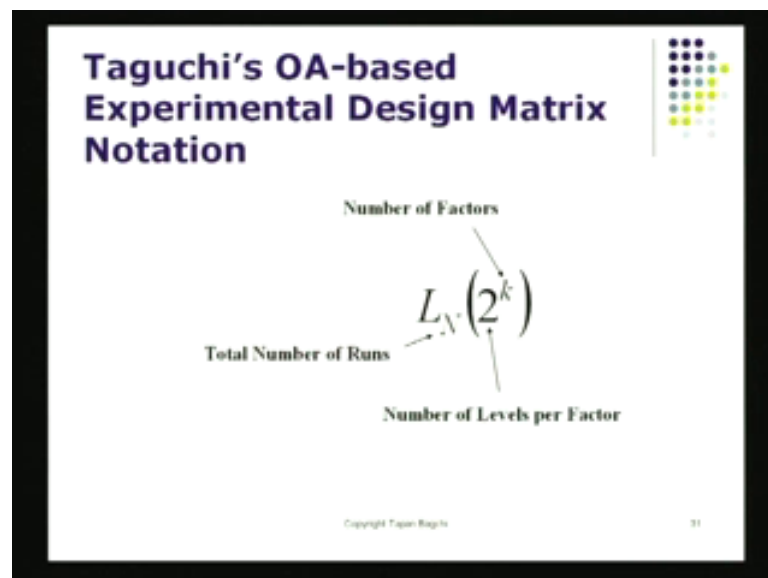


Six Sigma
Prof. Dr. T. P. Bagchi
Department of Management
Indian Institute Of Technology, Kharagpur

Module No. # 01
Lecture No. # 34
Robust Design

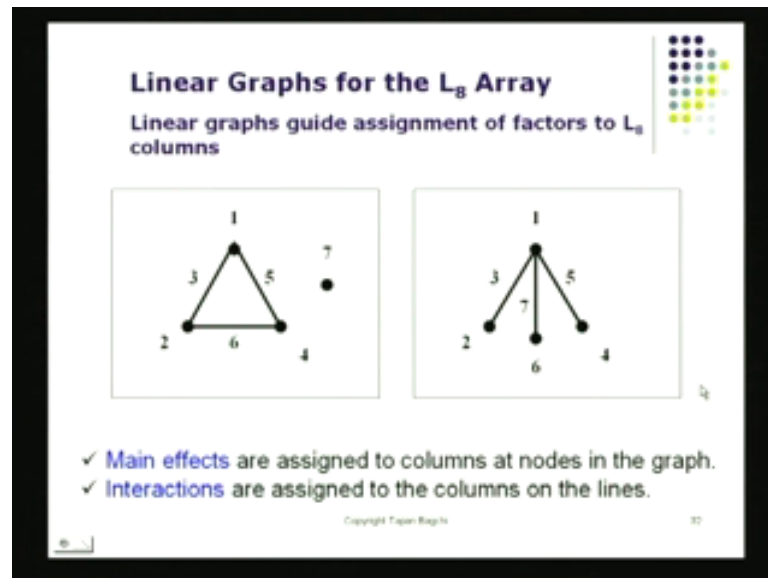
Good afternoon, this is the second part of our **design** discussion on Robust Design. And this is continuing from the previous lecture, lecture number 33 that was on the basics of Taguchi methods. What we will be doing now is discussing the method for achieving a robust design that is, what we will be illustrating in this particular lecture.

(Refer Slide Time: 00:50)



Before I go further let me just **you know** tell you something about the notation that is used commonly. Generally speaking most of Taguchi's experiments those are based on orthogonal arrays. Orthogonal Arrays O A in short, those are **you know** given in matrix notation and the notation is usually written like this L N and then in bracket 2 to the power k. And these are actually let me just explain what these are, L is of course the **notation** notation of the O A matrix itself. Number of runs are specified by this letter N then number of factors those are specified this letter k and the levels that you use for each of those factors that is denoted by what we call 2, this number 2.

(Refer Slide Time: 01:50)



So, that is actually L_N and in brackets two to the power k that is like one notation that is used for orthogonal arrays. There is another arrays another notation that is also used and sometimes of course, we got something associated with each array each O_A that is called the linear graph of that particular array. For example, look at the triangle **in the in the** on the left it is showing here column for **certain certain** it has got certain nodes and certain branches, the **arcs actually** arcs and nodes both specify the mark of the column.

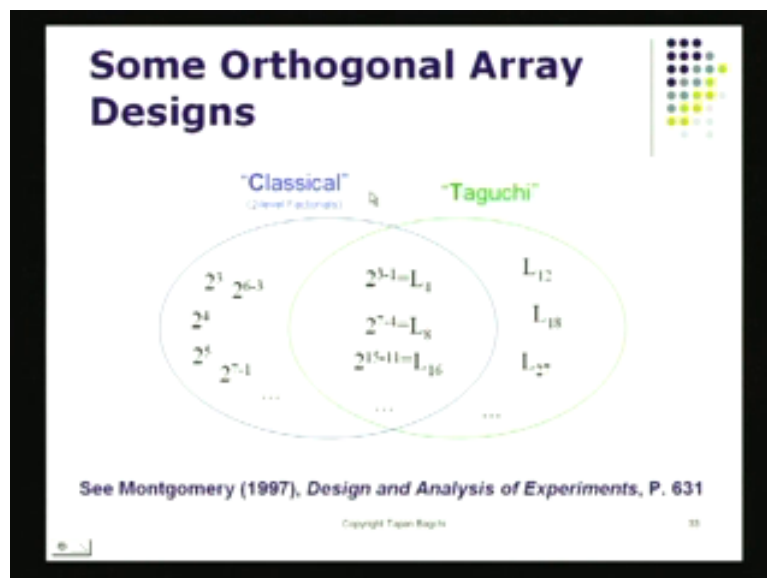
So, **there** there is 1 node, that is specifying column number 1 and another node is specified column number 2; then between them we have got column 3 assigned. Similarly, I have got column 4 here and column 5, what this is really saying is the interaction of some factors that I have assigned at column 1. And another one at column 2, the interaction between those two will be noticeable in column 3.

Similarly, if I have assigned factor **at setting one** at column 1 of the orthogonal array and another factor at column 4 of the orthogonal array; then column 5 of that array is going to represent the interaction between 1 and 4. So, will be the interaction between factors 2 and 4 would be found by doing calculations using the data, using the notations in column 6. Column 7 stands by itself and I can apply I can use that, so here I can study the interaction of 1 2 3 factors with each other that is like three different two level factors. This column is standing by itself and I can assign **another different** another factor there **to to** conduct my experiment.

So, in all I will be able to study seven factors, out of those seven factors basically I will be looking at 1 2 4 and 7 as main effects. 3 is actually an interaction effect between 1 and 2, 5 is interaction effect between 1 and 4 and 6 is the interaction effect between 2 and 4, that is (0) it is done.

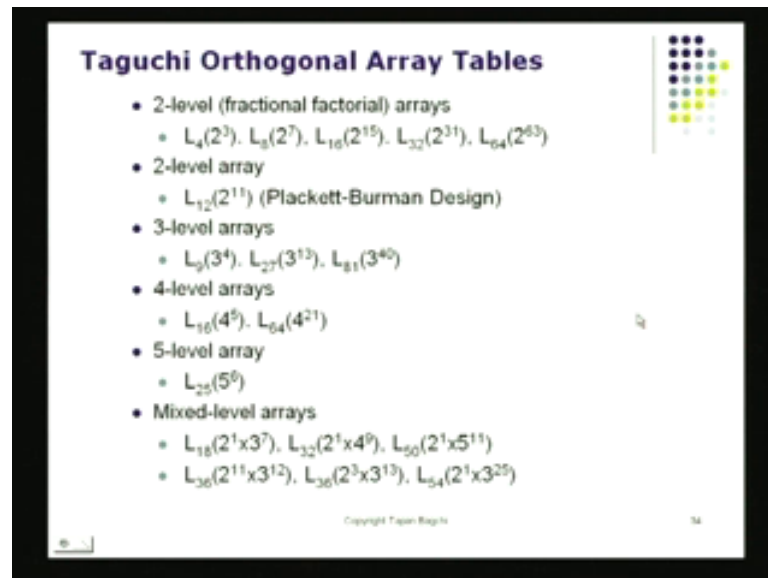
I could do the assignment column assignments slightly differently, if I assign factor 1 to column 1, factor 2 to column 2, factor 3 to column 4 factor 4 to column 6 or a here b here c here d here I will be able to study the interaction between a and b by looking at column 3 and so on and so forth (Refer Slide Time: 00:04). So, these are like linear graphs that make it easy for us to recognize, what are the interactions are going to be; and thereby I can do my calculation to find main effects and also some interaction effects.

(Refer Slide Time: 04:38)



There is clearly a correspondence between the classical two level factorial designs and Taguchi design. Taguchi designs are just been popularized by Taguchi that is why they are called Taguchi matrix. They are identical to classical matrix and Taguchi has provided some additional matrices also that is what one of the things Taguchi is done and I picked it up from a book which is there. The design and analysis of experiments book by Douglas Montgomery in 1997 edition, if you look at page 631, it will show you this representative there.

