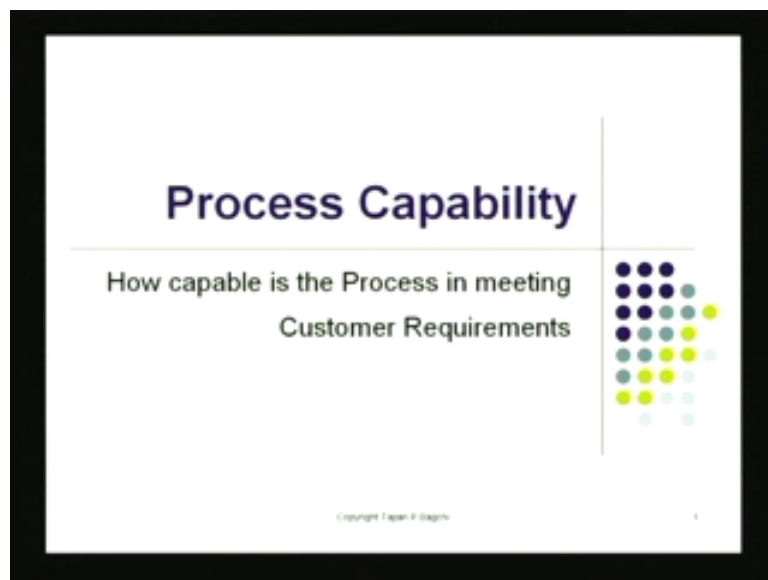


Six Sigma
Prof. Dr. T. P. Bagchi
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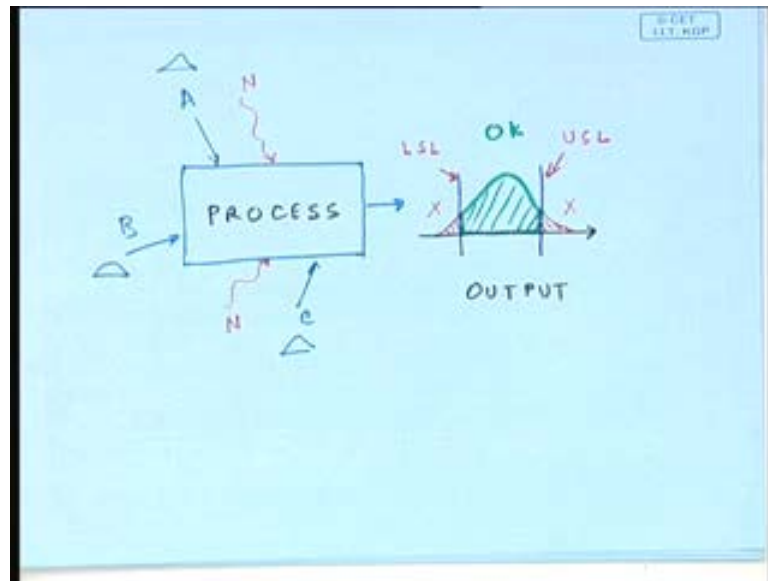
Module No. # 01
Lecture No. # 24
Process Capability

Good afternoon; we resume our lecture on six sigma and in particular, today's lecture is the fourth lecture in the set of the three different other lectures that I had on statistical process control. This is the very important topic; the one that we are going to be discussing today this is called process capability as you see in the slide. There the title slide it talks about the capability of the process to be able to satisfy customer requirements; that is what basically this is.

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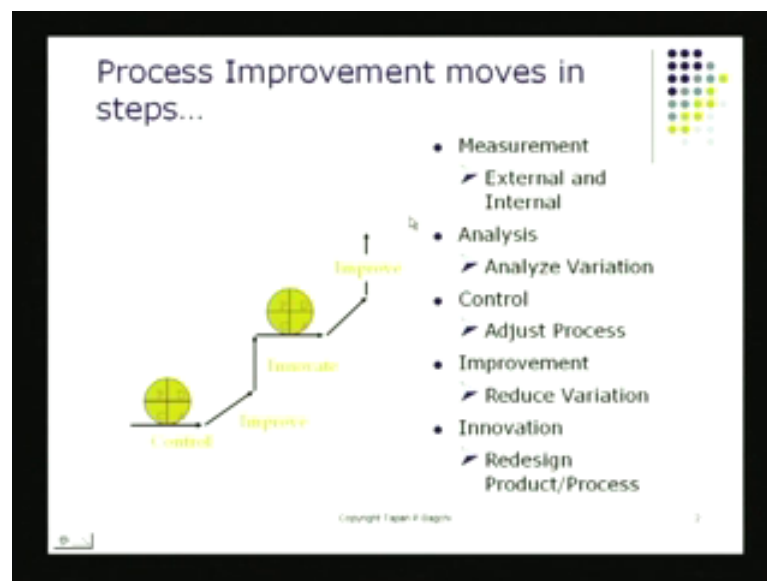


Now, if you look at the diagram that I have on the paper, here is a process and you can actually see the process. It is a box for us and it is improvised by various factors A B C Ds are process control factors. Then of course, we got noise factors which we are not controlling but, they are nevertheless there. They are part of the environment which is causing the variation in the process. as a result, what you will find is when you look at the output, the output does not turn out to be a single value. The output itself is also variable; the process quality characteristic that you see there the output actually shows quite a bit of variation and that sometimes goes even beyond the limits that is specified by the customer. For example, this is the upper spec limit USL and this is the lower spec limit that is the lowest value of quality that the customer is willing to live with. In between of course, the part is that part is...

So, **if I** if my production is all within these two limits there then, there is no problem with the process but, if it turns out if it has got variations like this when part of the output falls below the lower control limit or part of it falls above the, beyond the upper specification limit I say that the process is not capable of satisfying the customer 100 percent of the time. And, this can happen not only because of the variations on noise factors which you are not controlling but, it is also very possible that the process variables themselves; they also have some variation.

