Six Sigma
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Module No. # 01
Lecture No. # 06
Review of Probability and Statistics-II

Good afternoon again, we are back in the session if you remember as we ended the previous session, we are discussing tossing of coin, so here I have got a coin and I am going to spin it a little bit, and you will notice it picks up something, and this get turnout to be a head. I try to do the same thing again I try to turn it out in different, my god it turnout to be a tail, now just just a chance.

Now, different types of events such as, tossing a coin or rolling a dice, any of these things, different different events they have different characteristics. Let us try to understand, what different types of characteristics are there, if these events occur in their life. Unless you understand that, we would not to able to combine, we would not be able to calculate what we call, the probabilities characteristics that we not able to do.

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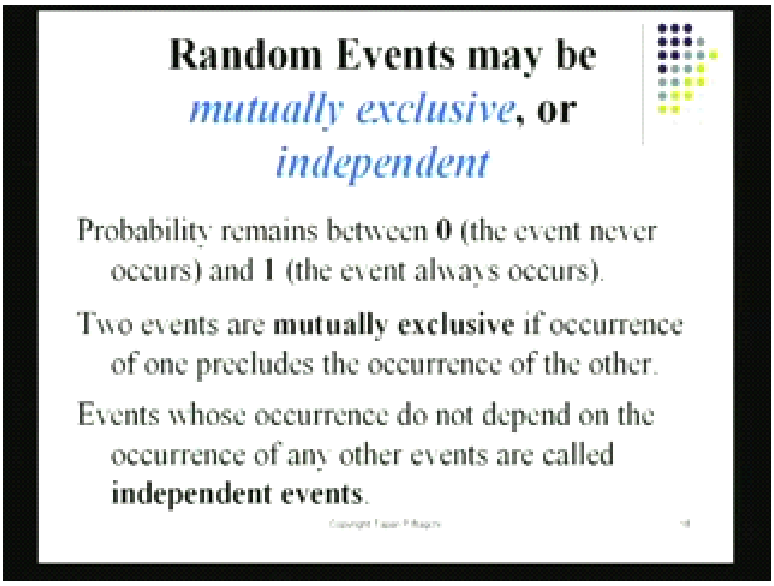

Let see how we do that, I have two simple categories, I have one kind of one kind of event which, I called mutually exclusive event and I have another set of events, which I called independent events, these are shown on the slide here. Let us try to define what they are, I must thought out by saying that, the probability of any event could be, it has to be between 0 and 1, you cannot you cannot have the probability of an event to be greater than 0 or less than 0 or greater than 1 that is just not possible, it has to be restricted between within 0 and 1.

So, probability is a measure that always stays between 0 and 1, 0 actually means that the event will never occur and one is that the event will always occur, which is clearly not the case of tossing a coin. Now, just take a look at mutually exclusiveness, two events are set to be mutually exclusive, if the occurrence of one (()) roots that is prevents the occurrence of the next. And now in this you know the coin toss situation, the events are going to be either it is a head or it is a tail; so either I have a head or I have a tail, these two events are mutually exclusive what happens, if one occurs the other will not occur, these are mutually exclusive. Then, there are other events possible those events are called independent events, what are independent events? One independent event could be I toss a coin here, and somehow the light goes out.

Now, they might have occurred, by chance they might have occurred together, but it does not mean that, one is dependent on the other, there are so many events that take place for example, it is raining in Calcutta and in New York there is a traffic accident or like last night there was an earthquake in Andaman and (()) there was a book fair being held in Delhi; there is no these are, these students have absolutely nothing to do with each other, they are totally independent of each other. So, what we talking about there are two types of events and these determine basically are we going to add the probabilities or we are going to multiply them or whatever we will do.

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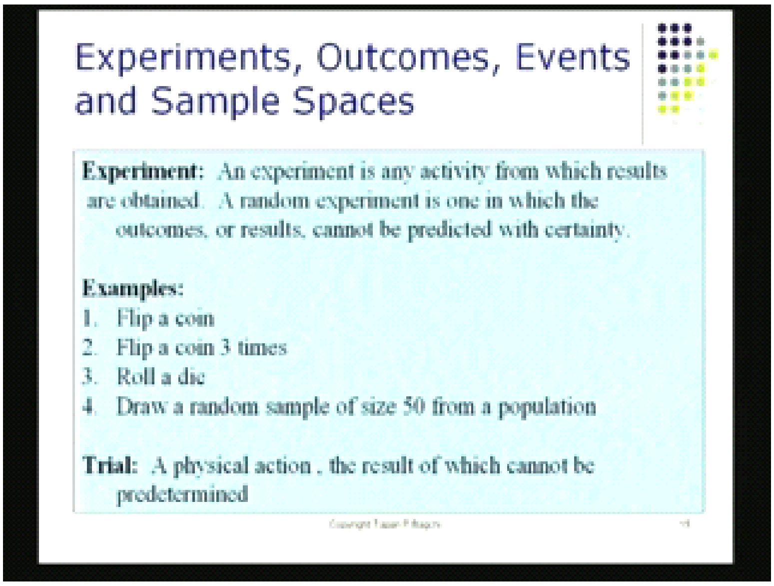

What is an experiment? An experiment actually is something that produces an outcome for example, flipping a coin is an experiment, flipping a coin 3 times is also an experiment, any experiment from produce an outcome. Generally speaking experiments are driven by many different factors, so we call them random experiments. Rolling a die similarly, an experiment it has got, its got outcomes and the outcomes are random and drawing a sample of size 50 of products it could be it could be for example, pens produced by a factory by machine, drawing a sample of size 50 from a full range of production full days production that also is a is the example of an experiment.

What is a trial? A trial is the physical action, the trial is for example, in this case, when I got this coin in my hand, one trial is flip the coin and let the let the coin falls somewhere, and looks at the output, that is a trial. And it turns out that this trial may be different, the outcome of this trial may be different from the outcome of another trial, again I flip it, there is some other outcome there, when I flip it again each of these are trials each of these are trials, so trials lead to outcomes, trials are the steps involved in producing the output.

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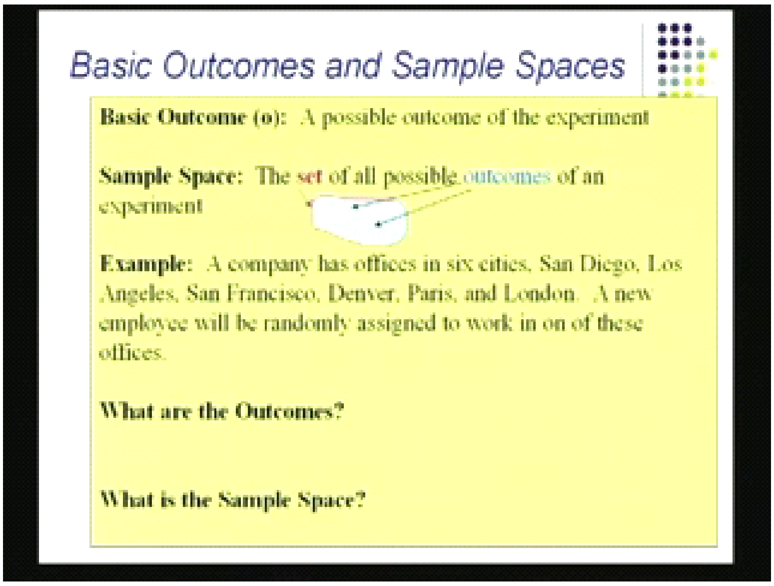

If you look at all the outputs together, these set that is concern, that comprises all the different outputs, that total set is called the sample space, so here in the diagram I have got this little blue area, which is like a little lake there; this is actually outcomes are inside and the set itself is this sample space, no outcomes can be outside the sample space, I define the sample spaces in such a way, that is comprises all the different outcomes. So, there are lots of examples given here, and the outcomes are there for example, those are given here in this particular instance.

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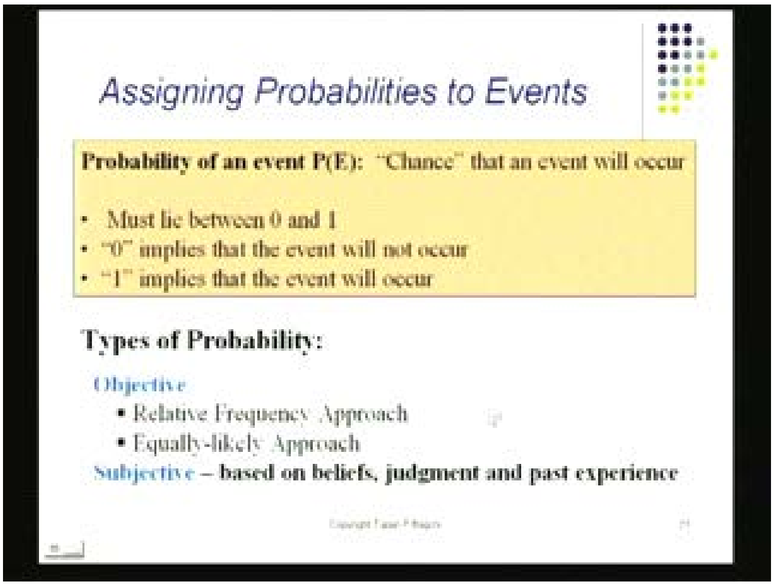

I assign now, probabilities to events for example, we know from our experience the chance of the event being a head is 0.5, the chance of it being tail is again 0.5, these are probabilities, that are being assigned to those events. Again what is the rule by which, I will be assigned these probabilities, the assigned probability value must be between 0 and 1, number 1; 0 will imply that the event will never occur 1 will imply that it will always occur.

And of course, many events are never certain or never totally uncertain. Therefore, there will have some value of P, probability, there will between 0 and 1, what types of estimates we have for probabilities, there is something called the objective estimate of probability, and this is based on relative frequency.

So, for example, somebody again came along with this coin and he had this coin in his hand and he wanted to make sure this coin is purpose used in betting between me and him, its I am going to bet you a 100 rupees based on the based on the results of this one outcome. I have to now, then decide should I play with this coin or should I tell him no there is a big chance of losing money therefore, I am not going to be playing, and before that I have to find out basically is this a fair coin for that what I do, I toss it 200 time and I count the actual number of heads found.

Let us say the actual number of heads found in 200 trials turns out to be 85, which actually means, I am next to find the head than a tail when it comes to tossing using this particular coin, here I have got frequency here I have got frequency, what is this?

