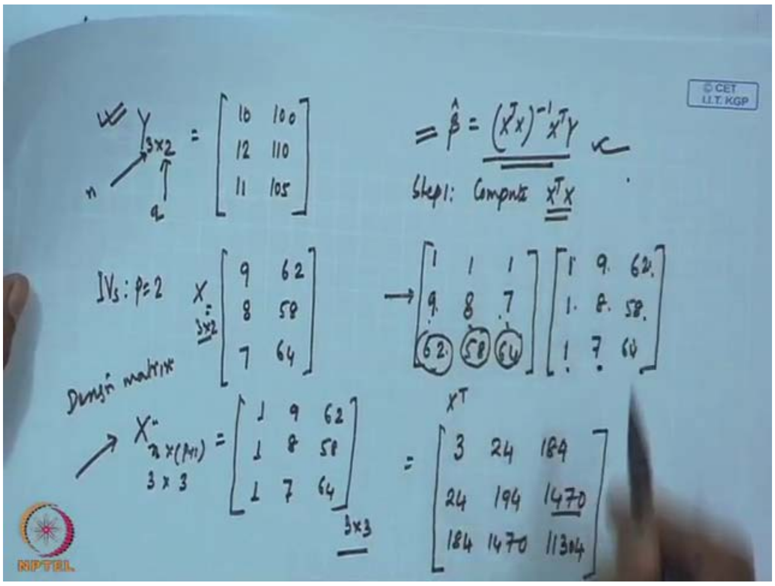

From design matrix point of view, it is 3 cross 3 matrix. We are using this beta cap equal to X transpose X inverse X transpose Y, that is what is the estimation formula for beta. So, our step one comprises X transpose X and we got this value. Then step two you want the inverse of this X transpose X, this value is like this. Your step three is you have to find out X transpose Y.

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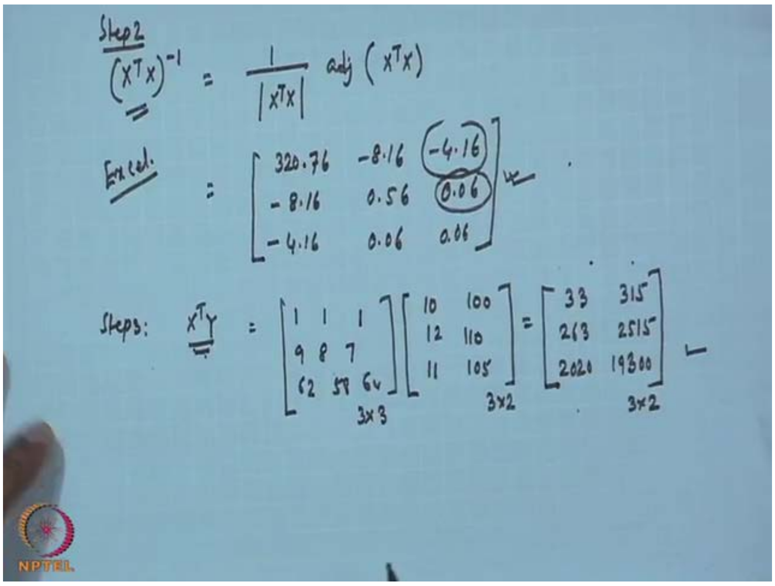

Because, you see that X transpose is X transpose is 3 cross 3 and Y is 3 cross 2. So, you will be getting a resultant matrix of 3 cross 2, this is X transpose y. Then your step four is what is required? You have to multiply that two to your step four that X transpose X inverse X transpose Y, this 2 matrix you multiply.

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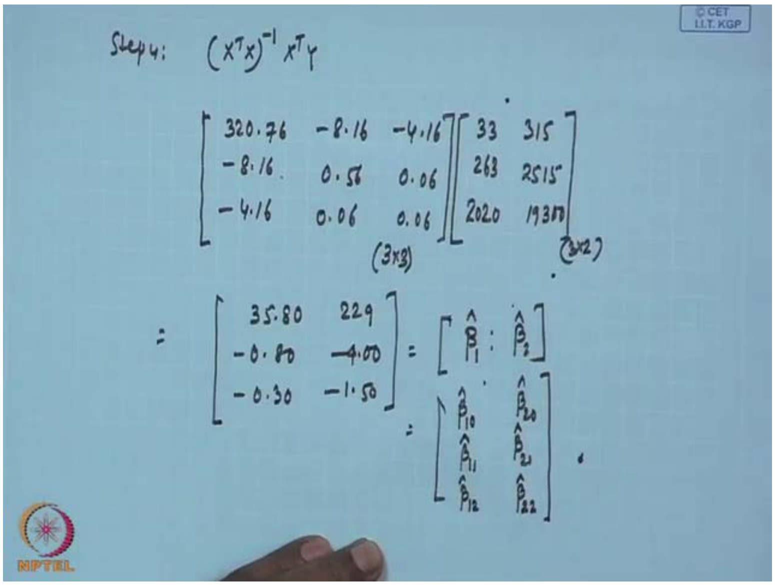

Then, what is our X transpose X inverse? X transpose X inverse is we have found out here. So, your X transpose X inverse is you write down that is 320.76 minus 8.16 minus 4.16 minus 8.16, 0.56, 0.06, then minus 4.16, 0.06, and 0.06. Then what is our X transpose Y? So, our X transpose Y is 33, 263, 2020, 315, 2515, 19,300 this is the resultant matrix. This 3 cross 3, 3 cross 2 we will be getting a 3 cross 2 matrix. That value if you again use a software or you go by mat lab or excel, what you will get?

You will get this multiplication of this 2 will give you 35.80 minus 0.80 minus 0.30. Then here it is 229, then minus 4.0, then minus 1.5. This is what is your beta matrix, this is what is your beta 1. If I say that beta 1 cap and beta 2 cap, which is nothing but your beta 1 cap 0 beta 1 1. So, beta 1 2 then beta 2 0 beta 2 0 2 is affected by 2 1 and beta 2 is affected by 2. So, what you have got? Then you have got two equation, if I go by the logic that Y is X beta plus epsilon where Y is here. In this case Y 1, Y 2 and your here X is here X 1 and X 2 and beta is here, that beta 1 0, beta 1 1 cap, beta 1 2 cap, then beta 2 0, beta 2 1 cap and beta 2 2 cap.