So each of the process variables that you set at the set point they may also have some variation and because of that these variations they impact the process and the result is you got some inflated variation in the output of the process itself. Process capability: this phrase is the measure of, how will a particular process be able to satisfy or fall within the upper and lower specification in this? That is what really process capability is to.

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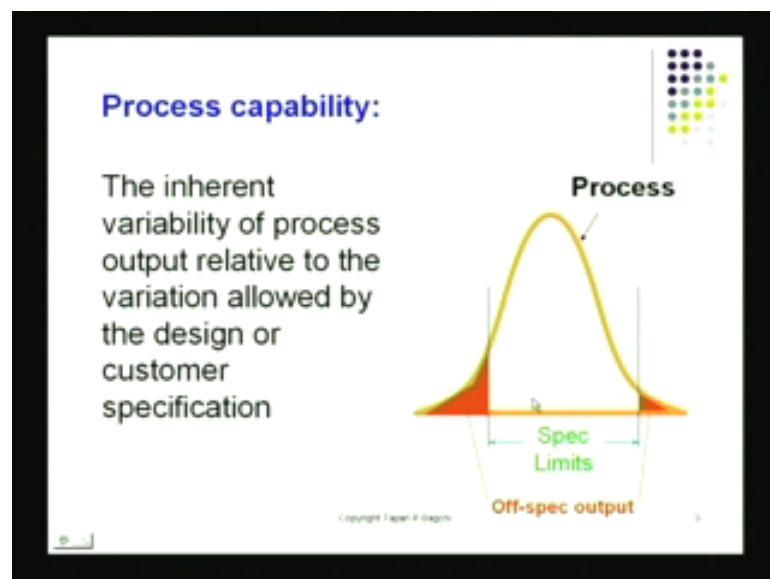
Let us move into the lecture. here, we realize and we know by this chain that if you are talking about process improvement, process improvement generally moves in steps. You know in the six sigma frame work we of course, have DMAIC. DMAIC is defined; measure analyze improve and then control DMAIC. any problem solving actually any problem solving procedure generally moves along these paths there and the same thing I have for process improvement I have measurement. Then, I have analysis then I have control; then I have got to improvement done and then of course, innovation. You need innovation otherwise, you will be stuck wherever you are.

If you look at the steps that these are steps, that this is following I have control and improve that will be like for example, if you have taken care of measurement and analysis then you will be controlling and you will try to improve to a high level of quality and to get there generally, the break through procedure would be through

innovation. That would come through a process like design of experiments for example, and again you would stabilize with your PDCA cycle. You stabilize around a certain level of quality; from there you might like to compete a bit better and you would like to improve the process and that you do.

So this in fact goes on and on and on it moves like a cycle you have control then improve innovate then again control improve innovate and so on. That is how you will be moving along one of the reasons for doing this is generate to try to improve the process capability of any process that you are operating. If you are not doing it your competition for sure is doing it this competition generally a good competitive he benchmarks its products against the best that is there in the market place and it tries to offer value. Value is what you offers for the price that you offer in you will always try to improve upon the value that is offering and he'd like to he try his best to try to out compete other people.

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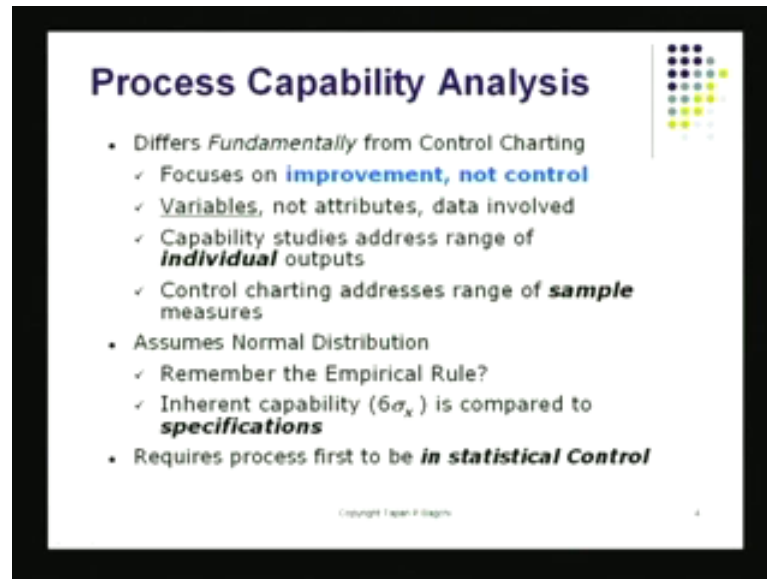


Let us try to get a visual picture of this notion of process capability as I showed you earlier the process will have a lot of variations coming in because of variety of reasons. Some may be due to the control factors which themselves are although they are set at set points they may not exactly stay there. As a result of that, you will end up with some variation that is introduced by the process control variable themselves because, those

themselves they did not hold on to their set points. Then of course, in addition we have to other factor that you are not controlling and these are the noise factors as you saw in the diagram there is a noise factor here and **there is a noise factor here and** these are all operating independent of whatever else is going on there.