(Refer Slide Time: 05:10)



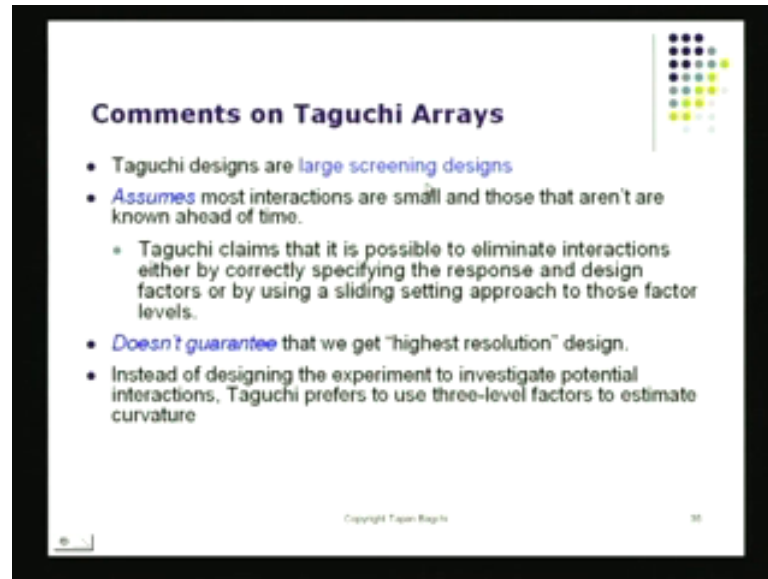
The slide, titled "Taguchi Orthogonal Array Tables", lists various types of orthogonal arrays. It includes a decorative graphic of colored dots in the top right corner and a copyright notice at the bottom. The content is as follows:

- 2-level (fractional factorial) arrays
 - $L_4(2^3)$, $L_5(2^7)$, $L_{16}(2^{15})$, $L_{32}(2^{31})$, $L_{64}(2^{63})$
- 2-level array
 - $L_{12}(2^{11})$ (Plackett-Burman Design)
- 3-level arrays
 - $L_9(3^4)$, $L_{27}(3^{13})$, $L_{81}(3^{40})$
- 4-level arrays
 - $L_{16}(4^5)$, $L_{64}(4^{21})$
- 5-level array
 - $L_{25}(5^6)$
- Mixed-level arrays
 - $L_{18}(2^1 \times 3^7)$, $L_{32}(2^1 \times 4^9)$, $L_{50}(2^1 \times 5^{11})$
 - $L_{36}(2^{11} \times 3^{12})$, $L_{36}(2^3 \times 3^{13})$, $L_{54}(2^1 \times 3^{25})$

Copyright Taguchi Systems
34

And of course, there are large number of other arrays which are also useful, they are also useful as for running Taguchi experiments. And something here we got to remember is you remember we were not looking at interaction therefore, Taguchi arrays are basically screening designs; they are basically let you see very quickly, which factors are important and which factors are probably not so, important. That is something that can be done quite easily using orthogonal arrays. I assume when I am using Taguchi's arrays, I assume that most of the interactions are either small or they are not present at all and it does not guarantee that we get highest resolution design; that means, we are not able to see a lot of interactions that also should be in some cases important.

(Refer Slide Time: 05:18)



Comments on Taguchi Arrays

- Taguchi designs are large screening designs
- Assumes most interactions are small and those that aren't are known ahead of time.
 - Taguchi claims that it is possible to eliminate interactions either by correctly specifying the response and design factors or by using a sliding setting approach to those factor levels.
- Doesn't guarantee that we get "highest resolution" design.
- Instead of designing the experiment to investigate potential interactions, Taguchi prefers to use three-level factors to estimate curvature

Copyright Tapan Raycha

Let us move now to this concept of robust design. Something I have got to tell you is when we are talking about robustness we are talking of variability of response; and we got to remember that this variability is produced not because the design variables are floating all over the place. But because environmental factors these are the factors we are not controlling they are impacting on the system and **and and** causing it to vary in terms of its performance; that is like something that is happening.

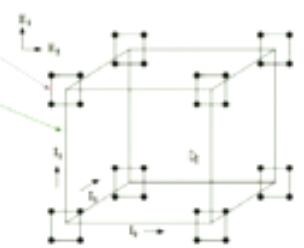
When, I am using the product **out in the** out in the field. Under the influence of environmental factors only. In addition to of course, the design factor which are there. When this is happening what I have to do is to create a robust design, I have to somehow conduct experiments that look at the variations done on the design variables; and also variations done on the noise factors, because the two together they really determine the total effect the total response.

And total response, in design is partly controlled by design variables and partly by the environmental factors; what we have to do is we have to choose these design variables in such a way, that the system becomes insensitive to effect of noise that is something that we have to do; if you are able to do this we will end up with a robust design.

(Refer Slide Time: 07:15)

Taguchi's Robust Design Experiments

- Taguchi advocated using *inner and outer array* designs to take into account *noise factors* (outer) and *design factors* (inner)
- Design factors: I_1, I_2, I_3
- Noise factors: E_1 & E_2
- Objective: Maximize response while minimizing its variance



Copyright Tapan Rayati

Let us see how we do it what Taguchi suggested was that he constructed two arrays two matrices, one will be the inner matrix plus that is the green matrix, the green matrix is inside the big box. The big box is the design matrix this is the one that has got I_1 I_2 and I_3 , three design variables. And with that we are able to basically generate we are able to generate eight different types of prototypes; if each of these factors is basically looking at two levels. If you are if you are manipulating **these variables** these different design factors at two levels each I will end up with the these eight possibilities. Two cube that is eight that is the construction of the inner array, the outer array basically what it does it brings in noise now also in a controlled way.

So, here we have got two noise factors in this particular example we got two noise factors E_1 and E_2 , they are the environmental factors and with them I am able to generate, if there are two if there are two levels each I am able to generate a little square. Thus like a tiny matrix that will try to shake this prototype. The same matrix I will be using here, so here what I am doing is I have got a product, I have got a prototype and I am going to be shaking, I am going to be shaking it with the environmental factors.

So, if I have got a product in my hand **I will try to** I will try to impact it using environment, environmental factor which in this case could be a physical force it could also be temperature it could be dropping it and so on and so forth. Any of those things even the quality of the components those could be considered to be noise factors. When I

bring in design factors which are inside and I impinge it with design, with environmental factors the variance of this is the response of this is going to vary. I have to discover such a design which will be one of these prototypes.

See, there are eight prototypes here prototype 1, prototype 2, prototype 3, prototype 4, prototype 5, 6, 7 and 8. One of these is most robust and that is what I have to discover for that what I have to do is I have to shake these prototypes each of them with comparable noise conditions when I do that I will be able to find the one find the prototype that is going to be the robust one and that is someone I will try to commercialize.

(Refer Slide Time: 09:33)

Example: Robust Design OAs of Starter Motor Parameter Design

Outer array: battery voltage, ambient temperature

Inner array: armature turns, gage of wire, ferric content of alloy

Run #	Inner array			Replicates →				Starter torque	
	I1	I2	I3	E1	2	3	4	Output MEAN	Output STD.DEV
1	-1	-1	-1	75	86	67	98	81.5	13.5
2	1	-1	-1	87	78	56	91	78.0	15.6
3	-1	1	-1	77	89	78	8	63.0	37.1
4	1	1	-1	95	65	77	95	83.0	14.7
5	-1	-1	1	78	78	59	94	77.3	14.3
6	1	-1	1	56	79	67	94	74.0	16.3
7	-1	1	1	79	80	66	85	77.5	8.1
8	1	1	1	71	80	73	95	79.8	10.9

Copyright Tiger Team

Let us see how this is done in real life we have a situation here, where we are trying to you know this is like starting your car early in the morning. When a battery hopefully is fully charged and of course, that voltage the voltage of charging, and the voltage of the fully charged battery it. If it is used for a whole it will is partially charged. If it that is reduced that is going to reduce its voltage then of course, I have got ambient temperature you know very well on cold days in **when when** the temperature outside is cool it is not very easy to get all the juice that you want from the battery.

But the starter motor it better crank at better turn the full engine with all its **(())** and that should happen with minimum sensitivity to two things, one the battery voltage within some region; and also ambient temperature within some region. If you are able to do this we got a good starter design, what did they do (Refer Slide Time: 10:40)? They had three

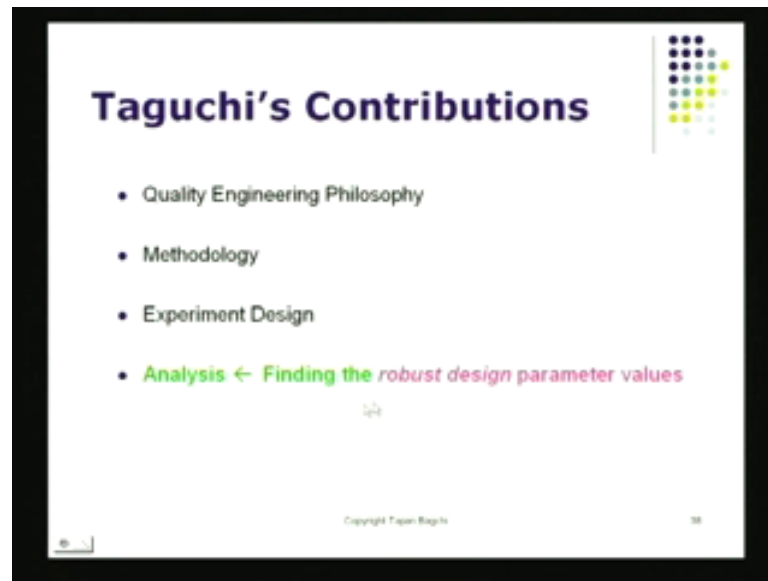
design variables, armature turns on the starter motor, gage of the wire used and the ferric content of the core.

These were three design variables that they could play with they wanted to get a starter motor design, they wanted to reach a good starter motor design and as far as environmental was concerned battery voltage would be one which could be at two levels low and high. Ambient temperature **ambient temperature** also would be low and high. Then, they beat one prototype which was at low level of I 1, I 1 is one of the design variables armature I 2 would be gage of wire also kept at low level and the ferric content of the core that was also kept at low level. They ran this trial and they made sure that **while these results** while the results were evaluated the motor itself the motor, that you produce the prototype itself was subjected to these noise conditions.

