Applied Multivariate Statistical Modeling
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Lecture - 22
MLR- Sampling Distribution of Regression Coefficients

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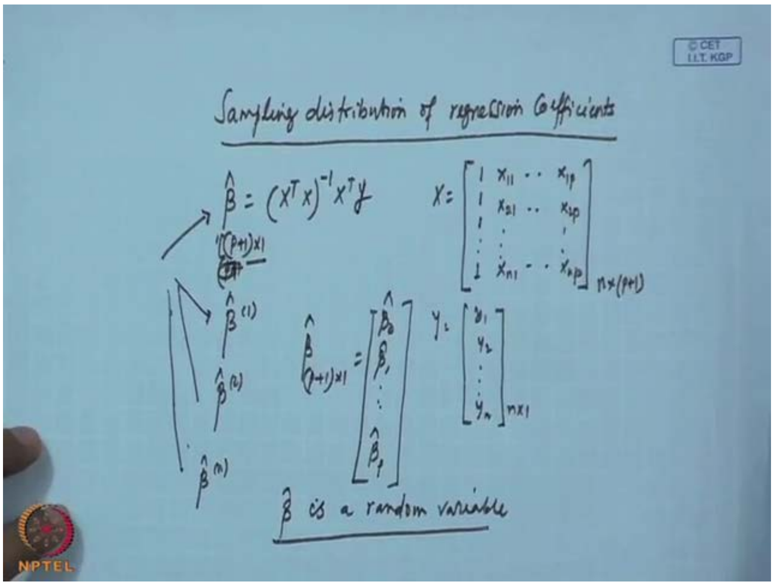

Today our discussion is sampling distribution of regression coefficients. In last class, we have estimated beta coefficient like this x transpose x inverse x transpose y, where x is the design matrix, first column is all 1, second column will be the first variable data, x 11, x 21 then x n1. Similarly, last one will be the pth variable data x 1p, x 2p, x np and y is the n observations, this is n cross 1, this is n into p plus 1. And the, this one beta, this will be definitely p cross 1, p plus 1, p plus 1 cross 1.

So, if you collect one sample you get some value for beta cap, if you go for another sample your beta cap will be different. For example, sample one beta cap maybe 1, sample two beta cap maybe for sample two, like this if we go for n sample so beta cap n so you will be getting series of beta values. And please keep in mind that, beta is or beta cap is p plus 1 cross 1 vector, that is beta 0 cap beta 1 cap like this, beta p cap.

So, beta cap is a random variable, is a random variable and we want to know its distribution and that is the sampling distribution of regression coefficient. And then using the distribution of beta cap we will, we will derive confidence interval and other things.

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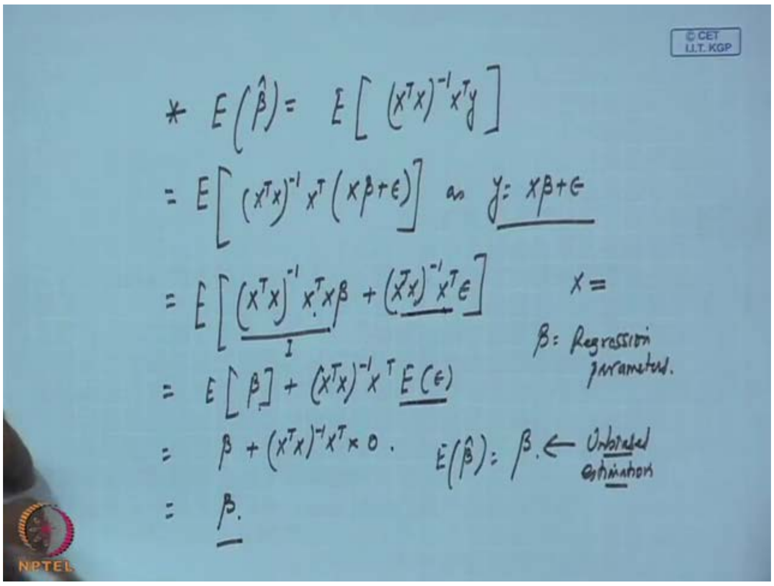

So, for a distribution you require to know what will be the expected value of beta cap, we want distribution of this. So, we first required to know the expected value of beta cap, this will be expected value of beta cap is, x transpose x, inverse x transpose y. So, we can write this as, expected value of x transpose x, inverse x transpose now, you all know y is x beta plus epsilon, as y equal to, this is the regression equation. So, if we do little more manipulation, this will become x transpose x, inverse x transpose x beta, plus x transpose x, inverse x transpose epsilon.

Equal to now x transpose x, inverse x transpose x symmetric matrix, inverse symmetric matrix we get it identity matrix I. So, this will become I s, ultimately expected value of I into beta is beta plus, as x data already collected so x transpose x inverse x transpose this is a fixed quantity. So, we will keep it out from the expectation operator then we are writing expected value of this.

Now expected value of beta is beta because beta is the regression coefficients, these are regression parameters or coefficients from the population point of view, that is constant plus expected value of error is 0, that is our assumption. So, you will get x transpose x inverse x transpose into 0 so this will become beta so it says that expected value of beta cap is, the beta that is unbiased estimation. So, in addition now first we want, we have taken this one, that expected value of what is the beta cap, we want to know also the covariance matrix of, covariance matrix of beta cap.

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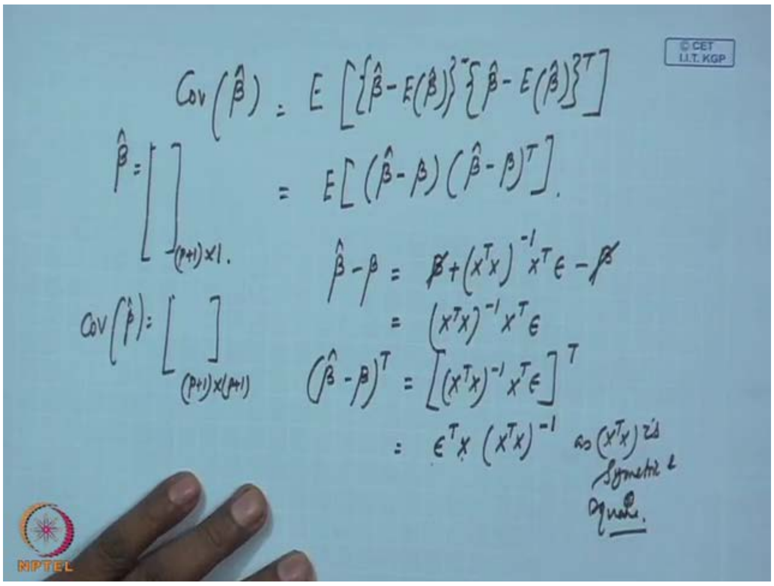

So, we want to know covariance of beta cap this will be expected value of, all of you know that beta cap minus expected value of beta cap, whole this is p cross 1, now this into beta cap minus expected value of beta cap, this transpose. Because, covariance matrix of beta means it will be beta cap is a p plus 1 cross 1 so our covariance matrix of beta cap that will be a matrix of p plus 1 cross p plus 1. Now, we can write that this one, expected value of now beta cap minus expected value of beta cap is, beta into beta cap minus beta transpose.