So, they will also be active and they will also try to impact the variability of the process itself. When that happens, return into the slide there the screen here we have where we end up with what we call some inherent variability of the process. we realize that every process is going to be variable. Unfortunately, what happens, customers as customers we have our specific requirement and these specific requirements are communicated and they are actually captured by market research and those would be called what we call specification limits. This is kind of the tolerance band within which the product would be treated to be and if it goes beyond that tolerance band the product would be considered to be off spec. That is exactly what we see there, when we look at the output there these red regions. These two red regions one is to the left the other is to the right these two indicate basically the different points which are below either below these specification limit or above the specification limit. These products if they are offered in the market price no one is going to buy them because, they are beyond the specification limit that is something you got to remember. Therefore, it becomes very important for us to worry about a process that is delivering products that fall beyond these specification limits; they are below the lower specification limit or above the upper specification limit it is very important that we design processes and operate them in a manner that keeps the total process inside this specification limits all the time. Such a process will be called a capable process; if a process goes as it is operating naturally, if it goes if it produces output that goes beyond the upper and lower specification limits, we will say that the that particular process is not capable.

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Process Capability Analysis

- Differs *Fundamentally* from Control Charting
 - ✓ Focuses on **improvement, not control**
 - ✓ Variables, not attributes, data involved
 - ✓ Capability studies address range of **individual** outputs
 - ✓ Control charting addresses range of **sample** measures
- Assumes Normal Distribution
 - ✓ Remember the Empirical Rule?
 - ✓ Inherent capability ($6\sigma_x$) is compared to **specifications**
- Requires process first to be **in statistical Control**

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We will try some quantitative measures of this as you go into this; what does process capability analysis do? It actually focuses on improvement it tries to improve process capability of a particular process. Any process that you bring about it will always have some variation. What we have to do is we have to make sure that that variation does not take the output beyond these specification limits.

Now, such a process will generally be under the influence of factors which cause too much variation. One of the task in process capability analysis is first of all to obtain a measure of how good that presents as this, as this capability is if that turns out to be too large. If it turns out that there are parts of process that are producing output which are going beyond these specification limits; we have to tighten the process that means, we have to go back.

If you look at the procedure diagram, in this diagram there are these factors A B and C. perhaps, they have been set in such a way or process on them are not good enough to keep this process well within the specification limit. When you would look at the output of the process perhaps there are too much variations in these and perhaps there is too much noise. Also, these are the reasons why my output may be beyond the specification limits and my process capability may be poor in trying to measure in trying to quantify

process capability.

What we do is, we will look at the individual output values. We look at the individual output values; so perhaps for example, it could be the x_1 x_2 x_3 . These are measurements produced that are coming out that are on the items that are coming of the production line you measure some quality dimension it could be distance and it could be length, it could be weight, it could be viscosity. It could be any of these things; any of these characteristics that you can measure. Once you measured it you go out here and you compare that quantity; you compare that quantities, value to the tolerance if it turns out that the measured value is beyond the tolerance. That means, you produce something that is off spec which we call in industry, we call it off spec.

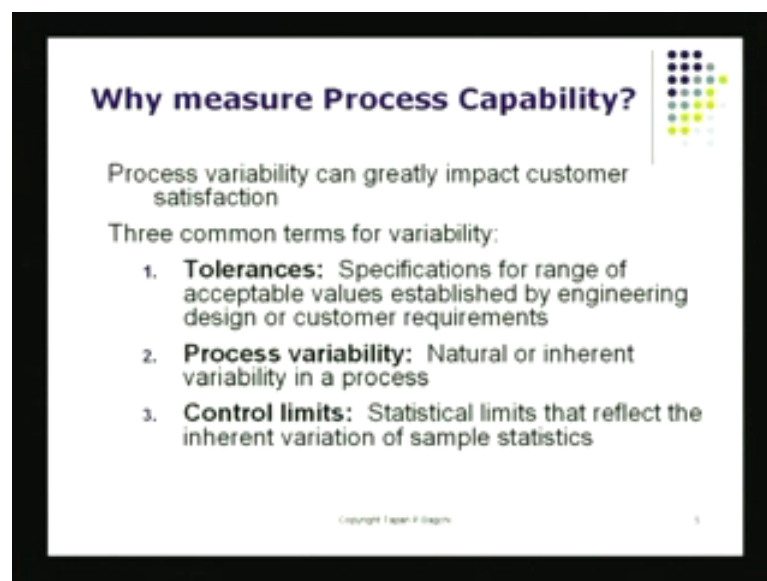
Our goal is going to be to try to understand factors that are causing this wide variation in the process and are producing products that are beyond this specification limits. So, the process capability study is actually they really look at the range of the individual outputs. In fact, the output that I show here the output that I show here these are the plots of the individual items. So, I have got various individual items they have been plotted on this and it turns out some of those are below the specification limit here on the lower side and some are beyond the upper specification limit but, these are all individual measurements. I am not talking about \bar{x} , I am not talking about r ; I am talking about raw x values that is what I am talking about. That is what I have plotted; in fact, that is the quantity that I need to measure in large quantities large numbers to try to see what that distribution looks like.

If that distribution turns out to be $(())$ and a lot of good junk of it is beyond these specification limit, I will say that the process is not capable. I cannot go out to the market place and compete in the market place in the open market place with a process which is producing products that are generally off spec and that are good fraction of this production that is off spec in trying to measure process capability. We make an assumption, we assume that the output is going to be normal of course, we could work with any other distribution but, the convention has been that we have assume the output to be normal.

In fact, most of the stuff if you measure them if you look at measured data measured data generally follows the normal distribution that just makes our life little easier when it comes to quantifying process capability the other thing that we would also like is we would like the process to be under the influence of chance cause on their we do not want the assignable factors to be disturbing the process when I am trying to make a measurement of a process capability.

So when I think of measuring process capability, it is very important for us to realize that the process must be under what we call statistical control. That means, it is under the influence of the ()... are small vibrations and so on and so forth. Those are the parts; those are the causes which do not shake the process too much. Any factor, any item, any cause that are actually would shake the process we first have to stabilize the process. We first have to make sure we have removed all the assignable factors and the way to do that is run the process under the process control using control charts. Perhaps, the x bar chart or the r chart and then remove as many of those factors as possible. Once you have done that what is left is the process that is operating under the influence of chance causes on their...

