Applied Multivariate Statistical Modelling
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Lecture - 13
Multivariate Inferential Statistics (Contd..)

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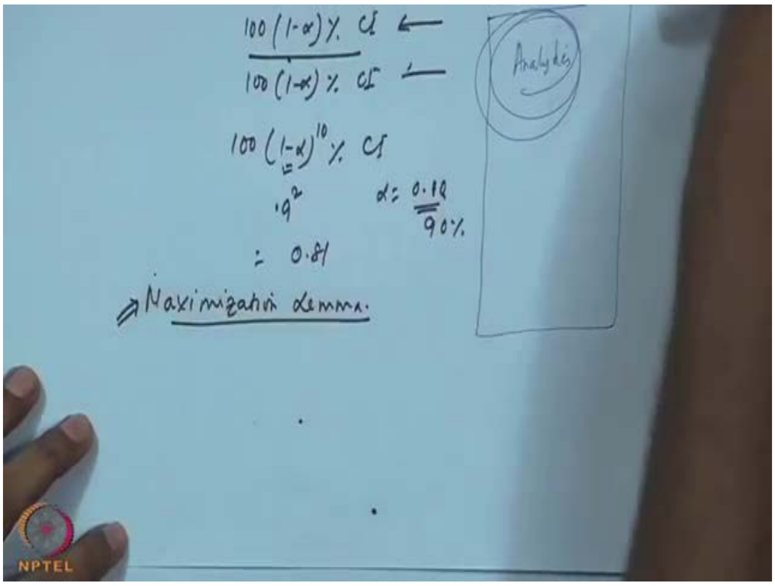

We will continue that our maximisation lemma to find out the simultaneous confidence interval this one.

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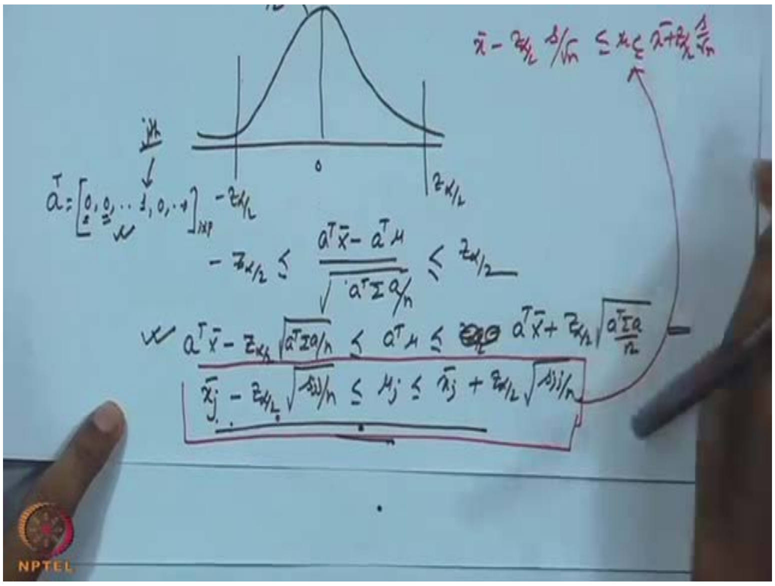

What I told you that this quantity let a transpose X bar minus this one, if you select something like this for a, you will get like this, and which is basically like a univariate interval. And your problem will lie here that you will not get simultaneously 90 percent or 95 percent confidence interval, you will go you require some modification.

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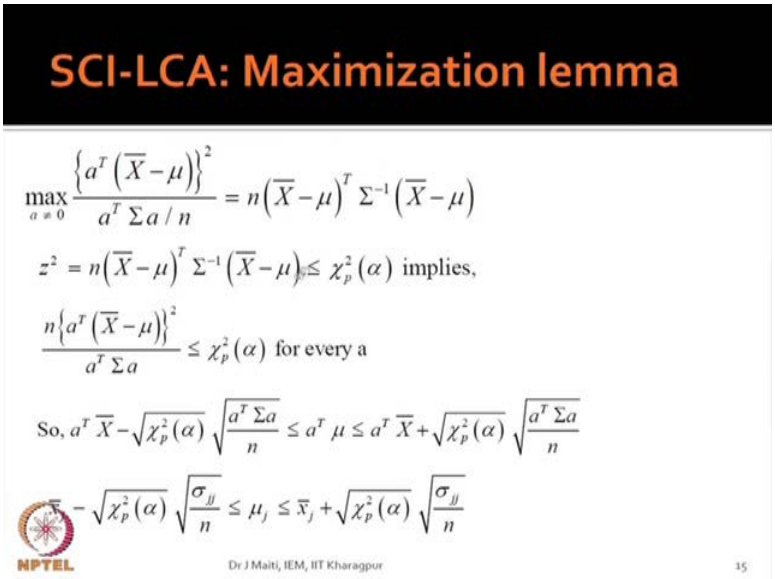

And that modification is given here by using maximization lemma that Z square n X bar minus mu transpose. And this quantity follows Chi square distribution. Now, with this modification you write the earlier equation what have you written earlier you have written a transpose X bar minus Z alpha by 2, and this less than equal to this less than equal to this. So, this Z alpha by 2 will be replaced by square root of Chi square p alpha.

Student: ((Refer Time: 01:50))

What sign?

Student: ((Refer Time: 01:51))

You see this one a transpose X bar minus Chi square this square into this variability component then a transpose mu less than equal to a transpose X bar plus the same thing. What is the problem?

Student: ((Refer Time: 02:17))

Before also same thing I have written. This minus this less than equal to this less than equal to this plus, this is plus.

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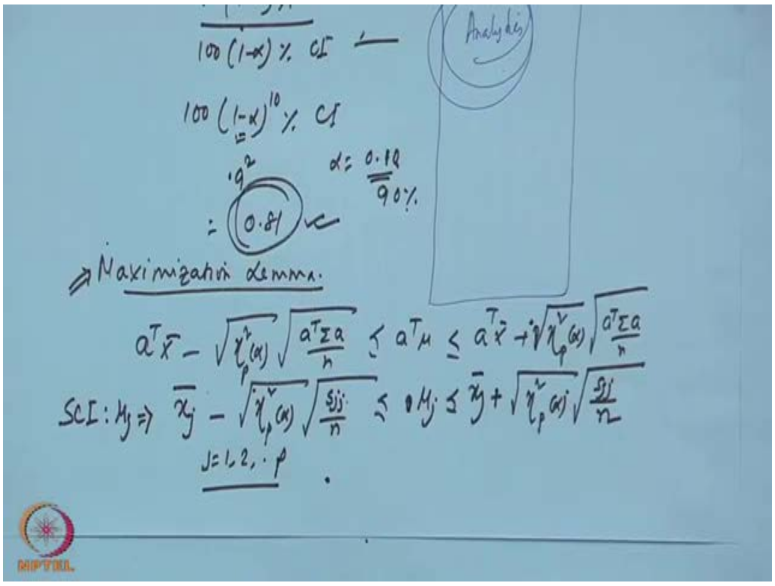

This change is like this a transpose X bar minus root over Chi square p alpha. Then the same thing you write a transpose this divided by n less than equal to a transpose mu less than equal to a transpose X bar plus again you write the Chi square p alpha. And same thing that a transpose this by n. Now, if you select a similar manner then your individual confidence interval will be X bar minus Chi square p alpha square root of again S j j by n less than equal to mu j. This will be mu j less than equal to x j bar plus Chi square p alpha square root of S j j by n.

This is the interval simultaneous confidence interval for mu j and if you change j j equal to 1 2 p you will be getting for all other variables. What is the confidence interval, it seems very difficult or easy. So, when you read that book you read in this approach what way I am presenting here, otherwise you will find it very boring at the first time when you see the Johnson and Wichern, you will find that I will never read this book. That is the purpose of this class purpose that I am making it as simple as possible, so that you will all feel like reading that please.

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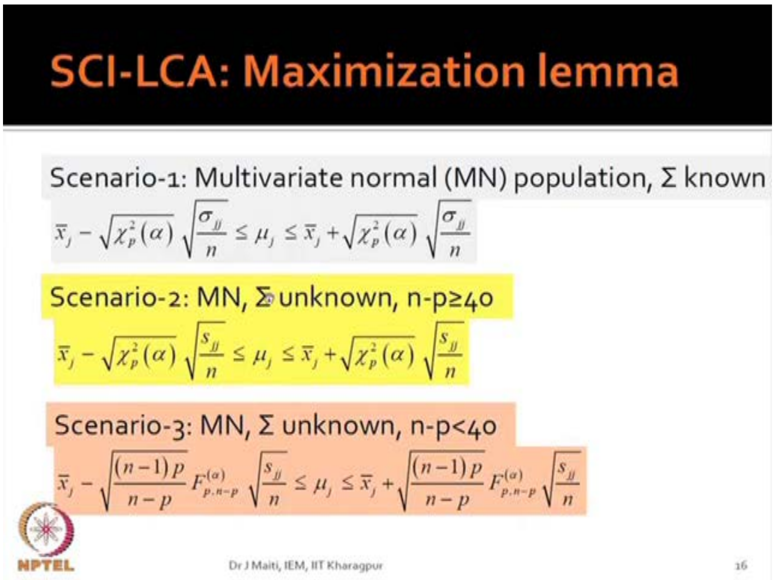

So, now the 3 scenarios we have seen earlier you tell me are you facing problem here simultaneous confidence interval scenario 1 sigma is known. So, as sigma is known you are using sigma j j because if sigma is known your it is better to use that sigma why we will go for some variance. If your sigma unknown and large sample size you are using S j j you are also using Chi square distribution. When it is small with small sample size and m n multivariate normal plus sigma unknown you are using f distribution. Can you find out the confidence interval for the problem given you develop, I am giving you 5 minutes time although I know that 5 minutes time is very big for us, but you please develop.