So, let us find out what is the value of beta cap minus beta so what, the value of beta cap if you see here, when we have described this one, for expected value of this one is this, this final means ultimately, you got beta plus this quantity. This is beta plus this so I can write this one using this, that beta plus x transpose x inverse x transpose epsilon, that is we can write for beta cap minus beta, this beta is there. So, beta, beta cancel out this quantity becomes x transpose x inverse x transpose epsilon then beta cap minus beta this transpose, this will become x transpose x inverse x transpose epsilon transpose.

So, when you next transpose it will be just the reverse order so epsilon transpose x transpose x transpose is x and x transpose x inverse, that will remain because symmetric matrix that will remain this. So, epsilon transpose x, x transpose x inverse as x transpose x is symmetric and definitely square also. So, then if I use this, if you use now this operator, expectation operator here so we can take one more page.

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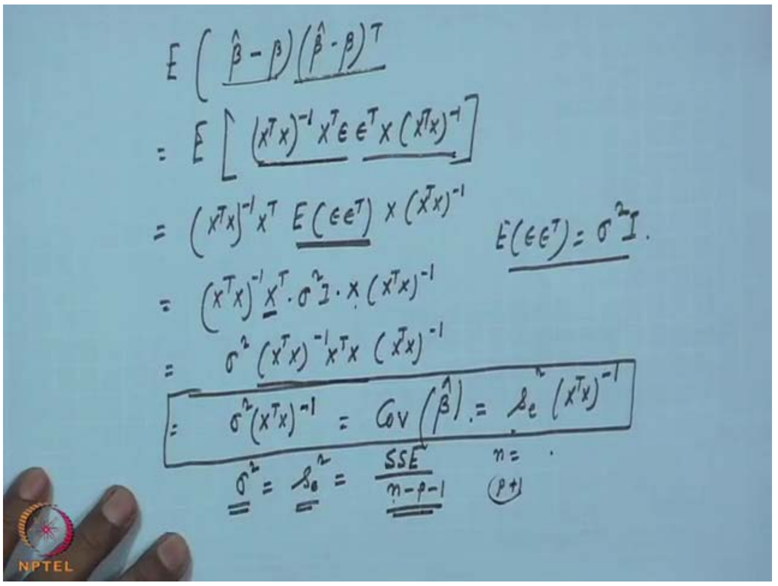

So, expected value of beta cap minus beta, beta cap minus beta transpose, this is nothing but expected value of, that is coming x transpose x inverse x transpose epsilon then for this one, epsilon transpose x, x transpose x inverse, this is coming. So, for this we have written this portion, for the other one written this portion. Now, again what we will do then we will just bring out the fixed values like x transpose x inverse x transpose then expected value of epsilon, epsilon transpose then x, x transpose x inverse so what is the x this one.

Student: ((Refer time: 10:48))

Covariance matrix of error term, now covariance matrix of error term we say that, equality of variances across the x observations. So, that will for y and that will go to the place so ultimately expected value of epsilon, epsilon T this will become sigma square into I, plus there are n observations for.

Student: ((Refer time: 11:18))

Same, that is from the assumption it is coming. So, this quantity will become x transpose x inverse x transpose sigma square I, x into x into x transpose x inverse. Now, sigma square is constant, I is the identity matrix that will go. So, finally it will be like this, sigma square x transpose x inverse now, here is x transpose here is one more x, then x transpose x inverse, tanmay ok.

So, what is this quantity then x transpose x inverse x transpose x.

Student: ((Refer time: 12:07))

This is again I so sigma square I is sigma square so this will become sigma square x transpose x inverse, this is what is covariance of beta cap. So, then how do we get the value of sigma square, sigma square is also not known. So, sigma square will be S E square that mean, the, from the error whatever value you get, from there you will calculate the variance component. And this will be, SSE by degrees of freedom for SSE is, number minus parameters estimated.

So, we can write then that covariance of beta cap is S E square, x transpose x inverse. So, you will not get this sigma square value, these are population value so from sample error you are able to calculate S E square. This S E square is nothing but SSE by n minus p minus 1 and this n minus p minus 1 is coming because there are n is the sample size, number of parameter to be estimated is p plus 1. So, this degrees of freedom is lost, when you are calculating the errors so this is the case.

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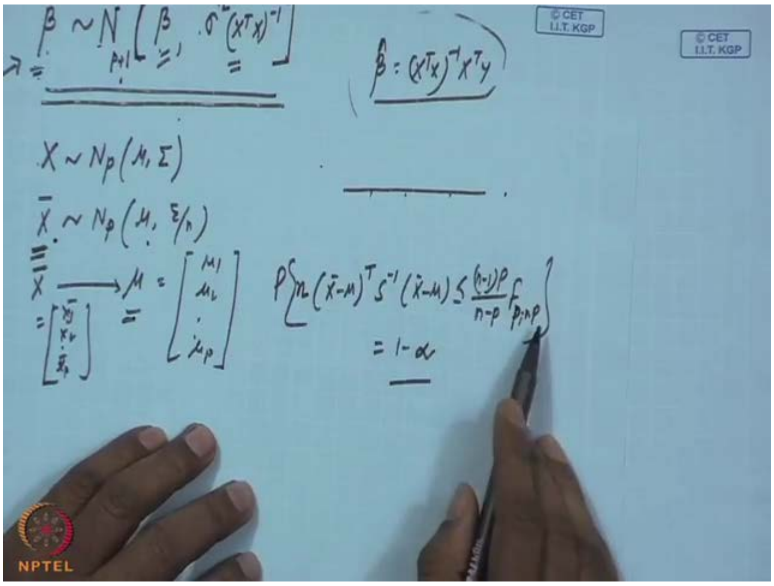

Then we can say that, beta cap is a random variable, is a random variable with mean vector beta and covariance matrix we can write this way, sigma square x transpose x inverse. This is correct, what will be the distribution, this distribution will be multivariate normal, this is the assumption, we have started with that data comes from multivariate normal. And the way we have estimated beta, beta cap is x transpose x inverse x transpose y so here what happen because of this multivariate normality assumption, this will also become multivariate normal. Now, this multivariate normal what will be the

Student: ((Refer time: 14:47))

So, N actually in the regression the error is the key point, key issue, y is definitely multivariate, y is here one variable so normal actually what is happening here, in this particular, in this regression, multiple regression case. So, you have a different sets of x values then you are calculating y values and everywhere there are, there is error, this error is, ((Refer time: 15:29)) N error will be there. So, that error component also will go for multivariate N cross N so we are here, we are estimating p plus 1 parameters. So, beta is, we are assuming that this is multivariate normal with p plus 1 and this is, what is the distribution, this is the distribution of beta cap.

Now if this is the case now, we will go back to our some earlier lectures where, we say if x this is not one to one relation, but to get some clue for related to beta, for some more derivation. We have seen that, suppose if x is multivariate normal with mu and sigma then we calculated x bar and that one is also multivariate normal, mu and sigma by n. So, similarly this we have taken this data from multivariate normal and we have calculated some statistic, x bar is also statistics here also, we calculated some statistics. And for that statistics we found out the, mean vector and the covariance matrix and like x bar covariance matrix and mean is also, calculated. So, in the same manner it is beta is multivariate normal.

Now, let what happened we have also described that what is the confidence region, can you remember, confidence region for mu where, mu equal to mu 1, mu 2 to mu p, that we have discussed earlier. And with the help of x bar, we found out where x bar is x 1 bar, x 2 bar like x p bar, we have found out this confidence region of mu. And if you can remember you will find out that, we have found like this that n, x bar minus mu transpose S inverse x bar minus mu, this will become less than equal to n minus 1 p by n minus p, f p n minus p, this one probability of this equal to 1 minus alpha.