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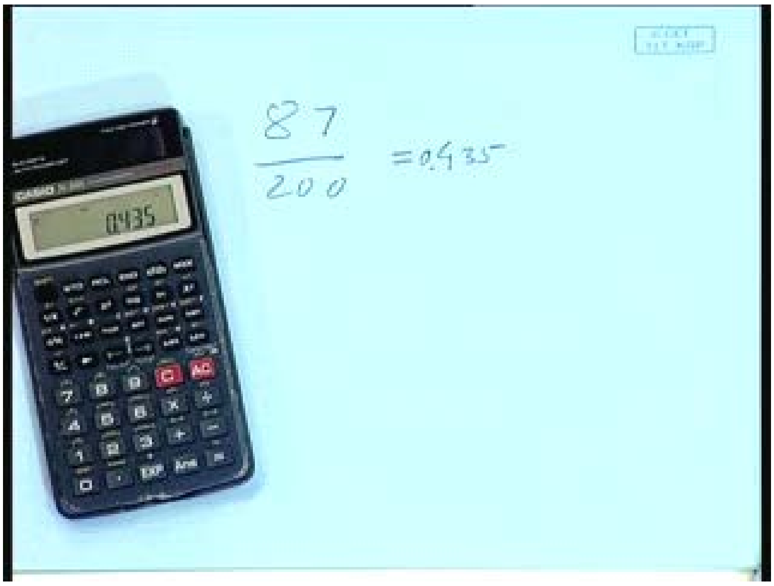

87 divided by 200 and that in your thing if you wanted to do the exact calculation, you would say 87 divide by 200, that will turn out to be 43, 0.435, that is the probability of you know a coin of the type that has got this defect giving a head, its only we will call it 0.435 that is the probability for finding a head with a defective coin.

Now, let us say I take a different situation, in which case I go by belief and most of us we are use to the rupee coin, and we know that the rupee coin is reasonably balanced because, we may have flipped it a few times, we might have actually seen, what happens, and what we found is if we do it do it 200 times, nearly 100 times it will be head, nearly 100 times it will be tail. Therefore, I can make a subject suspend there, I can probably say well the coin is, this coin is fair is 50-50, this is now based on belief belief is quite different from actual counting, actual counting of heads in a number of trails that is going to give me what we call the, objective estimate of probability, and based on my belief if I give a number that is going to be subjective estimate of probability both are used, it all depends.

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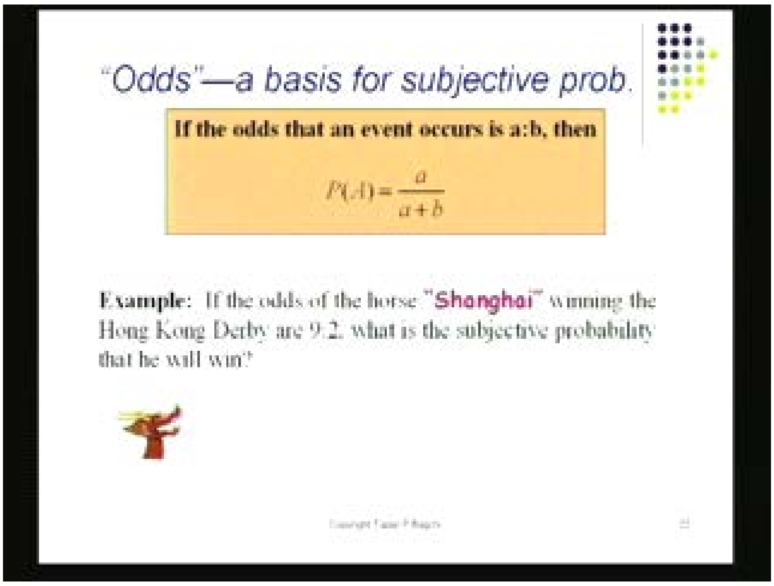

On when we are able to find a number and we are not able to find a number based on our experience, let us try to see let us try to see a situation, when we have we have the odds given to us and the odds are given to us by observing again and in this case, let us say the odd is in Hong Kong as you probably know, like in Bombay we have got a race track, in Hong Kong also there is a race track, there are various horses and the horses are named Beijing, Shanghai, Hong Kong and so on.

And those are raced in the evening and there is a particular horse and people are saying that the odds of that particular horse winning the Hong Kong derby, is 9 to 2, 9 to 2 basically means if the horse ran are total of 11 times, 9 times, the chances are that 9 times out of those 11 times its going to win. Now, if odds a given as 9.2 and somebody says, please tell me what the probability is I just did that for you, and it is sort of like this.

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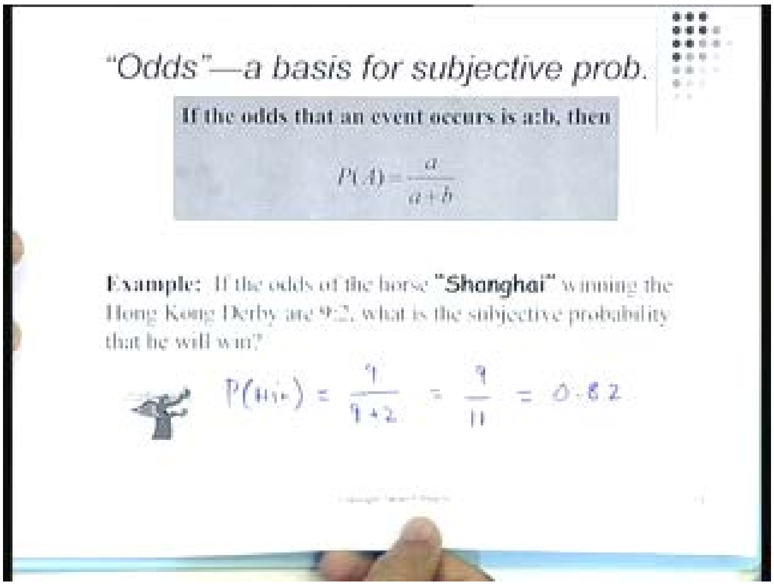

I have here the probability of probability of this particular horse winning, this 9 is the chance of winning divided by the total number of trial chances available, which is 9 plus 2; so 9 divided by 11 which turns out to be 0.82 that is the (()), that is the probability of this horse which is named shanghai winning, the derby this is one particular example.

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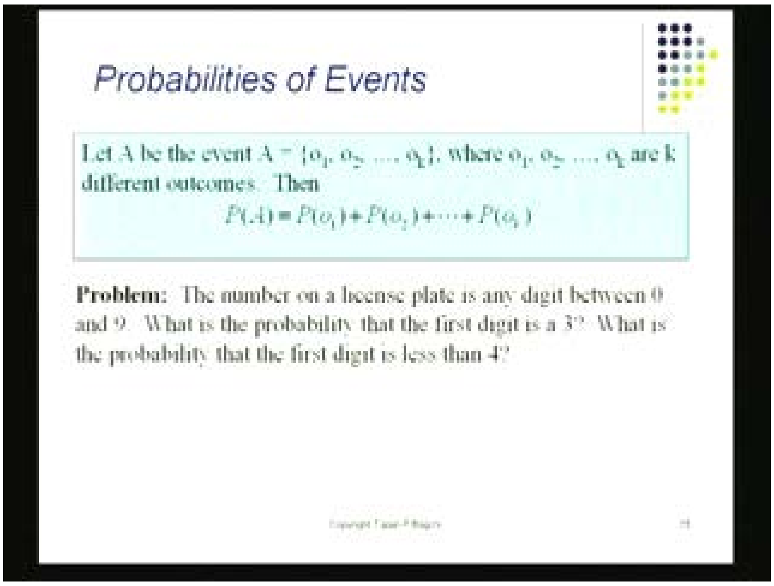

I i can have another example, and I am going to bring a bring a another example now for you, and it goes like this I have as you see in the slide, I have events which are O 1, O 2, O 3, O 4, O 4 and so on so forth. These are going to be various outcomes k different outcomes that occur when the event when the event A occurs, any of these occurring O 1, O 2, O 3, O 4 any of the occurring would signify, that the event A has occurred.

If that is so for example, I could have a number of boys walk in to the room, I would have a number of boys walk in to the room and each of them are at is at distinct person, so I call a particular name and somebody walks in, then it is the next person, next person and so on. If they are, if only boys enter then the event that a male has entered this room, that event has actually occurred if any boy walks in, if by chance a girl walks in of course, the event that a male person has entered the room that event will not have occurred.

Now, if there are x number of, they are k boys standing outside what is the chance of my event A occurring, when A is the event that actually sees a male has entered the room, that is going to be the chance of any one of those k boys entering and therefore, to be able to do that I act the probabilities of the first person entering, first boy entering plus the second boy and remember these are all disjoined these are all disjoined that means, there is nothing common between boy 1 and boy 2.

In fact, let us take a look let us take a look at at an example, the number on the license plate is a digit that is between 0 and 9 you know, just a single number we are looking at and that is between 0 and 9, how many different possibilities are there, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 possibilities are there.

What is the probability that the first digit is 3, you got strings on numbers on the license plate, the very first one could be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 what is the chance that the first digit is 3, you can see very easily that the first digit turns out to be 1 divided by 10, because there is one tenth chance, there is one tenth probability that the number 3 would have appeared in the first position.

Now, suppose I asked the second question which is, what is the probability that the first digit is less than 4, less than 4 that means, 4, 5, 6, 7, 8, 9 these are excluded; so what are the digits that could lead to the first number that is less than 4, it could be it could be if you see the answer here.

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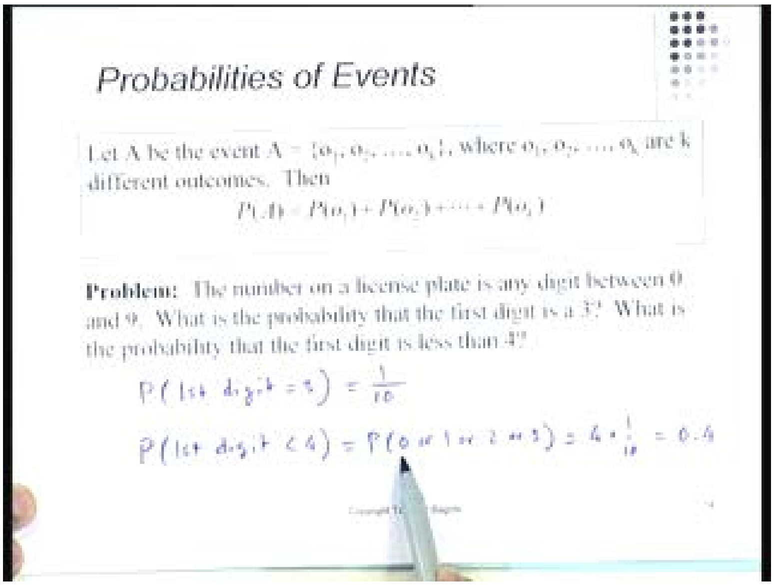

It could be that the number is 0 or it is one or it is 2 or it is 3, now because there are 10 numbers, any one of them will have the probability of appearing a 1 by 10 and there 4 such events and they are all independent, and it turns out in that case I can add their probability, and the probability of the first digit to be less than 4 is going to be than 0.4 quite easy quite easy to work it out.