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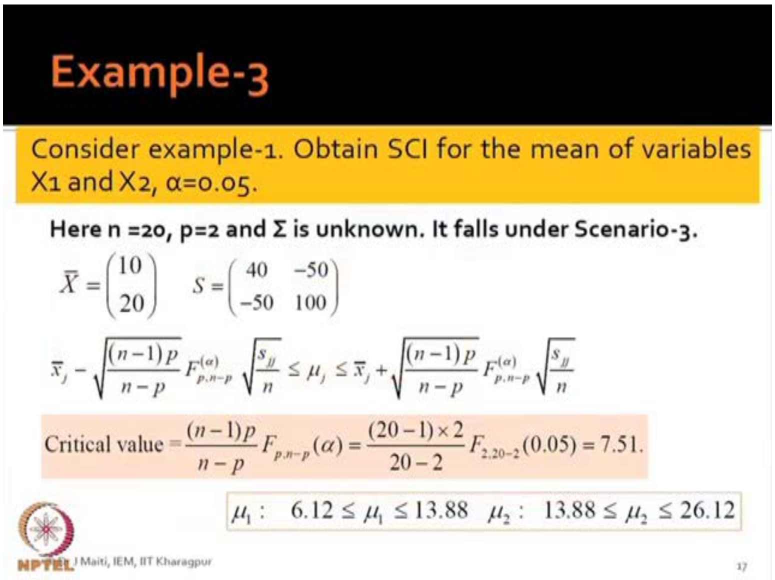

What I say that you have to develop simultaneous confidence interval for the mean of the variable x 1 and x 2 given alpha equal to 0.05 everything is the answer is with you. How do you go about it everything is given here you see x 1 bar is 10, x 2 bar is 20. And your S11 is 40 and S22 is 100 and you require to know n minus 1 p by this quantity, what will be the value. Earlier we have already seen that this quantity the critical value is 7.51 just you put into this equation.

So, 10 minus 7.51 that square root into square root of 40 by your n is 20, less than equal to mu 1 less than equal to 10 plus this quantity. I am sure that possible and you will and you see that these are the intervals.

Student: ((Refer Time: 07:35))

Which one?

Student: ((Refer Time: 07:37))

Whole square root including F p n minus p alpha, this is whole square root. So, I think right hand side find out for mu 1 and left hand side for mu 2 right, this side people find out for mu1 left hand side. Please find it out for mu 2 cross check x 1 it is then x 2 will also be. So, we will go to the next slide I hope that you all will be you have already found out the mu 1. So, mu 2 also it is possible not a big issue.

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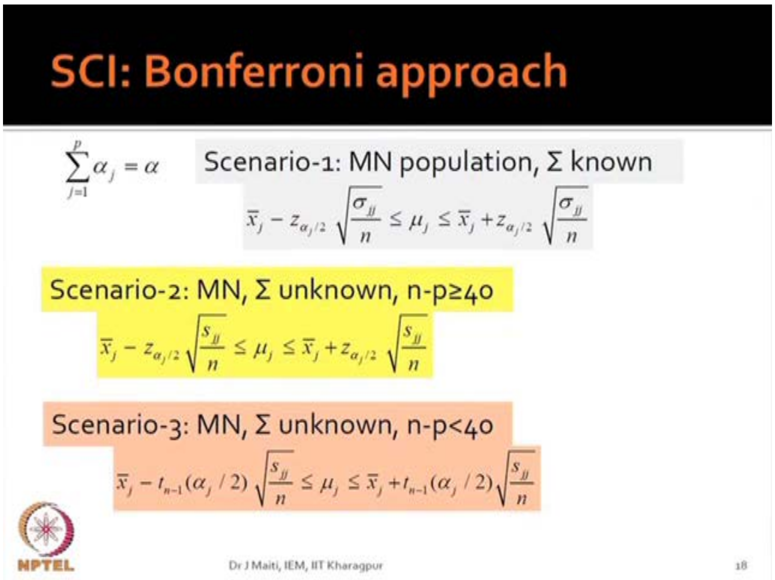

Then what is Bonferroni approach, Bonferroni approach is different than maximization lemma.

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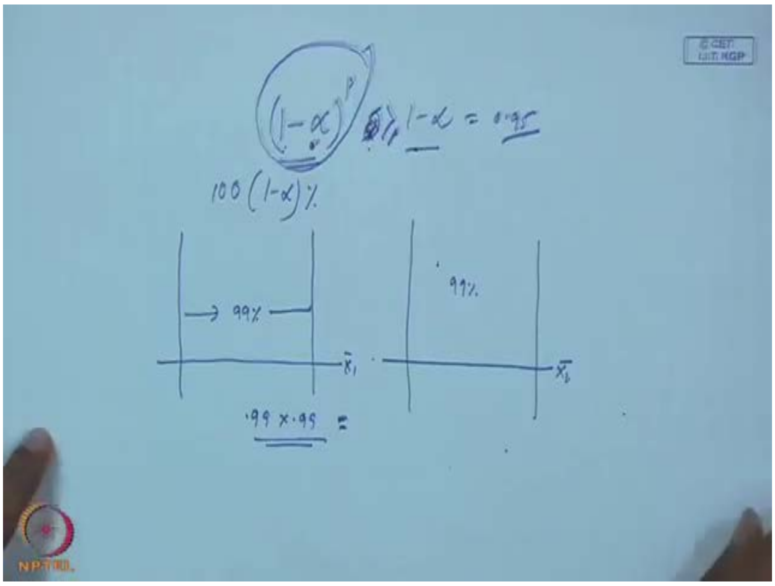

Bonferroni approach says that our sole problem is when I have large number of variables I am not getting this one. Now, this quantity definitely is less than 1 minus alpha because alpha 1 minus alpha is always less than 1. So, what Bonferroni says that if we want to find out collectively 1 minus alpha into 100 percent confidence interval. So, you tighten the individual interval you are getting me. Like my first variable individual interval, let it be 99 percent you are finding out. Then second variable also you find out the individual interval 99 percent then what will be the total if you multiply 9.99 into 0.99.

What will be this quantity? You just check it will be also a large quantity if my interest is to get 95.95 percent. Probably, it is more than 0.95. So, you have to choose judiciously so that the ultimate this one minus alpha to the power p will can be like this. What I can say it will be always like this, this should be greater than that is my requirement. As a result he has given a formula access that you can give equal weightage to each of the variables or you can give differential weightage.

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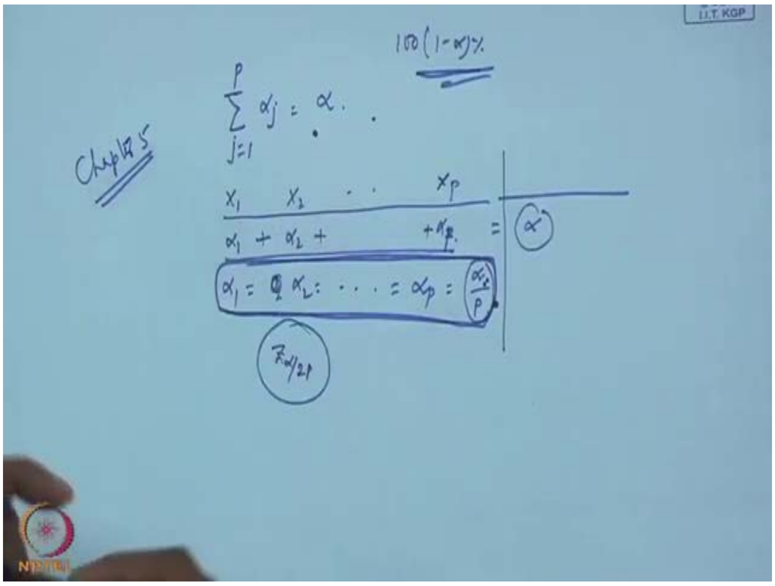

So, let alpha is like this my j is 1 to p, there are p variables. And I am giving each variable with alpha j way, that will be alpha. What is happening her. So, you have suppose p variable x 1, x 2 to that x p variables are there. Now, this is alpha 1, alpha 2 you are giving probability value for each like this alpha p. Now, my question is this plus this plus this, this is alpha. If you give all variable equal weights then this is your alpha p then this will be alpha by p, if you give equal weight to each of the variables. So, there are p variables your total should be alpha you will be getting alpha by p.

And what will be this alpha, that all depends on what is your 100 into 1 minus alpha percent confidence interval. And what I can say you have to check this one. Now, let us see that using this for different scenario, what are the confidence interval? You see the first one and you tell me whether you are comfortable or not.

First one we have already seen that is Z alpha by 2. Now, instead of Z alpha by 2 you are writing Z alpha j by 2, and here also Z alpha j by 2. Now, if you give equal weightage then this will be Z alpha by 2 p, when your this condition satisfied. This condition satisfy, this will be Z alpha by 2 p, this will also be Z alpha by 2 p. So, here you are using Z here also you will be using Z, but here your sigma is unknown large sample size. Your S will replace that sigma absolutely no problem. Third one instead of F you are using T distribution. So, my request to you that to chapter 5 of Johnson and Wichern you please go through. And see that this derivation as well as whatever explanation given there, if there is any problem please report to me.