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Why measure Process Capability?

Process variability can greatly impact customer satisfaction

Three common terms for variability:

1. **Tolerances:** Specifications for range of acceptable values established by engineering design or customer requirements
2. **Process variability:** Natural or inherent variability in a process
3. **Control limits:** Statistical limits that reflect the inherent variation of sample statistics

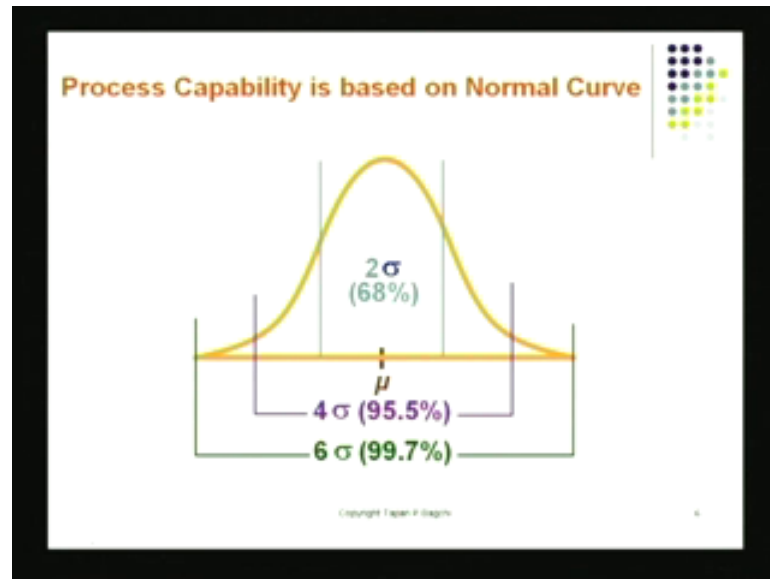
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So, that is like one of the assumptions when we go out and start measuring process

capability, couple of terms we will be using here. One is of course, tolerance and tolerance is the difference between the upper and lower spec limit and if you look at my diagram here, these specification limits here is denoted by this line there. That is the upper spec limit and this is the lower spec limit. This range is the tolerance; this actually implies the range of quality characteristic values that the customer is willing to live with this is rather important for us to realize we got to find out what that tolerance is that is given by the customer the second thing of course, is the variability and again you can notice the variability there that is a notion of touch really. Basically, the plotting of all the output that I have measured without controlling anything, without basically doing any kind of screening and any kind of censoring and so on; we just take that raw data and we plot them up completely and that would reflect now. The variability in the procedure itself that is also (()) very important; we got to remember now whenever we are dealing with process capability we are going to be using specification limits not control limits.

So, yes we use control limits to stabilize the process we use control limits on control charts to try to stabilize the process which means, we want to remove any assignable cause that might be disturbing the process. That is something we would like to be able to do but, once we done that the process capability measurement will in no place utilize control limits that it will not do. So, please do not be confuse between these two types of limit one is specification limits these basically specify the range or the tolerance that the customer is willing to live with that is the range of quality characteristics that is acceptable to the customer. On the other hand, if I am trying to control the process, if I am trying to do control using statistical process control, I shall be interested in what we call control limits. Then, these control limits would be imposed on these two statistical charts one is the x bar chart; the other is the r chart when I am making measurements. I could do of course, the same thing with p chart or c chart any of those charts but, is use control limits only to stabilize the process only to bring it under what we call statistical control.

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Let us go on and take a look at the distribution itself. Here is the distribution and if you remember the normal distribution, the normal distribution is symmetric, has got two parameters controlling it. One is mu which shows the mean of the process; the other is sigma. Sigma indicates the variability of the process; so, there are two aspects of the output here. One is of course, where it is located central where is the central location of the process there is a central point about which the process data are distributed.

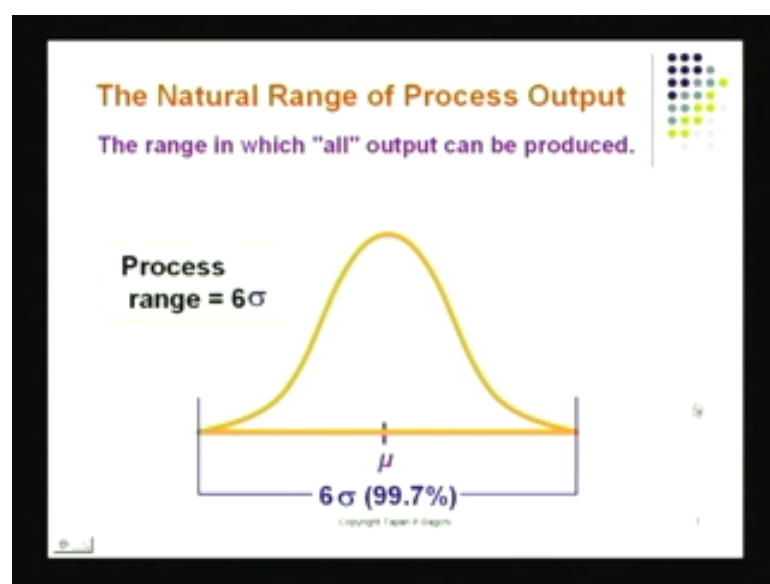
So, that is going to be our mu; the mean then of course, we also have to bring in what we call variability of the process and the parameter that indicates that very easily is our sigma - sigma is standard deviation and if you specify mu and sigma you defined everything that you need when it comes to defining a the normal distribution. Now, some other some other features also we utilize, now once we have acknowledge that the process is going to be normal distribution, normally distributed. We do a couple of things we can mark parts that will specify the exact area the exact area that will fall within a certain range that is beyond, that is above and below the mean value of the thing.