So, I took the same thing and I measured its torque, at these different conditions under certain conditions it gave me good torque in certain conditions it get me poor torque. And I calculated the mean of these responses for each of the noise settings and also the standard deviation of the responses. Notice something here in the conventional design task, I would have just probably looked at the mean I would have found probably the highest one.

And that would have would have been the one that I would have recommended for you know basically for commercialization. But it turns out that such a thing would not be **would not be** really a robust design **if a** if a the robustness could be further improved, because this standard deviation here was 14.7. If I look at another prototype the torque is slightly less, but it is much more robust it 14 here and say 8 here that is almost like half of it.

(Refer Slide Time: 13:06)

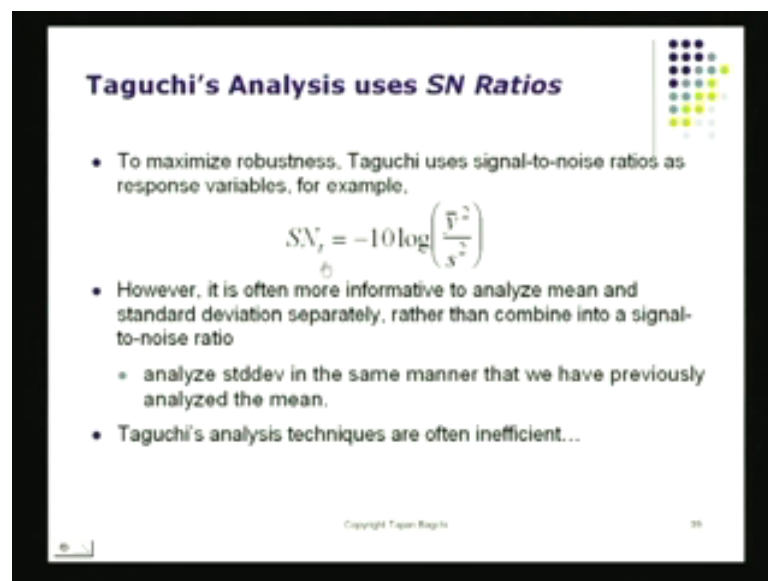


Taguchi's Contributions

- Quality Engineering Philosophy
- Methodology
- Experiment Design
- **Analysis** ← Finding the *robust design parameter values*

Copyright Tapan Bagchi 38

(Refer Slide Time: 13:40)



Taguchi's Analysis uses SN Ratios

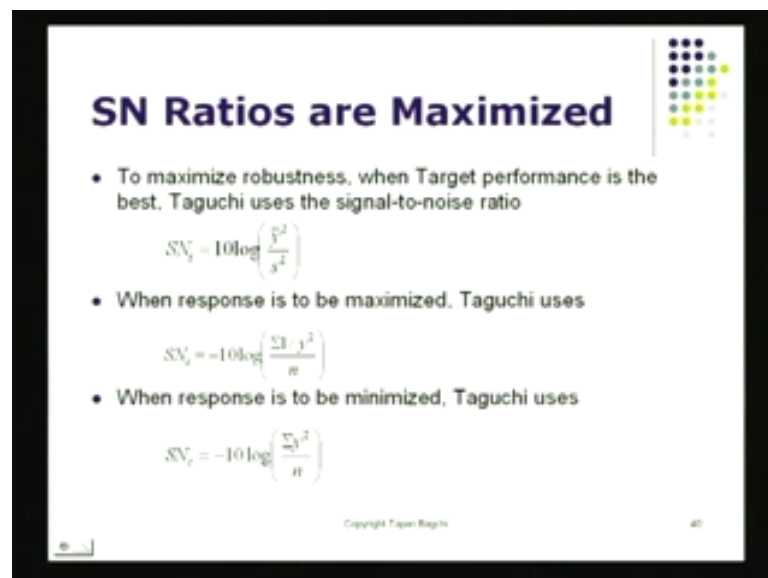
- To maximize robustness, Taguchi uses signal-to-noise ratios as response variables, for example,
$$SN_r = -10 \log \left(\frac{\bar{y}^2}{s^2} \right)$$
- However, it is often more informative to analyze mean and standard deviation separately, rather than combine into a signal-to-noise ratio
 - analyze stddev in the same manner that we have previously analyzed the mean.
- Taguchi's analysis techniques are often inefficient...

Copyright Tapan Bagchi 39

So, in fact, it turns out that the standard deviation of the output turn out to be much much smaller here. This of course, I could plot **this I could plot** and I could find the optimum settings for this and I could do that by doing this things and I will try to show that to you in a **in a** minute I will show that to you. Couple of other things Taguchi did he said you have done your trials **you have done your trial** you got the data, let me show your data analysis method. And of course here, there is a bit of controversy in the way Taguchi approached his data analysis method.

Taguchi being electronics engineer, electrical engineer, he went after signal to noise ratio because he had variability which was like noise and he had average which was like the signal, so he worked out a signal to noise ratio type of measurement. And that is what is shown here you see that little formula there signal to noise ratio it is shown as with this little formula minus 10 log y bar square divided by s square. This little formula here is an indication of this sensitivity of the **of the** system to noise, signal to noise ratio (Refer Slide Time: 13:36). This was Taguchi's approach what he did was he went along and he applied that to define various types of conditions under which you probably need different types of signal to noise ratio.

(Refer Slide Time: 14:10)

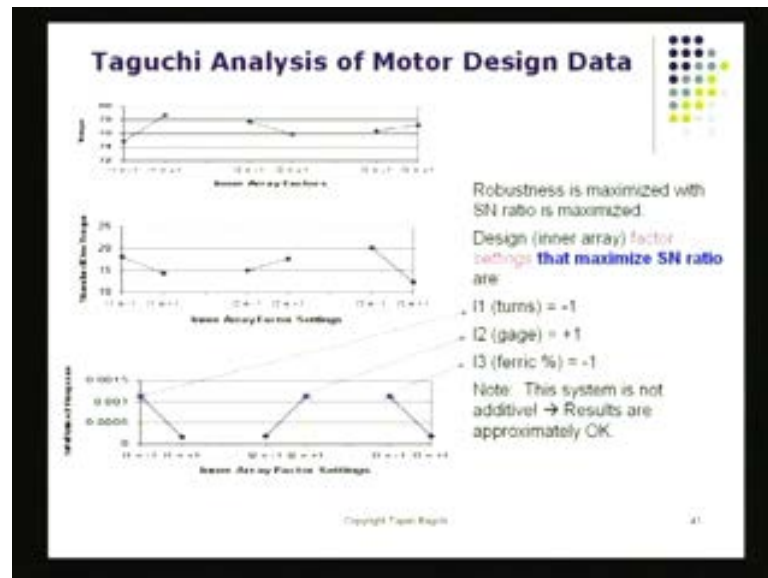


SN Ratios are Maximized

- To maximize robustness, when Target performance is the best, Taguchi uses the signal-to-noise ratio
$$SN_{\eta} = 10 \log \left(\frac{\bar{y}^2}{s^2} \right)$$
- When response is to be maximized, Taguchi uses
$$SN_{\eta} = -10 \log \left(\frac{\sum (y_i - \bar{y})^2}{n \bar{y}^2} \right)$$
- When response is to be minimized, Taguchi uses
$$SN_{\eta} = -10 \log \left(\frac{\sum y_i^2}{n} \right)$$

Copyright Tapan Kojan 40

(Refer Slide Time: 14:36)



So, Taguchi gave us three different formulas for evaluating signal to noise ratios, once your data was collected from your robust design experiments which is the inner array outer array experiment. So, if you had an experiment like this, if you had an experiment like this or if you had an experiment like this **you take the data** you take those data you would convert that into signal to noise ratios like this using one of these formulas. And that could be then be plotted and here what we have is the plot, first of all of the **average** average torque; then the standard deviation of the torque and them of course, the signal to noise ratio.

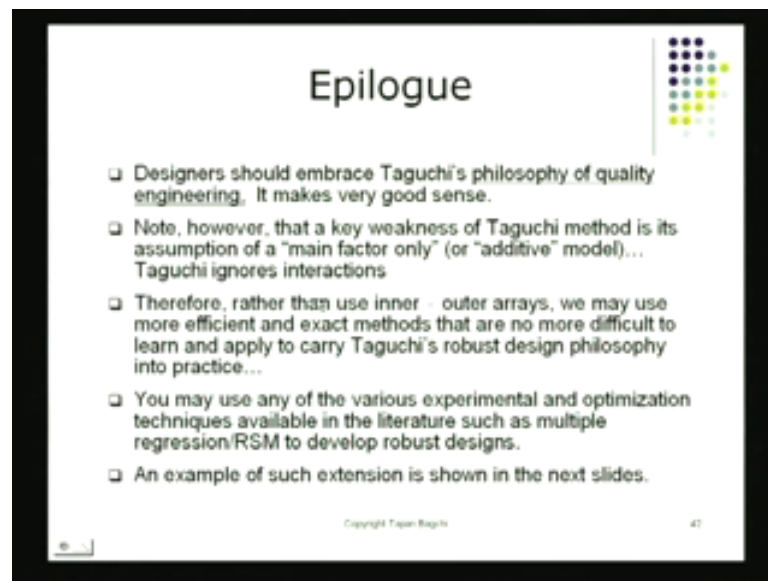
Taguchi's idea was if you maximise signal to noise ratio you are getting maximum robustness that is what he said, he said if you are trying to if you are maximizing signal to noise ratio, you are maximizing robustness that was his suggestion. And of course, here we have got the main effect of basically these different, three different control variables which we saw earlier. And let me just to remind you what those variables are they are armature turns gage of the wire and ferric content of the core.