This is the confidence region, we have found out with respect to that population mean vector. So, can we not find out the similar thing here with respect to beta, getting me? So, then it is not derivation what I am giving it is not derivation, it is just understanding that why how this confidence regions can also be calculated, with respect to beta cap which is multivariate normal.

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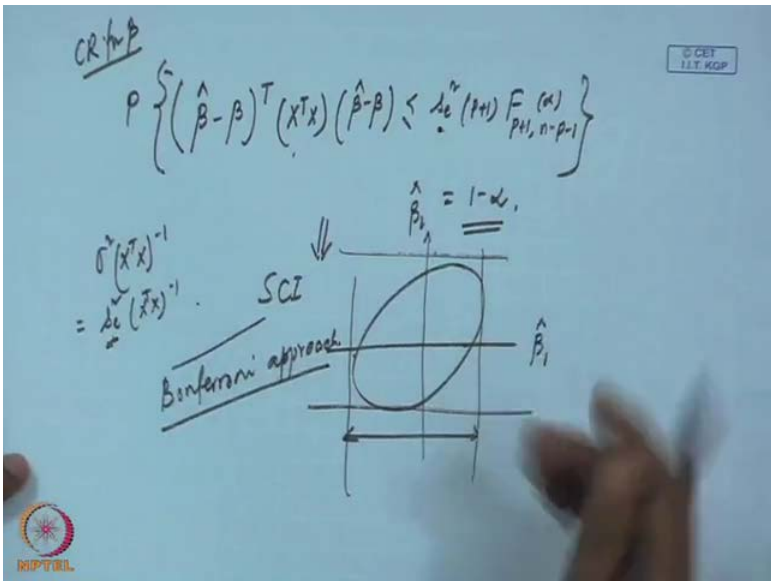

So, can we not write like this then beta cap minus beta that is, x bar minus mu similar type of things, this transpose, there was S inverse. So, S in, S is the covariance matrix so our covariance matrix is what, our covariance matrix is sigma square x transpose x inverse, which is we can write S E square x transpose inverse so what do we want inverse of this, that means x transpose by S E square.

So, if I write like this x transpose x, I am not writing S E square here, later on we will write S E square then beta cap minus beta. What distribution it will follow that is important, that also follows here, that this S E square it will come, again I am telling you it is not derivation. Derivation maybe something, some different way people will come, but this is the way we will understand for our application.

So, this is S E square p plus 1, F p plus 1 n minus p minus 1 alpha probability that, this quantity will be less than this is 1 minus alpha. So, this S E square what is coming out here by this, we are keeping here and the rest of the things are like this, but there are some modification, with respect to the parameter, number of parameters to be estimated. So, this is the confidence region for beta, but we are not we, what we will do with the confidence region. So, parallely then what you require to do, you require to go for simultaneous confidence interval, understood tanmay.

What I am saying that, beta cap is multivariate normal, beta cap is a statistic, earlier we have seen x bar, multivariate normal that statistic with x bar, we created the confidence region for mu. Here, with beta cap we want to find out the confidence region for beta now, this is the formulation which can be used. Then from confidence region we have gone to simultaneous confidence interval, for two variable case you have found out that when, multivariate normality will be coming that something like this, you got confidence ellipse. So, from this ellipse to, you want to go to the sides what is this interval confidence, what will be this side that is true for beta cap also. Suppose, this is beta 1 cap and this side it is beta 2 cap same thing, nature is same.

So, for simultaneous confidence interval I think you have, we have seen two approaches one of them is Bonferroni approaches. So, this Bonferroni approach is, approach is easier for us and it gives good result also, we can use Bonferroni approach. And in regression, whatever it is found that individual parameters when you test and you will find out that, interestingly this situation will arise that, if I go instead of beta cap.

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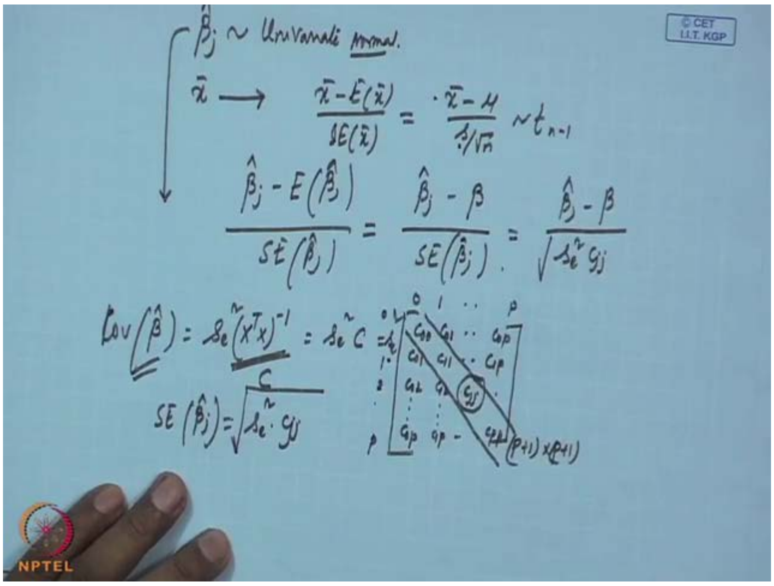

If I go for beta j cap, what will be the distribution of this, this will be univariate normal, one variable so this will be univariate normal. Now, depending on sample size what will happen, we will go for, when we have taken x bar, if you can remember we created something like this, that x bar minus expected value of x bar, divided by standard error of x bar. Can you remember, this one is nothing but x bar minus mu by s by root n, when sigma is not known.

So, this follows t distribution with n minus 1 degrees of freedom now, for beta cap let us do the similar thing, that if I write that beta j cap minus, expected value of beta j cap divided by standard error of beta j cap. What will be this value, this value will be beta j cap minus beta divided by what will be the S E standard error of, beta j cap that you require to find out.

How do we find out this one, see you know that covariance structure of beta cap is already given to you, this one is nothing but S E square x transpose x inverse. So, if I write x transpose x inverse as C then this is S E square into C. So, this can be written like this, that S E square will remain then how many, what is the size, size is p plus 1 cross p plus 1.

First one is 0 then 1 like this up to p here also, 0 1 up to p, first one is C 00 then C 01 like this C 0p then C 01, C 11, C 1p. So, then here it will be, it will be, this is 2 so C 02 so like this C 0p then C 12 like C 1p. So, in this manner you will calculate then C pp somewhere, the jth one will be C jj understood. So, what I am saying that x transpose x inverse is a square matrix of the order p plus 1 cross p plus 1 and that we are writing in terms of capital C and each element is C jk now j stands from 0 to p k stands from 0 to p.

So, the diagonal elements of this when multiplied by S E square will give you standard error of beta j. So, this will be S E square into C jj and off diagonal will give you the covariance, between the parameters estimates. So, you can write now that, beta j cap minus beta by, you can write this S E square this is standard error [FL], as it is standard error, we have to give the square root. This is standard error so we have to give the square root and this component is the variance component, off diagonal elements are variance so S E square C jj.