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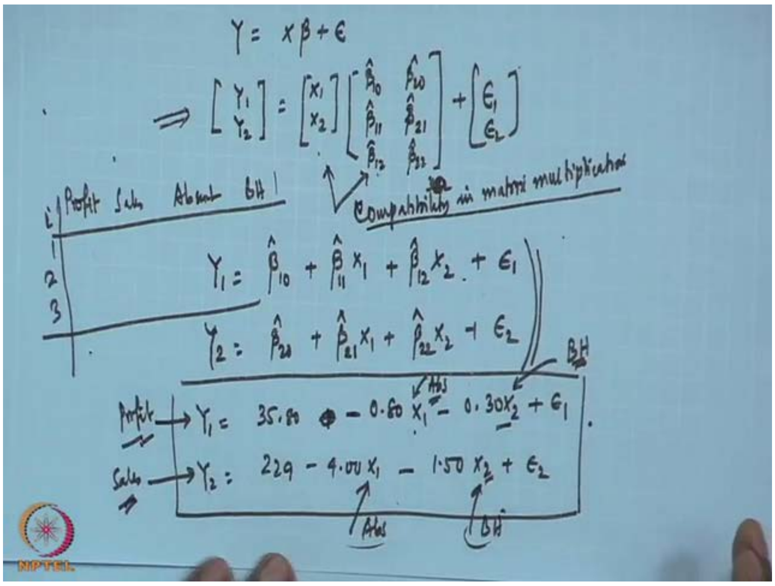

So, if I write like this, this is basically 2 cross 1, this one is 3 cross 2. So, in this manner it will not work, what is required plus epsilon that will be there. So, you have to think from matrix, that compatibility in multiplication matrix, multiplication. So, what we will do then? we will not write like this straight cut, if I write Y 1 equal to beta 1 0 plus beta 1 0 cap beta 1 1 X 1 then beta 1 2 X 2 plus epsilon 1. This one epsilon 1 and epsilon 2 then your Y 2 is beta 2 0 cap beta 2 is affected by 1, X 1, beta 2 is affected by 2 X 2 plus epsilon 2. So, that means if you write like this beta this row wise and then this side X 1 and X 2, so this into this plus this into this, so that way it will get.

So, here I told you that matrix compatibility is required, this is the resultant equation. So, in our case you can write that Y 1 equal to 35.80, then minus 0.80, X 1 minus 0.30, X 2 plus epsilon 1. Your Y 2 is 229 minus 4.00 X 1 minus 1.50 X 2 plus epsilon 2. This is what is the example data? Actually, what I have done? I have taken these profit sales, then your absenteeism and breakdown hours from the city can data. We have taken only first three observation 1 2 3 just to go for computation, because you will be applying software. So, you require to use more data points not three data points. There will be lot of problem with small data set, I will show you.

So, if this is the case then this Y 1 is nothing but profit is nothing but your absenteeism X 2 is nothing but your breakdown hours, Y 2 is sales. Then this one is again absenteeism and this one is breakdown hours. So, in multiple regression although this Y side Y 1, Y 2 their profit and different variable, but you see X side they will be same variable for both the equations.

So, if absenteeism increases profit will go down, if breakdown hour increases profit will go down, if absenteeism increases sales also will go down, if breakdown hour increases this also go down. So, that mean the relationship will be negative and that is coming from this example also, but as we have taken only a small amount of data set it may, so happen. That it will give different results means conceptually just opposite, so but do not worry, this type of situation may occur. Because, of your problem in collecting data primarily, the starting design problem that you have to take care.

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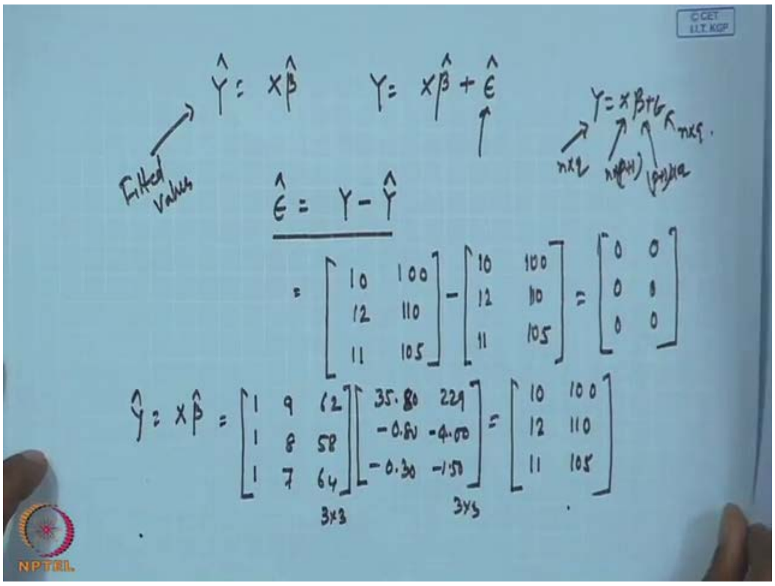

Now, once you have the equation that Y cap equal to X beta cap, this is what is your fitted values. So, your original observations are Y equal to X beta, if we write like this plus epsilon cap, because from parameter estimation point of view you have to estimate this epsilon also. Then from this 2 we can write epsilon cap is equal to Y minus Y cap Y minus Y cap, that is also you require to find out. So, if this is the case, what you require to do? So, your Y variable is Y 1, Y 2, so 10, 12 and 11 and Y 2 is 100, 110 and 105. So, what you require to do? Now, you require to find out what will be the Y cap values.

Now, Y cap values is Y cap is X beta cap, now X is 1, 1, 1, then if I go to the data set original data set that is 9, 8, 7 then 62, 58 and 64. So, this one is 3 cross 3, then what is your beta? Beta cap we found out I think that 35.8, 35.80. Then your minus 0.80 minus

0.30, then 229, 4.00 minus this is minus, 1.5. So, this is your X beta, so when you have written this X Y equal to X beta plus epsilon, please keep in mind we are writing in terms of data matrix. Now, if here it is n cross Q then it will be n cross P plus 1. Then this 1 is P plus 1 cross your Q and this will be n cross Q. That is why in the earlier format what we have written, that not in terms of equation variable on equation, but from the observation point of view you have to write.

So, this into this is also 3 cross 3. So, ultimately if you write down this one, what you will find out? Surprisingly, that 10, this will be 12, this will be 11 this will be 100, 110, 105. So, what you are getting? Exactly same value of Y. So, if I write here then if I write here, then 10 then 12, 11, 100, 110, 105 what you are getting? 0,0,0,0,0,0. Why this is happening? This is happening, because if you recall the multiple regression case, there what we have seen?

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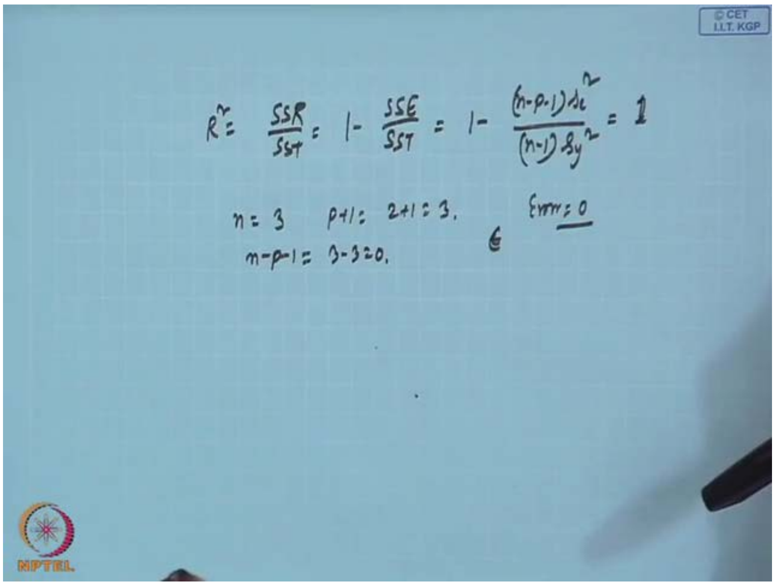

There we have seen that when R square equal to S S R by S S T. Then we have written 1 minus S S E by S S T. Then we have written 1 minus your n minus P minus 1 into s e square by n minus 1 into s y square, you can remember this. Now, here in this case in this case if I think from the that parameter vectors point of view for everyone. Suppose, if you go separately for Y 1 multiple regression for Y 2 multiple regression, your estimate value remains same.