But, I did use some rules and one of the rules I used here was, when the events are disjoined, when for them to occur together for the for them to occur together, and we have to look at that compound event now, the first event occurs and the second event and the third event and so on, I like to add the probabilities and this is what I did in the other case.

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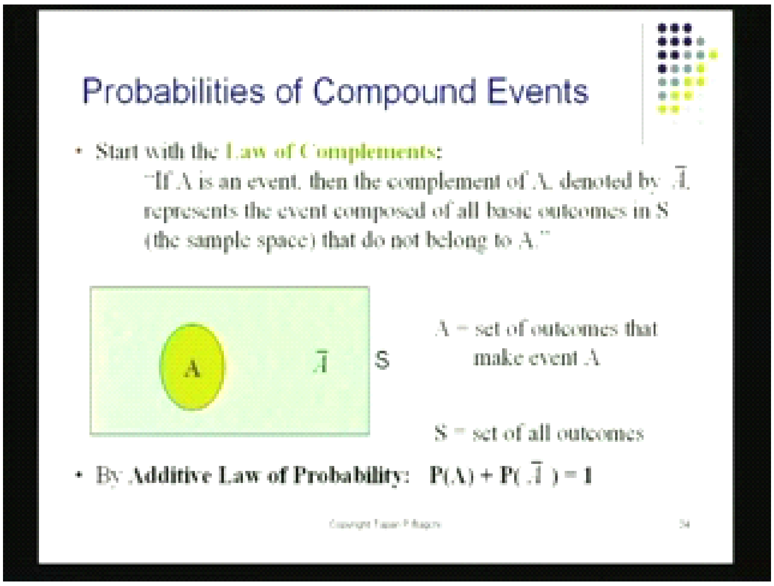

Then there are set of things which are called compound events and the simplest example of a compound event is, the event A and its complement like you look at the yellow area or a kind of the greenish, yellow area there right in the middle which is the event A that is the space if any outcome appears inside also event A has occurred. If any event occurs that is not inside A, also the complement of A has occurred or A bar is occurred these sample space S really is the set of all the outcomes, some are inside A and some are outside A.

And therefore, the probability of the event A occurring plus the probability of of the event A bar occurring, if we add these two it should turn out to be one, because then nothing else is possible, either it will be A or it will be A bar and the sum total of this is going to be 1, this is the additive law of probability and what is distinct of A and A bar they are disjoin.

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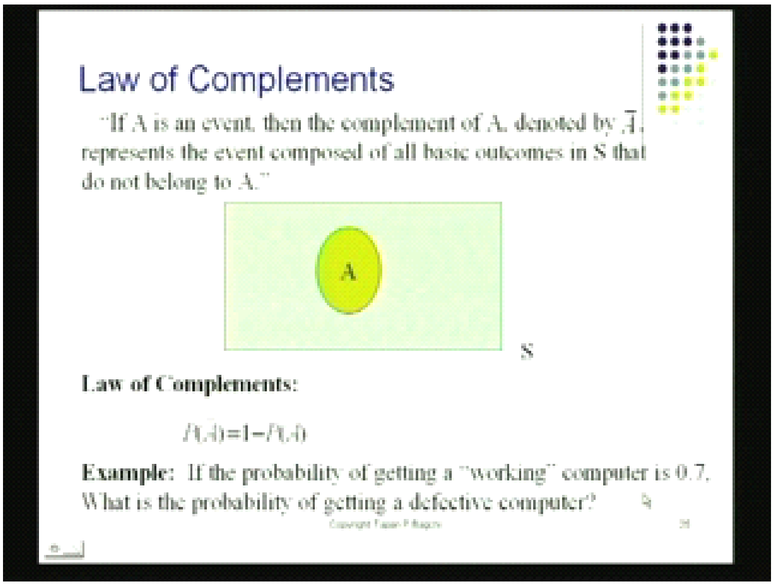

In fact, if if this is the story then of course, I can find probability of A bar which is simply going to be 1 minus P A, if I know P A, if I know this probability I can find the complement, the probability of its complement which is A occurring. And let us take an example, a very simple example and the example is this the chance of getting a computer working is 70 percent, 0.7 is the probability of of a computer working.

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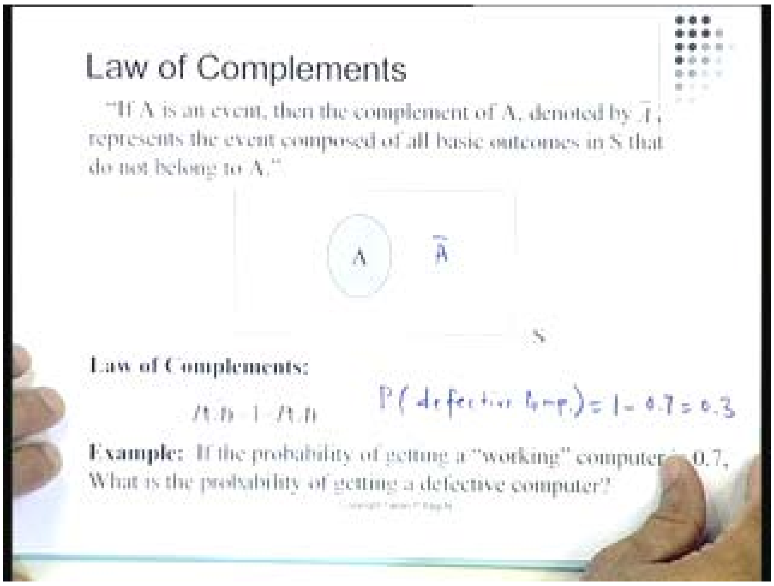

And then what am I talking about, I am talking about the chance of my getting A defective computer, now notice working computer and defective computer they are complements of each other, it is a chance of the computer working is 0.7 obviously, the the chance of the computer not working is 1 minus 0.7, which is equal to 0.3. Which is, what we have shown here, if you look at the outcome here, this answer here it actually tells you, that now the complement has occurred, so I have got a I have got a defective computer in my hand, this is like less take a look at some of these others also.

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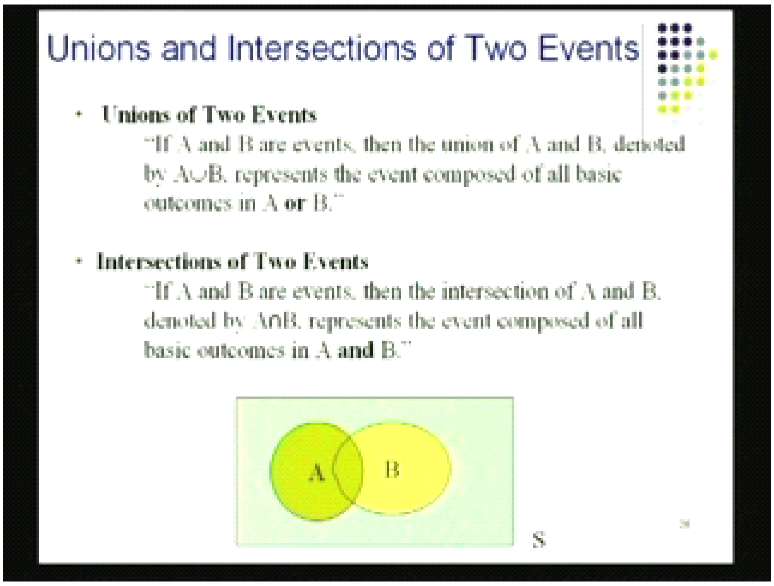

Unions and intersections of two events, unions are those conditions when the outcome of outcome can be either in this set set A or it can be in the other set set B, then I say if if the if the event has occurred in the union of A and B, it has either occurred here or occurred here.

So, anything that appears either within A or within B, we belong to what we call the union of A and B notice here A union B, I put my cursor right there A union B is basically the any outcome that falls either in A or in B, so I actually designate that by saying the outcome is actually either in A or in B obviously, it could be in both that also is alright.

When I do that, I worked out what we call the union of two events, what about the intersection of two events, its going to be only those outcomes that come right in the middle that means, they are belong to A and they also belong to B, so these events here which are inside A and B these are these are going to be now at the intersection of A and B.

A and B are two different events, A and B, A is the result of certain outcomes, certain outcomes together they constitute A and certain outcomes together again they will constitute B, A and B together, they form the union and whatever is common between A and B that form to intersection of A and B. And that is what we have right in the middle, what we have in the middle here what we have in the middle here is basically the intersection these these elements belong to A and also they belong to B.

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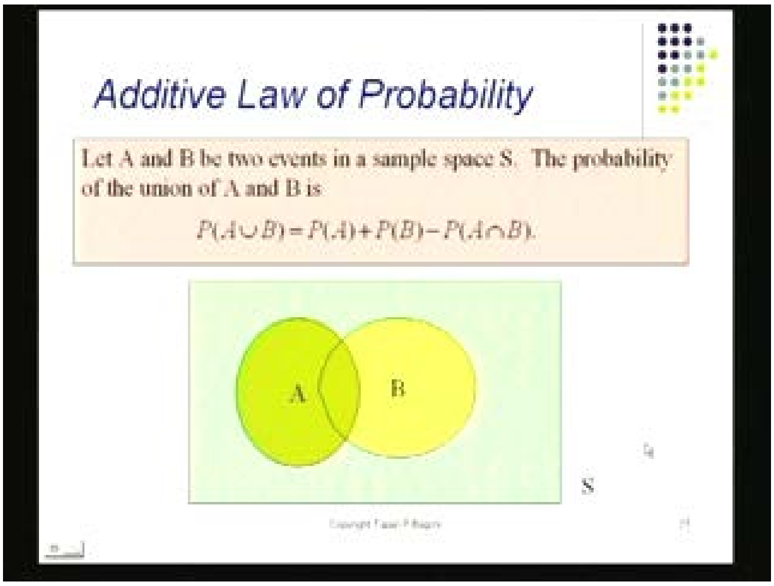

Now, we come to what we call, the law of adding probabilities and this is very very important, its very important first to appreciate that this is now, the beginning of building A kind of a calculus or kind of a adding rule, multiplying rule, subtracting rule and so on, with probabilities. And these cannot be, the probabilities cannot be added and subtracted like numbers, because they are not numbers, they have the number has appearance, but in themselves they are not counts, they are chances, they are the odds, they have the chances.

This is a very very important rule the first law, the first law is called the additive law of probability it says the probability of A union B is equal to whatever belongs to A the chance of that which is P A, and whatever belongs to B is now, union union is therefore, either in A or in B that way it will work, but if you just added P A and P B or we adding this whole area and also adding this whole area, then this area will become once in A and again it will come once in B.