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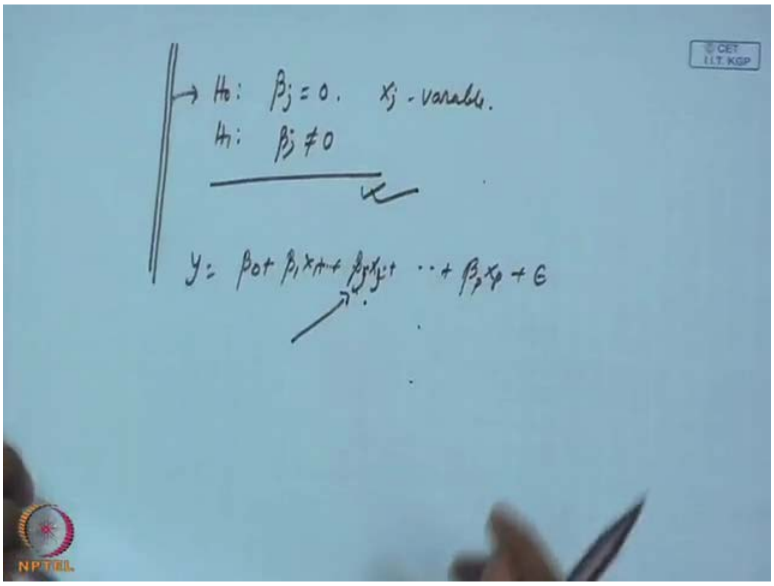

Now, under the assumption of null hypothesis H0 that, beta H0 our null hypothesis that, there is no effect of the x j variable. Now, what do I, what do you mean our regression equation is like this, beta 0, beta 1 x 1, beta 2 x 2 dot, dot, dot beta p x p plus epsilon. Somewhere there is, suppose this is our beta j x j what we are saying, if x j has no relation with y, other way x j does not contribute in explaining the variability of y. Then we can assume that beta j value is 0 so this is our null hypothesis when you test the individual regression parameters. Then alternative hypothesis is beta j not equal to 0 so if we cleared this what will happen ultimately then under null hypothesis this quantity that, beta j minus this will be 0, beta j equal to 0 so then by square root of S E square C jj getting me, difficult.

Student: No, Sir beta j minus beta that one

Beta j cap minus beta j, all are beta j here, all are beta j, beta j because we are talking about the jth variable only. Now I will come back to this, this estimate that test part later on now, let us see that we will just, what we are developing that confidence interval part.

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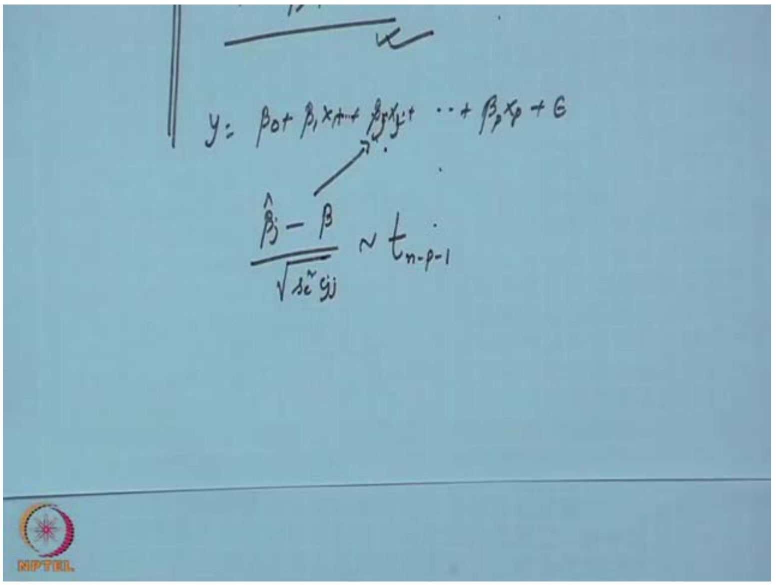

So, beta j minus beta by square root of S E square C jj, this will follow T distribution with n minus p minus 1 degrees of freedom.

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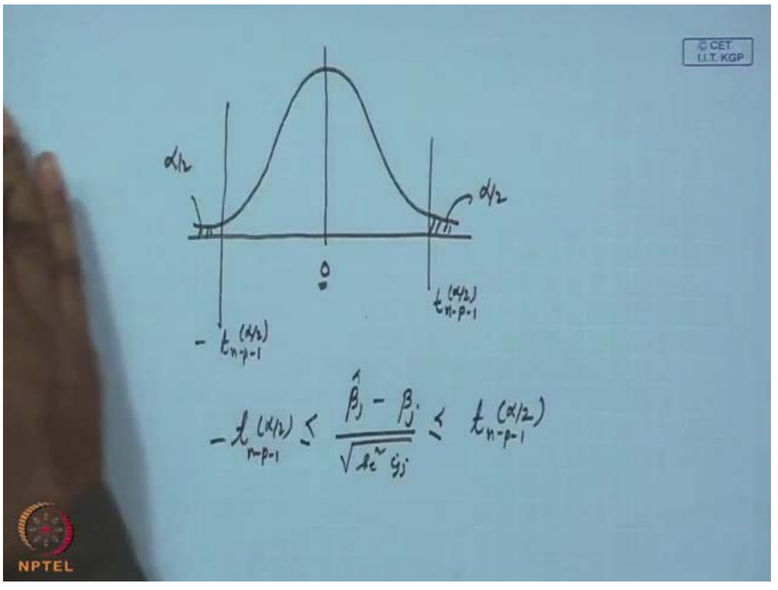

So, if this true then we can write like this, this is my T distribution this is 0, this 1 is t n minus p minus 1 alpha by 2, this side is alpha by 2, this side also alpha by 2, this is minus t n minus 1 minus n minus p minus 1 alpha by 2. This value and please remember that, we are talking about beta j only under null hypothesis this is, this is [FL], null hypothesis case we will not discuss now. So, then can we not write down now, with respect to this, this quantity will be beta j cap minus beta by root over S E square C jj, less than equal to t n minus p minus 1 alpha by 2. And here minus t n minus p minus 1 alpha by 2.

Student: Sir, beta j

Beta j.

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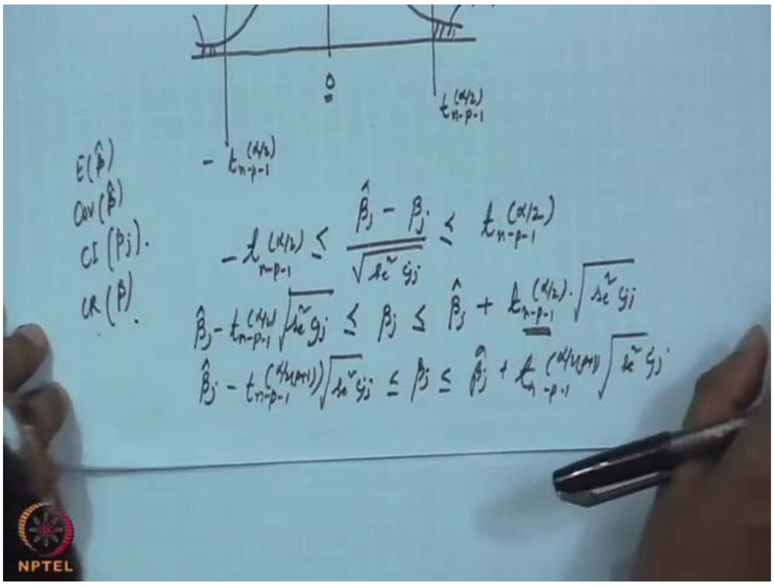

So, now manipulate little more so what you require to do then you want beta j here this side and this side less than equal to and less than equal to, this quantity will come here as beta j cap plus t n minus p minus 1 alpha by 2 into S E square C jj. And this side it will be beta j cap minus t n minus p minus 1 alpha by 2 root over of S E square C jj. This is from usual individual the t distribution, but again you see that, it is n minus p minus 1 degrees of freedom is, much less compared to n minus 1 in our original x bar case.