If I go 1 sigma beyond mu that is like this point is now going to be mu plus 1 sigma and this point is going to be mu minus 1 sigma this difference here the distance between these two is 2 sigma and that covers approximate 68 percent of production. If your

output is going to be normally distributed; if it is normally distributed then within plus sigma and minus sigma you will have 68 percent of production. If you go beyond this if you go to 2 sigma limit on the right hand side and 2 sigma below the average below the mean there that will cover about 95 percent of the total area that is under the distribution; that is the distribution of the output that is there if the output is normally distributed.

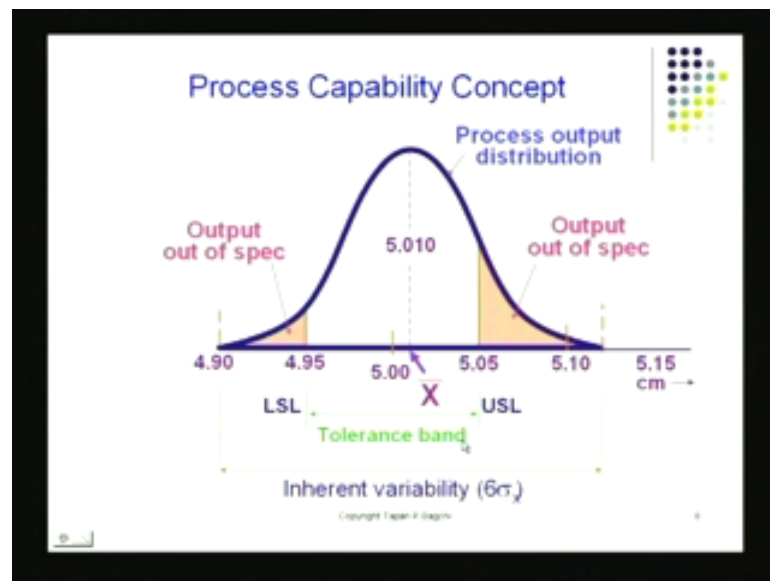
If you go to 3 sigma, the range that is covered between mu plus 3 sigma and mu minus 3 sigma that total area turns out to be 99.7 percent. So, very little is left beyond that. In fact, these turn out to be measures that we utilize when we do our process capability calculations. So, something for us to remember is if the distribution is normal then within plus and minus 1 sigma i will have 68 percent of the output. If I expand that range to plus and minus 2 sigma i will have 95 percent production that will be in that range that mu range plus minus 2 sigma and if I make that tolerance even wider if I make it 3 sigma that is if I go mu plus 3 sigma would be the upper limit and mu minus 3 sigma will be the lower limit. Then, the amount of output that is inside the plus 3 sigma and minus sigma range that is going to be 99.7 percent this is the property above the normal distribution. This is the property of the normal distribution and we utilize this when we do our C pk calculations.

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Take a look at the full range now the full range that covers almost all the output almost all the output is now contained with a mu plus 3 sigma and mu minus 3 sigma this 6 sigma range pretty well covers almost the total output. If the output is normally distributed that is something for us to remember again and of course, what we do is we go on and we sort of try to see what happens in a real process.

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Then if you remember, the best place to put your production is to put the average value right on target. This gives you an accurate process if your average equals, if the average value of output coincides with the target of the process. That is a requirement you end up with what we call an accurate process a precise process is going to be one that is going to have a tight distribution. A precise process always at tight distribution and we have some means by which we can specify these different characteristics accuracy and precision.

Let us see how we do that. I have a process here and this process has a target which is at 5.00; that is the target being there. What we have to do is, keep the target there and look at your real output the real output. Actually, if I average the whole thing are all real output \bar{x} that is at 5.010, notice the difference between 5.010 and 5.00. This actually indicates there is a problem with accuracy of the process; there is an accuracy there is some difference between the target and what the average quality

characteristic is that the process is delivering. So, here the first problem that we have is we have a some problem of accuracy. Then of course, there is something that I would like to find out from customers and I say regardless of whatever the process is doing what are your requirements. Then, the customer comes along he says well the target is correct I want the quality characteristic to have the value of 5.00; my tolerance on the upper side is 5.05 and my tolerance on the lower side is 4.95 and that is going to be my tolerance value. if you supply output if I supply products that are in this range, I will be pretty happy.

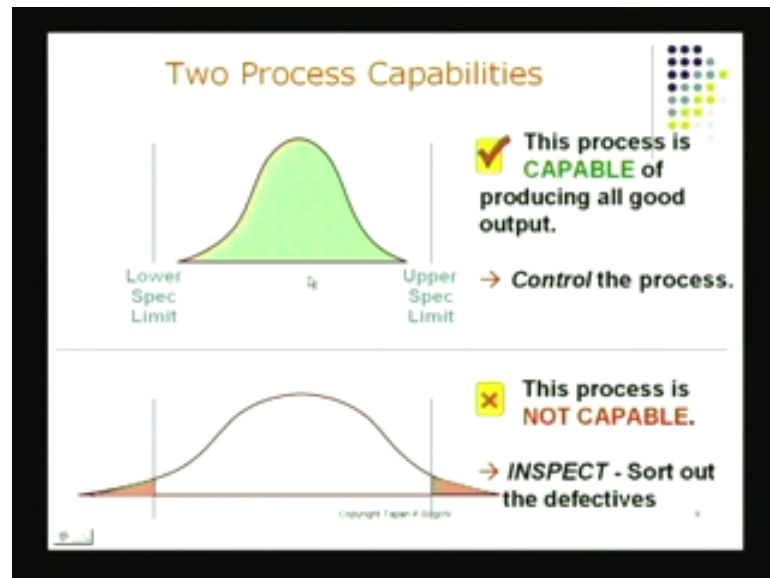
Of course, I will be most happy if you supply it right of the target but, I tolerate things that are I will accept things that fall within the upper spec limit and lower spec limit and this is my tolerance band; this would the customer conveys to you. Now, come back to production come back to the process remember the process that I showed I showed you a process when I drew the diagram here is a process the process is what I am operating I will manufacture I am operating this process my output may not exactly we fit for the customer to use one hundred percent of the time. Let us take a look at what is happening there the target has been given as 5.00 and the tolerance band has been given as 4.95 to 5.05; these have come from the customer.