Once you did that once you plotted this you could see the main effect of each of those things and it turns out, this guy has the largest effect this particular factor has the largest effect on torque the higher the better of course. But, if you look at this standard deviation it turns out this design also is a good one; it gives you a lower standard deviation I am

getting high torque and **lower** lower standard deviation ideally this is the one I should be using.

But, Taguchi said what you should do is use the signal to noise ratio and I end up with some other factors here. There is some controversy here and many people of course, say that instead of using signal to noise ratio you should be using other approaches to achieve robust design.

(Refer Slide Time: 16:14)



The slide is titled "Epilogue" and features a decorative graphic of colored dots in the top right corner. It contains five bullet points:

- ❑ Designers should embrace Taguchi's philosophy of quality engineering. It makes very good sense.
- ❑ Note, however, that a key weakness of Taguchi method is its assumption of a "main factor only" (or "additive" model)... Taguchi ignores interactions
- ❑ Therefore, rather than use inner - outer arrays, we may use more efficient and exact methods that are no more difficult to learn and apply to carry Taguchi's robust design philosophy into practice...
- ❑ You may use any of the various experimental and optimization techniques available in the literature such as multiple regression/RSM to develop robust designs.
- ❑ An example of such extension is shown in the next slides.

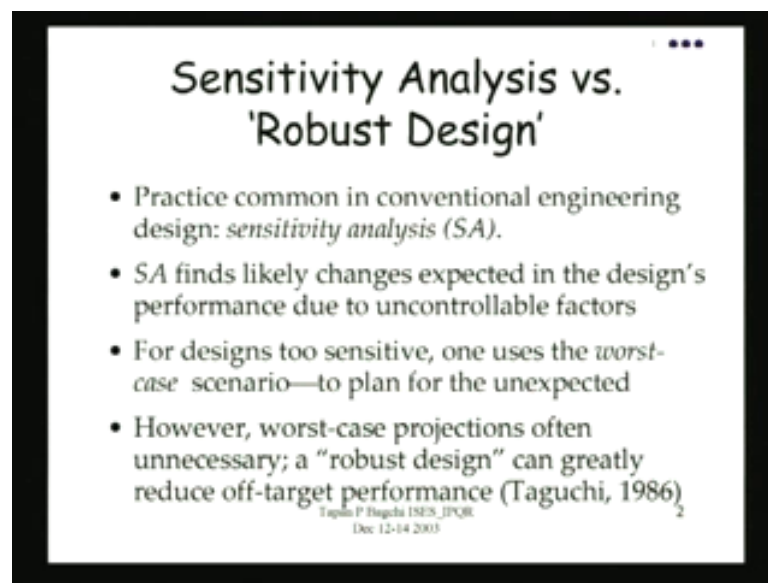
At the bottom of the slide, there is a small copyright notice: "Copyright Tapan Bagchi" and the number "47".

Still his philosophy stands it is very important us to realize Taguchi gave us many new things one of course, is that being at the target is the best that is number 1. Number 2 you got to make the product robust this is number 2 it is **very very** important. Number 3 to conduct experiments, you should use orthogonal arrays many times these turn out to be quite alright; you do not really need fancy interactions to be incorporating analysis.

Then of course, the fourth contribution that he made which is where some of the controversy **is** is his recommendation that you should be using signal to noise ratio. I have also if done lot of Taguchi experiment several engineering experiment that involved the Taguchi method, what I normally try to do? And this is of course, in print also it is in publication is **I try to drive** I try to drive performance to target and I get a contour; I get a contour of design combinations design variable combinations that give me performance on target. Then, what I do my second step is I go on that contour and I try to find the point that has got highest robustness.

So, I am achieving target value, achieving target value for the performance and also I am achieving robustness by minimizing standard deviation of that, standard deviation of variation I do not use signal to noise ratio that is my personal choice. But of course, you know the **we** we have got a lot of choice here. Once we know the purpose once we know the philosophy that is all we really want to **how we do it** how we do it in detail is pretty well left **left** to you and left to me and that is what you will find most of the time.

(Refer Slide Time: 18:06)



Sensitivity Analysis vs. 'Robust Design'

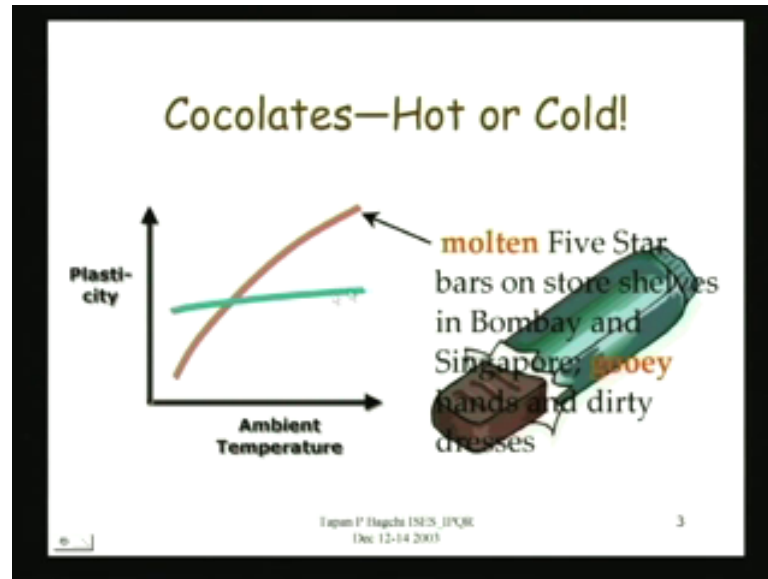
- Practice common in conventional engineering design: *sensitivity analysis (SA)*.
- SA finds likely changes expected in the design's performance due to uncontrollable factors
- For designs too sensitive, one uses the *worst-case* scenario—to plan for the unexpected
- However, worst-case projections often unnecessary; a "robust design" can greatly reduce off-target performance (Taguchi, 1986)

Taguchi P Taguchi ISEIS, JPCOR
Dec 12-14 2003

Now, there are cases when we have we have gone a little bit beyond where Taguchi took us, one of which is called multi objective robust design. Now, to get there let me remind you of what **we started with** we started with for example, robust design as a philosophy and this is different from sensitivity analysis; because here you are also trying to incorporate Taguchi's philosophy that things should be on target. And then they should be robust if we just do sensitivity analysis let us say I completed the design of a product, then I vary one or two factors **one or two factors I vary**.

And I see the impact of that on the on its performance what that really does is it just gives me one factor at a time type of sensitivity. I really I am not doing robust design type of experiment what I am really doing sensitive analysis.

(Refer Slide Time: 19:24)



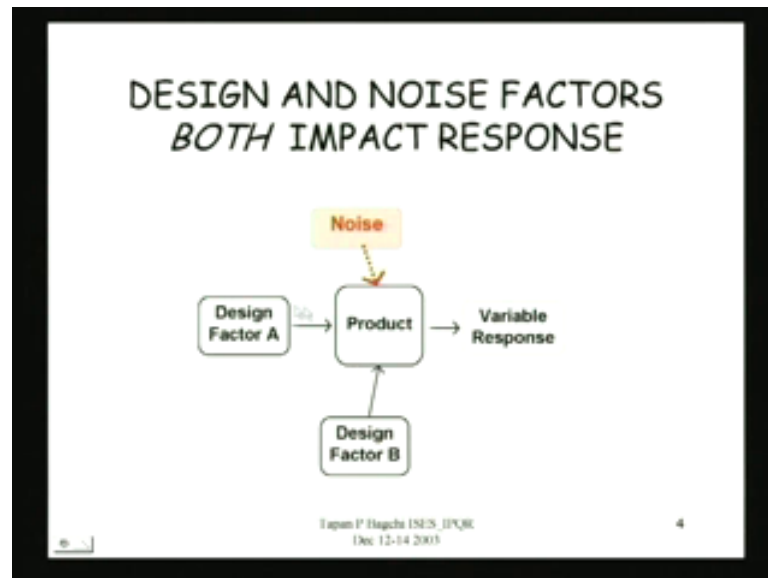
So, this is something you got to remember if you do it the Taguchi way if you use the inner array and the outer array. Perhaps what you could do is then see on what settings I am getting target performance; and which among all the designs that give me target performance I am getting maximum robustness if you are doing that you are doing pretty good. Once you except this philosophy the method that you use is pretty well up to you, but please make sure performance is on target number 1 and number 2 you get robustness (Refer Slide Time: 19:19).

It turns out that there are examples that we can go back to remember the chocolate experiment. The red bar the red chocolate bar would be the molten molten chocolate those are the ones that that kids would like to probably have, but mom's would complain because they have to wash their shirt. And of course, kids come back with all the gooey stuff in their hands that is also something mom's do not like; on the other hand if the design is done in a such a way that the design is tropicalized, which really means, it is made robust with respect to the environment in the tropics that is in bombay or kuala lumpur or singapore or somewhere.