So, what we have discussed so far, we have discussed ultimately, we have found out that expected value of beta cap, we found out covariance of beta cap. Also, we have found out the individual confidence interval for beta j also, you know now that, what is the confidence region for beta, with the help of beta cap. And from there using Bonferroni approach, you can using, yes, you can use this equation, only thing is that this will not be alpha by 2, it will be alpha by 2 into number of parameters p plus 1. So, that mean this will be beta j minus t p minus 1 alpha by 2 into p plus 1 square root of S E square C jj, less than equal to beta j less than equal to beta j cap plus t n minus p minus 1 alpha by 2 p plus 1 square root of S E square C jj.

So, now let us solve one problem with this whatever, we have discussed so far, we will solve one problem.

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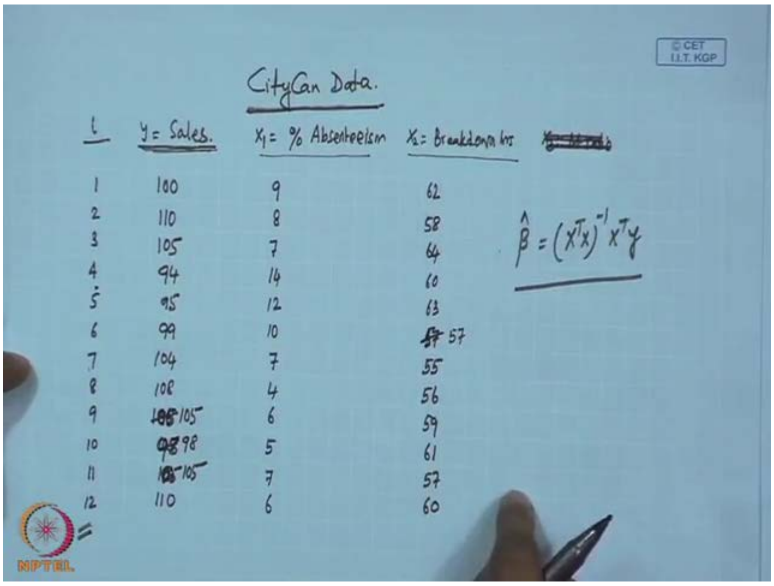

We will solve the problem earlier we have discussed that, city can data. We will consider the sales volume y, this is the data set 12 months data we have collected and x 1 absenteeism and x 2 breakdown hours, these two independent variables we are considering. We are not considering m ratio for this explanation because the computation will be much more, but you have to collect all possible data, for all possible relevant variables and then you have to develop the regression equation. With respect to this, what I want to know I want to calculate beta cap so first work is beta cap you calculate, this will be x transpose y. So, we will not go for calculation now.

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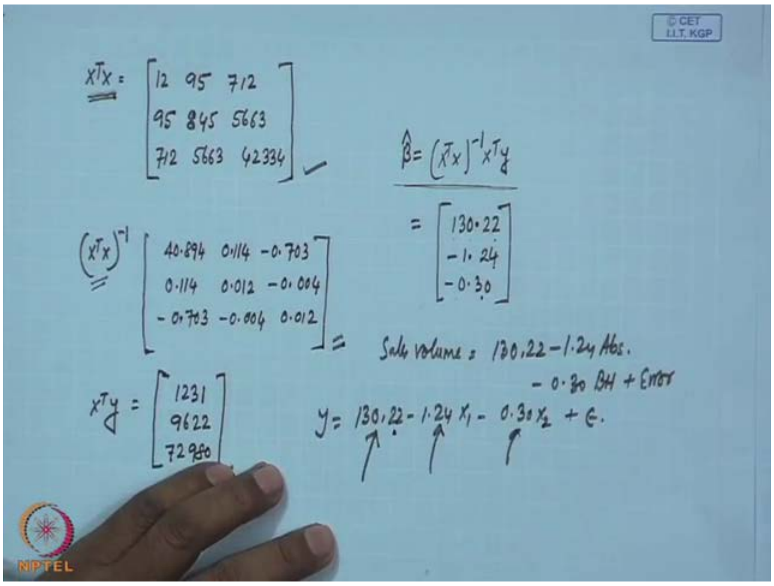

Because already we have seen that, this is the data matrix x transpose x, the conversion from x to x transpose x, these are the values, x transpose x inverse is also computed and this is the values. And x transpose y is this value and if you calculate this, then your values are minus 130.22 minus 1.24 minus 0.30. So, that means your sales volume equal to 130.22 minus 1.24 absenteeism, minus 0.30 breakdown hours let it be BH, plus some error will be there. So, other way we can write y equal to 130.22 minus 1.24 x 1 minus 0.30 x 2 plus epsilon.

Now, what do we want, we want that whether this 130.22 is really 130.22, it is not 0. You want to see that, what about 1.24 what about this and two ways we have seen, that one thing is that we will go for hypothesis testing then using t test we will see that, they are significant or not. Second one we have said that, you create the confidence interval for each of the origin population parameters.

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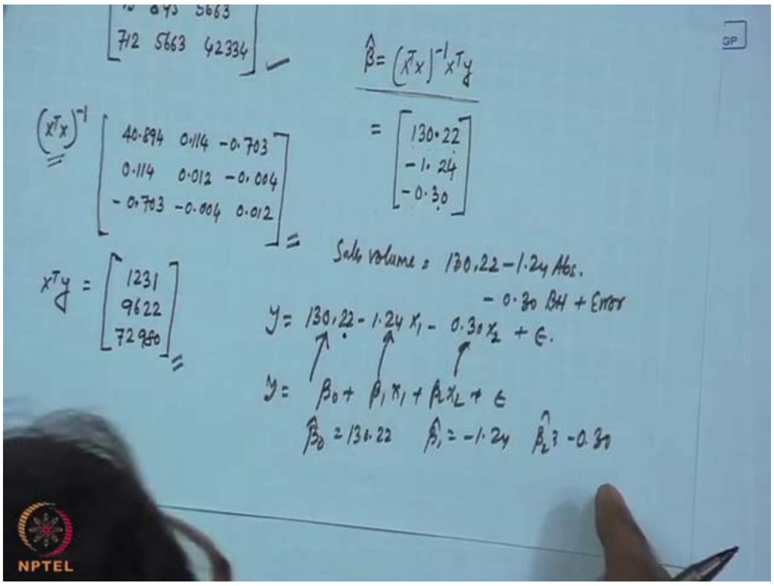

Here the population parameter is like this beta 0, beta 1 x 1 plus beta 2 x 2 plus epsilon, this 130 beta 0 cap is 130.22 beta 1 cap is minus 1.24 and beta 2 cap is minus 0.30. So, whatever we have developed so far now, that we have seen here with respect to this, what I want to do now, we want to find out the interval, confidence interval for, let for beta 0.