Therefore, it will become actually something that is more than A plus B, so what we have to do is, if you have to really look at the probability of A union B, we have to subtract this area which is not being counted twice, once it has been counted using A when I am counting P A it is there, A union B, A A intersection B is already in A, and A intersection B is also in B.

Therefore, I have to take it out its been counted twice when I write this expression here, when I write this expression here P A plus P B is already got this guy, this A intersection B counted twice in within them therefore, I subtracted once. So, what is the law now, it says (()) probability of A union B is equal to P A plus P B minus the intersection of A and B, minus the probability of intersection of A and B that is the additive law.

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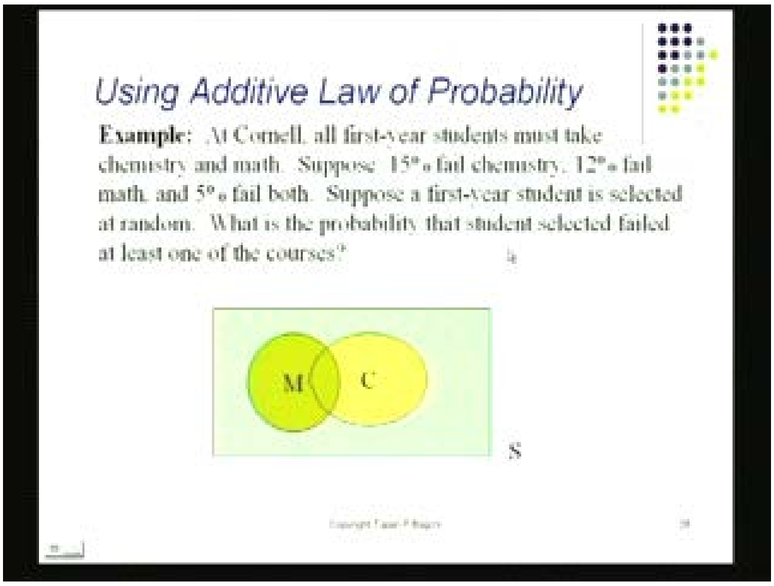

And let us take an example, let us take an example at Cornell this is a school where the first year students they must take chemistry and math, this is a requirement for all first year students entering Cornell. So, first 15 percent fail in chemistry, these kids apparently they came not a from, not such a strong high school and they manage to fail in chemistry, and 12 percent manage manage to fail in math, so some people failed in chemistry and some people failed in math, but there were 5 percent that failed in both.

So, here I have got this A situation, situation A is failing in chemistry, situation B is failing in math and some people they have failed both in math and also chemistry; if this is so if the numbers are given like this. Suppose, if first year student is selected at random I walk in there and I asked someone to stand up, and I ask him what about your performance in the school, what do you think is the chance that the student selected would have failed in at least one of the courses that means, he should have failed at least in either in math or in chemistry or perhaps in both math and chemistry.

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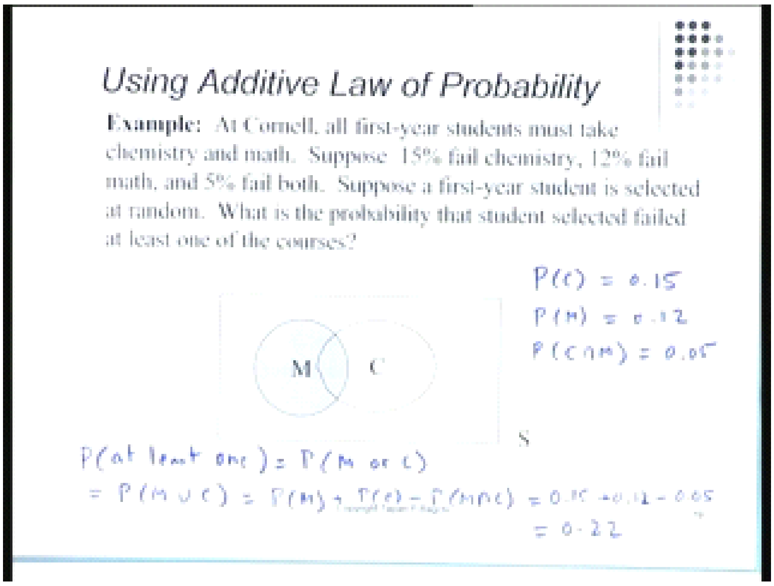

Let us try to see how we calculate that, notice here we are given that the probability that the person fails in chemistry is 0.15, we are also given that the probability that the person fails in math is 0.12 and the fact that the student can fail in chemistry and also in math is 0.05 that is given to us, and what we are talking about is the student failing in any of these courses, either in math or in chemistry or both.

So, for that what I need to do is, I need to work out the union of all these things how do I work it out, I write this formula there at least one really means, either in math or in c of course or in both. So, I do this union, I do math unions chemistry, I end of probability of math plus the probability of chemistry minus the probability of M intersection C and this I find out, because these numbers are given there are find 0.15 plus 0.12 minus 0.05 and they are (()) to be 0.0 0.22 that is the probability that the student would have failed, in at least one of the two courses, he could have failed in both and that is also counted within this (()), so this is how, this is like how I calculate unions.

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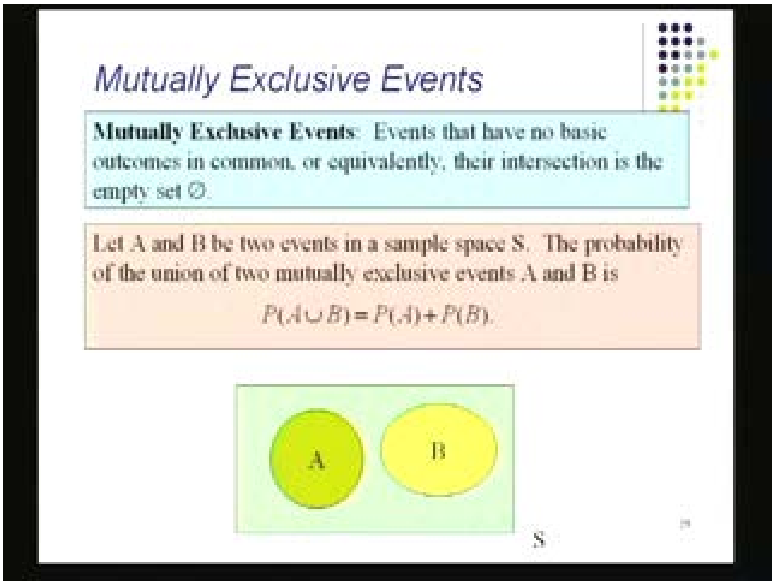

Let us move on here move on here, is there is something called mutually exclusive events, and these are tricky events, what kind of events are these, it turns out mutually exclusive events, the preclude the preclude the occurrence of the other one, remember the coin toss experiment. In the coin toss experiment, head and tail these outcomes are mutually exclusive, since the event head occurs, tail will not occur or if the event tail occurs head will not occur, this is something that actually is controlled by the nature of the process, and the nature of the object that we are looking at, and how we are observing things.

So, there are many examples of mutually exclusive events you know when the two cannot be even partial intersection intersecting, if that is so if you look at the union, if I have to say that the probability I tossed a coins a few times and I wanted to add up the probability of either A or B, either head or tail, if I just wanted to do that how would I do that, I do it by the union formula, because A and B basically says A or B, if there is nothing between them notice here, there is nothing between them that means, the intersection is 0, it is the same old formula that we using earlier.

But, there is nothing inside the the the the section that is common between A and B it has got null null means nothing is there inside that, if that is so my old formula which was written as this formula A union B and I had here, A union B given as P A plus P B minus P intersection C.

If A and B are disjoined, if A and B are mutually exclusive, which is like in this case there is nothing common between them, the expression will then turn out to be P A union B which is like A or B, it will be P A plus P B that would be the probability calculation for this, so for mutually exclusive events at the simple addition.

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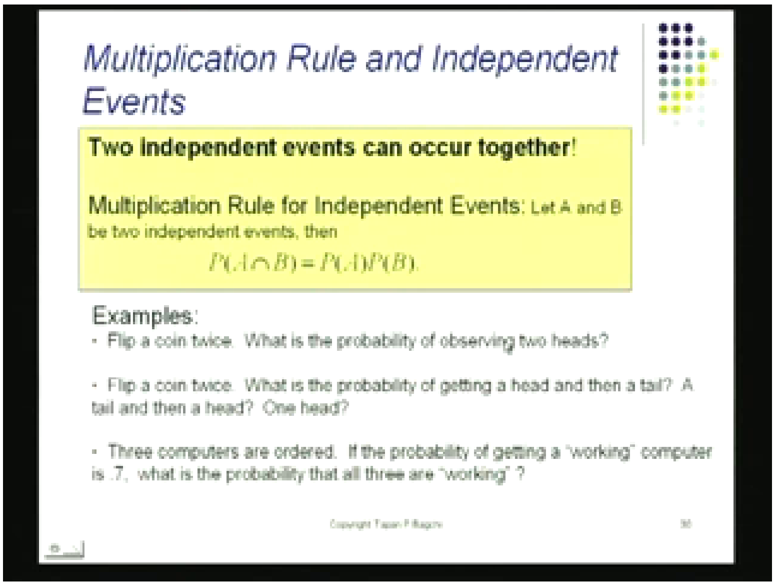

Let us say, take a look at two independent events, can they occur together, yes indeed two independent events can occur together like there was a earthquake here, yesterday at night there was a earth quake and of course, in Delhi they opened a some fair that event also took place, so there was a certain probability that it could have occurred yesterday or any other time, it turns out if I am looking at events the occurrence of the earth quake and also the occurrence of the, opening of the fair in Delhi, these are independent events, they did not influence each other in any way.

One could have occurred on its own, the other could have occurred on its own, there is no real dependency between them if that is so and if I am looking at their joint occurrence this word is very important, the the the word is called joint occurrence joint occurrence this is very important thing and joint occurrences can occur certainly events can be can be independent, but they can occur together it is a coincidence, but they can occur together and therefore, there is a joint probability also possible.

Like for example, there are lot of examples given here, flip a coin twice, so I take a coin again I take a coin and I flip it twice what is the probability of occurring of observing two heads, I i toss the same coin twice, now the first toss has nothing to do with second toss and the second toss is nothing to do with the outcome of the first toss remember this, then the event that I am interesting is observing two heads that means, the first first occurrence is a head and also the second occurrence is a head, both the first and the second they have the same outcome, but they are independent believe me.

So, what is the chance of their occurring together, it will be multiplication of the probability of finding head in the first toss, and the probability of there being a head in the second toss, and there of course, all we doing this multiplication I will be doing probability A intersection B and that will turn out to be probability A and B in probability language it will be P A times P B, so when I did the calculation, I will just describe some events here.