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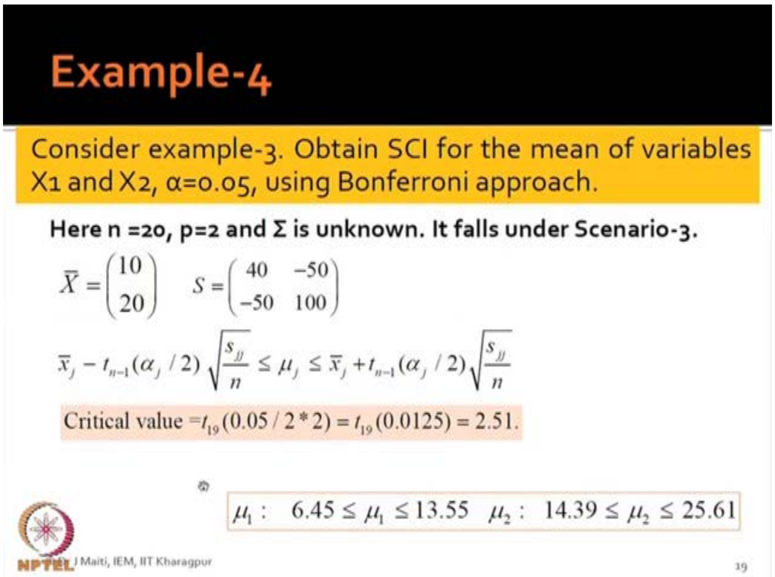

Then what is essentially the difference between Bonferroni and this approach maximisation lemma, which one is easier to you Bonferroni, easy way just you divide the total alpha 2 different variables, collectively the sum will be alpha. Now, there is a problem example four consider, example three obtain simultaneous confidence interval for the mean. So, t n minus 1 alpha j by 2 that is t 19 0.0125, how we have got 0.0125 because our alpha is 0.05 then there are 2 variables. Then alpha by 2 is 0.025, again the 2 tailed further divided by 2 you are getting 0.0125. This value is this and once you put into this equation all these values you will be getting 6.45 less than equal to mu 1 less than equal to 13.55.

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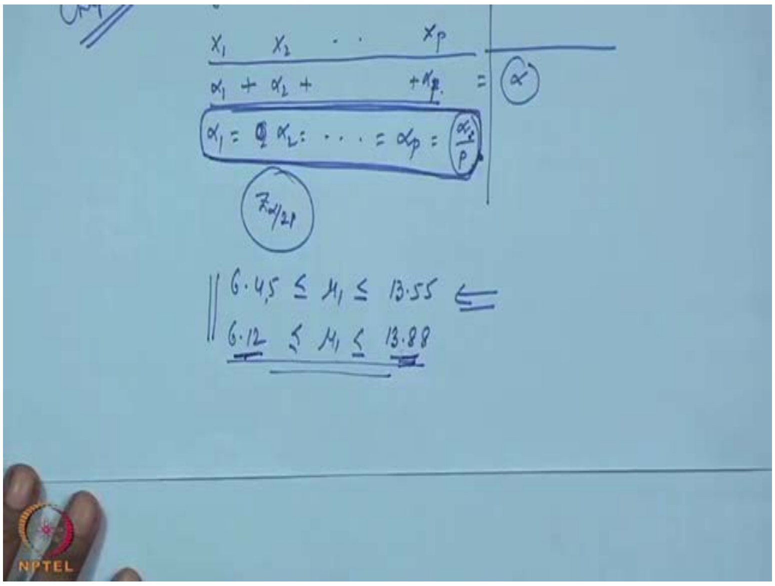

So, I am writing this 6.45 less than equal to mu 1 less than equal to 13.55. Using maximisation lemma, what was the interval we got?

Student: 6.12 less than equal to mu 1 less than equal to 13.88.

There is little difference almost this 16.12, this one is less than this, this one is more than this. So, little difference is there that difference will always be there because you are using different alpha values and here it is T distribution there it is F distribution.

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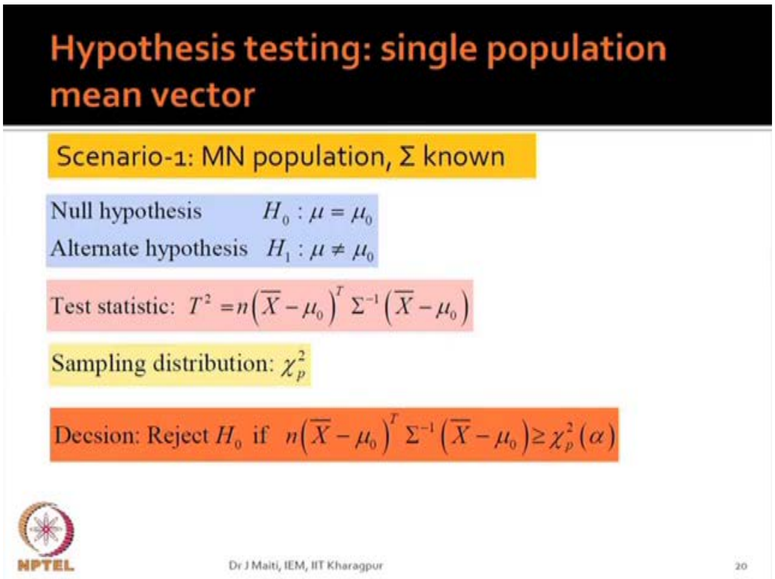

Now, hypothesis testing, this is the last topic today what is hypothesis. I told you in the last class yes or no.

Student: ((Refer Time: 16:51))

That is yet to be proven, any statement and definitely it should be a good statement it is not that, I am going there you are also going there this type of statement scientific statement. Any scientific statement that that is yet to be proven that is your hypothesis. And you have seen hypothesis in univariate case also.

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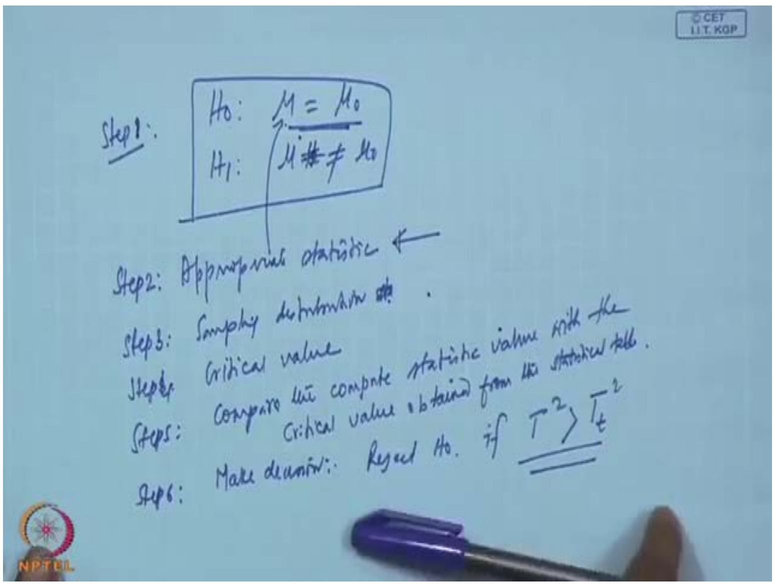

Now, in multivariate case what will be your null hypothesis. When you are talking about single population that related to mu we are talking about. So, mu equal to mu 0, what is my alternative hypothesis mu naught equal to mu 0. Very simple what is this mu?

Student: ((Refer Time: 17:54))

This is mean vector please remember whenever we are writing like this in multivariate domain although we are writing mu many a time you commit mistake. Here you think that this is 1 mu that scalar quantity it is not like this. So, in hypothesis testing the steps are find out the null hypothesis and alternative hypothesis that is your step 1. What will be your step 2, what I have given you in univariate case you have to find out the appropriate statistic. So, find out appropriate statistic then what will be your third step.

You must know the sampling distribution know sampling distribution of that statistic. Then what is the fourth step you definitely compute the value of the statistic from sampling distribution of the statistic you find out the critical value. Then your step 5 is compare the computed statistic value with the critical value obtained from the statistical table. You will use table and appropriate statistical table, then your step 6 is make decision. What will happen if your computed statistic is less than the tabulated value.

Student: ((Refer Time: 20:04))

Null hypothesis cannot be rejected. So, decision is reject null hypothesis H 0. If in this case we are saying T square that computed is greater than that particular tabulated value you have to see that what is this T square tabulated value. It all depends on the distribution, if it is Chi square then you have to use the Chi square value here, if it is F distribution go for F distribution like this. See this slide your first scenario Chi square distribution and this is the case reject H 0, if this quantity is greater than Chi square p alpha. See already we have computed all those values. In the earlier example you have computed this value also you have computed I think that this value we have not computed mu 0 is given now.

One of the issue here you must remember that whenever we are talking about sampling distribution of a statistic like this T square is this. We are saying that when null hypothesis is true these follow these statistics follow this sampling distribution. Please never forget this one. So, if you change your null hypothesis to something like this that mu less than mu 0, then you will not get the appropriate distribution because it is not developed maybe developed we do not know, but what we know here that is our null hypothesis is mu equal to mu 0. Then only this quantity's square this is Chi square distributed where T square is this where sigma inverse is there. So, please do not change your null hypothesis.

Student: ((Refer Time: 22:09))

Actual means there are two hypothesis in this case null and alternative hypothesis. Null hypothesis is someone who is it is a statement, which is not yet proven that hypothesis you are talking about. There is no actual hypothesis something like this it is basically it is a proposition you do not know what is that true things. You are trying to prove it based on data analysis. So, there are two hypothesis null and alternative hypothesis.