When I collect some output data and I plot, it gives me this dark curve there which is you can recognize it is a normal distribution; more or less a normal distribution alone. They hold a good junk of that normal distribution it falls below the lower specification limit so this would be call out of specification these products are going to be out of specification these would not be acceptable to the customer if I look at the outside if I look at beyond the upper spec limit again I find there is a whole bunch of output there that is again beyond the beyond the tolerance of the customer.

So again, these are again out of spec; if a lot of my production is like this and this I cannot compete in the market place with the process that I have that is operating like this and that actually means that the process capability of this process. This blue process of this dark blue process is not so good, is actually not so good it turns out that the inherent variability this is the natural variability of process that is wider than the tolerance band. Therefore, in the language of process capability the process is not capable of taking care

of the customer requirements 100 percent it is not.

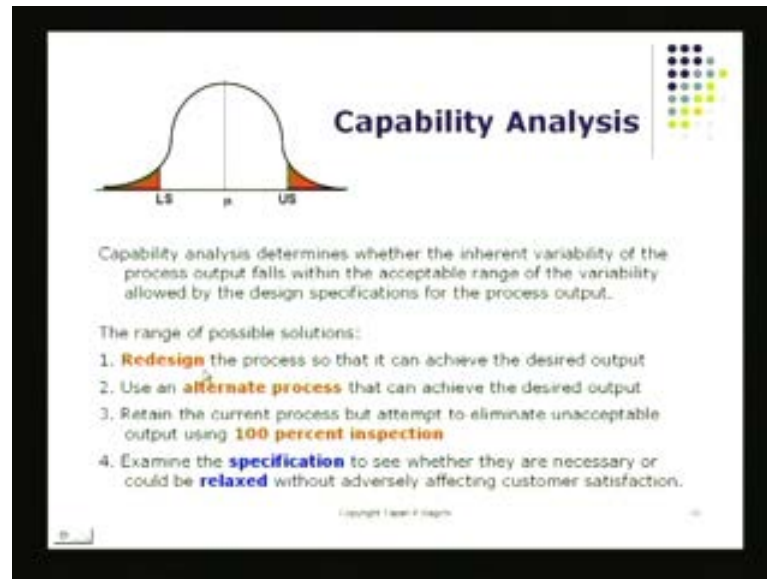
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Let us take a look at two processes; one is a perfect process. I show the tolerance here the upper and lower specification limits and look at the natural variation of the output its well within the tolerance. So, this is the good process this is what we call capable process it is only within the tolerance the natural variation of this process is only within the tolerance look at this process by contrast the tails are beyond the tolerance limits which actually says if I use this process to basically support by a market place, there will be many occasions when there will be complaints from the customer because I find some tails of it that are beyond the beyond the tolerance limits and this process is therefore, not capable.

So the green process is capable but, this lower process is not capable this is just a give you an idea of what we are talking about when talk of capability the process to be capable has to be wholly within the tolerance the same picture is shown here.

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Now, what do we do? What can we do if we got a process like this? This is the process that again is as you can see right away - the lower and upper specification limits are shown and there are red tails which are like unacceptable out there what do we do well we could redesign the process we could change those factors A B and C. Remember, these factors A B and C? These are process control factors perhaps also there is a lot of noise that is causing the inherent variation plus, we could redesign this process we could probably redesign this process and bring it wholly within the tolerance limits and the tolerance limits as indicated to you those are shown right there.

That could be done by redesigning the process; that would be one strategy. Suppose, this was the machine that was like an old machine and there is no way I can redesign or I can fix the machine then of course, I will have to go for alternate process that is like another option that is option number 2. That is also a possibility if I am going to be competing at the same market place; that is also going to be one of my one of my requirements I shall be using an alternate process to supply into this market place. The third approach is going to be why do not why produce the way I produce? But, I sort the output between good and bad what I do is I make some deliver inspection I inspect everything that comes out of the process and I reject the parts from shipment that are either below the lower specification limits or above the upper specification limit. I supply parts or I

supply products that only fall within the tolerance band and throw away the parts. Perhaps, I scrap the part or I rework the parts that are beyond this spec limit; that is going to be expensive again. And, the other thing of course, is I go and plead with the customer please widen your tolerance band if I did that with the process that I have here if the customer is kind enough if he would widen the tolerance band then these red area will reduce in size that could be my four strategies certainly not a good strategy at all the best strategy is really to redesign the process and that means you have to identify factors which are causing this wide variability.

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Process Capability Ratio C_p

Process capability ratio = C_p

$$C_p = \frac{\text{Specification width}}{\text{Process width}}$$
$$C_p = \frac{USL - LSL}{6\sigma}$$

Motorola Corporation uses Six Sigma management.