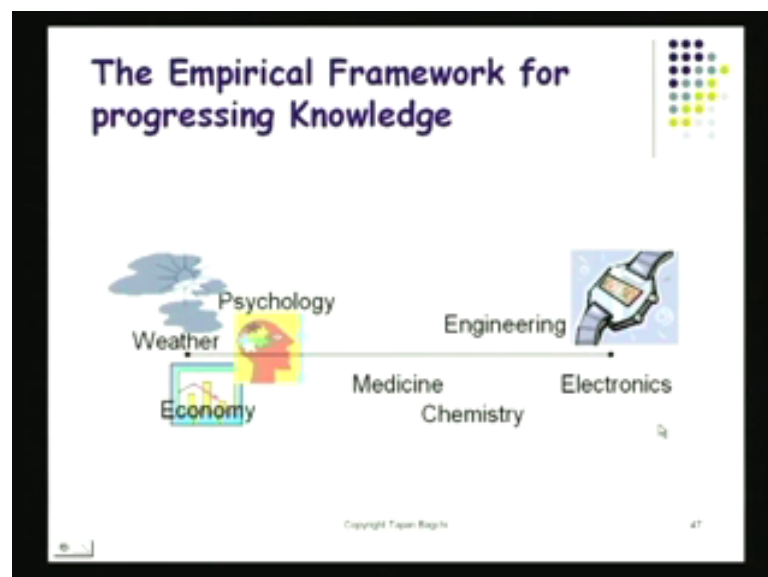
Then of course, what you have to do is you have to play with the recipe, once you play with the recipe make sure you choose the recipe that gives you that right target that is the kind of kind of shall I say plasticity. That kids are going to be happy with the real consumers of these chocolate bar that is number 1. Number 2 you also got to make it

robust for that you may have to change the you have to play with the recipe and these are recipes the this playing with the recipes can be done in controlled setting using design of experiment, so that is like something that is like one approach.

(Refer Slide Time: 20:30)



(Refer Slide Time: 20:58)



Let me show you another approach I have noise I have noise of course, noise factor is there and there are design factors a and b I can combine these different factors I can produce different prototypes. I can subject those prototypes to noise and I can get I can really then do my robust analysis robust design analysis I could do that. This is like

something that, I can do **I can do** quite easily, but just take a look at this diagram that I showed you long time back. What we are really trying to do is we are really trying to see are there frameworks available.

Or can we construct frameworks were by we could do this empirical work perhaps not using O A's because O A's do not really give us the exact optimum setting O A's probably do not do that, O A's probably it is not very easy for O A's to reach exact the exact optimum or even near optimum not. So, easy they have to do multiple experiments multiple rounds of experiments is there some method that can actually short cut this path can take you a little closer there. So, basically what we are doing is we are saying on this march toward knowledge and in trying to do this empirically we may not exactly always use the O A type of method. Let me show you a method that actually works quite well.

(Refer Slide Time: 21:48)

The Taguchi Experiments

- Taguchi advocated using *inner and outer Orthogonal array designs*
- Objective: Maximize Satisfaction while minimizing its variance

Taguchi P. Taguchi ISEIS, IPQC
Dec. 12-14 2003

5

The diagram illustrates a 3D orthogonal array design with three axes labeled x_1 , x_2 , and x_3 . It shows eight points in a cube, each representing a different combination of the three design variables. The points are connected by lines, forming a cube. The axes are labeled x_1 , x_2 , and x_3 .

Taguchi experiments would have required you, if you got three design variables to build these eight different prototypes, **see** see eight chocolates bar there each of those chocolates bar would have would have then to be shaken up using temperature variation and perhaps humidity variation.

And you will probably have to see **which is which of these parts if they** which of these bars first of all would meet the target performance that is the **target** target stiffness. That I can chew on without too much trouble number 2 I will also have to find out those designs that give me the target performance, **those designs** among those designs is there

one that has got the smallest sensitivity to noise this is the one that is going to be what we call the robust design.

How do we achieve that well Taguchi's approach would have been this would have been conduct the inner array experiment. Bringing the outer array shake each of these chocolates and then workout the robust designs; that would be one way.

(Refer Slide Time: 22:51)

Robust Design search by GA

- GA replaces *inner array*
- *Outer array* replaced by Monte Carlo
- Objective: Keep response on target while minimizing its variance

The slide features a 3D scatter plot with axes labeled I_1 , I_2 , and I_3 . Several green, pill-shaped objects are scattered throughout the plot, representing different design configurations. A small grid of colored dots is visible in the top right corner of the slide.

Taguchi P Taguchi ISES, IPQR
Dec 12-14 2003

6

The alternative approach is this, what are we trying to do **try to try to** think a little bit what we are trying to do when we are trying to do the inner array outer array experiment. In the inner array, we are searching the design space **we are searching the design space**. In this case what we have done we search certain spots in the design space and that we have done by taking two settings for each of these design factors who ended up producing eight prototype.

There is an alternative approach and this approach comes from the inspiration that is given by Charles Darwin. And the person who works this out in good detail was John Holland and **he** he came up with a technique called genetic algorithms; that is also search approach and that is quite different from the design of experiments approach. What you there do is you **(()) like** like nature for example, nature produces through its genetic chromosome crossovers and mutations it producing various types of products, products are actually living things.

Some of these are pretty fragile they are not able to adapt to the environment and they die off these species die off. On certain occasion these species turn out to be pretty robust, those are the ones that they survive they are adapt to the environment and they live on for millions of years. We know about turtles we know what is cockroaches they have survived the on sort of so many things other things have adapting themselves they have modified their genes chromosomes and so on. And they have been able to basically survive in the thing there proteins are not quite different from the original proteins and so on.

Now, this model is also there this is also a search we are creating various different designs in subjective those to the environmental impact. Now, what do we do how can we utilize in our engineering design, we will create these prototypes **create these prototypes** as per GA guidelines Genetic Algorithm guidelines and then is subject each of these prototypes to what we call Monte Carlo simulation of the noise (Refer Slide Time: 24:46).

Some producing these prototypes I do that by g a and these prototypes would then be a combination of **some design some design variables** some design variable settings then I shake them up using Monte Carlo. I shake them up using Monte Carlo where the randomness is coming representing basically the effect of noise, because noise is something I am not going to control, so I am using a Monte Carlo sort of framework to produce this shaky noise.

And I subjecting each of these prototypes that have been produced using g a to that thing if I do that, I have got shaking going on for each of these prototypes; and it is not very difficult to generate **to generate** a robust design using this approach also and its its been done (Refer Slide Time: 25:26). And this also is in publication this also is in print and therefore, it can be seen by anyone. What the G A does? It may start with a population that is like this is flying all over the place an you may have no hope really to basically come up with the good design using this. But G A goes through iteration it evolves just like darvin pointed out that, we have these species; and occasionally we will have crossovers, we will have you know sexual interaction and so on and so forth.

And with that we will be able to produce new organisms and some of this organism will also go through some mutation because of radiation from cosmic radiations and so on.

They will go through some **some** some changes and the genes would and the chromosome would change the position of the genes would change and so on. When that thing takes place the proteins manifested also will be different, so you end up with **new new** species. These new species they are then subject to the environmental factor there will be the impact of the environment those that survive they go on to become a new specie. And those that do not survive they die off and that is what is one explanation of how Darwin thought evolution had taken place.

(Refer Slide Time: 27:16)



We are trying to do the same thing there, so this is like a noisy it is just a random creation of species. Then we subject them we subject them to the environmental things and we keep the ones that are good. And then now I have got a smaller collection of candidate solutions, candidate robust design, again I subject these to basically noise and I select the ones that survive the effect of noise.

How do I go from generating one to generation two? It is through these interaction between the chromosome. So, I will take the tail of one I will take the head of the other and I will combine the two I will create a new organism. And the new organism will turn out to be half of mother half of father or a one third of mother two third of father and so on and so forth.

If this is how new species are created if this is how new chromosomes are created, those then can be tested by putting them into **in** in the **in in** place of the environment,

subjective environment. And then of course, if they survive **they** they lead on to a new species I will have descendents that will be like **they** they are well adapted and so on and so forth. Then, those would be surviving in there if they do not then of course, we got problems there.

So, this is like one way where I could substitute that orthogonal array which was there in this in this whole design by a scheme that is now using genetic logic genetic interaction logic and you are able to do that (Refer Slide Time: 28:11). Let me give you an example here is an example, this is an example of a engineering design this is engineering design that was done. And it had a lot of complications and I will give you an example of you know give you some examples of the kind of complications we got into.

Most of **you you** know particularly those who are engineers they have played with what we call strain gauges. Strain gauges are things which are really stuck on the bodies of various things for example, if I wanted to find out the deformation of this bottle as I am touching it I am deforming it and with that the diameter is changing or something else is changing. And that thing can be actually detected even if the deflection is very small you can detect that with a little **with a little** strain gauge that you that you mount on the side of this.

I can do that mounting various parts wherever there is a deformation expected, we can put our little strain gauges there and that is how I really measure, I subject this object of this body or this part to some sort of a pressure, some sort of a stress. And that stress is going to cause some deformation my little **my little** strain gauge if it is mounted on the side of it, I can run wires from it. And I can look at the resistance of that strain gauge and when it deforms; obviously, the length of the wires inside the strain gauge those are going to change and therefore, resistance is going to change.

And therefore, the manifest voltage, if I am putting a steady current through it that the manifest voltage is going to change of if you are applying the same voltage current is going to change.