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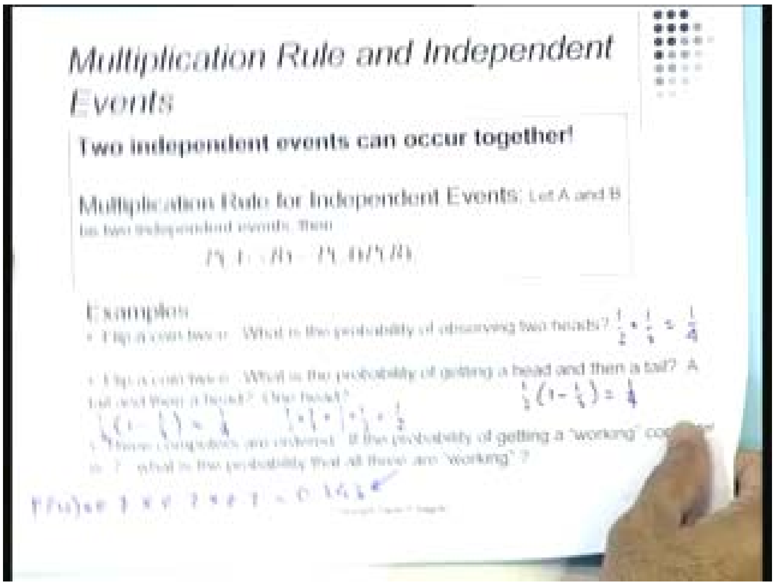

One event or set to be like this, what is the probability observing two heads and I have written it here and I hope you can get close to the screen I read them, this is going to be half for the first first being head, first first toss being head and half of the second toss being head, and the multiplication of the two terms to be 1 by 4 that is the joint probability of by finding a head in the first toss and head in the second toss.

Now, let us look at the second event flip a coin twice and you can read the event very clearly, when it come down to your slide here, the second question says flip a coin twice what is the probability of getting a head, and then a tail, now I am talking about getting a head in the first and the tail in the second.

Now, this is occurring in a very strange way it turns out I have got a head in the first toss which is 1 by half and tail in the second toss which is 1 minus 1 by half and by chance it also happens to be 1 by 4, 1 by half multiplied by 1 by half and do not confuse this one with the previous 1 by 4, that we had this is the probability of this particular event which is like getting a head in the first toss and the tail in the second toss; this will not be so if the coins for a symmetric, if the coins did not give you 50-50 chance of getting a head the the numbers here, would be different.

It also says here I tossed it twice, what is the chance of my getting one head, one head how will I get one head, the first toss the first toss could be a head and the second toss could be a tail, then I will get one head or the first toss could be a tail and the second toss could be a head this is the other one, so I have got two ways to get one head head and tail or tail and head. The first one is head and tail, I already calculated the probability for that head and a tail just 1 by 4 and the tail and the head it turns out to have the same probability, if that is so I have one quarter plus one quarter my answer turns out to be half.

So, half is the probability of doing two tosses and finding at least one head, find exactly one head, then let us took take a look at the third example, there are three computers ordered, three computers have been ordered and the probability of any one of them working, working being in a working state that means, I boot it and it boots up its 0.7.

Now, what kind of event is this, think of this, three separate computers have been installed do they influence each other they do not unless of course, the power supply goes off or something and if there is a common power supply then of course, all of them will not work. But, suppose they happen purchased in a way they come for different place perhaps, and the number we have given is 0.7 is the probability of any one of them being defective, what is the chance that all three will work, these are independent events, the first one working the probability of the first one working is 0.7, given to us.

The chance of the second computer working is also 0.7 taken by itself again given to us, the chance of the third one working is also 0.7 given to us therefore, in an event when all three are working and these are independent events, it is going to be 0.7 multiplied by 0.7

multiplied by 0.7 which is, what I have if you look at the bottom of the sheet here, if you look at the bottom of the sheet here, I have got 0.7 that is for the first computer to be working, 0.7 for the second computer to be working and 0.7 for the third computer to be working.

And if I put my calculator here, I do 0.7 multiplied by 0.7 multiplied by 0.7 equal to that is 0.343 that is what I get in the answer there, this is like I could do this, because the events were independent and the rule that I used here is the rule that is right in the middle of middle of the sheet here. Probability of A intersection B, when A and B are independent is going to be P A times P B again this is how I begin to multiply multiply different probabilities, this is like one very handy way to multiply different probabilities.

Let us keep going here, let us take a look at some other events also and for that what I do, I will define the events slightly differently and this is going to get slowly more and more complicated.

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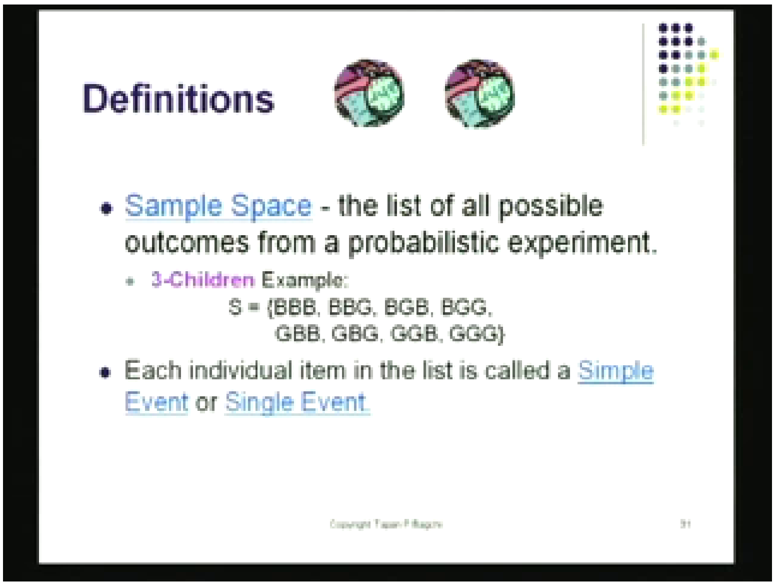

Let us look at the slide; the sample space is such that I am looking at 3 children being born, are they being born in the same family, so they have a sequence in which they have been born. Now, what is the sample space, what is the set of all the different outcomes, the first child could be a boy or a girl, so for the first child I already have, either a boy or a girl as the possibility for the first child, the second child could again be boy or a girl, the third child could also be a boy on a girl or a girl.

Now, sequence is employed here just that these two possibilities are there for the first child, this two for the second, these two for the third. So, this is like how many do we have here, I have got 2 multiplied by 2 multiplied by 2, I have a total of eight different possibilities for by having 3 children, which are like either it could be boy boy boy or boy boy girl or girl boy boy and so on and so forth.

If I do that, if you look at the sample space the sample space is shown in the screen here, 3 children I have got the first outcome as, first possible outcome is boy boy boy, then boy boy girl, then I have got boy girl boy, then I have got boy girl girl, I have got girl boy boy, I have got girl boy girl, girl girl boy and girl girl girl these are the only possibilities, no other possibility is there; I have exhausted all the possibilities when any one of them can be either a boy or a girl, these are of course, simple events and these are actually they can also be called single event.

Now, that was how I defined these events, so I defined them to be single, simple, atomic event that is what they are, so whatever the family has the outcome is it is a simple event they have a boy, then they have a girl and then have a girl again, it is an event, let us try to see, if you to make it more complicated.

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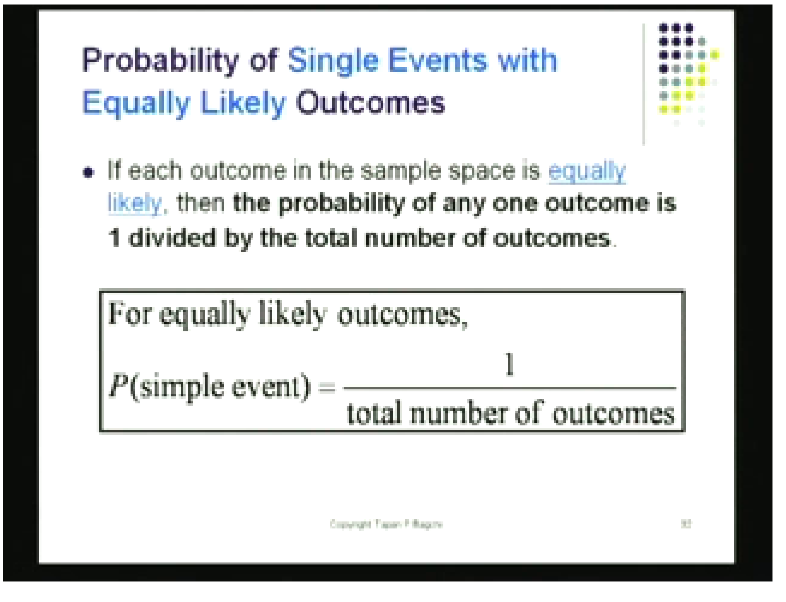

To try to do that what we will have to do is, we will have to find the probability of a simple event, remember we had eight outcomes we had a total of eight combinations possible, if eight combinations are possible then any one of them occurring, if they are all equally likely is 1 divided by the total number of outcomes, so the total number of outcomes in this case is eight therefore, the chance of any those simple events boy boy girl or girl boy boy or boy boy boy and so on. Any one of them will have the same probability and that is going to be equal to 1 divided by 8, so that is what I get, when I end up with equally likely events.

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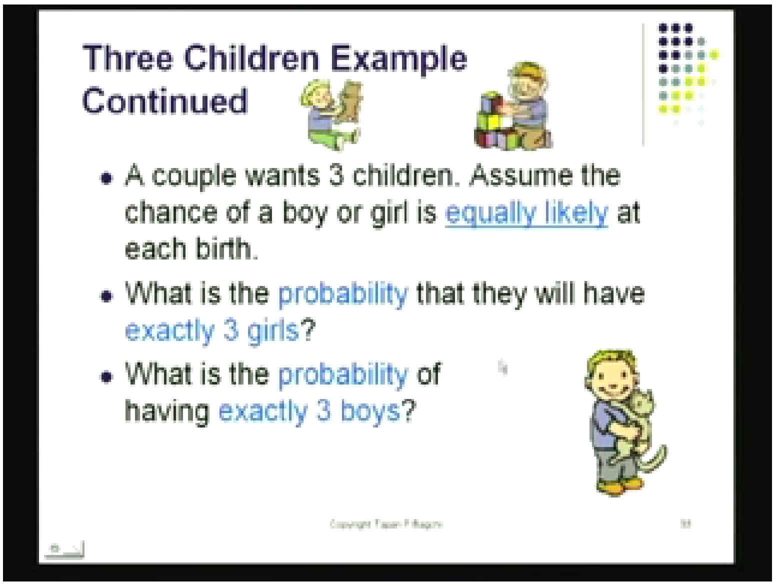

Let us take a look at this, what is the probability of having exactly 3 girls which outcome will lead to this is going to be g g g, and what is the total number of outcomes possible eight, this is only one outcome that is possible within the total set of eight outcomes. So, it is going to be the odds for my finding g g g, in the variety of different possibilities of they are having 3 children is going to be 1 divided by 8, and what about the probability of exactly 3 boys, again it is going to be the same.