For Motorola, $C_p = 2$

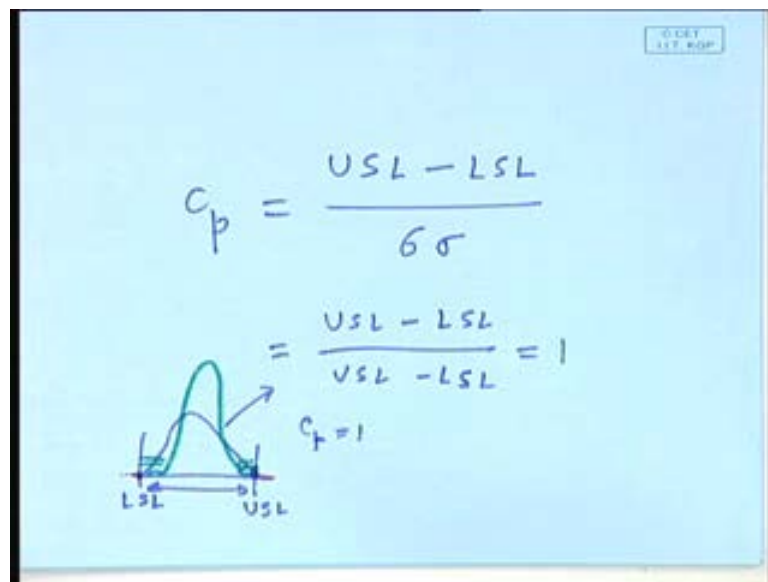
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Let us now see how I measure process capability how we end up quantifying process capability is defined as follows it is C_p . C_p is the notation that is used **C_p is the notation**; C_p is called process capability ratio it is a simple measurement of process capability there are other measurements that are more complicated than this but, this is the simplest one. It is used a lot by industrial C_p is just a ratio of specification width which is a tolerance divided by process width which is six sigma.

I have got C_p equal to upper spec limit minus lower spec limit that is the tolerance given by the customer divided by 6 sigma and this sigma is my process variability. Remember, I had process variability; I am just going to bring that up again. I am going to bring up

this is my notice here. The 6 sigma limit there this is my process variability. So, this would be in my denominator when I look at my Cpk formula. My Cp formula gives me 6 sigma in the denominator; I have got tolerance of these are numerator and I have got in the denominator I have got six sigma this ratio would be called Cp. It turns out if a process is six sigma process then your process has such good precision and your centering is so good that this Cp number turns out to be 2 for your process. Let me give you one or two other examples.

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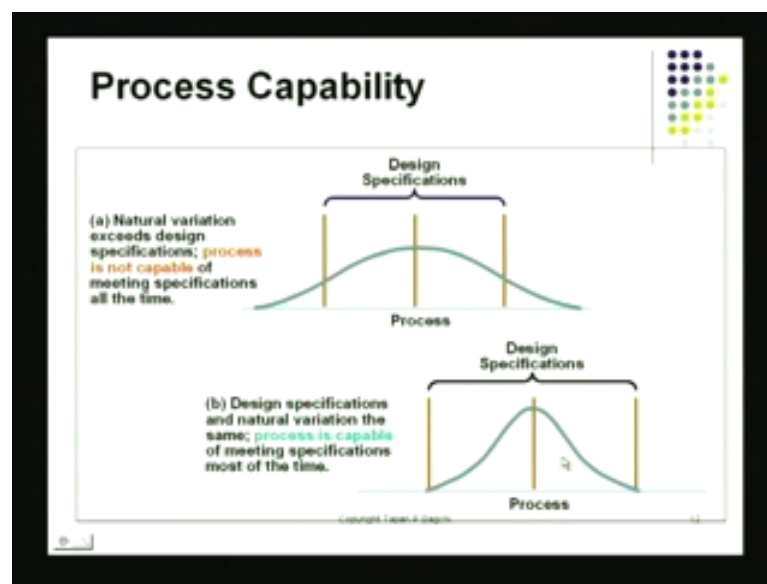
Suppose, you have a tolerance limit that fits exactly the 6 sigma quantity, so what we have here? If I would write the formula there I have Cp equal to and I put down here upper spec limit minus lower spec limit divided by 6 sigma and suppose, this sigma was exactly equal to this then I will end up with upper spec limit minus lower spec limit divided by upper spec limit minus lower spec limit and this turns out to be a very neat formula, this gives me a Cp equal to 1.

Now, this is the process where the tolerance and the distribution of the process; they match exactly. This process as a Cp equal to 1, is this a safe process? It is not a very safe process because, there are chances that occasionally you will end up with some processes. Some parts that are beyond this specification limit a further for this what we

have to do is, we have to make this tighter. So, we should in fact try to get a process. We should understand the causes of variation; we should try to get a process that is wholly within the limits like this. So, there is some room here and also there is some room here; so, real danger of going beyond the control, beyond the specification limits, I have the upper specification limit here, upper specification limit there and lower specification limit there. Those are my my tolerance band; this is my tolerance band and of course, then within that my good process is such is got some slack left on both sides.

So that, this one is really it has it has hardly any chance of going beyond these lower and upper specification limit in Motorola's case when they reached there, when they produced a process that had a 6 sigma of quality. That means, it produces part that was only 3 or 4 part per million; their C_p measured turns out to be 2 turn out to be 2.