As I told you that finally, the error term that will be the difference here? What is happening? What is the n? n is 3. We have taken three observations what is P plus 1, 3, 2 plus 1, 3. So, then if i put here n minus P minus 1, that is 3 minus 3, this is becoming 0. So, your R square becoming 1, R square will become 1. When epsilon that will be 0. the error will be 0, completely 0. So, we should not be happy with under such condition, this is an indication that, model is not at all fit model.

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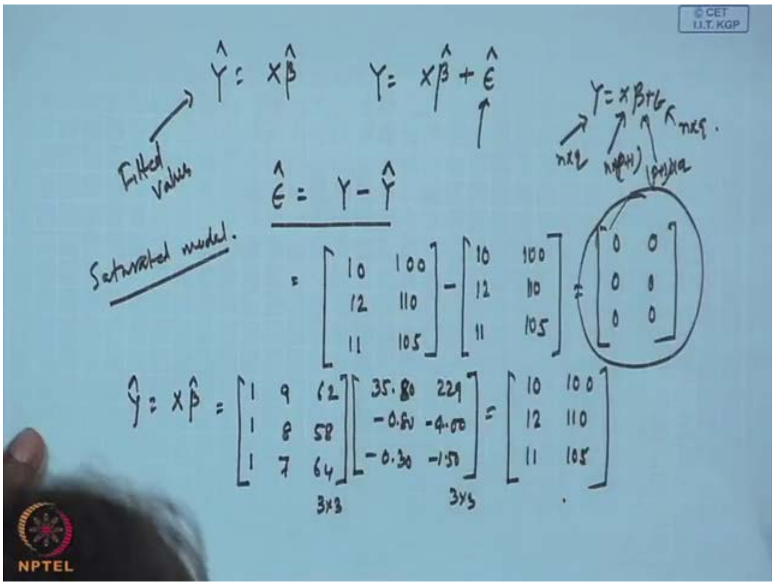

Although, we say R square 1, this is a saturated model. What is saturated model? The number of observation is equal to the number of parameters to be estimated model case, always you will get the perfect. Estimate perfect fit that for regression purpose this is not desirable. So, once you have developed, this that your equation is ready. Then what is the next? Next is you have to have sampling distribution of beta sampling distribution of beta cap getting me.

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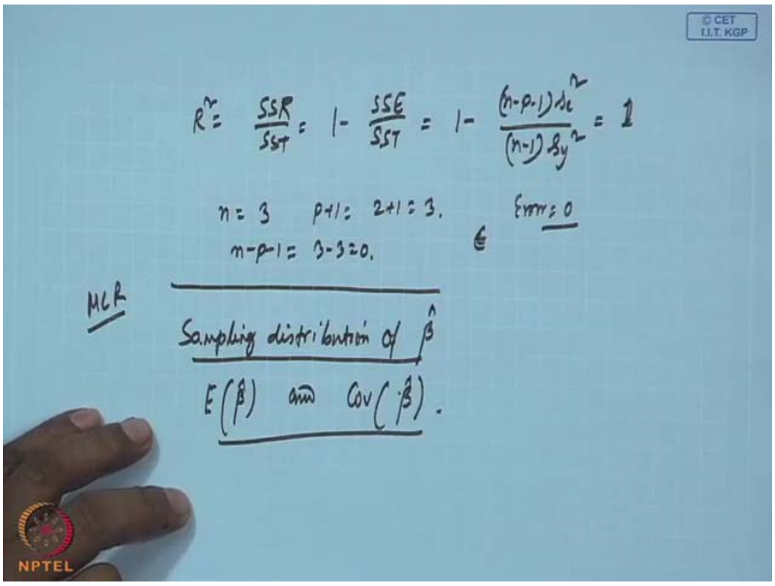

So, similarly, in M L R also we have found out the sampling distribution of beta cap. So, here what is required to be known? If you want to go for that sampling distribution of beta cap, then you must know what is the expected value of beta cap. You also require to know, what is the covariance value of beta cap? Thi s 2 we require to estimate. So, now let us see what is the expected value of beta cap expected value of beta cap? We can write that expected value of X transpose X inverse X transpose Y, because beta cap formula is this.

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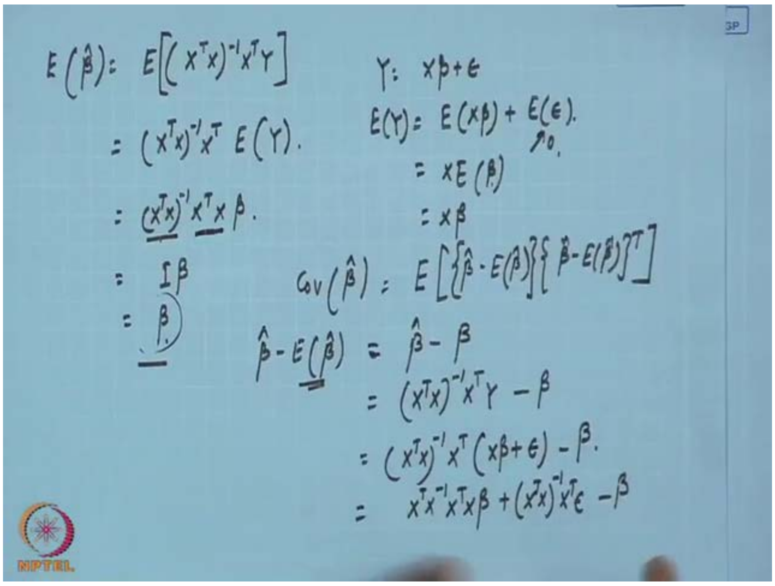

So, X is fixed here, so we can take X transpose X inverse X transpose out of the expectation, it will be expected value of Y. Now, all of you know what is the expected value of Y, see expected value of Y, suppose Y is X beta plus epsilon. So, if I say that what is the expected value of Y? Then expected value of X beta plus expected value of epsilon, this is zero. So, this one is X expected value of beta, beta is constant. So, it is X beta, so I will put here X transpose X inverse X transpose X beta. Again, you see X transpose X inverse is X transpose X is symmetric square matrix. This transpose into this, this will be an identity matrix. So, I beta we can write this is beta.

What you require now? You require covariance of beta covariance of beta cap. So, this can be written like this expected value of beta cap minus expected value of beta cap, can we write like this? This into beta cap minus expected value of beta cap this transpose, we have seen earlier this one. Now, what is beta cap minus expected value of beta cap expected value of beta cap is beta? So, this can be written like this beta cap minus beta. So, beta cap is X transpose X inverse X transpose Y minus beta.

So, then you put here X transpose X inverse X transpose Y is X beta plus epsilon minus beta. The resultant quantity will become X transpose X inverse X transpose X beta plus X transpose X inverse X transpose epsilon minus beta and this quantity is I. So, this is beta, so beta minus beta cancel out that one.