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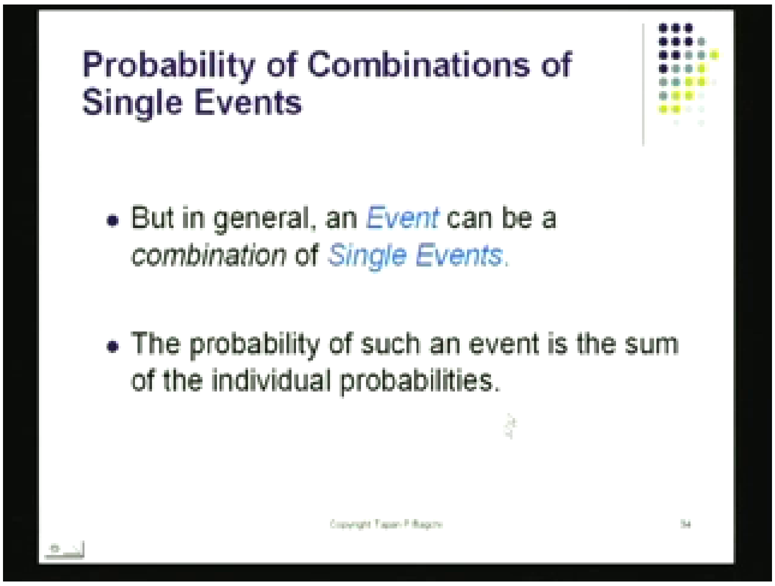

But, let us try to make it more complicated, we can make it more complicated and we will slowly try to do that, by but in general actually when we got events, the events themselves can be a combination of simple events, we can have simple events and they combine in a way they end up with more complex events. And the probability of such events is going to be in certain situations, the sum of the individual probabilities and we will see when it occurs, this is not true all the time.

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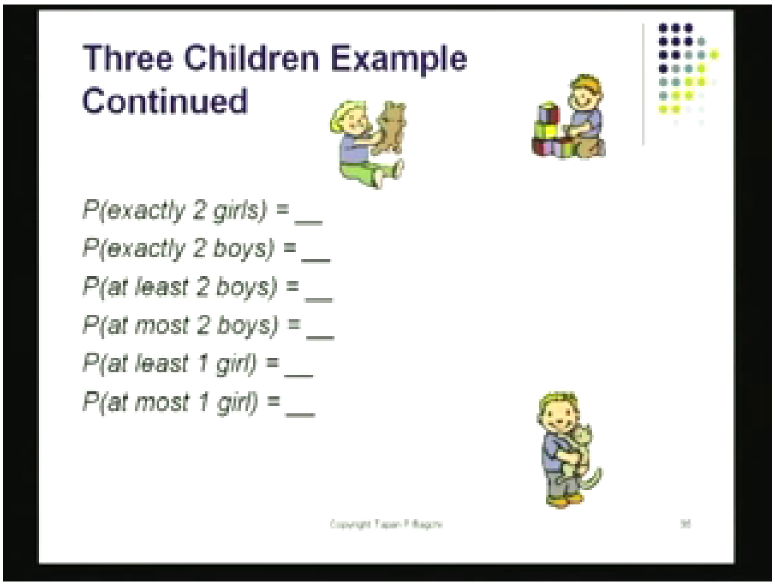

But, this fellow occur we have got some kids here, and the kids are being born and the probability we have to work out some of these possibilities, and we will see how we work them out, we will actually doing the calculations soon work them out, the probability of we are finding or finding exactly 2 girls think About this, how will I find exactly 2 girls, what are the possibilities. Let us try to write them down, and I am going to write them down, write here.

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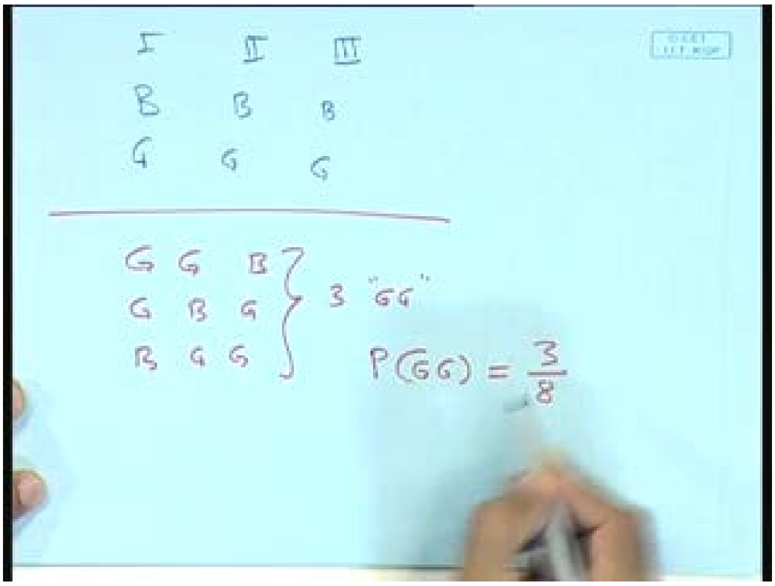

Exactly 2 girls, so how could I have them girl girl boy just like one outcome that will give me 2 girls, girl boy girl, boy girl girl is there any other way I could have 2 girls, can you think of any other way, when they are 3 kids is there any other way I could construct a a a family that will have 2 girls, it is not possible.

So, it turns out they are 3 different ways I could have g g, out of how many different possibilities the total number of possibilities is eight therefore, the probability of g g turns out to be 3 divided by 8 and of course, we could do we could very exact and we could work this out. Let us take a look at similar similar situations, when I get other other questions and we will try to work out the other questions also, take a look at take a look at for example, exactly 2 boys exactly 2 boys is again the same situation, boy boy girl, boy girl boy or girl boy boy and these are again three possibilities only.

So, if therefore, again probability of finding 2 boys to exactly 2 boys in a family of 3, family of 3 kids is going to be 3 divided by 8, at least 2 boys, now this is the interesting situation here, what are the possibilities let us try to work this out, let us try to construct the clear picture and then we will come back and take a look at the situation, I am looking to find at least 2 boys.

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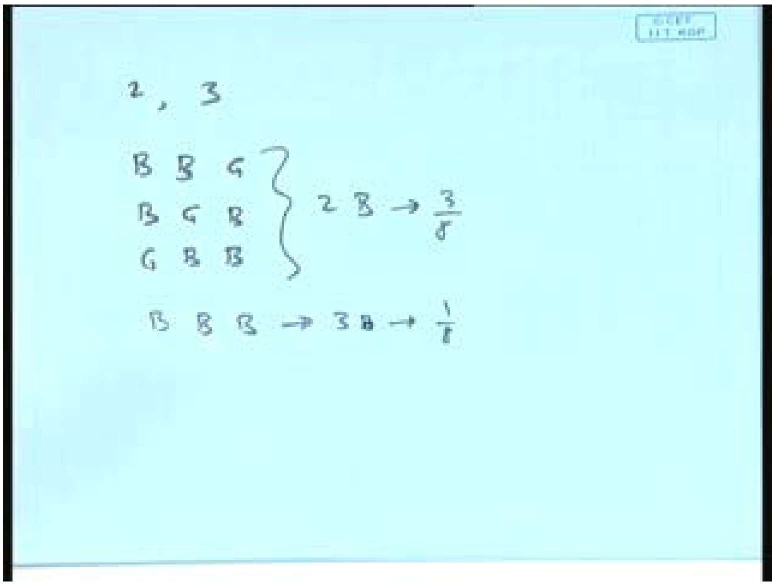

And let us try to, at least 2 boys means, I could have 2 boys or I could have 3 boys, for 2 boys I could have boy boy girl, boy girl boy, girl boy boy and what about 3, how could I have 3 these will give me 2 boys and what about what about 3 boys that happens only if I have got boy boy boy, this will give me 3 boys.

So, together at least 3 boys this will implies I have got 2 boys or I have got 3 boys, in this case what I will do, I will work out the probabilities for this, the probability for this turns out to be 3 by 8 and the probability for this turns out to be 1 by 8, when I add them together I get 4 divided by 8, which is the answer, when you look at look at the answer for this. We end up with 4 divided by 8, so when you are looking at a situation, when you are looking at, at least 2 boys you are looking at 2 boys or 3 boys, and you got 4 by 8 at most 2 boys at most 2 boys, what could be the count of boys here 2 boys or 1 boy still it is at most, is at all, what about no boys?

So, they are three possibilities here, 2 boys, 1 boy and no boys, no boys is the easy case g g g, what about one boy, boy girl girl or girl boy girl or girl girl boy just 1 boy, then of course, you got the 2 boys situation. Because, I am looking at, at most 2 boys, so that will be boy boy girl, girl boy boy or boy girl boy, if you add them all up it turns out to be

7 divided by 8, 7 divided by 8, then you look at this interesting situation, we should try this before you see the answer, you should try this, the probability of probability of finding at least 1 girl.

So, of the total outcome that I have here, remember on the sheet here I have got the total all the different outcomes possible g g g, g b g g b and so on so forth. All these guys how many of these have no girls, because I I should have at least 1 girl, so to exclude from this, those events that lead to no girl, what is that situation its b b b that is the only outcome, that does not have any girls in it.

And what is the chance of that occurring 1 by 8 therefore, what is the complement of it which is the which is the probability having at least 1 girl, it is 1 minus 1 by 8, which is 7 by 8 and at most 1 girl and this I leave to you and I leave to you figure out, how I get at most 1 girl that I will I will just move on here.

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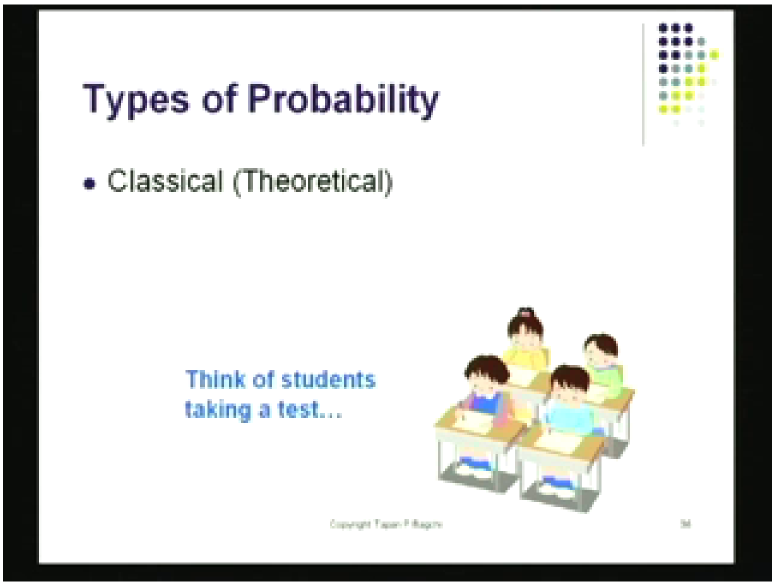

Think now of students, will be doing some more experiments and what kind of events are we going to be looking at now, now we are going to be doing a little more complicated situation, a little more complicated calculations, let us start doing this, what we call what we call you know there is something we call the theoretical estimate of probability or the relative frequency estimate a probability.