So, it is going to be possible for me to detect the changes in resistance as caused by deformation of the strain gauge itself because it is mounted on the side **on the side** of a body. The strain gauge is mounted on the side of a body and this mounting will cause the gauge also to change its resistance by the deformation that might be going on with the

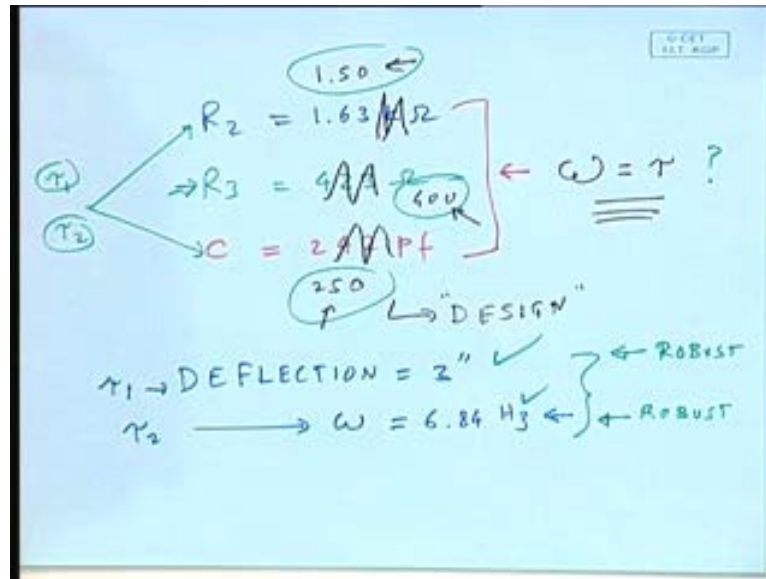
with the body with the main body itself. And by reading a galvanometer, I can then find out whether what is the strength of that deformation, what is the strength of the various things I can found out; because I have converted now the deformation of the stress into a deflection.

Now, this is all fine this is basically the principle of designing strain gauges, but there is a problem here, because the strain gauge is very sensitive. It is very sensitive to slight changes here and there and suppose you are doing the experiment and you lean on the table, or somebody walks by or somebody drags your furniture or something like that. All those vibrations also get into that strain gauge and the deflection and the and the and the reading that you are finding there is been a steady chain suddenly it fluctuates it fluctuates this way that way and so on.

So, what you have to really do one of the things that you must do when you are using a strain gauge is to somehow dampen or reduce the effect of these low frequency changes. When some person is walking by or some person is leaning on it these are not high frequency fluctuations. These are the one that actually are pretty low frequency and we should have a cut off, we should actually what we have to do is, we have to create an electronic filter that will look at a signal that is coming out of the strain gauge

And it should remove those it should remove those unwanted variations those are being caused by people walking and people leaning on the table and so on so forth. Those are the vibrations that should be cut out of the system this can be done if you design what we call a electronic filter and you interface that filter between your source of the electrical signal, which is the strain gauge and where the reading is taking place. If you put a filter in between then only the kind of signals that must go through all the way to the reflector, only those would go through the others would get basically chopped off at the filter.

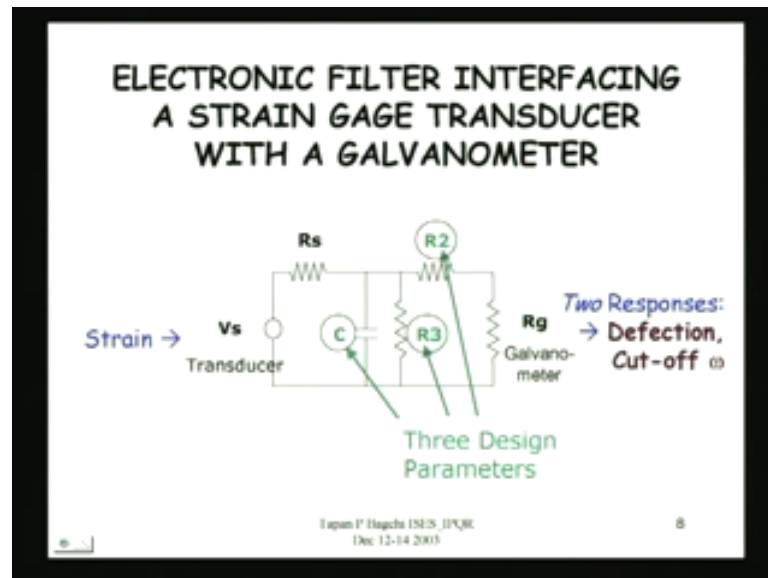
(Refer Slide Time: 33:42)



So, the filter does a pretty decent job, now these filters very fortunately you know we have enough theoretical development in the electrical sciences, were we can design a factor to exact specs. But, that will lead be to a particular cut off frequency, you give a cut off frequency of let us say 10 cycles 10 hertz's or 50 hertz's or a 1000 hertz or something like that. If you do that the electrical engineer is going to sit down he is going to be able to do some calculations, and with that he would be able to come back and say pick your components and those are going to be components inside that electronic filter. Pick them to have these values and your filter is going to work like a charm those calculations are possible.

And I can come up with an exact design that will give me the exact cut off frequency that is there. But even I we know that when calculations are done it is very possible that you will say some resistance R_2 must have a value that is equal to 1.63 kilo ohms. He may also say I need **a I need** another resistor and that is going to be R_3 and that is going to have 423 ohms that is the requirement there. And I need a capacitor and the capacitor must have a picofarad capacity that is going to be let us say 242 here, that is going to be the thing. Now, these are **these are** the specs for the components of the electronic filter and you see the electronic filter on the screen here.

(Refer Slide Time: 34:26)



I have got R 2 and I have got R 3 and I have got C, these are three design components, these are three design parameters that I can play with; by playing with this I can come up with any target cut off frequency. So, if I have got a cut off frequency some cut off frequency ω , I can pick these in such a way that this ω can be made equal to exactly whatever you want. And we will see just in a minute what the target value is going to be, so there is going to be a some target value and that target value can be met with these things (Refer Slide Time: 34:48).

Now, suppose as I was saying to you suppose this has been done an electrical engineer is sat with his calculators. And he went through the formulas and so on, and he did all his manipulation and he came up with R 2 equal to so much, R 3 equal to so much, C equal to so much and he was able to give you a design. This is sir this is your design. So, these are the designers recommendation. This is the design. Now the problem is this, if I go to a store and if I say please get me a resistor that is 1.63 kilo ohms, I will never be able to find because that is not one of the standard parts. If I say give me a resistor that has got the resistance of 423 ohms tough luck, I cannot find that on the shelf anywhere. In the catalogue I can also not I cannot find a capacitor that has got the value of 242 picofarad I just cannot find it these are not standard parts and that actually means, I will not be able to meet this target also.

So; that means, I have done this design on paper, but I am I am not able to actualize that I am not able to realize that in real life I am not able to do that. Now suppose somehow somehow I did something by which I was able to allow some variation here, some variation here, some variation here, some variation here (Refer Slide Time: 36:18). And stay pretty close to this I allow for a variation here, I allow for a variation here and I allow for a variation here. So, maybe the nearest one is 1.50 maybe the nearest one is 400 maybe the nearest one is 250. If I get these components if I get these components notice three different components if I get these three components and let me just mark them in green this one is one this one is the other one and this one is the third one.

Suppose I get these components will I still get close to this will is still get my performance close to this well luckily Taguchi's robust design says yes you can. You got to be slightly lucky because generally speaking engineers can start with a design that is like this the exact design that will give me the exact performance and make the design robust for that you will probably have to manipulate certain things. And you can then allow this value to be slightly different from 1.63 that is 1.50 here instead of 423 it will be 400 there, instead of 242 it will be 250 here (Refer Slide Time: 37:30).

You can allow these changes and still this will stay there provided this performance has been made robust with respect to the choice of these values there. If you could do that you got a robust design you you created a robust filter. That will be give you cut off frequency that you want and the same time it will allow you to go to the market and buy the components that they have as standard component this is like a big jump in in terms of the utility of what is there available to us. At the same time you are guaranteeing you're staying pretty close to whatever we wanted to do which is like staying close to the target at which the cut off frequency should be.

(Refer Slide Time: 38:21)

Let us see how we did that did that we again go back to the slides there and I have got now two responses one is of course, the deflection; that is the on the galvanometer I got to have a certain kind certain amount of reflection that is giving to me as 3 inches that is like specified. So, I have got one target that says deflection deflection equal to 3 inches. And then I have got cutoff frequency that also has been specified to me and we will find out in a minute what that is?

It turns out this has to be 6.84 hertz that you can see the pretty low sort of thing and **this** **this** is generally kind of vibration that you get **when your** when there are people moving around and so on you will get a vibration like this. And this 3 inches is the pull is the full deflection of the galvanometer these are the two targets that we got. So, you got tau 1 here and you got tau 2 here I got to make sure as Taguchi said I got to make sure I get this performance on target tau 1 and this performance also on target tau 2; and you got to make sure the whole system is robust.

(Refer Slide Time: 39:40)

The two Responses to be made robust

targets

$$\omega = \frac{(R2 + Rg)(Rs + R3) + R3 \cdot Rs}{2\pi(R2 + Rg) \cdot R3 \cdot Rs \cdot C} = 6.84 \text{ hz}$$

$$D = \frac{|Vs| RgRs}{Gsen[(R2 + Rg)(Rs + R3) + RsR3]} = 3.0 \text{ inches}$$

⇒ Two constraints leave **one design parameter** free

Taguchi P Taguchi 15125, IITKGI
Dec 12-14 2003

Let us see how we go about doing this it turns out as you if you get into electrical engineering you can find you know you got those conditions; there you got these design values, these different component values. If you do circuit analysis you will be able to derive the expression for omega you will also be able to derive the expression for d the deflection. These are not very difficult to do for electrical engineers and then of course, what I do is I set the first one omega is set equal to 6.84 and I set the other one equal to 3.0. Notice here the degrees of freedom I have I have three different design variables in my hand that I can pick anywhere I want; and **I have to meet** I have to meet two target values with these two target vales I can manipulate now my design variables are the same one; I have got R 2 R 3 and C.