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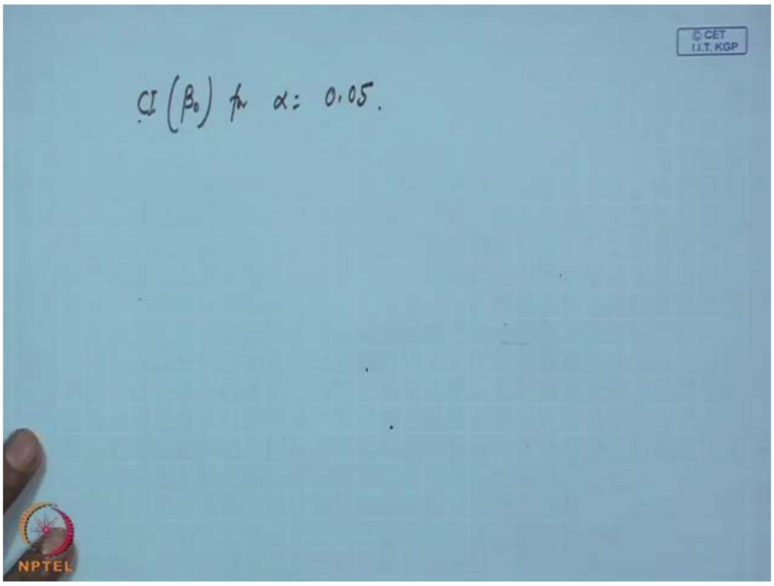

So, you find out CI beta 0 for alpha equal to 0.05 so what are the things you require for this.

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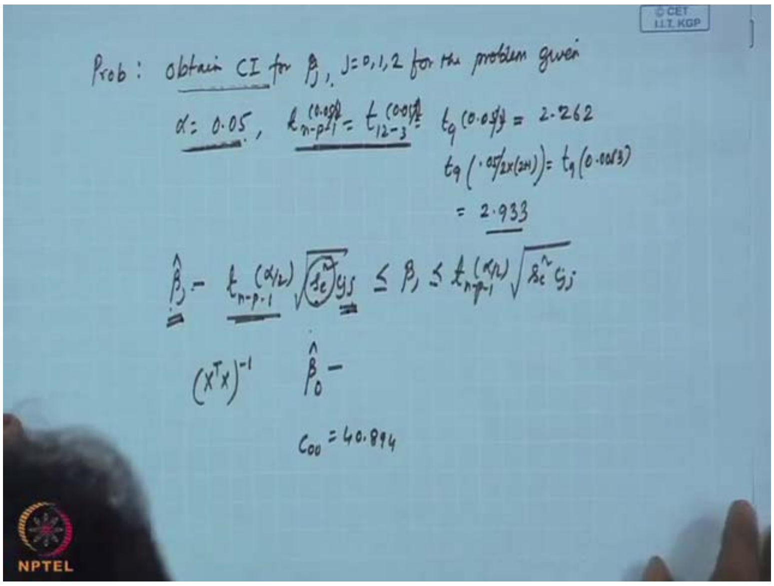

If you want to calculate this, what are the things you require as I written here, obtain confidence interval beta j, j equal 0 to 1 for the problem, what is given now. Alpha equal to 0.05 you require to know this quantity, you have already seen. You have already seen the formulation that beta j cap minus t n minus p minus 1 alpha by 2 square root of S E square C jj less than equal to beta j less than equal to t n minus p minus 1 alpha by 2 square root of S E square C jj. So, here our beta cap is beta j cap is known, we want to know this value S E square we have to compute, C jj we also require to know and like this now, how do we get this values.

So, if you require to calculate c j, what is this C j,j you require to know x transpose x inverse and I have already given you that x transpose x inverse is 40.894, this matrix you will be computing this matrix. So, that mean the diagonal elements are your variance part first one is for beta 0, second one is for beta 1, third one is for beta 2. So, if I go for beta 0 then beta 0 cap minus we will write this 1, but ultimately for beta 0 that C 00 will be taken C 00 is 40.894, this is our C 00. So, second one will be your C 00, third one will be your C 22 so you will take form this matrix this values, but you also require to know S E square. You are knowing this value fine, but what will be the S E square value correct so S E square, you have to find out error, that error you require to find out.

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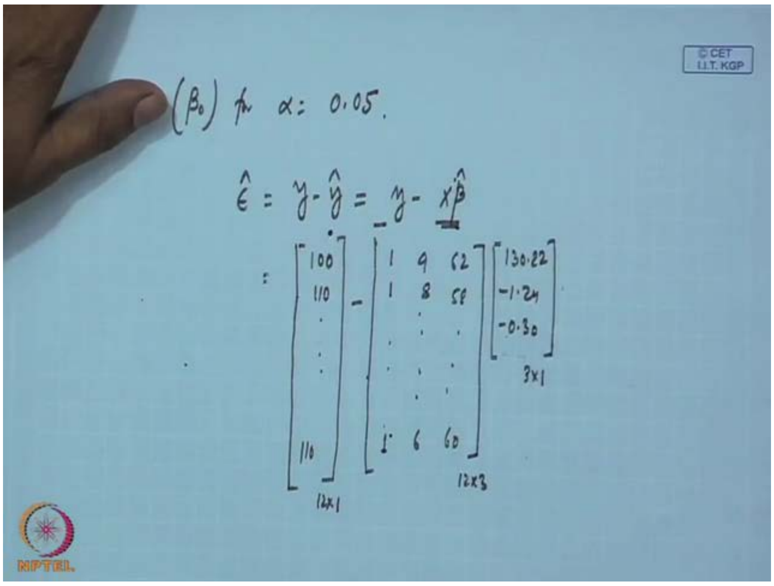

So, for this you require to find out epsilon cap, epsilon cap is y minus y cap, which is y minus x, I think beta cap, getting me? Now, if we go to the original data, original data so this is y now, using the formula x beta cap, you have to find out y cap. Now, then this will be nothing but if we write like this, I am not writing all values, some values I am writing 110.

So, like this finally 110, this value minus x value you have to write, what is your x value. When you compute, your x value will be all 1 then 9, 8 like this then 6, 62, 58 like this, 60. This is your x value so this one is 12 cross 1, this one also 12 cross 3 then your beta cap value already is there, what are those values, beta cap values you found out, estimated values 130.22 minus 1.24 minus 0.30 . So, this is 3 cross 1 so the resultant value will be like this.

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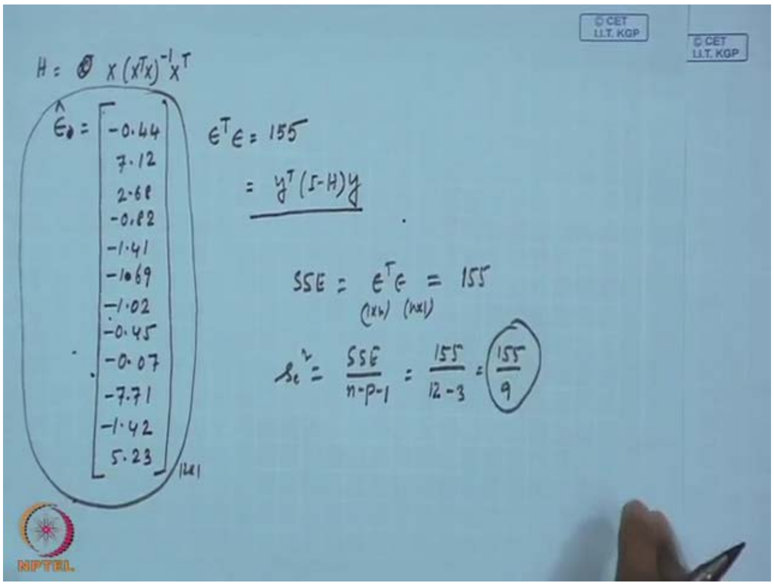

Beta cap will be like this, that is 12 cross 1, I required to compute this all those things, we compute and the difference will come like this. So, once you get this that means you know SSE, SSE is epsilon, transpose epsilon. So, it will be 1 cross n, n cross 1 that one value, this value if you compute it will be coming around 155. Because of rounding error, there might be here and there some error, but it will be around 155 then what is S E square, S E square is SSE divided by degrees of freedom, n minus p minus 1. So, we can write 155 by n is our 12 p plus 1 is 3 so 155 by 9. So, 155, this 9 is known that means what are the things known to you, now, what you require to compute. Here you require this.