The theoretical estimate basically utilizes all these adding subtracting and so on and so forth, and they also utilize certain principles, calculation principles, computational principles, those are utilized in working out probabilities by theory and that is the classical approach to find the probability.

Then there is of course, what we call the relative frequency approach to find the probability and this happens when it count the interest count the interesting event, count the occurrence of the interesting events, and this could be done either you observe passively or you do some experiments and from that you figure out, what the frequencies are you count and find out what the frequencies are.

Let us think of a situation, when there are kids who are taking a test, so these kids here they have been they have been ask to take a test and these test of course, are not very easy test, so me of them fail the test.

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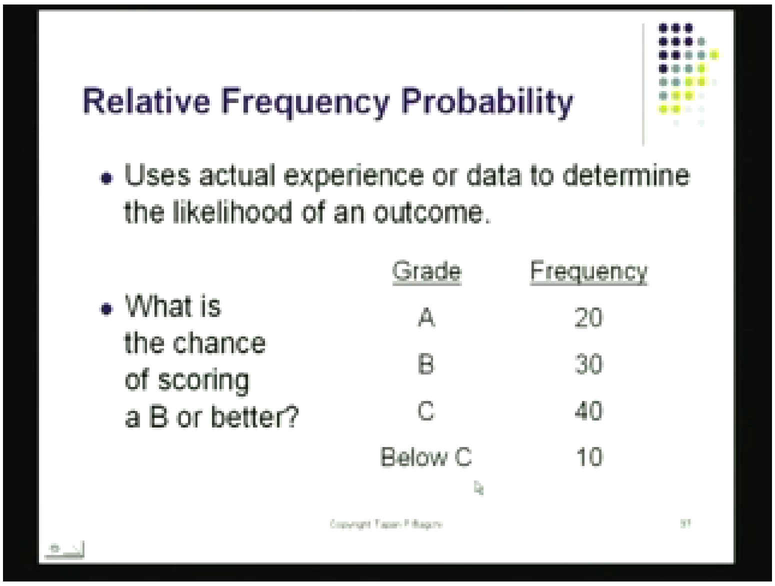

Let us say, what we done is we have given them grades at the end of the test we have given them grades, and here we are using some actual experiment actual experience and therefore, we got some real data collected here, if you look at the data collected it turns out, how many students are there 20 plus 30 plus 50 plus 40 that is 90 plus 10, 100 kids.

It turns out that 20 kids scored A, 30 scored B, 40 scored C and 10 went below C, what is the chance now of scoring B or better, B or B or better what are those events take a look at the events, B or better would be scoring A or scoring B that would be B or better or what is the frequency there 20 plus 30 divided by the total number, which is of 100, so that turns out to be in this case to be something very straight forward 0.5 and I had just written it down.

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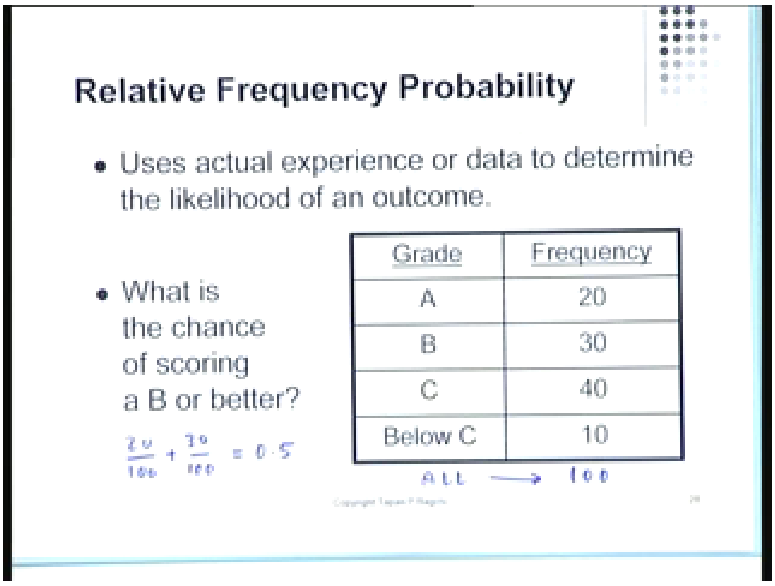

I have got this total calculation there, and you can really see that the calculation shows you first, the total count of all the boys or all the students and then I have got here scoring B or better some got, so many 20 percent score B, 30 percent score C. So, it turns out overall 50 percent or 0.5 is the probability of a randomly picked student to a found B, to a found or better, either B or better, that is like this thing, let us try to make this still more complicated.

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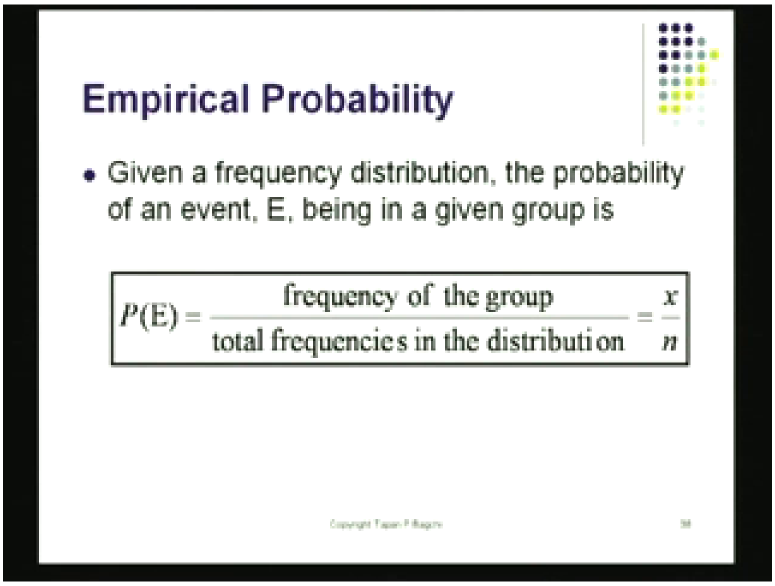

So, here what did I do, I just used empirical probability, empirical is based on real data whenever, I talk about empirical work I actually mean, you actually go out and observe and you write down the numbers and you do some calculation, you utilize your calculator and from that you do the calculation; if you are doing this you are doing empirical work. You know if you are using it only on the basis of subjectivity or some calculation you might be using some classical principles of course, both approaches could be taken, it all depends on what the situation is like, if there is no way to calculate real data to to observe real data, you may have to do some subjective assessment of what that probability is.

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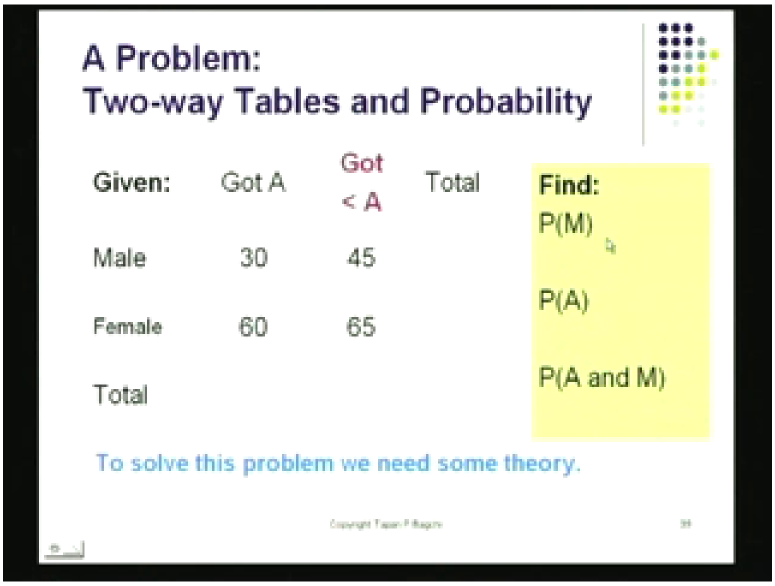

Let us took take a look at this another problem here, I have constructed what we call it two way table, and notice here the two way table is kind of tricky, I have male and female students on one side and I have students who scored A or who scored less than A shown on the other side, so here this is more like a little matrix, I have one-dimension the rows here, are marked male and female and the columns are marked A and less than A got less than A.

And then here I have got, here I have got basically these some total of those who managed to get A and they were male there were 30 of them managed to get A and 45 could not make A out of males and for females 60 manage to get A, a 65 of course, scored less than A. What I need to do is, I need to solve this problem, when I will first pose the problem of course, and what is the problem, the problem is this find the probability of a male having you randomly approach a student, what is the chance you will be a male, what is the chance you would have scored A that is probability P A randomly pick student, has he scored A and what is the chance that he is scored he scored A and also he is male, what is that chance?

Now, to be able to do that we need some theory, we will have to really see this I could not do by just counting or something I will have to work out some theory, just by doing what we did here, we would not be able to do that. So, let us see what what kind of theory you utilize and I am going to show that to you by showing you first the solution to this, and then we will explain the theory to you.

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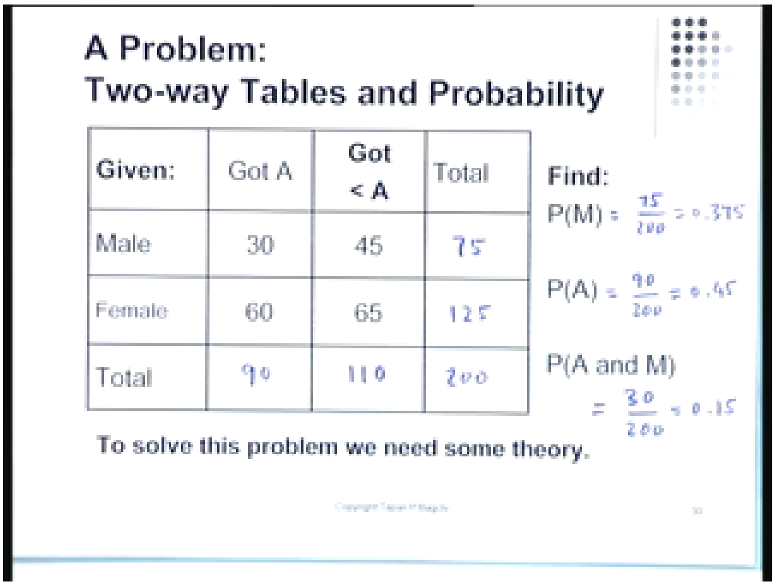

Notice here, we have the solutions the to to approach the solution the first thing, we did was found out how many total students took the test, it turns out 30 plus 45 or total of 75 male students took the test and a total of 60 plus 65, 125 females took the test. So, the total number of students who took the test is 200 and now, if you look at how many got that is 110 plus 45 plus 65 they manage to score A out of 200.