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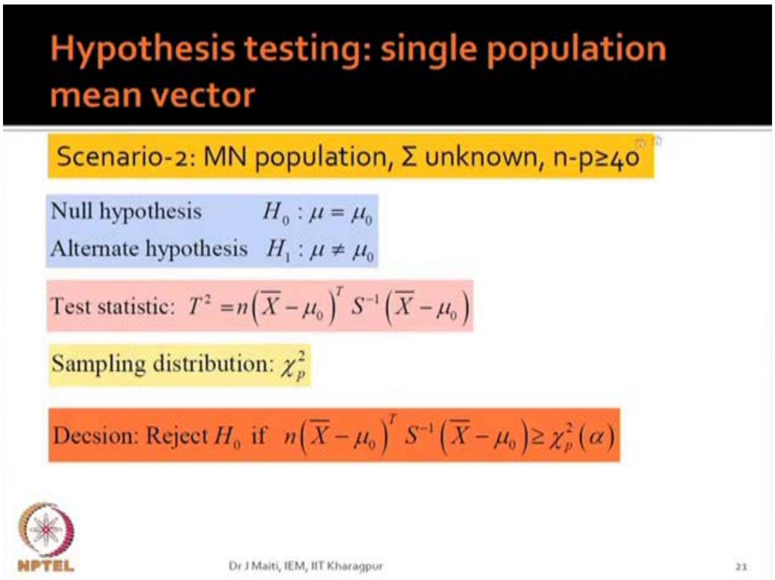

In case of population your scenario 2, your test statistic is this distribution is this and reject like this. So, do not look at the equations you just think the message what I am giving you that is more important because whatever, I am giving to you ultimate aim few things.

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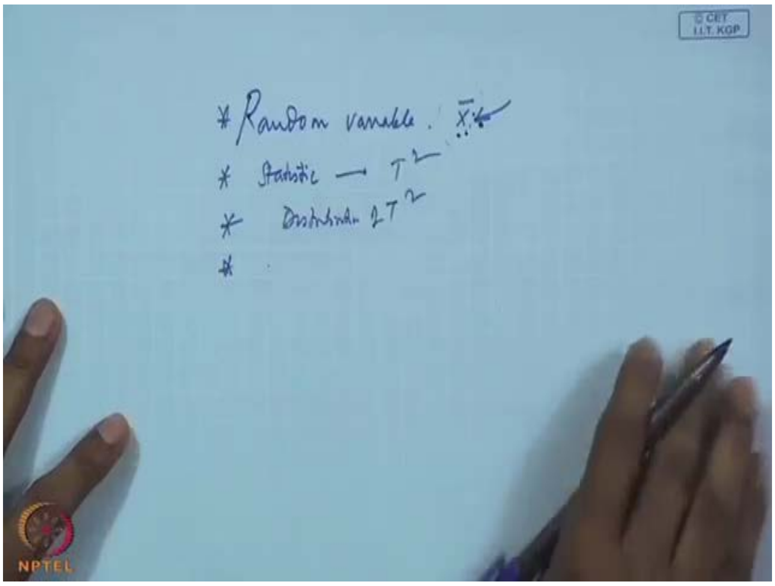

That must be remembered by you one is what is your random variable. For example, X bar is also a random variable. Then you are creating statistic using appropriate statistics using this random variable that this also is a statistic, but for finding out some distribution you may not get distribution of X bar. You may get distribution of something else here x distribution is also available, but for computational purpose we are converting into Z or T square or something like this.

Then you must also know the distribution of this statistic. Here it is distribution of T square and then you have to find out that what are the values. And this is the steps whether it is T square distribution Z distribution Chi square distribution T distribution all cases things to be followed exactly the same. So, do not forget that these equations may be big, but these equations are no way hampering your understanding.

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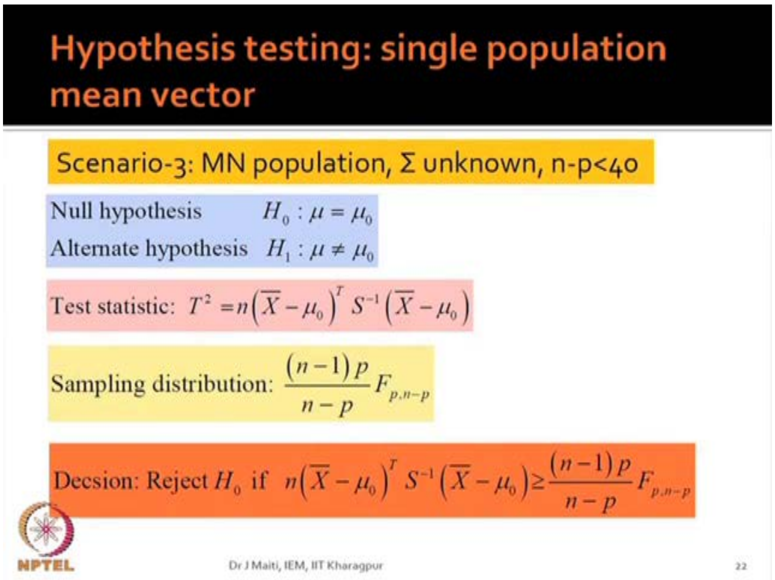

Then scenario 3 you all know that you will be using the sampling distribution F. So, again your decisioning will be like this.

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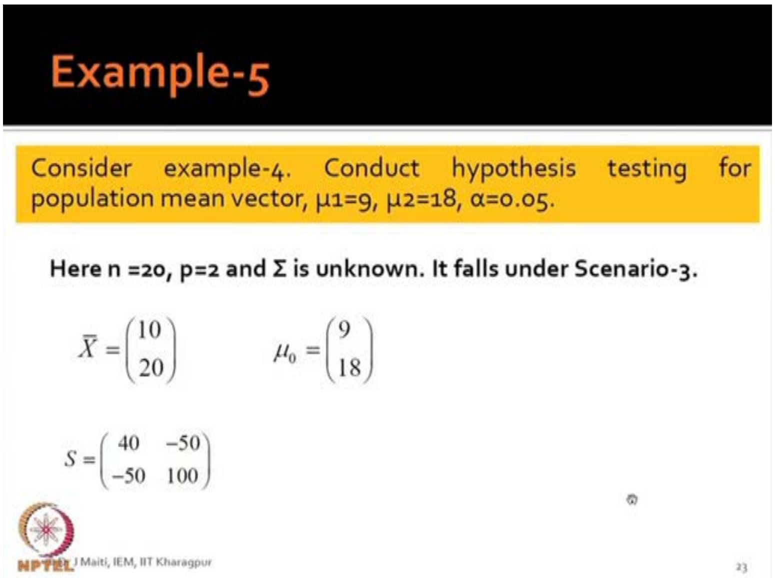

Now, if this is the case then one problem again consider example 4 conduct hypothesis testing for population, mean vector mu1 equal to 9 and mu2 equal to 18 alpha equal to 0.05. See X bar is given mu 0 is given what is your null hypothesis here, mu1 mu2 equal to 9 and 18. So, you write down this X bar is 10 20 the same problem I am discussing that mu 0 is 9 18 and S minus 40 minus 50 100 and this is the calculation.

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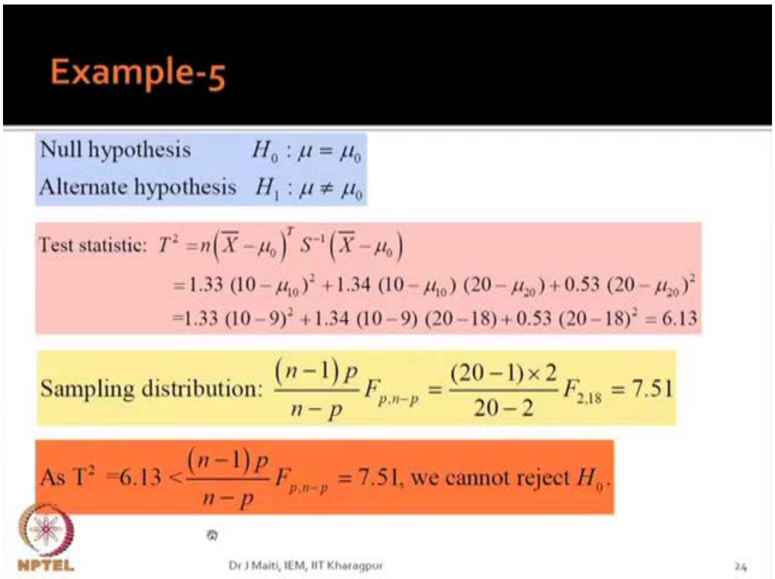

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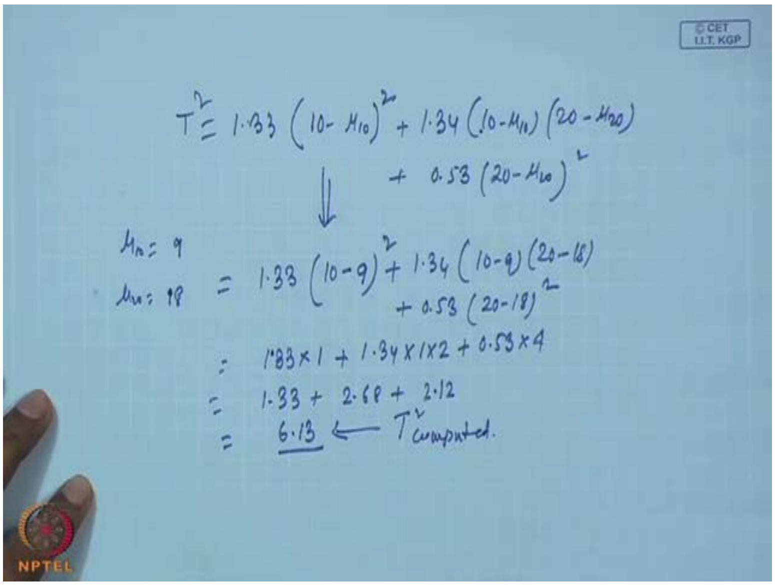

I think you all know that we have already find out that the t square is 1.33 10 minus mu 1 will be mu1 0, so mu 10 square 1.34 10 minus mu 1 0 20 minus mu 2 0 plus 0.53 20 minus mu 2 0 square. What we have found out that mu 1 0 this 1 is your 9 mu 2 0 we have found out it is your 18 put into this equation. If you put into this equation this is 1.3 10 minus 9 square 1.34 10 minus 9 into 20 minus 18 plus 0.53 20 minus 18 square you are getting a value 1.3 three cross 1 square that is 1 plus 1.3 4 into 1 into 2 plus 0.5 3 into 2 square that is 4. So, what will be this value 1.33 plus 2.6 8 plus 4 into 3 12 2.12. So, this is 6.1 3 correct. So, this is what is computed T square T square computed from data what is your tabulated value?