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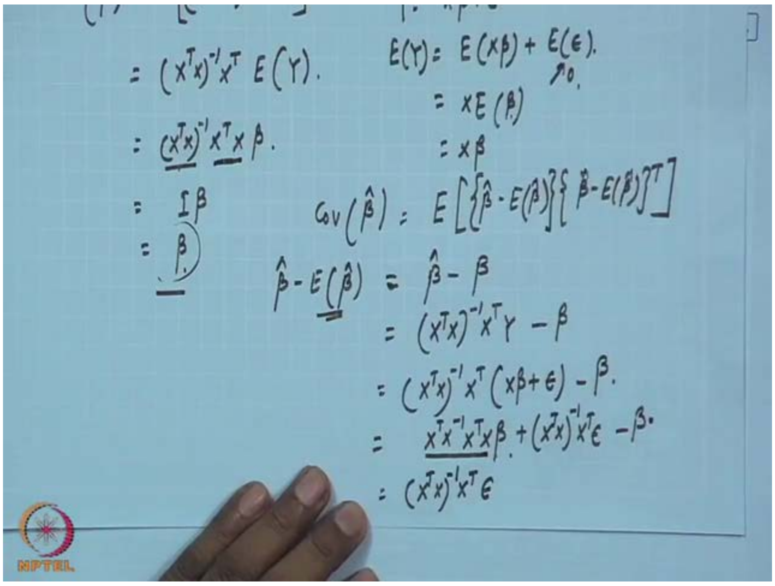

So, ultimately you are getting X transpose X inverse X transpose epsilon. So, your beta cap minus capital beta is this.

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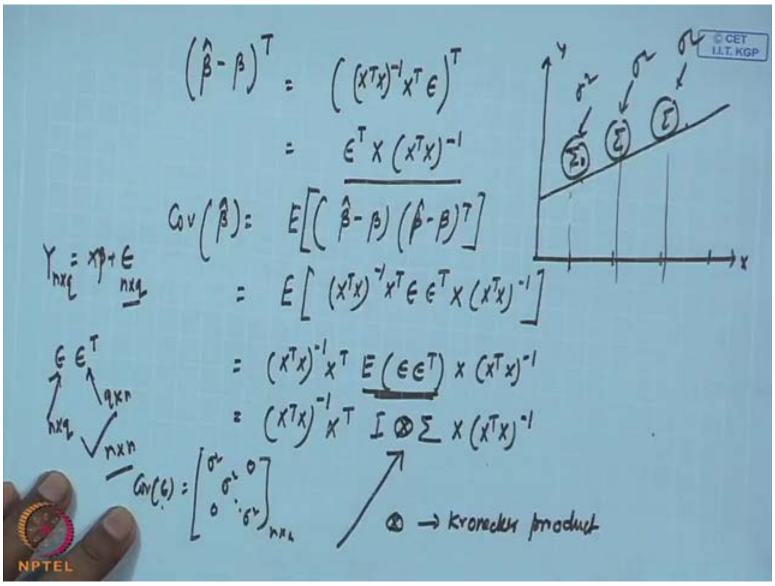

Then our covariance, then what will be the second one? beta cap minus beta transpose. So, this one you can write X transpose X inverse X transpose epsilon transpose. Now, this will be just, it will reverse way epsilon transpose, then this X will come. Now, X then X transpose X inverse is X transpose that remains same, because of symmetric matrix.

Now, this is the quantity, now what is our covariance of beta cap? That mean expected value of beta cap minus beta cap minus beta transpose, that was the formula. So, I will write this, here our beta cap minus beta is X transpose X inverse X transpose epsilon. The transpose of it epsilon transpose X transpose X inverse. So, as usual we will bring the X factor out of the expectation operator. Then we will get X transpose X inverse X transpose expected value of epsilon transpose X, X transpose X inverse, this is fixed value does not require the expectation operator here.

Now, question is what is epsilon epsilon transpose getting me? Now, if you if you see the equation Y is n cross Q equal to X beta plus epsilon, where epsilon is also n cross Q. Now, then epsilon epsilon transpose this will become this is n cross Q. This one will become Q cross n then the resultant will become n cross n, getting me? Resultant will become n cross n. So, if you can remember the one of the assumption, what I have told you in the last class of multivariate regression, that I said that across this X the different across X observations. When you see the Y value, ultimately we say that covariance structure, the same covariance structure.

This covariance structure with respect to what? With respect to the Y, there are Q Y variables. So, ultimately what will happen? Then this quantity will become like this this one is X transpose X inverse X. Then this one will become into I that there is one operator called Kronecker, this quantity will become like this. So, this symbol is known as Kronecker product. So, into X transpose X inverse this will be the case.

Now, why this here n cross n? Ultimately, what will happen for every dependent variable for Y 1 n cross n for Y 2 again n cross n for Y 3 n cross n Y Q n cross n. That is why again what will happen ultimately, if you go by the multiple regression mode there you will find out that for every here if it is 1 Y variable only, this sigma this will not be sigma. Then it will be sigma square. Now, when we create the covariance matrix of epsilon for multiple regression also we get n cross n, but here everywhere the diagonal element will be sigma square. Because, for every element of the that error that the variance is sigma square and off diagonal element will definitely become 0.

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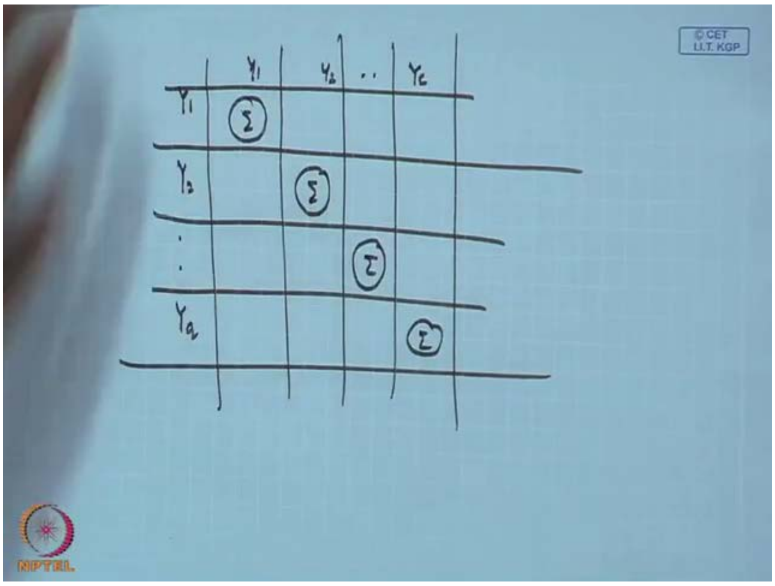

So, here in this case what is happening? In this case it is happening like this suppose, I have Y 1, Y 2. So, similarly Y Q this side also Y 1, Y 2 like Y Q. So, see what is happening here? Here you are getting Y 1 versus Y 1. This case also you will be getting, what is this? This is you will be getting here sigma. Here also you will be getting sigma, here also you will be getting sigma, here also you will be getting sigma. So, that is why this total, this error term, this Kronecker alpha is coming, because of Kronecker product is coming. Because, of this, so you will get this type of terminal.