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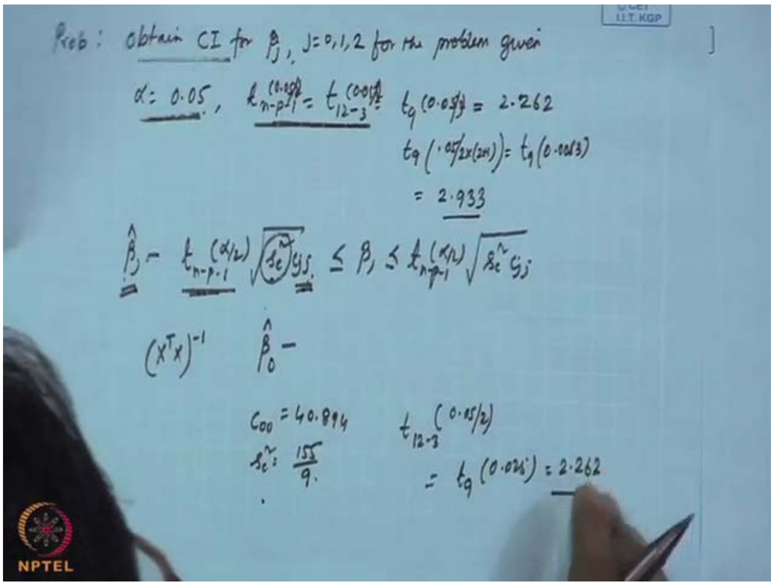

So, our S E square is 155 by 9 C jj is known, S E square is known now, we require to know what will be the t value, t 12 minus 3 into alpha, let it be 0.05 by 2. So, that mean t 9, 0.025, this value if we see in the table, it will be 2.262. Now, let us put all the values here.

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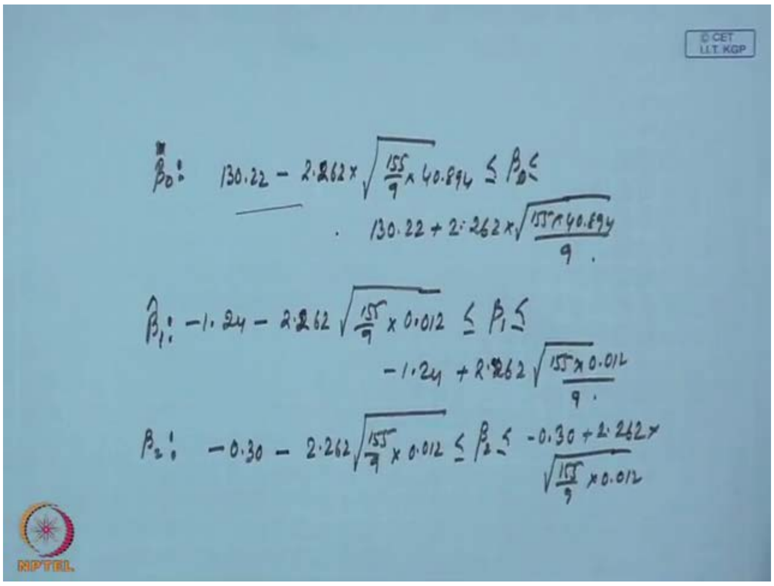

So, our beta 0, our beta 0 is 130.22 minus t 9, 0.025 minus 2.262 into square root of 155 by 9 into cj, C 00 is 40.894 that less than equal to beta j, less than equal to, you will be getting some 130.22 plus 2.262 into square root of 155 into 40.894 divided by 9.

Now you have to see that, what is this value how much it will come, some range we will be getting. Similarly, if you want to use it for the second coefficient beta 1, it will be minus, this is for beta 0 cap, beta 0 this is for beta 1 confidence interval, that minus 1.24 minus 2.262 square root of 155 by 9 into C 11. Now, C 11 value is how much C 11 value is 0.012 less than equal to here, it is beta 0, the second one is beta 1 less than equal to minus 1.24 plus 2.262 square root of 155 into 0.012 by 9.

So, beta 2, for beta 2 what will happen beta will be minus 0.30 minus 2.262 square root of 155 by 9 into, I think the C 22 also same value here, 0.12 less than equal to beta 2 less than equal to minus 0.30 plus 2.262 into 155 by 9 into 0.012 you will be getting all the intervals. Now, if any interval content 0 then that parameter is not significant. This is in nutshell what is basically, what we will talk about that sampling distribution of, sampling distribution of beta cap, sampling distribution of beta cap, but similarly what will happen.

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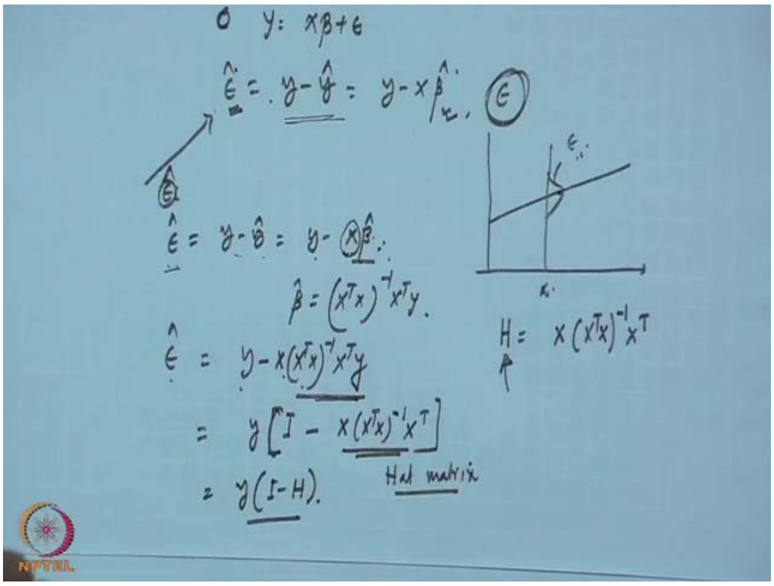

Suppose, you are, you know that y equal to x beta plus epsilon. Now, also we know that this one is y minus y cap that means, y minus x beta cap, you may be interested to know what is the distribution of this. How do I find out, can we not get this now the, is the, this distribution similar to epsilon please, when we define in the population domain, we say this is epsilon. If it is x i then epsilon i the distribution now, you are finding out epsilon i cap, for this case. So, similarly that is why I am saying, that epsilon in general that cap will it be same the, as the population epsilon, it will not be the same.

So, here because this distribution, the distribution will be governed by this beta cap, what you are estimating here. So, little bit some clue I am giving you here, this is y minus y cap equal to y minus, what he says that I think x beta cap we have used earlier, y minus y cap y. y equal to x beta and x beta cap suppose, I want to write down instead of beta cap, I will write down x transpose x inverse x transpose y.