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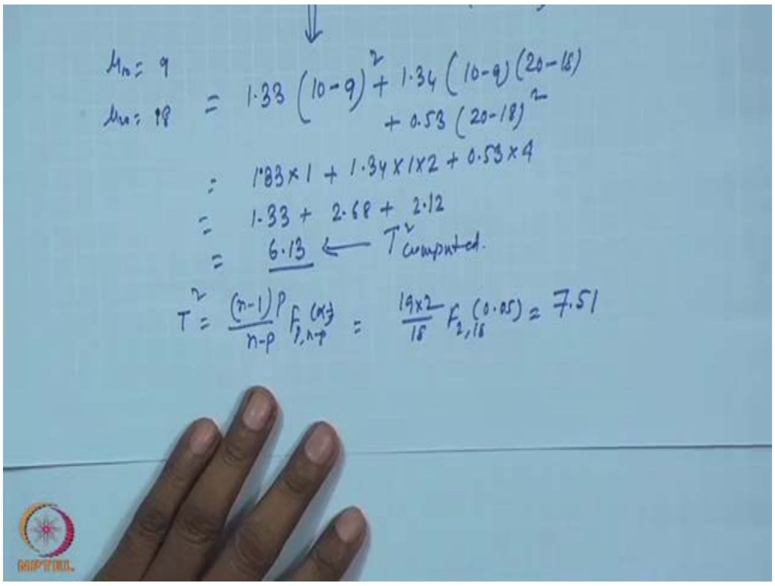

We say it will follow n minus 1 into p by n minus p F p n minus p. And you have to find out some alpha value alpha is 0.05 like alpha. So, that we have already seen that 19 into 2 by 18 F 2 18. If you write it 0.05 this resultant quantity is 7.51. So, reject yes or accept why reject.

Student: ((Refer Time: 28:18))

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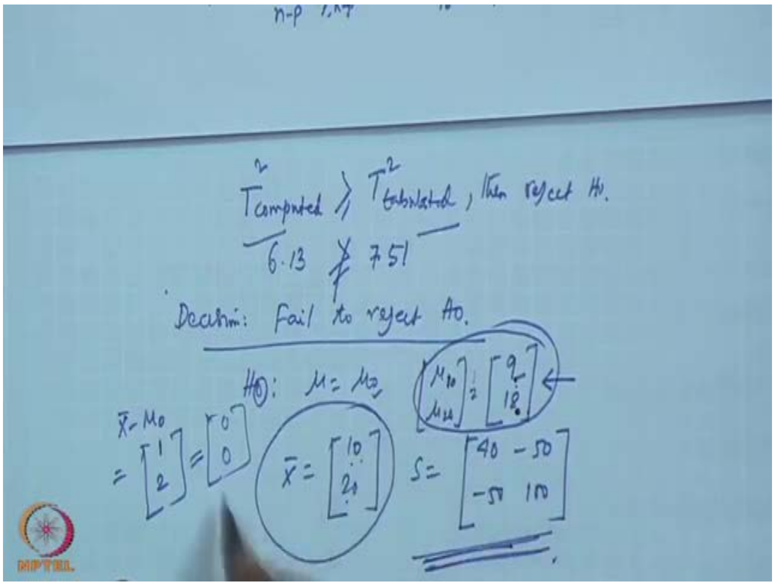

You will accept or reject? If T square computed is greater than equal to T square tabulated then reject H 0. So, our computed one is 6.13 is it greater than equal to 7.51. So, our decision is fail to reject H 0.

Student: Sir, what is the null hypothesis ((Refer Time: 29:13))

Mu equal to mu 0.

Student: ((Refer Time: 29:15))

No your null hypothesis H 0 mu equal to mu 0 what we are saying then suppose I have written 0 like they are basically mu 1 mu 2 that 0 stands for null hypothesis. That this one is 9 and 18, this is your null hypothesis what you have X bar equal to 10 and 20 and you have S that is what is 40 minus 50 minus 50 and 100. And now, you know what is the population correct statistics and based on this T square computed T square tabulated both is computed. And you see if you see this value what is X bar 10 20. Now, you are creating one hypothesis null hypothesis like this mu 1 0 and mu 2 0 this just to give this 0 notation here 9 and 18.

So, 10 minus only 1 and 2 that difference X bar minus mu 0 that difference is 10 minus 9 is 1 and 2. The statistics based on this sample covariance matrix when you are computing like this you are not finding out this one is similar to 0, 0 getting me that difference what you have assumed and coming there is not much difference.

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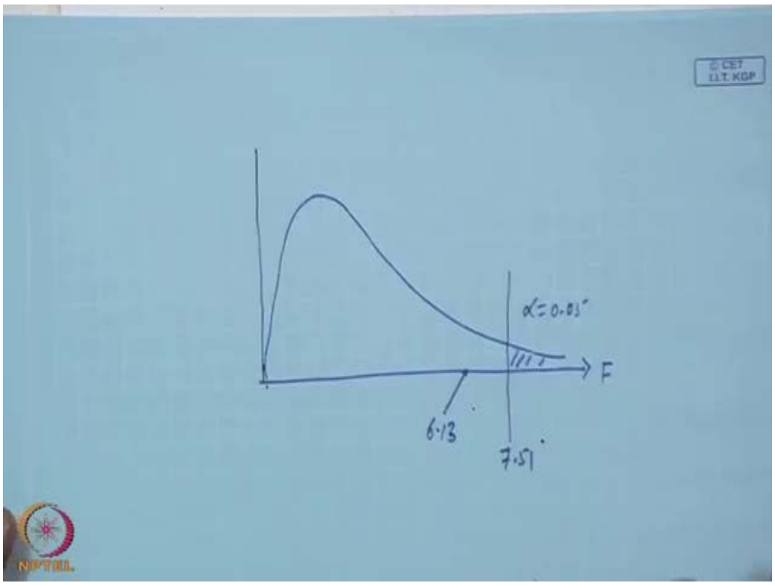

Actually what is required to know here your Chi square distribution I think we have taken F distribution. F distribution this is your distribution when you are taking alpha value is 0.05 this is what we are getting here 7.51. And your value is coming somewhere here 6.13 it is within this interval you cannot reject the null hypothesis.

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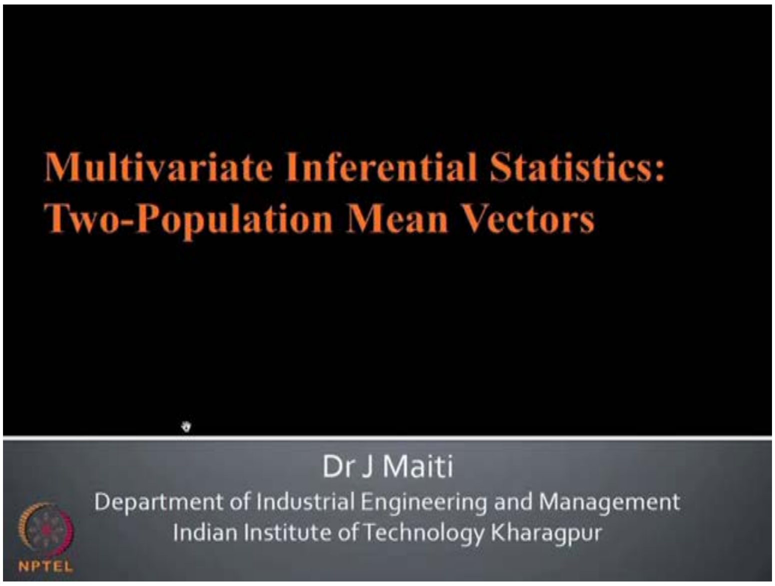

Now, what will happen if you go for difference in two population means, getting me how do you proceed. You have done in univariate case yes or no, we say that.

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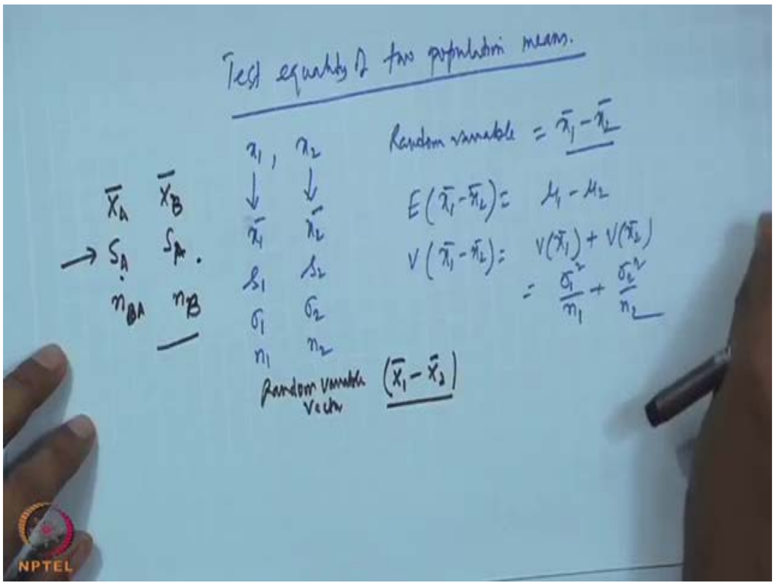

We want to test the equality of two population means I think you please remember your univariate case. There are 2 variables x 1 and x 2. And you computed x 1 bar and x 2 bar. You also computed s 1 and s 2 you maybe knowing the sigma 1 and sigma 2. Depending on the situation you have created a random variable which is x 1 bar minus x 2 bar. You recollect your lecture on test of equality of 2 population mean under univariate situation, this is random variable.