These are three different variables **design** design variables I can pick I can choose these in such a way that I meet this target; and also I meet this target that I could do because of

this equation the equations which are there tell me right away. It is possible from me find the values of R_2 , R_3 and C in such a way that I need this objective when I am satisfying this target requirement. And also I am able to satisfied this objective when I meeting the reflection requirement. Notice here I have three variables to play with I have got two constraints and therefore, **one constraint** one design variable remains free and that is the one I am going to exploit when I am seeking the robust as I am going to exploit that and I will show in a minute how I do that (Refer Slide Time: 41:22).

So, I have got two constraints and therefore, I get one design factor free what I did was the next thing that could be done was you build these prototypes. And of course, you do not really have to physically build them you can get a circuit simulator like for example, there are many different circuit simulators available. These are software where you can specify the circuit you can specify the conditions spice is one and with that you can get a performance simulated; you can get the simulated performance.

You can find out by putting certain things for R_2 , R_3 and C what target you are going what **what** value performance you are going to get you can derive that using spice. You can do the same thing for this one also is not very difficult to that. So, I am able to simulate that I am also going to be able simulate noise because of by using little monte carlo I i can simulate that noise. Remember I showed you the example where the different you know chocolate bars were produced on paper of course, and they were disturb using Monte Carlo simulation. So, we got the effect of noise and effect of that all those we could do.

Now, what is it that we are trying to do what is your objective our objective is two fold. One keep our performance on target. So, we got to meet this requirement we also got to meet this requirement. In addition we got to make this one robust hence also this one robust these two have to make robust. Now, something I have got to remind you of is when we talk of robustness you are talking of minimizing variance that is what we are looking for we are trying to minimize variance; this variance being cause by noise.

In our case the noise is coming from not getting the exact values of the design variables that I would like to have **in my in my** in my electronic filter. In place of 1.63 I am getting 1.50 kilo ohms and in place of 423 ohms I am getting 400 (Refer Slide Time: 43:15). So,

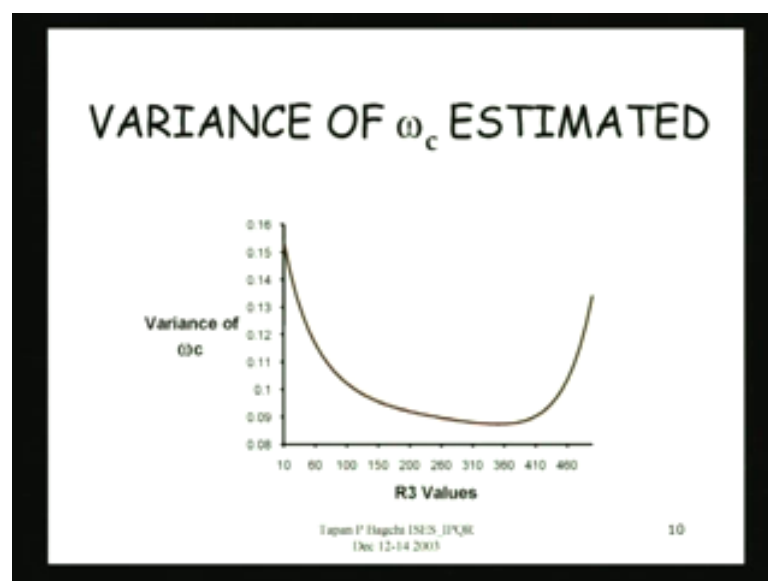
there is; obviously, noise variable there a noise component there you know noise component there.

And I have to same thing here in place of 242 picofarad I am getting 250 there is also noise component there. These are can simulate **there are can simulate** in spice some otherwise I can do Monte Carlo to do that what will that do? What will give me a variance for this target this performance and also a variance for this performance.

This is what has been plotted here **this is what has been plotted here and here** what we have done we fixed we chosen R 2 and C in such a way that we get the two targets tau 1 and tau 2. Because these are simultaneous equations I have been able to get **have been able to get** this target and that target met by selecting a appropriate R 2 and C. The one that is left we is this one this is the one; I am going to be manipulating to try to minimize variance and let me show you what we saw.

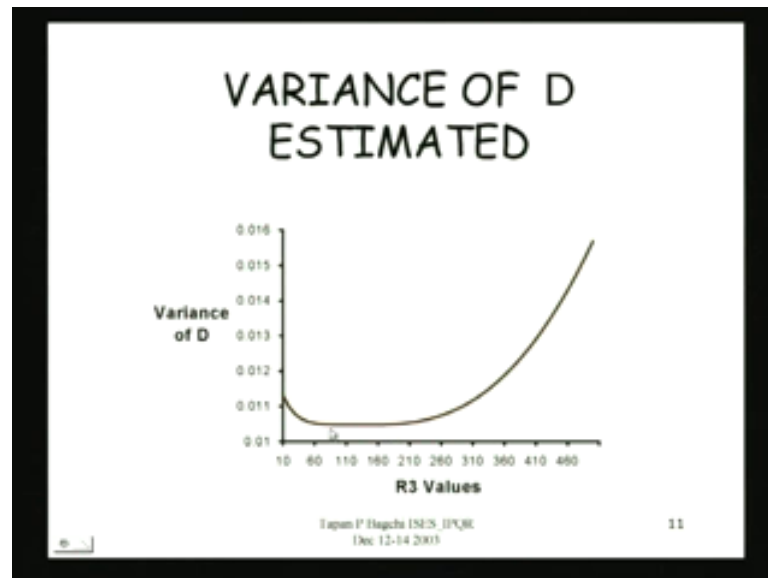
What what was seen when we track the variance of omega c omega c is now our cut off frequency the variance of that corresponding to different values of R 3 because R 3 was in different variable. Remember for a given value of R 3 we could choose C and R 2 in such way that we meet the two targets **we meet the two target** tau 1 and tau 2. So, that was given that was that could be taken from granted. Now, all we wanted to do was we wanted to minimize the variance of **omega three** omega c the cutoff frequency.

(Refer Slide Time: 44:55)



And that could be done by just by doing a little search there we found around R 3 value of 360, we got the best design that is the most robust design that give us the minimum variance and this variance of course, was simulated by Monte Carlo.

(Refer Slide Time: 45:26)



We looked at then the deflection and the variance of deflection and that also turn out to be quite quite low in certain range of r 3 values the only problem was if we look at the value of R 3. The base value for R 3 that would be around 360, there and the base value of R 3 that would be around 110 some of there 110 120. When we try to minimize the variance of R 3, these two did not coincide one was here the other was here. So, that means, I really did not have enough degrees of freedom to make both the both the objectives. What was the two objectives variance of d deflection also variance of omega. We could not make both of them simultaneously minimum that was not possible what we do?

(Refer Slide Time: 46:27)

Original Taguchi method here is ineffective!

- Note that the filter design problem cannot be tackled using Taguchi's "two-step" procedure
- Effects of R2, R3 and C on ω_c (or even D) are not separable (additive)
- Further, one-response-at-a-time optimization does not produce Pareto-optimal designs
- The two plots did produce two "best" robust designs, but none is "globally optimum"

Taguchi P Taguchi ISEIS, IYQR
Dec 12-14 2003 12

While there is a **there is a** think to recognize here, if we applied the old two step method of Taguchi we would not be able to reach this point. We retained his philosophy what we change the technique we change the technique for doing this. One we did that we reach what we call it globally optimum design and let me just show you how this was done.

(Refer Slide Time: 46:36)

Genetic Coding of the Problem

<u>Decision Variable</u>	<u>"Chromosome" builder</u>
R2	010010010
R3	100010011
C	001101010

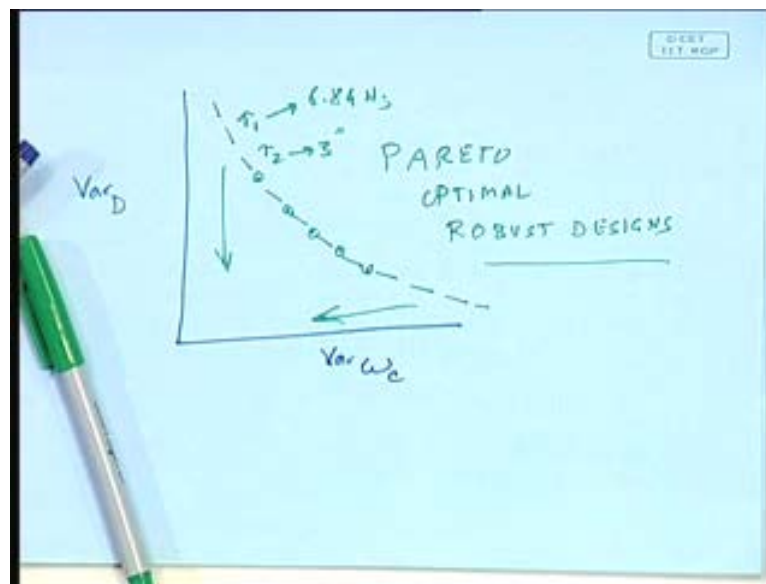
Population size kept constant through "survival of the fittest." Progenies produced through *reproduction*, mating by *crossover*, and *mutation* of some "parent" chromosomes. GA parameterized by DOE. *Nondominated Sorting Multiobjective GA* used.

Taguchi P Taguchi ISEIS, IYQR
Dec 12-14 2003 13

We used a technique that is available now to people who are working with multi objective problem and they are doing this using genetic algorithms. We using G A now to create robust designs. And the technique is called non-dominated multi objected G A

the non-dominated sorting multi objected G A is a special kind of G A, I will I am just going to show you the flowchart for it this is the kind of genetic algorithms it is special genetic algorithm it is different from the old Holland Goldberg type of algorithm. It is special one and there are couple of books now in this area they are available. And with that what you are able to do is you are able to generate what call pareto optimal solutions I will give the idea of pareto optimality just one second.