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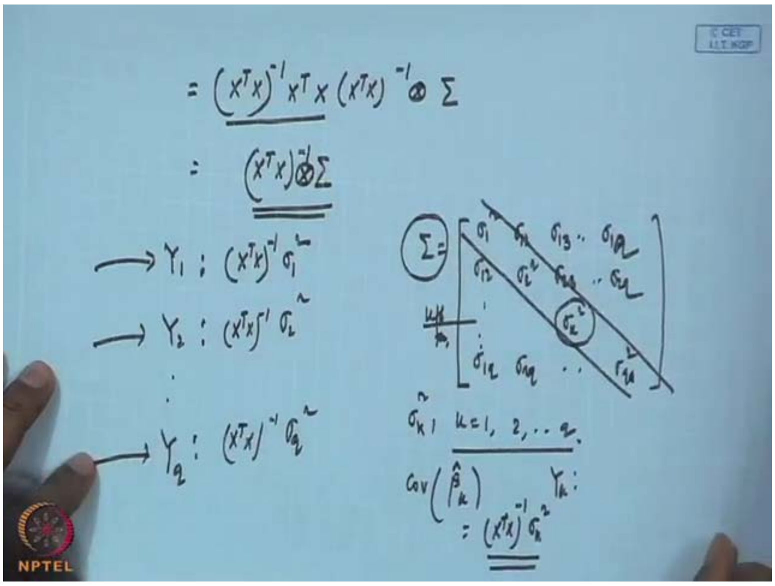

This type of relationship, which will be if you further simplify, X transpose X inverse X transpose X x transpose X inverse. Then this Kronecker product this, now X transpose X inverse X transpose X is I. So, ultimately that will not be thereM, so X transpose X inverse this, but there will be this Kronecker product. Why this for every Y X transpose? X will be multiplied to every Y. Here this one will be X transpose X inverse, here it will be mine X transpose X inverse. Here it will be X transpose X inverse, everywhere it will be multiplied. Ultimately, what we want to say that ultimately that for this one from, now you think of the situation like this.

Suppose, here we are talking about individual Y vis a vis X, then if we go by the Y that expected Y value. Therefore, every observation what will happen? Every observation that sigma value will be equal, every observation of X. Now, Y sigma will be equal and that is true for all other Y values also.

So, as a result this is fine, but when you will be using for individual Y like Y 1 like Y 2 like your Y Q. So, you have to be very judicious that what we will be using, so you will be using sigma. I think I have given you earlier also sigma is sigma 1 square sigma 1 2 sigma 1 3. Like this sigma 1 Q, then sigma 1 2 sigma 2 square sigma 2 3, like sigma 2 Q. This manner sigma 1 Q sigma 2 Q like this sigma Q, Q means sigma Q square. Somewhere, there will be sigma k square for the kth term.

So, if I go by individual Y variable for the kth variable this diagonal element kth variable. Here I think you can still remember that we used in univariate case X transpose X and sigma square. So, that means sigma 1 square X transpose X inverse sigma 2 square. So, like this X transpose X inverse sigma Q square, when you talk about individual dependent variable part.

Now, that mean I must know that sigma 1 sigma k square k equal to 1 to Q. If you know this then you know the variability part that covariance of beta k, getting me? What I mean to say that, if I consider Y k variable that case the covariance of beta k. We are saying that, it will be X transpose X inverse sigma k square, that we are saying that is what we have seen earlier also. So, but question is that sigma is not known capital sigma is not known. So, how do we understand? That what is this sigma, but you know this epsilon cap is known to you.

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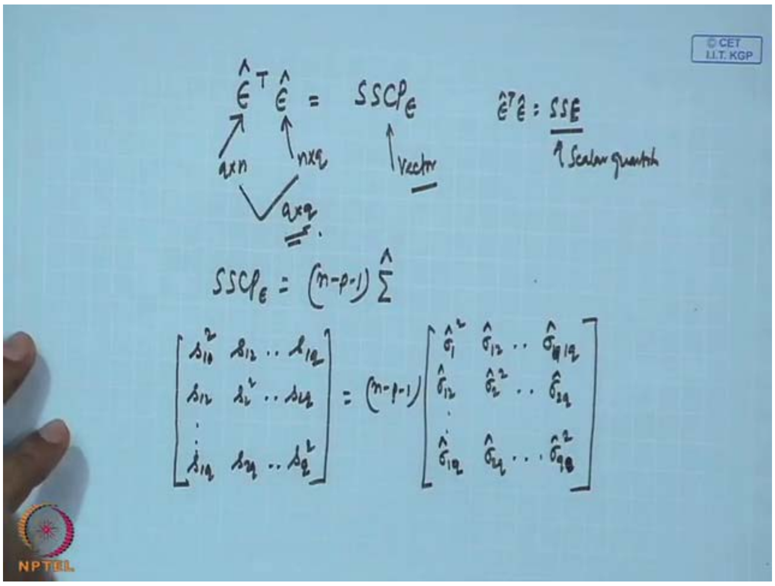

Now, if I make epsilon cap transpose epsilon cap, then what is this value? This value epsilon transpose will be Q cross n and epsilon will be n cross Q. So, this quantity will be Q cross Q. So, this is what is your S S C P error part instead of in M L R you have written this one epsilon transpose epsilon. When you have done you said S S E this you say S S E. It was a scalar quantity, but here this is Q cross Q matrix that is why it is a vector quantity getting me and this value. So, S S C P E this value is n minus P minus 1 into capital sigma, getting me?

Now what we will do if we write like this S S C P E if i write s 1 1 or s 1 square let we write s 1 square s 1 two s 1 Q, because this is Q cross Q. Then s 1 2 s 2 square s 2 Q. So, like this if you write s 1 Q, s 2 Q, s Q square, so this is n minus P minus 1 into sigma 1 square that cap, then sigma 1 2 cap. So, like this sigma 1 Q cap sigma 1 2 cap sigma 2 cap square sigma 1 2 to two Q cap. So, like this sigma 1 Q cap sigma 2 Q cap. So, like this sigma Q, Q cap means sigma Q square cap. Correct? So, if I take one value here, which is which s for the kth dependent variable.

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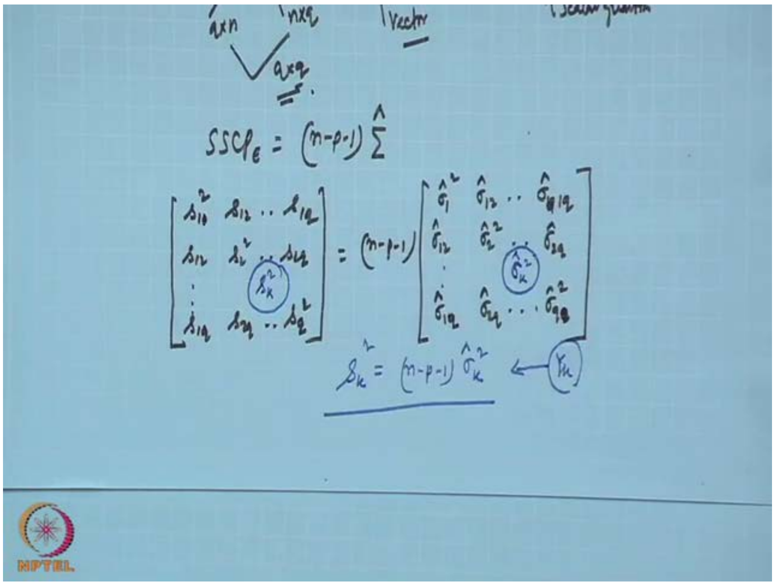

If we take a value here s k square, then this corresponding that population case that is sigma k square, but we are estimating this one getting me. Then what is the relationship is s k square equal to n minus P minus 1 sigma cap k square. So, ultimately your when you are saying k, that mean we are talking about Y k, variable Y k. You are going back to you multiple regression case. Now, what you require now? So, you know what are the things you know? You know The things what is given to you, now that beta cap point estimate.