Now, this random variable there will be expected value that is mu 1 minus mu 2. And there will be variance you have seen that this is variance x 1 bar plus variance x 2 bar. So, that 1 we have seen that this is variance means sigma 1 square by n 1 plus sigma 2 square by n 2 where n 1 sample from population 1 and n 2 sample from population 2. What is multivariate counterpart of this, getting me what will happen? Your random variable will be a random vector x 1 bar minus x 2 bar. This is your random variable is this variable vector. And your population is like this.

So, what happened your x 1 bar from the first population x 2 bar from the second population again these all are p variable case. Then you have computed S 1 you have computed S 2 what are those these things covariance matrix for population 1 covariance matrix for population 2. And you know that suppose the first one is n 1 or n 2. So, I will give a different notation I will give a and b, because I have developed in like this a and b. That slide I prepared using a and b notation. I am asking you find out the expected value of x 1 bar minus x 2 bar. Last bench here that side idea any idea, what I said expected value will be this for univariate case multivariate case also will be the same thing, but vector notation.

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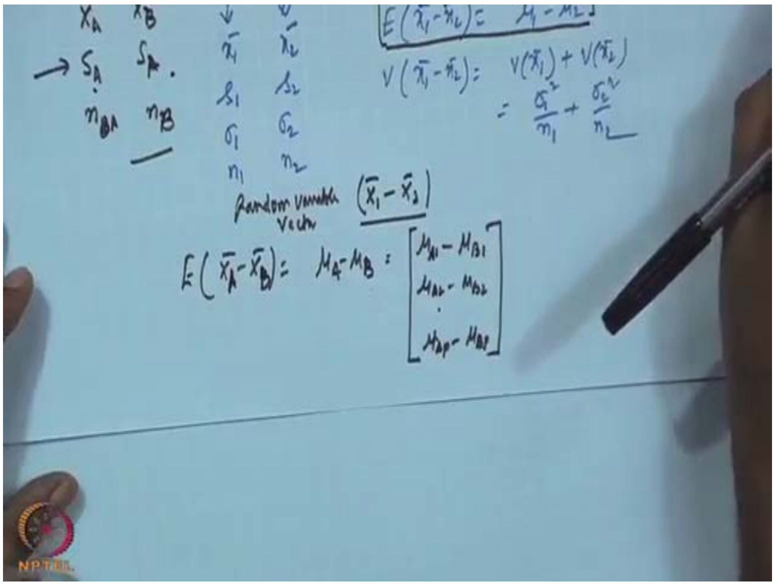

So, that means expected value of x 1 bar minus x 2 bar that is nothing but mu 1 minus mu 2. How many variables are there are p variables. So, this will be a quantity like this suppose mu 1 1 minus that I think I will give a then that will be better. I told you X bar minus this bar mu a minus mu b then this is mu a 1 mu b 1 mu a 2 minus mu b 2 like this mu a p minus mu b p. So, those who are absent in univariate class for them it is difficult to grasp. And you will never learn because you have missed the vital issues already.

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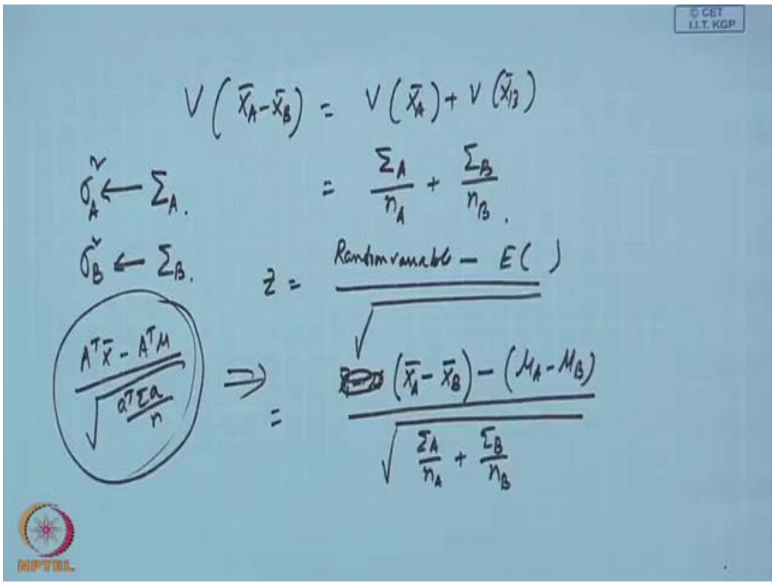

Then what will be the variance part X a bar minus X b bar. That will be variance of X a bar plus variance of X b bar. I have shown you in multivariate univariate case variance of this is this. Forget about how this derivation and all these things are coming because we are not statistical expert we are users. So, then if sigma square is the variance for the univariate case we all have seen that capital sigma is the counterpart in multivariate case. So, if it is a, this will also be a. If this 1 is b I will go by this. So, this 1 will be sigma a by n a, because you have seen earlier that from population around n 1 and n 2 is divided I am now denoting by a and b and this will be your sigma b by n b.

Now, you think what will happen I want to create Z. What is the approach Z approach is you first write down the random variable minus it is expected value divided by square of its variance. So, I can write down here very clearly what is my random variable X 1 bar minus X 2 bar. So, let us write X 1 bar X a bar minus X b bar minus expected value means mu a minus mu b divided by you can write down the variance component sigma a by n a plus sigma b by n b.

Now, see this is in the multivariate domain and you have already seen in univariate case. That similar things you have created when I have written like this a transpose X bar minus a transpose mu by that square root of a transpose sigma a by n. This is for only one population case that means similar and analog has to be created for difference also. Please remember here is one random variable vector that is X 1 bar minus X 2 bar, although you are taking sample from two populations. Finally, you are creating 1 random variable vector X 1 bar X a bar and X b bar X a bar and X b bar. So, the same principle will be applied now. Only thing is that one more component added by denoting sigma a sigma b you are later on will be adding S a S b something like this essentially other things remains same.

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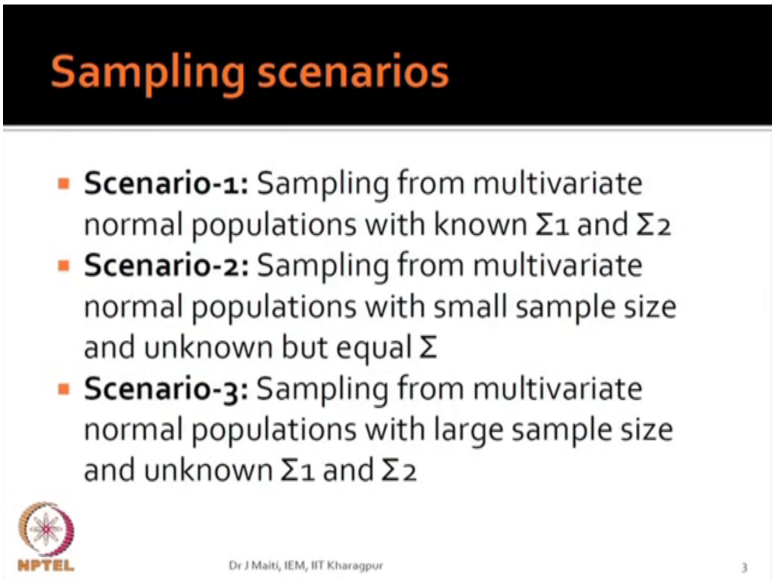

So, you have similar situation here. You are sampling from multivariate normal populations with known sigma 1 and sigma 2. You ask some question otherwise it will be one sided game. Go through this slide first, is there any problem in understanding the three scenarios.

Student: ((Refer Time: 40:52))

Independent, so scenario 1 scenario 2 and scenario 3, here first one is like this that sampling from multivariate normal population known sigma 1 and sigma 2. Sampling from multivariate normal population with small sample size unknown, but equal sigma, and sampling from multivariate normal population with large sample size and unknown sigma 1 and sigma 2. We will discuss the second one because that one you will we will be using more frequently.

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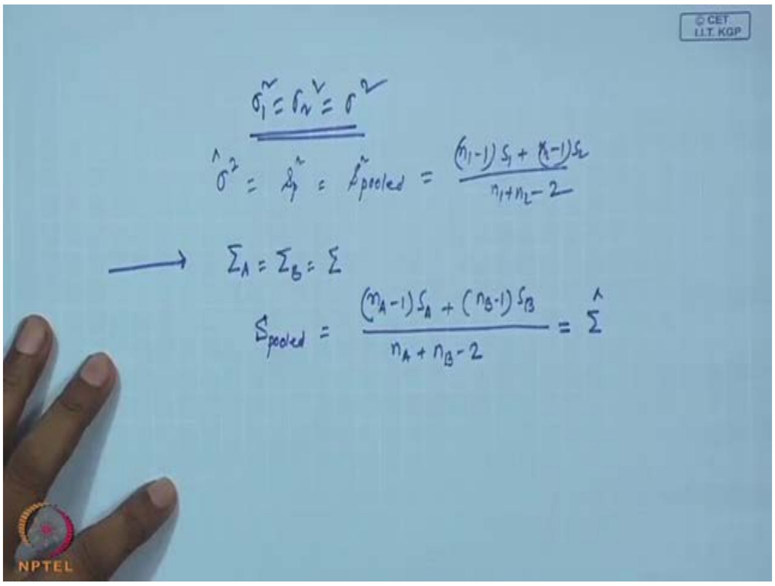

Now, in univariate case again you recall that we said that if sigma 1 square equal to sigma 2 square equal to sigma square then we have used certain estimate of sigma or sigma square. What is this we say S p square can you remember that we said S square pooled. I think you can remember there we have used that n 1 minus 1 S 1 plus n 2 minus 1 S 2 by n 1 plus n 2 minus 2. Here we will use the same thing here what is our case sigma a is sigma b equal to sigma this is our case. We will be creating similar S pooled covariance matrix and that will be n a minus 1 S a plus n b minus 1 S b divided by n a plus n b minus 2. So, that this 1 is our estimate of sigma.