(Refer Slide Time: 47:36)



Let us I have got two objectives and the objectives are given like this I have got objective one and this could be for example, omega c and not quite omega c, but the variance of omega c. Variance of omega c is one objective that would like to minimize and this is variance of d the deflection which you would also like to minimize. So, the good place to be is here that is why we will get the **most most** robust design, but in actual fact it turns out because we got only R 3 to play with.

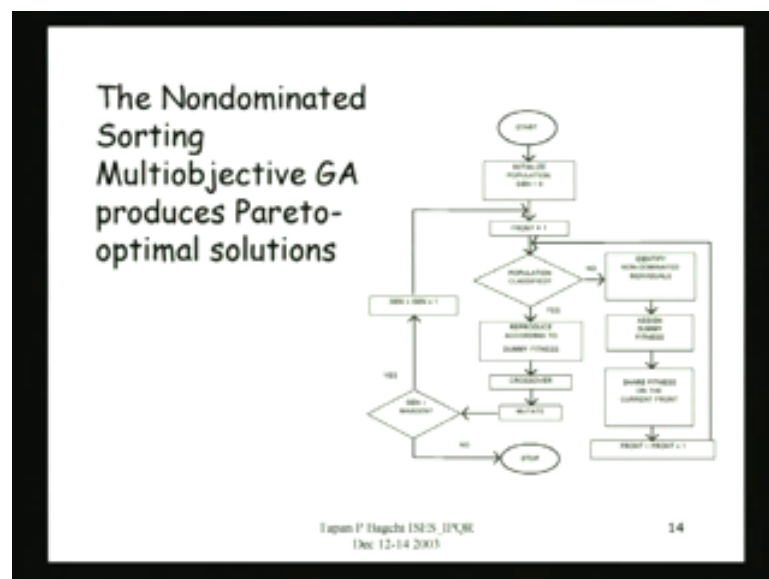
If we change R 3 you get different types of different combinations of various omega 3 various omega c and various d. Now, these can be found using this multi objected G A. Notice any of these **any these** points is a design; that means, these are **these are** actual designs that you can create, you can fabricate these designs. And those designs they are the once that are pareto optimal P A R E T O optimal robust design. Pareto optimal in the sense that what is happening here is it will go out and **it will give you a** it will basically

give you a design that is going to be best that is possible **best that is possible** when you vary R 3.

And also it is best that is possible when you try to manipulate the other variables. So, I end up with minimizing this as best as I can, so I am coming some coming this way and also I am trying to come this way. And these are the best design if we go below this your target value will not be met. So, therefore, this is the point, these are the designs that give you the target both tau 1 and tau 2, **tau 1 and tau 2 tau**.

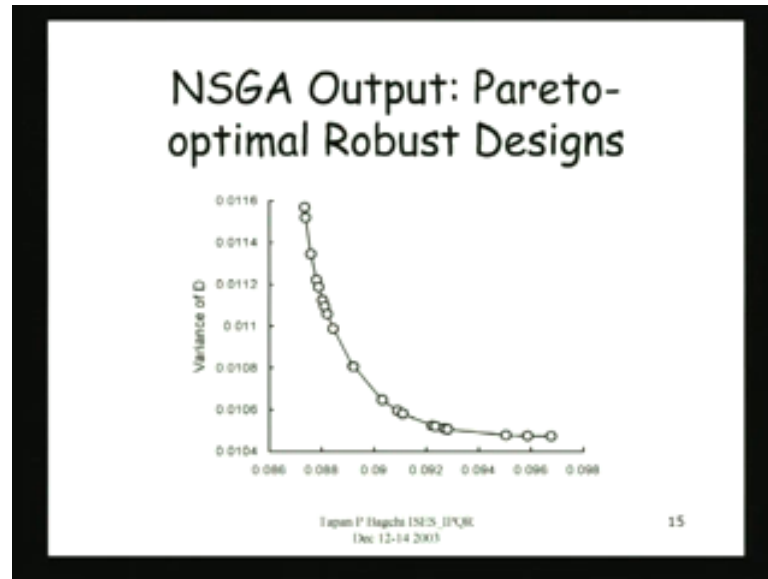
One remember was 6.84 hertz and tau two was 3 inches both of these would be met on all of these points. But is first variance is concerned you cannot really get better than this. Identifying these is done by this multi objected G A. The multi objected G A will give you these front we start some here then gradually bring it down all the way all the way all the way and ended up with these design there. These are the design that are pareto optimal design and let me just show you what they look like.

(Refer Slide Time: 50:08)



This is the logic of the genetic algorithm, the multi objective G A that actually produces the pareto optimal solutions that **that** is possible to be done by this. And when you look at solution they turn out to be like this.

(Refer Slide Time: 50:20)



These are the best designs as far as meeting the target is concerned we are unable to meet the first of all the first target τ_1 and the second target is τ_2 both I am able to meet one is with regard to cutoff frequency the other is with regard to deflection. And the third thing that it has done **it has minimized to the extent possible** it has minimized to the extent possible the variances and these variances are caused by noise where did the noise come from? It came from not finding the exact settings of the part the exact values of the parts that came out in my calculation. For example, when I did my calculations I had four twenty three and I had 1.63 and I had 242 I could not get these in the market place. Instead I got these things. This design can use these parts give you both of these targets and also minimize the variance; and we have been assisted here by the genetic algorithm. That is how we have been able to produce **these** these designs.

(Refer Slide Time: 51:50)

Sample Numerical Results

R3 (Ω)	R2 (Ω)	C (farad)	Avg(ω_c) (Hz)	Var(ω_c)	Avg(D) (in)	Var(D)
311.7881	22.43919	0.000462	6.841476	0.08788	3.00349	0.0112
156.2765	157.7785	0.000434	6.844165	0.09586	3.00323	0.0105
306.0652	25.66875	0.000458	6.841552	0.08803	3.00347	0.0111
195.026	111.5562	0.000424	6.8434	0.09281	3.00325	0.0105
225.7759	82.24873	0.000426	6.842843	0.09110	3.00328	0.0106
195.1129	111.4653	0.000424	6.843399	0.09280	3.00325	0.0105
299.6366	29.40162	0.000454	6.841641	0.08821	3.00345	0.0111

Tapen P. Hagecha MS, PhD
Dec 12-14 2003

This is a **very very** powerful method, now the proof is obviously, in the **(())**. So, what was done at that point was we looked at **we looked at** the, we saw these designs and picked up few of them we picked up about 15, 20 of them and we fabricated these design. And we fabricated these different strain gages or the electronic filters and these low frequency filters after they were fabricated we only needed to pick some values of R3 some value of R2 and some value of C. These are the performances we got we got the average which is should be at 6.4 we got some of these values and average d also we got some of these values.

And you could really basically we can choose among these and you could get them as close as we wanted **to to** what was available in the market place. You got plenty of choice there, so we have choice there you got a choice there you got a choice there we got to make sure you use this combination or this combination or this combination.

These are the combination where you must stick by and therefore, you will be getting this performance on target and also this performance on target. Look at this number it should be 3 inches I have got 3.00347 pretty close to three inches you will be able to see that on the galvanometer you also would be pretty happy if the kind of frequency was something like 6.841 that is not so bad at all. These were done and the variance were also measured and we ended up with it and **this this** method led to a very good design.

(Refer Slide Time: 53:08)



Some of the references are give here and you can probably read them one the thing the particular method that I described to you those are in this particular **in this particular** reference there and they are number of other methods which are also shown here. So, **to** to wrap up the story what we have done, so far is we looked at first of all Taguchi's new concept, how he viewed quality he said quality has got to be one the target that is number 1. Do not manage quality by looking at specification do not try to do that. So, when you are producing a part make sure the dimensional of the part is on target number 1. Number 2 is got to be make sure that you your performance turns to be robust, it is very important because then not only you will have minimum loss cause to society, but also the product is going to be reliable in the old sense, but robust in the new sense.

Robust subjected to someone subjected to various types of variations that is number 1 number 2 of course, and then of course, the third thing that he suggested to us was use of orthogonal arrays. This really cut down the number of trials that were required because in the old days it used to be full factor design. And that is actually design of experiments could not become very popular. Even now even in six sigma trials many times we are going with O A's, because O A's give us the **full** full control and a pretty decent understanding of factor effects very quickly that is like something that six sigma people can also use and they do use **very** very frequently.

And the last thing that Taguchi also says was the idea of signal to noise ratio he said that should be the approach that should take to try to optimize things. And of course, I have shown you that we can go beyond that we can hold on to the philosophy of Taguchi, which is like be on target and create your robust design, but as far as the method is concerned you can bring in your own method. Once you agree once you accept **the the** objectives of being on target and producing robust design you can bring in any kind of optimization method, any kind of simulation method any kind of exponential method, exponential planning method.

And the particular example that I showed you use genetic algorithm produce robust design and these are pretty robust these are pretty decent robust design. These have been fabricated and they have been cross checked to see that yes they do and did a big thing.

Now, this completes the little section that we had for Taguchi methods and robust design for you. It is part and parcel of the six sigma method that we have following which is **(())** again to remind you **define** define the problem, measure the deviation from what the customers are expecting, so produce some measurements readings there analyze try to understand the capability of the system and make some measurements also makes some proper measurements. So, that you can you can **(())** in the analysis and that analysis may require some statistical work. Then of course, improve **and improve** will require for you to handle some technical design of a experiments that is like something very basic. And then of course, the last piece is control once you have identify the optimum setting for the six sigma process make sure the process stays there. And you could do using SPC Statistical process control to keep the different variables on track this is was a was a pretty major step. And of course, we have a few more things to talk to you about under our series of lecture that I continuing on six sigma. So, hopefully we will be seeing you soon and just tune in again thank you very much thank you.