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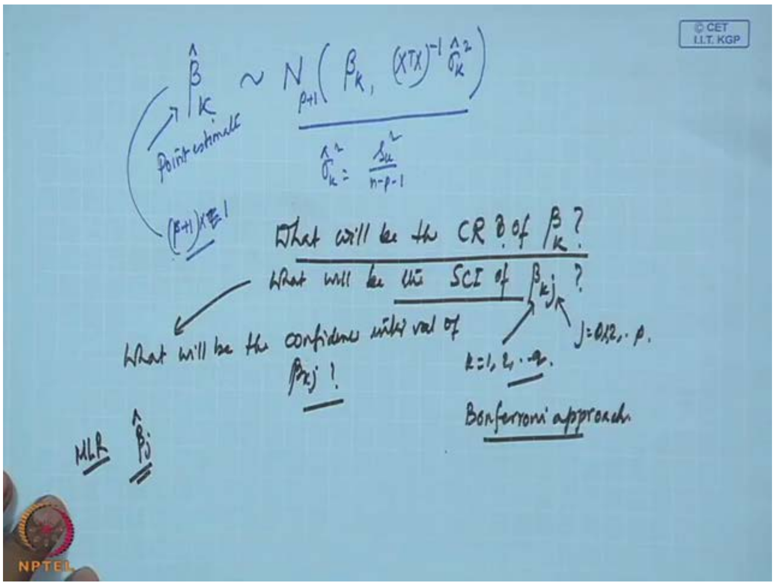

This is my point estimate it is known and I know that this also followed, that normal distribution. What type, what normal multiple or multivariate beta cap? beta cap is what is this beta cap n cross P plus 1 cross Q multivariate case. Now, what happened? I am writing here beta k, then we are going back to this M L R case. So, that mean this will be 1, so then this will become P plus 1.

Now, P plus 1 then this beta k into X transpose X inverse sigma k square. Correct? The sigma k square will be estimated sigma k square will be estimated. Ultimately, what will happen? Then that sigma k square you will be replacing by like this sigma k square is s k square by n minus P minus 1, where s k square is the kth element of the S S C P E matrix.

Now, question comes that, what will be the confidence region? Because, when we are coming to the multivariate domain first is confidence region. Then what will be the confidence region of what? Confidence region of beta k. Here we are going back to individual variable beta k, that we have seen in M L R. What will be the simultaneous confidence interval of beta k j? See that j stands for number of j equal to 1 to P 0 to P, all the regression coefficients related to X variables k. k stands from 1 to Q then comes that what this 2. Then comes that, what will be the confidence interval of beta k j.

Now, if we recall if you recall in M L R, that we have not gone for this confidence region, because of the robustness of regression. Also, we have we have described what is this. Also, we have described, what will be the simultaneous confidence interval or beta k j using Bonferroni approach. We have used Bonferroni approach. So, but what we have discussed there mostly beta j case, there k was that one variable, k was not there. So, in M L R we have discussed about beta j with respect to the estimate of this and you have seen that. We say that beta j cap minus expected value of beta j cap divided by standard error of beta j cap, follows t distribution that we have described. So, the same thing prevailed here.

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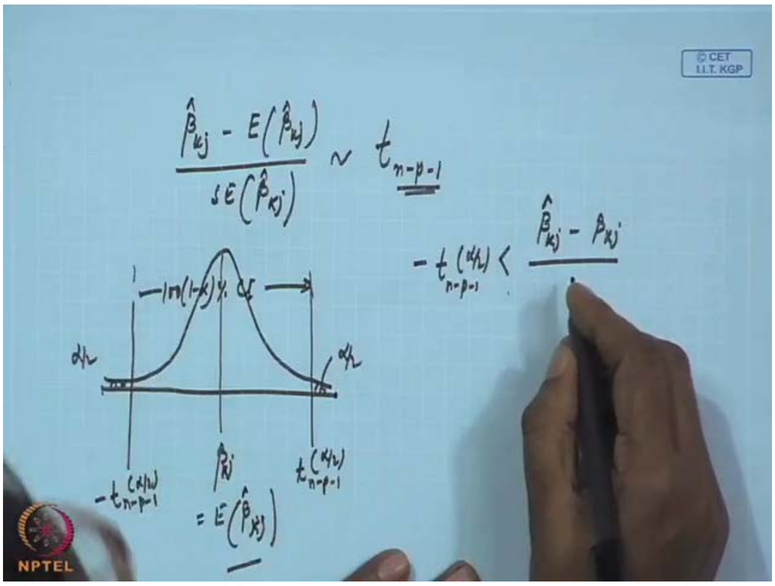

Also, same thing we can use here. We can write that beta k j minus expected value of beta k j divided by standard error of beta k j all cap. These are estimate one this follows t distribution with n minus P minus 1 degree of freedom. See here also the degree of freedom is also n minus P minus 1. Earlier, also we have seen the this n minus P minus 1 in M L R case also, because the X side remains same in M L R as well as this case.

So, then I think this does not require discussed here. Because, if you followed the my earlier lecture of M L R, then you will be able to find out that the same thing is given. So, what I mean to say suppose this one is minus t n minus P minus 1 into alpha by 2. This one is n minus P minus 1 alpha by 2, so this side is alpha by 2 and this side is alpha by 2. Then this is your 100 into 1 minus alpha percent confidence interval, for what? For beta j. What is this beta j? beta j is basically expected value of beta j. Here we are using beta k j, because k is coming from the dependent side expected of value of this.

So, you can write then you can write that, t n minus P minus 1 minus alpha by 2 less than, if I use less than equal to less than. Let us write less than then this will be beta k j minus beta k j beta k j cap minus beta k j by what is the standard error of beta k j. X transpose X inverse sigma k square, we found out sigma k square is nothing but s e square by n minus P minus 1.