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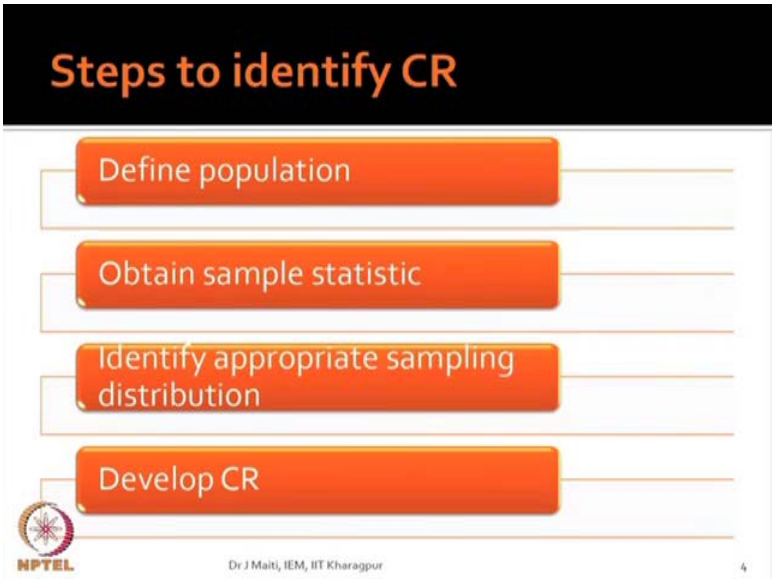

Now, let us see some of the slides first and we will explain then there is a same way that confidence region first followed by your simultaneous confidence interval, but please understand in totality we are repeating what we have developed so far in univariate your single population. Single multivariate population case we are repeating this one the random variable is changing if you do not understand this. So, I am repeating it also several times I am saying the random variable is changing accordingly the statistics that total formulation is changing. Otherwise the statistics that T square statistics again the a for Chi square distribution those things remain same. Your change is in terms of computation.

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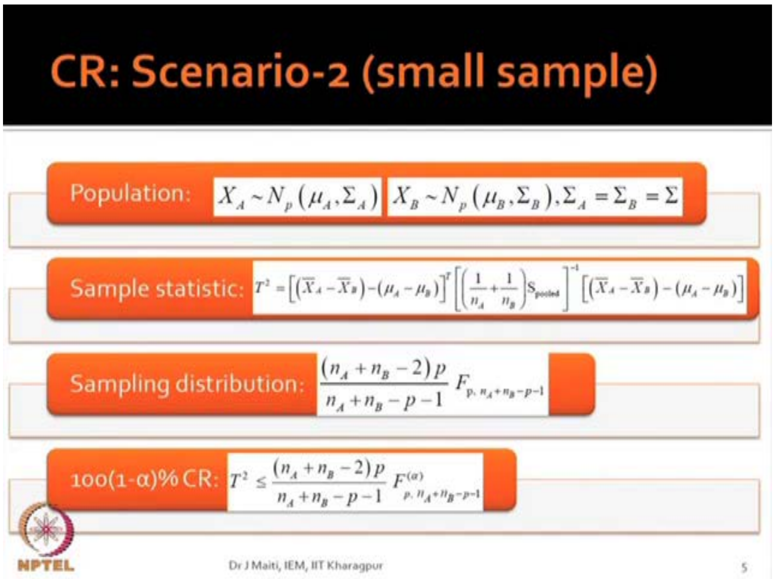

Here you see this slide and have a feel of it small sample case.

Student: ((Refer Time: 45:04))

All those things available in Wichern book, Junction and Wichern book, but then they have used some other notation not a b maybe they have used 1 and 2 something like this. What I am saying here my condition is like this that you are sampling from multivariate normal population. There are two such populations x a n p mu a and sigma a x b that is again n p mu b sigma b. And your population covariance matrix both the matrices are equal to sigma. And your sample statistics is what is my random variable x a bar minus x b bar, this minus its expected value please get the connection link.

We said the random variable minus its mean expected value. So, expected value of X bar a and minus X bar b that will be mu a minus mu b. In the single population case you have used the formula for T square X bar minus mu transpose. Now, no longer you are using X bar you are using the difference between two population mean vector. So, x a bar minus x b bar that is similar to X bar analog to single population that is X bar minus mean that transpose. Then followed by what happened there you have used S inverse here you have two population case show and sigma a equal to sigma b equal to sigma.

So, you are calculating the S pooled first. This S pooled is this 1 one by n a plus 1 by n b into S pooled, S pooled is the same equation what you have seen earlier. What is S pooled here n a minus 1 into S a plus n b minus 1 into S b divided by n a plus n b minus 2. So, you are writing this and you know that earlier we also derived in univariate case that 1 by n 1 plus 1 by n 2 that quantity will be there. So, this is what is the variance covariance part? So, you have to take the inverse of this covariance, then you square this 1 x a bar minus x b bar that random variable that is squaring.

Student: ((Refer Time: 47:54))

Single population.

Student: ((Refer Time: 48:00))

Yes, here n is here.

Student: ((Refer Time: 48:07))

It is coming here n a, n a plus n b because in earlier case what happened this is one. One by root n and then n you are taking other side variance part in univariate case, what is that sigma by root n scalar quantity that sigma by root n. That derivation is such that root n is going to the numerator, you are writing when you are going for totalling T square. What is happening they are also single population case that n is there and coming to that numerator and you are writing here n. In this case what is happening, this n is taken care of by 1 by n a plus 1 by n b. See this n component is coming because of variance.

In univariate case in multivariate case because of covariance when it is single population you do not have any problem this square n component will come here. And here it is the two population difference case. So, the variability will be taken care like this and this is the quantity. This quantity follows what distribution F distribution again, but you see some other part the two component is there. In earlier case it was n 1 minus n minus 1 n into p by n minus p what it is written here, n a plus n b minus 2 into p divided by n a plus n b minus p minus 1. That you have to remember otherwise if you want to derive it very difficult it will go.

You will forget that I will stop reading this multivariate statistics this one and see F p, p is the numerator degrees of freedom. And n a plus n b minus p minus 1 this is the denominator degrees of freedom and your confidence region will be T square less than equal to this. We will not repeat this I hope that you are understanding I am sure you are understanding, but those who are who were present earlier they are understanding.

I am sure whether you maybe it is maybe sometime difficult because so mean much of this terms mu a mu b n a n b all those things, but the concept the message must be clear to all those who were attending from the beginning. If not then please ask me question come to my room and again you get it clarified. And those who are not attending forget. That is the large sample and in large sample case Chi square distribution you will use this is for the confidence region.

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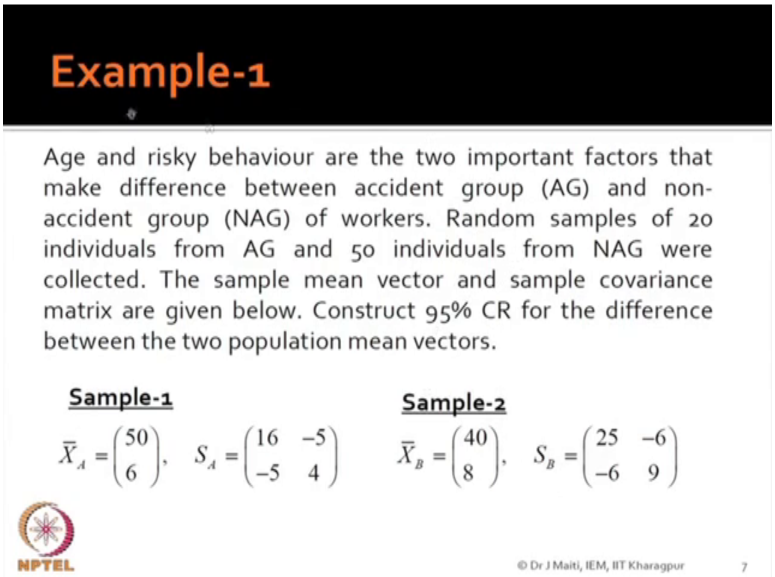

Now, this is one example and it is a the data you have just arbitrarily given, but we have done one study in some of the coal mines this is a real application I am telling you. This data are not from that real application, but similar application we have done we have found out that the management was facing problem with managing accident particularly using under control mines. They have gathered lot of data, lot of injury data means people who were injured and there are another group who were not injured.

Our aim was that whether there is difference in characteristics of that two groups. So, we have taken 24 variables and then 150 observations each from accident group from non accident group. Then that means this is a two group phenomena I want to test the mean differences between the two population. If there is difference then we have to find out what are the variables making the difference that type of study we have done. It is applicable to quality control also for example, you will find out in Johnson and Wichern one application is given in from the police department activities of view that part you please check. So, now suppose this is the case two population and you are getting X a bar and S a X b bar S b like this.