Those who those who got less than A, those who scored A they were like 30 plus 60 plus 99 manage to get A, so if I if I now look at this probability here which I am trying to find out, probability of A which is like the probability of A randomly pick student scoring A, that is going to be 90 divide by 200 and that is going to be 0.45, little calculation there.

And now, the probability that the randomly pick student is turning out to be a male, what is that, how do I find that, like total of how many males took place took the took the test and what was the total number of students taking the test, and that turns out to be 75 divided by 200 and being how sharp I am with my math, I will quickly put down 75 divide by 200 and I interpreting 0.375 that is the number, if 0.375 is a probability that is what I have to put down here, so I have got that thing, written down there 0.375.

Now, what is the other question, the question is they scored somebody randomly picked was male and also scored A, how do I find that for that what I do is, I search in the place what the count is available for both of those events, somebody scored A and also was male that turns out to be this space here, this is this is really the number that meets this condition and also this condition, note here this and is given and is intersection and that turns out to be that number turns out to be 30.

So, again what is do is, I divide 30 by 200 which in this case I do not have to use the calculator, so smaller of number just 0.15, that is the probability of a randomly pick student scoring A and also being male that is this. So, that is how we manage to solve the problem which I showed you on the screen here, I have this event here and we already have figured out each of these answers.

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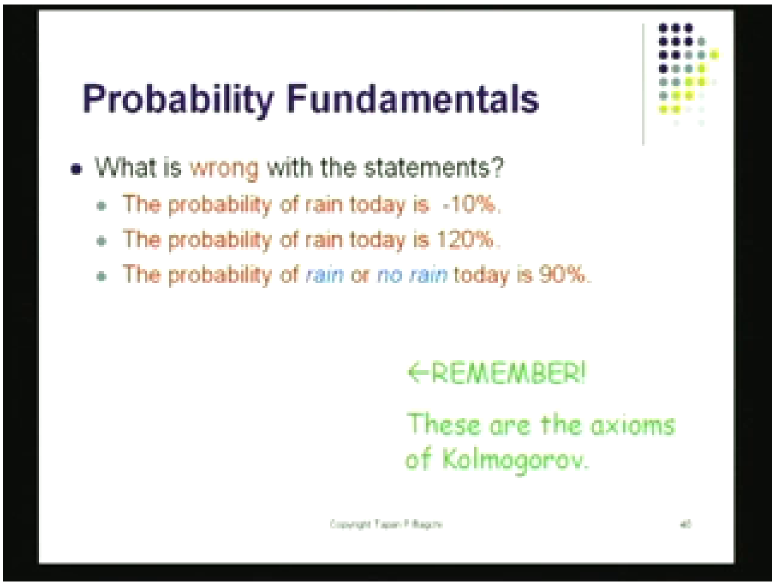

Let us say that you are talking to some people, and these were young students they really did know probability very well, and they were talking a little loosely and because, you are kind of the learned person, they wanted to talk to you in a language that would turn out to be that would turn out to be in in the language probability.

And they would make this comments, so the chance of raining today is minus 10 percent, what is wrong with this statement, this is the chance of raining today is minus 10 percent, it is very important for us to realize the probability can be never be a negative. So, there is some may be wanted to say 10 percent he wanted it to be low, just put that minus there, but be a little careful whenever you get talk to and if you a person is talking probability that number he gives you these must confirmed to certain condition it is very very important.

Similarly, if somebody comes along he says, sir the chance of there being rain today is 100 and 20 percent that means, the chance is 1.2 is that possible, can probability ever be more than 1 that is not possible, that is not how we have defined probability. So, therefore, again such a statement is not correct, not consistent the definitions of probability, what about this the probability of rain or no rain, whenever I am calling it or I am employing the motion of union and what about rain and no rain, what kinds of events are these, these are disjoined events.

In this case they turnout out to be complement of each other, if they are complement of each other they are some of, the some of the occurrence in the first event and the occurrence of the second event, it must add up to 1, 1.0, so the probability of rained or no rain, the chance of these two occurring, rain occurring there being rain or there being no rain, the chance of these two when added up that should equal exactly 1.0; it can never be 90 percent.

Because, what happens on the rest of the days, only two types of days are possible rainy days or non rainy days that is all, there is nothing else beyond rain and no rain, so that is why this again this statement again is wrong. If I want to remind you of what all things we did looking at, underneath the paper, underneath my book here, I have been hiding a little set of rules and what are those rules.

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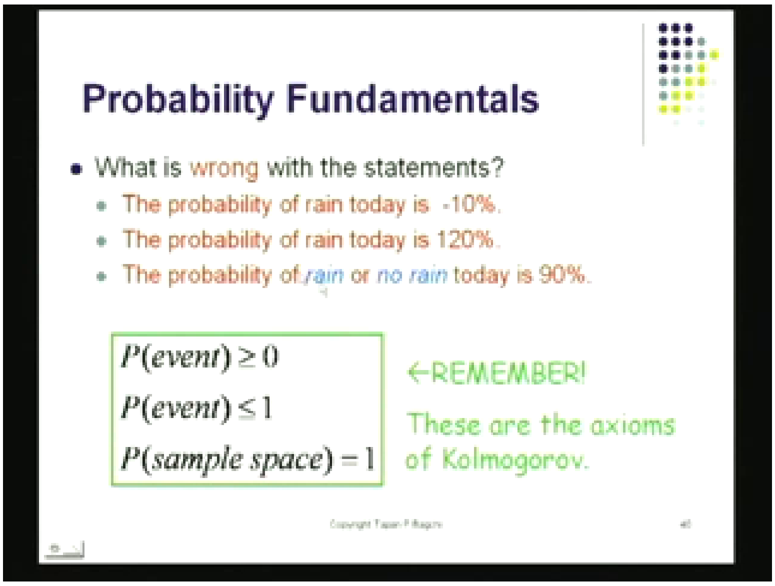

First of all, the probability of any event that we talking about must be greater than or equal to 0 that is condition number one, condition number two is the probability of an event must be again less than or equal to 1, at most it can be equal to 1, so probability of any event must be between 0 and 1. And the probability of the full sample space which in our case is rain plus no rain, the full sample space it has got two outcomes, one is rain, the other is no rain that must be equal to 1; these came from a Russian mathematician his name is Kolmogorov.

And all probability all probability literature is really based on these, if you take away these, these are building blocks, if you remove these bricks from beneath the whole building the whole theory of probability and everything it is all going to collapse. So, you got to make sure, whenever you get statements like the once that are written in red here, whenever you get these statements these statements must be valid against these conditions here.

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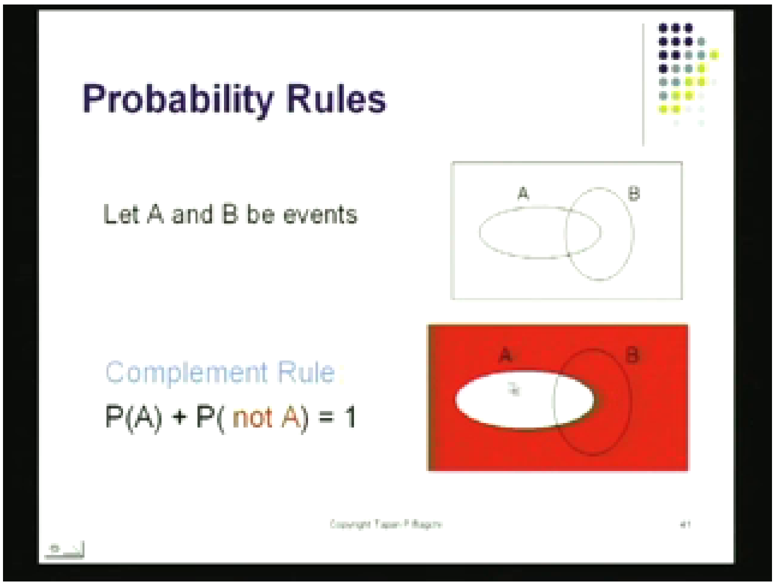

So, what are the rules, let us take a quick look at the rules, if A and B are there, if A and B are two events, then the complement rules says probability of A and the and the probability of no rain or no A that is going to be 1, so A is inside the white area they are and not A is the area that is outside A that is there, this is the complementary rule.

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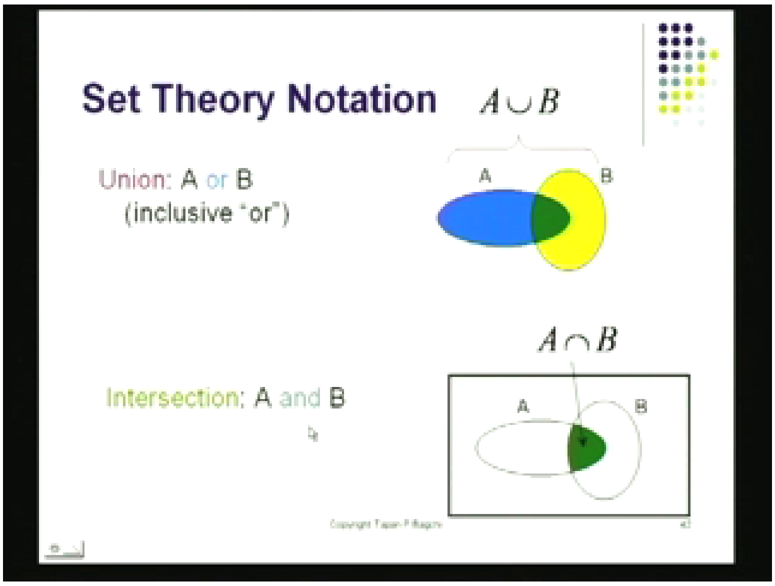

And what we have done here, we have used the set theoretic notation we have used the set theoretic notation, and we define union to be like this A or B and the other one is A intersection B in which case, the venn diagram, these diagrams are called venn diagrams.

It shows what is common between A and B and that is this intersection and this is to be very very clearly stated, you should whenever you define events and particularly when we define compound events you got to make sure you find very clearly is it a union or is it a intersection, if this is no then of course, given the probability of A and the probability of B you can construct what we call, the probability of any of these composite event, which is the A union B which in in which case we are saying either A or B would be, when it when I am talking union, when I am talking intersection I am saying A has occurred and also B has occurred, I will continue with this as we go along, thank you very much.