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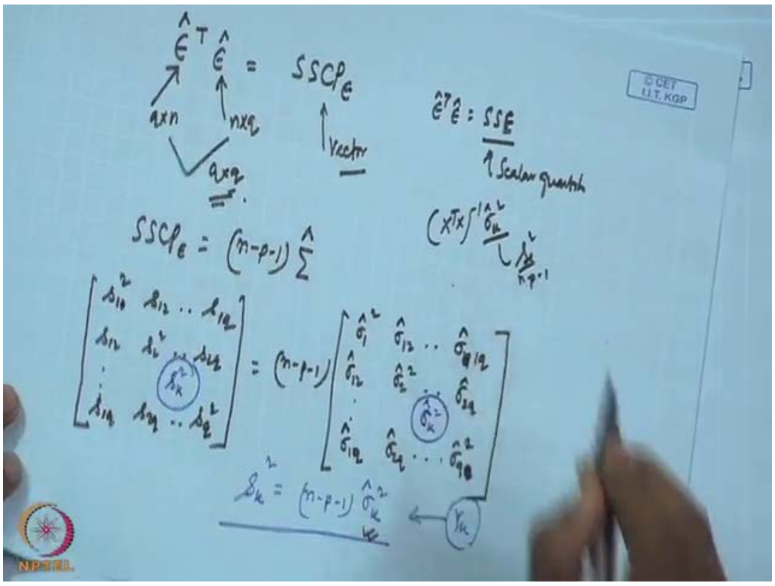

So, you can write down, I think I have given you somewhere here that X transpose X inverse then your sigma k cap square, which will become s e square by n minus P minus 1. Correct? So, that value will come here, that value will be coming here. So, I will not writing this. So, straightaway I am writing S E this, because of beta k j less than equal to t n minus P minus 1 alpha by 2.

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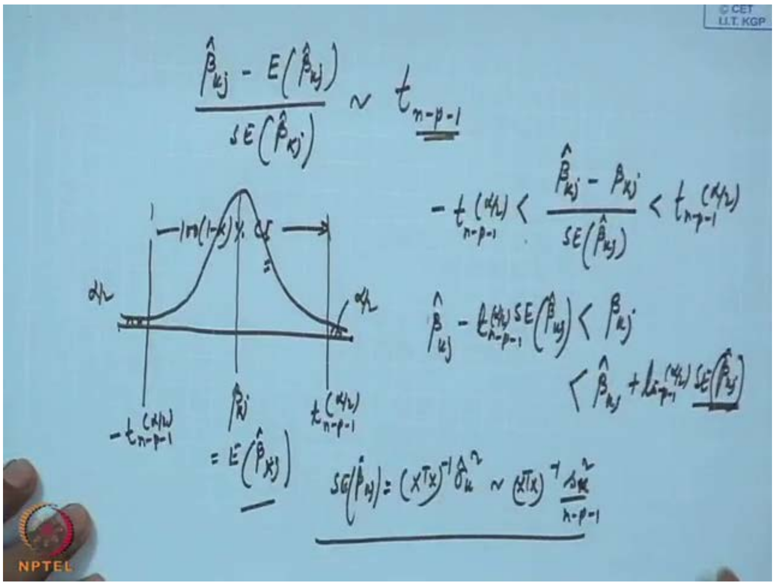

So, ultimately what will happen? Your confidence interval will become beta k j minus t n minus P minus 1 s e beta k j, less than beta k j, less than beta k j cap plus t n minus P minus 1. Definitely, alpha by 2 is there, alpha by 2 into standard error of beta k j cap. So, how do I know beta k j cap type, sorry standard errors? I told you X transpose X inverse sigma k cap square, which will give you X transpose X inverse s k square by n minus P minus 1. You check this one if there is mistake you report to me, I do not think there is mistake. So, ultimately this is your, now this one is your 100 into 1 minus alpha percent confidence interval. So, this one let us see that the result I have, if you go back. So, let us go to the data set initially we have considered.

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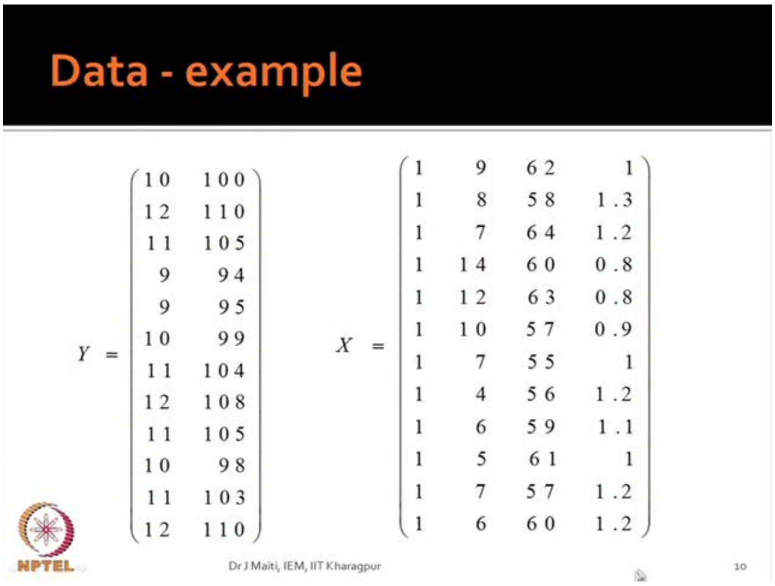

$$Y = \begin{pmatrix} 10 & 100 \\ 12 & 110 \\ 11 & 105 \\ 9 & 94 \\ 9 & 95 \\ 10 & 99 \\ 11 & 104 \\ 12 & 108 \\ 11 & 105 \\ 10 & 98 \\ 11 & 103 \\ 12 & 110 \end{pmatrix} \qquad X = \begin{pmatrix} 1 & 9 & 62 & 1 \\ 1 & 8 & 58 & 1.3 \\ 1 & 7 & 64 & 1.2 \\ 1 & 14 & 60 & 0.8 \\ 1 & 12 & 63 & 0.8 \\ 1 & 10 & 57 & 0.9 \\ 1 & 7 & 55 & 1 \\ 1 & 4 & 56 & 1.2 \\ 1 & 6 & 59 & 1.1 \\ 1 & 5 & 61 & 1 \\ 1 & 7 & 57 & 1.2 \\ 1 & 6 & 60 & 1.2 \end{pmatrix}$$

So, this is the data set, this one profit, this is your sales volume, this one is the constant term for X 0, this is absenteeism, this one is breakdown hours, this is m ratio. So, that mean 1, 2, 3 variables from the independent side we have considered. Two variables from the dependent side we have considered. Then what I have used? I have used S P S S, then S P S S result is this, getting me?

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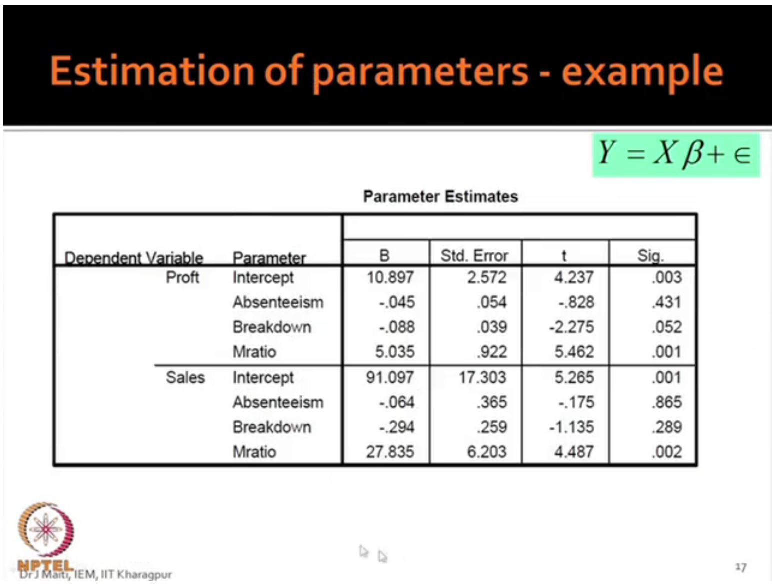