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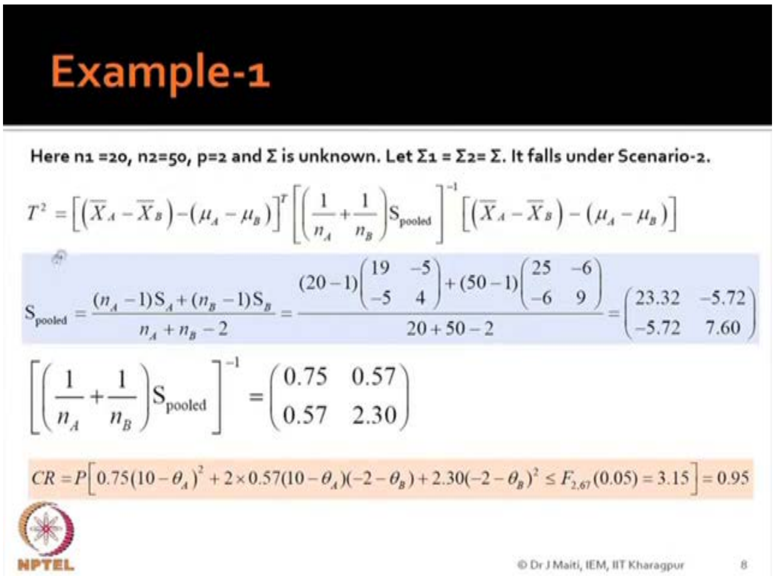

And then what you require to do you first find out the confidence region. Same thing you are doing with more computation nothing else. So, you are finding out T square then finding out your, what is the your sampling distribution and you are making this confidence region. And I ask you to practice very difficult if you do not practice it will it will be difficult multivariate is really complex, but if you go to the theoretical side then it is we do not have that background and but application side complex.

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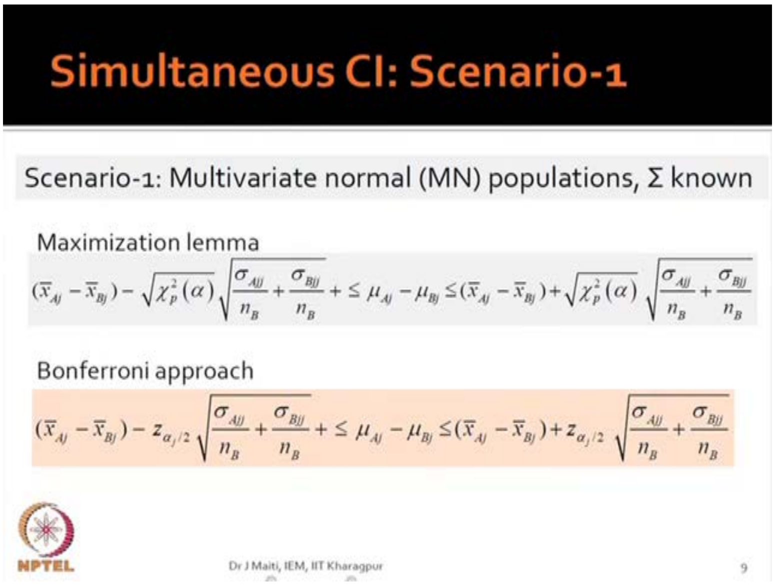

Now, we will use the same situation in maximisation lemma and the Bonferroni approach big equations. Now, see that maximisation lemma and Bonferroni approach the difference is coming Chi square and z.

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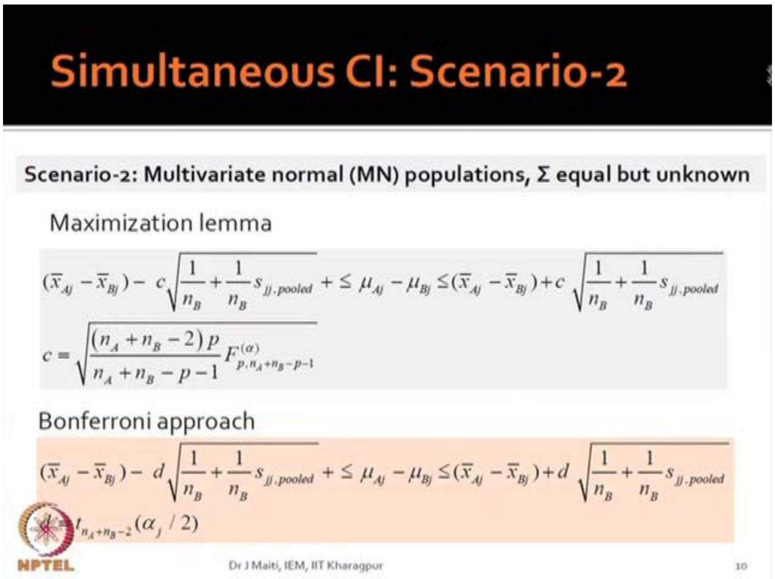

Then you are the second one unequal and unknown case. Here you are using this F distribution available in Johnson and Wichern you do not require to write.

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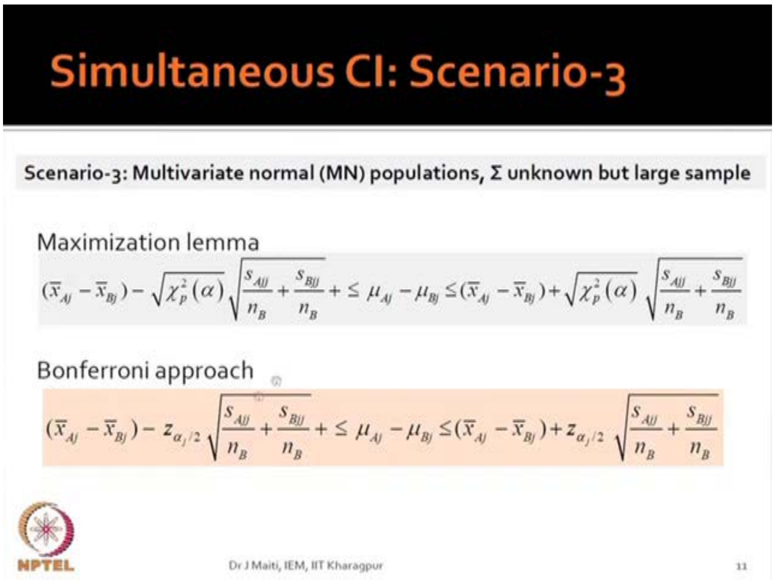

Third case large sample Chi square distribution, you create your example here.

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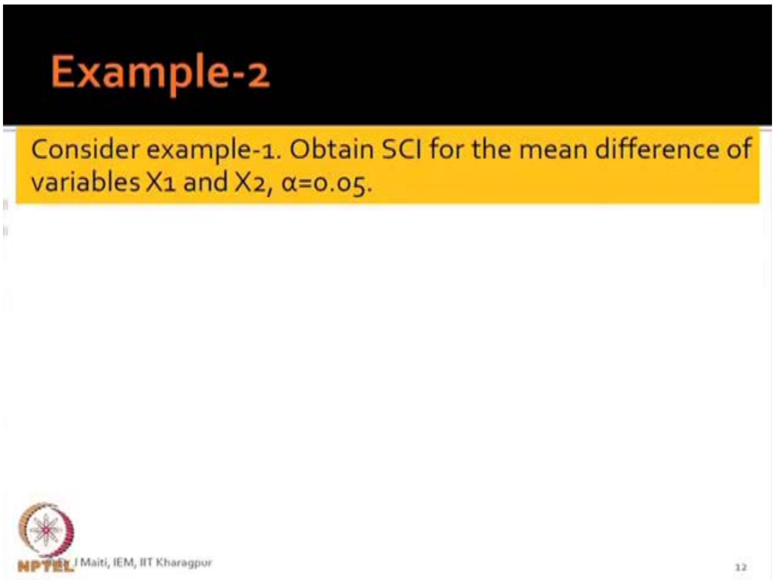

Next class I will ask you, there is no example I thought I am giving, but I find that no it is better do not give anything next class you will be giving one example for this. Arbitrarily I will be asking some of you.

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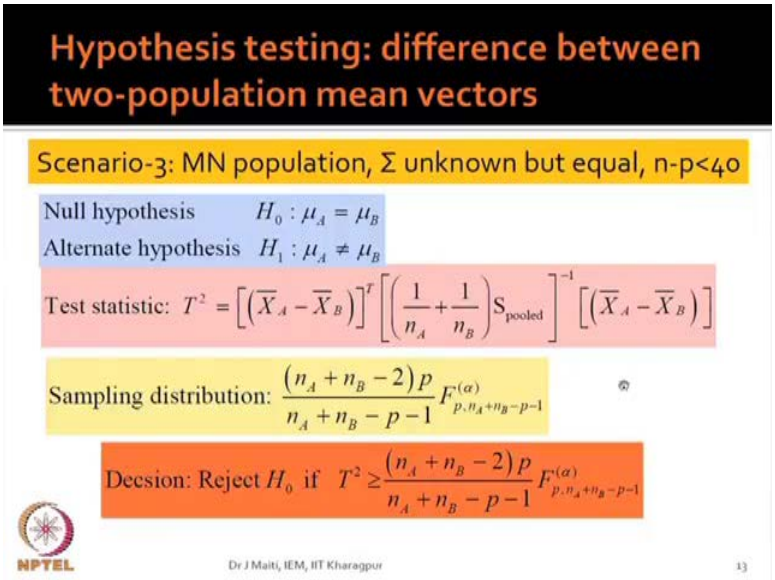

Then hypothesis testing, should I stop now hypothesis next class I will discuss or it is ok.

Student: ((Refer Time: 55:44))

So, you want one more hour on hypothesis testing similar. Then I think this is the end today. And next class tomorrow here for all of you.