So, then what will happen, this one will become y minus x transpose x inverse x transpose y, but one x is there. So, you write x here please, go through y epsilon equal to y minus x beta cap. So, I am writing the same, y minus x into beta cap is x transpose x inverse this. Now, this one I can write like this, I minus x, x transpose x inverse x transpose, this matrix is known as hat matrix.

So, hat matrix is denoted as H, H equal to x, x transpose x inverse x transpose. Hat matrix is very popular in regression and in diagnostics of MLR, multiple regression this hat matrix will be used. It is the projection property basically, this one is the projected one from the, that direct planes x plane and y plane so these are the projected ones. So, that mean, we can write this one as y I minus H so you require to your, I minus H into I think, I minus H into y. You have to see that the, this one matrix multiplication that comfortability.

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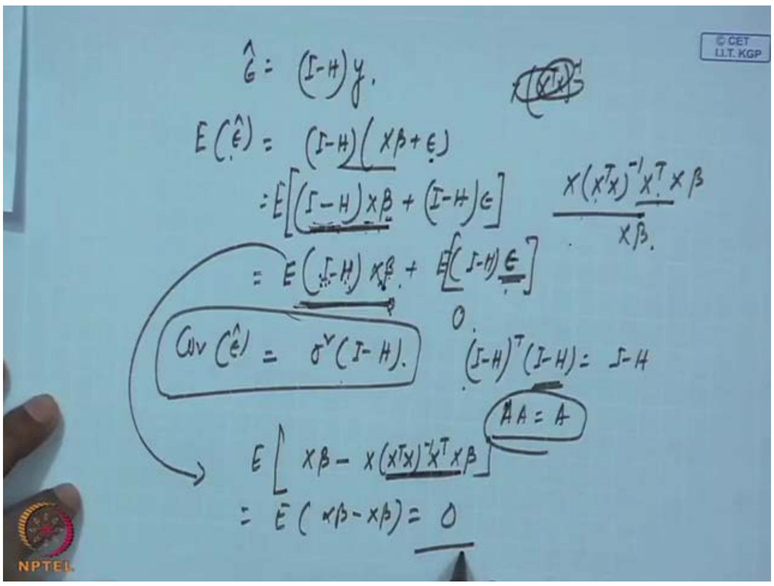

So, we can write I minus H epsilon cap equal to I minus H into y, I think this is comfortable one, this is comfortable one. Now what will be the expected value of this then I minus H, I will write again x beta plus epsilon here. See, this cap and this is not same so you can write this I minus H x beta, I minus H epsilon. You require to give an expectation operator here.

Now H is your x transpose x inverse now, x I think what we will define by as x, x transpose x inverse x transpose that is our H, these are all fixed values. So, that means I minus H is fixed values, x is also fixed value. Now, beta is constant so I can write this one, now straight way I can bring it to expected value of this, beta of this, I minus H, x beta. This is the, these are the fixed values and expected value of I minus H epsilon, what will be the this, what will be this one, expected value of epsilon.

Student: 0

That will be 0, what will be this one?

Student: It will be the fixed value

Fixed value. So, that means some value will be there so similarly you require to find out the covariance matrix of this. The resultant covariance matrix will be, I am straightaway righting the resultant part, this will be sigma square I minus H. So, I minus H, I minus H transpose those things will also come so I minus H suppose transpose I minus H, this will become I minus H only. Because, the property of this I minus H this is, this is a matrix which is known as idempotent matrix, that A into A equal to A. So, the resultant value will be this so you know the, this value, I think we have to, this one is x beta, if we just do little bit manipulation also, because I minus H is there.

Because, beta is there beta will come, this also will become 0, you just do manipulation what will happen this is x beta minus H into x beta [FL]. So, I will write x beta here so that means x transpose x, x transpose that is I, that will be x beta. So, that I minus H x beta, what will happening this quantity will become E, I am again writing x beta minus H is x, x transpose x inverse x transpose x beta. This quantity what I, so that will become x beta minus x beta so that so it is a fixed value definitely, but ultimately that fixed value is 0.

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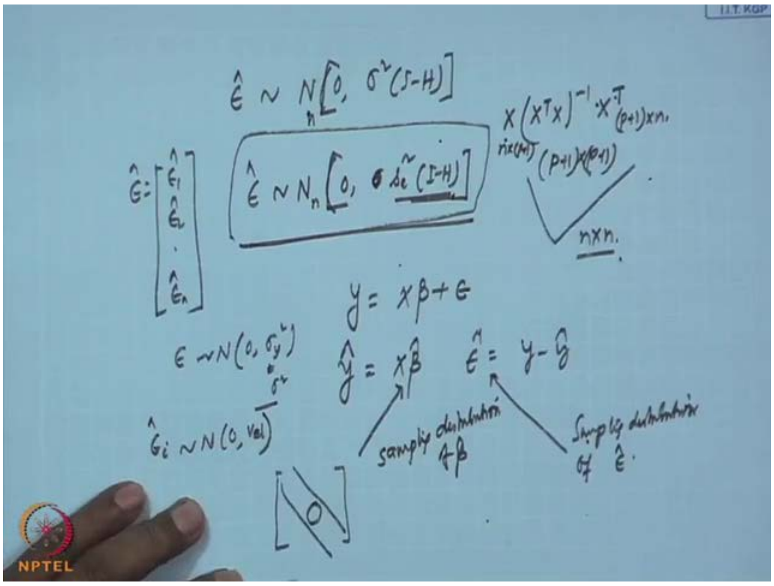

So our, now the distribution of this, this one is normal distribution with 0 and sigma square I minus H, will it be multivariate or univariate.

Student: Univariate.

Why? Why? What is, what is x transpose x inverse, this is p plus 1 cross p plus 1. If we multiply it with, what is this x is there then x transpose is there, what is x, x is n into p plus 1 and this one is p plus 1 into n. So, what will be the resultant quantity?

Student: m cross n .

m cross n? so this is multivariate normal with n so n you are getting epsilon cap, this you will be getting epsilon 1 cap, epsilon 2 cap like this epsilon n cap, all values. So, it is n, you can write this is multivariate normal with N 0, sigma square I can write S E square.

I think, you now, with respect to this y equal to x beta plus epsilon we have computed that, y this one is this one and epsilon cap is our y minus y cap. So, with related to this sampling distribution of, sampling distribution of beta cap also you now, you now know what are the mean and covariance. And this also, this is the distribution of epsilon cap so sampling distribution of, what is the learning here, apart from this beta all those things that, epsilon cap its distribution is not the distribution of epsilon. We have assumed the error terms, in our population domain this we say it is, normally distributed each error is normally distributed 0 mean and sigma y square.

And this sigma square,we have written like this, but you are getting when you are calculating the error terms, that we are getting in multivariate domain, you are getting a multivariate normal distribution for the error term. Now, if you take a particular suppose epsilon i cap, what will happen this definitely univariate normal with 0 and what will be mean 0 and what will be the variance component. Variance component will be this one so that particular variance component you have to find out because it will be a matrix of you have seen already, you have seen it is a matrix of n cross n.

So, you will be having n cross n matrix so ith term if I, if I can write variance v e, v e i, v e i that is the term, I am just, term I am writing here. I think the basic statistics part is very very important that is why, everywhere you are coming to some point when, the statistics to be, to be used. And that, the sampling distribution of the statistics is to be known otherwise, you cannot do this. Because many a times you will be using why t distribution? Why not other distribution and how regression parameters are becoming, following t distribution similar things will be there. So, we have covered up to sampling distribution, next I think we will go for goodness of it. Next class I think, tomorrow if possible.