Parameter Estimates

Dependent Variable	Parameter	B	Std. Error	t	Sig.
Proft	Intercept	10.897	2.572	4.237	.003
	Absenteeism	-.045	.054	-.828	.431
	Breakdown	-.088	.039	-2.275	.052
	Mratio	5.035	.922	5.462	.001
Sales	Intercept	91.097	17.303	5.265	.001
	Absenteeism	-.064	.365	-.175	.865
	Breakdown	-.294	.259	-1.135	.289
	Mratio	27.835	6.203	4.487	.002

Now, see the profit intercept 10.897 and its standard error that X transpose X inverse that c j j, that same formula. The standard error is 2.572 and what will be the t value? t value is basically under null hypothesis is true. That means h 0 beta k j equal to 0, then your beta the estimate by the standard error. So, 10.89 divided by 2.5, how much it is coming? If I multiply this into 4, I think this is coming, exactly.

So, see the intercept 4.237, t value is 4.237 means it is quite large value. The probability significance level is 0.003, so similarly absenteeism breakdown m ratio. You see ultimately, what is happening here that absenteeism is not significant. Breakdown is significant, but it is slightly higher than the 5 percent significance level, but still it should be it to be considered. m ratio is very significant from profit point of view, if you go by that sales point of view, suddenly when breakdown hours are also not coming into that significant case.

So, that means what I mean to say here I mean to say here that, you know beta these are the coefficients. So, this side is beta 1, this is beta 2 this is the standard error part. Then this is also standard error for this, now once you know this beta as well as its standard error. So, also you are in a position to find out the confidence interval, that if instead of confidence interval here it is giving the significance level as if h 0 is framed. That beta equal to true, but it is also possible to find out that this one, the confidence interval. You see this figure, that low of 95 percent confidence interval lower bound and upper bound this is given.

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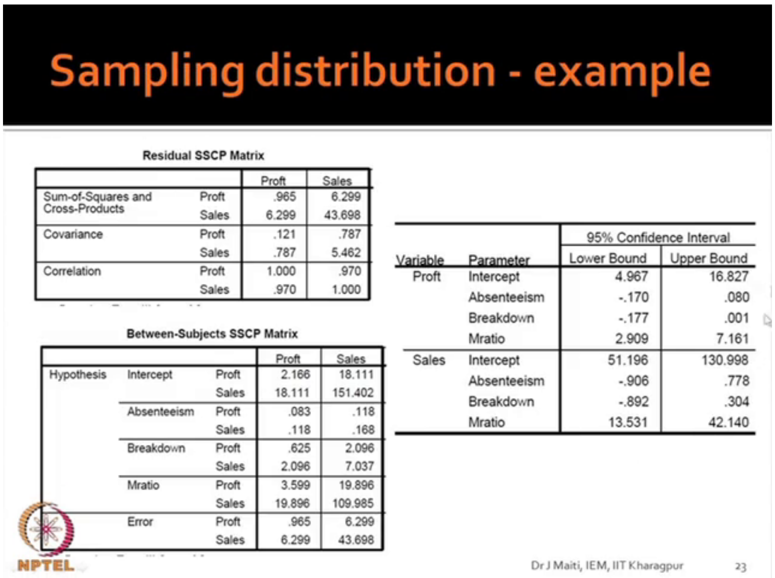

Residual SSCP Matrix

		Proft	Sales
Sum-of-Squares and Cross-Products	Proft	.965	6.299
	Sales	6.299	43.698
Covariance	Proft	.121	.787
	Sales	.787	5.462
Correlation	Proft	1.000	.970
	Sales	.970	1.000

Between-Subjects SSCP Matrix

			Proft	Sales
Hypothesis	Intercept	Proft	2.166	18.111
		Sales	18.111	151.402
	Absenteeism	Proft	.083	.118
		Sales	.118	.168
	Breakdown	Proft	.625	2.096
		Sales	2.096	7.037
	Mratio	Proft	3.599	19.896
		Sales	19.896	109.985
	Error	Proft	.965	6.299
		Sales	6.299	43.698

		95% Confidence Interval	
Variable	Parameter	Lower Bound	Upper Bound
Proft	Intercept	4.967	16.827
	Absenteeism	-.170	.080
	Breakdown	-.177	.001
	Mratio	2.909	7.161
Sales	Intercept	51.196	130.998
	Absenteeism	-.906	.778
	Breakdown	-.892	.304
	Mratio	13.531	42.140

If you see that S S C P matrix here it is given, if you see S S C P matrix that what I said that S S C P matrix. So, this is the first for the profit that is the S S C P that value and this one is your sales the diagonal elements. These are the S S E square s 1 square and s 2 square. These values are there and you know that n minus P minus 1 that value is known to you. You will be able to find out the variance standard error part, once you multiply it by X transpose X inverse. Just you check, because what I mean to say, I am not checked this by hands on calculation, i am not checked that.

You just check for one, that S S C P matrix is available and your X transpose X is also available, that inverse is available. You know that n P everything is given that I am ask you to check the what are the confidence bound you are getting, that interval part. That you check and I think you will get the same way it has to be. If I am not committing mistake in the theory and for this part particularly this sampling distribution of beta. That estimation as well as sampling distribution of beta, it is almost similar to multiple linear regression. So, you I think you would not face any problem here.

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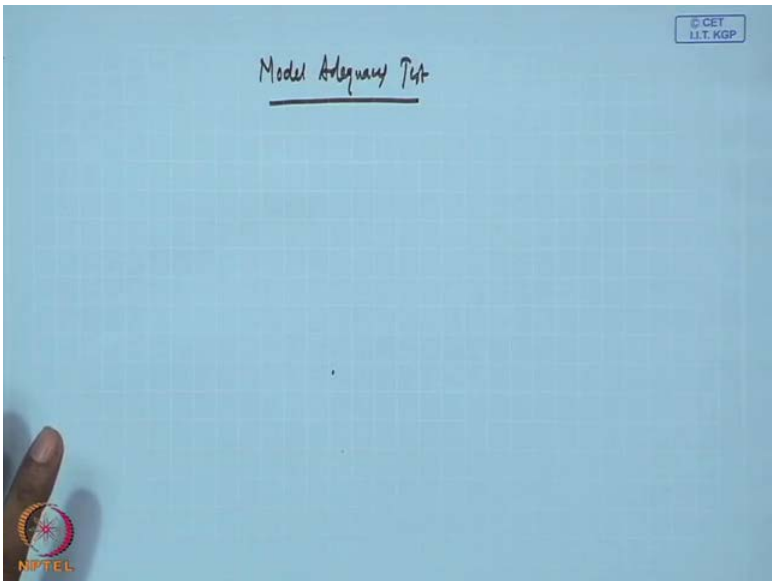

Your problem will be in model adequacy test, which we will discuss next. In detail we will discuss this model adequacy test in the next class.