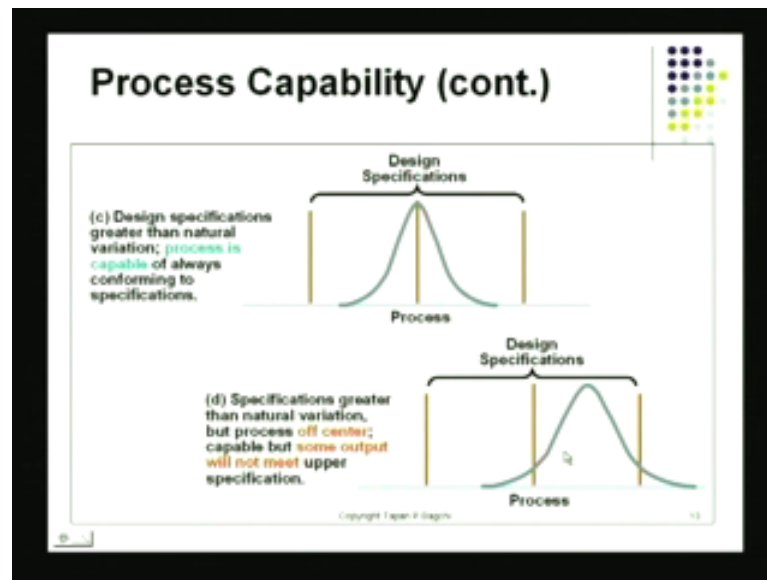
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Again, look at the idea of process capability, what are my design specifications? Design specifications are basically determined by what the customers require. So, I have got my upper specification limit there and lower specification limit there; if the process is centered, if the process is not capable of taking care of meeting these requirements, I will have some of these parts of these tails that would be beyond the specification limit. And, if a process fits exactly within say, the process is capable it is the C_p at least equal to 1.

Hopefully, it should have C_p that is more than 1 is always greater and certainly below 1 is not acceptable.

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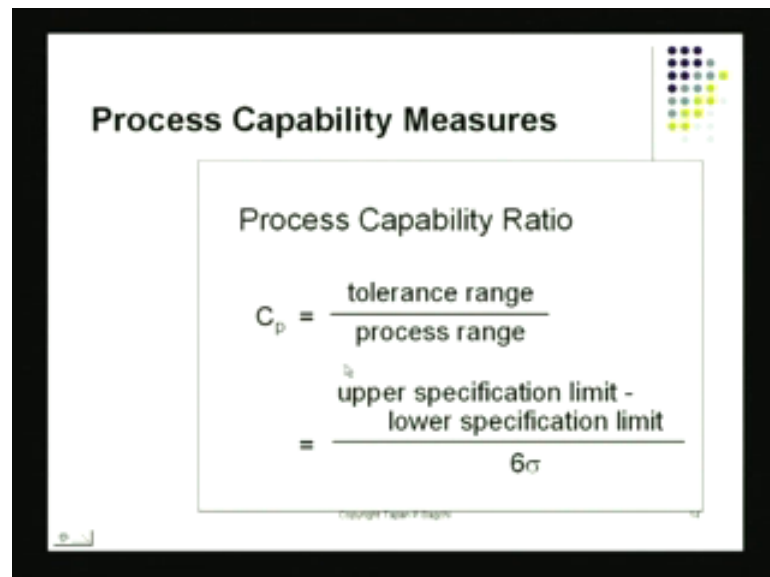


Look at a process here there are two processes they have two different types of problems there both actually are pretty tight processes this one is a pretty tight process as you can see this one also is not a very poor process in terms of procedure but, this process is centered no real problem there and it is totally capable this process here although the variation is not too wide it is after one side and therefore, it does have some part that would be that would be colored red. This part here, this tail here would be colored red what we have to then do is first of all just the old measure of C_p . Remember, I have the old measure of C_p which I showed here I used a formula that was upper spec limit minus lower spec limit divided by 6 sigma. Notice this formula does not incorporate the target anywhere this formula does not incorporate the target it has a variability sigma but, it does not in any place bring in mu. Therefore, this index is not capable of detecting a problem which is like the lower part of the problem here like for example, this lower part of the problem this problem will not be detected if you measured your process capability using C_p only.

What do we do then? What would we have to modify the measure of C_p ? We have to

measure change the formula that formula is changed from C_p to what we call C_{pk}. It is called process capability index and that will be done where I will be bringing in mu as well into the formula.

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The slide is titled "Process Capability Measures" and features a decorative graphic of colored dots in the top right corner. The main content is a box containing the following text and formula:

Process Capability Ratio

$$C_p = \frac{\text{tolerance range}}{\text{process range}}$$
$$= \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma}$$

Let us see how we do that. The definition of C_p was tolerance range divide by process range which was the upper specification limit minus lower specification limit divided by six sigma that is all that was C_p. What we do is we change that and there is just an example of how we do the calculation.

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Computing C_p

Net weight specification = 9.0 oz \pm 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_p = \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma}$$
$$= \frac{9.5 - 8.5}{6(0.12)} = 1.39$$

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Let us say that I am weighing certain things and the weight of the object the weight of the object is such that I have what we call a specification limit always and the specification limit is 9 ounce is plus or minus half an ounce so 9.5 ounce to 8.5 an ounce that is actually the tolerance of the customer; that is my range of specification if the process mean turns out to be 8.80. Let us say that I just keep a note of that I just keep a note of that and what we have done is we basically looked at the quantity which is here. I have here specification limits which is 9.5 minus 8.5 divided by 6 times sigma is also given here.

So in trying to find C_p I have never used my mean. I have never used the mean. I have only used specification rate and sigma value and that turns out to be 9.5 minus 8 y 0.5 divided by 6 times 0.12 and that turns out to be 1.39. Now, you may think this is this is pretty good because, this C_p value is greater than one which is saying there should be no tail end no red tail but, the problem is this my process itself is not centered this is not coming at 9.0 this is at 8.80.

So there is the displacement of the average process and if I show you the diagram corresponding to it is more like this. I have a tolerance standard target but, the average turns out to be somewhere else and that is not such a good situation to have.

Let us see what we do perhaps, what we should do is instead of misleading management by calculating only C p. We got to convey to them look gentlemen, there is a difference between the process mean that is being delivered by the process and what we consider to be that is the target that is really the best value of the quality characteristic. That is required by the customer to do this what we do is we change the formula we calculate process capability.

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Process Capability Measures

Process Capability Index

$$C_{pk} = \text{minimum} \left[\frac{\bar{x} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{x}}{3\sigma} \right]$$

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Now, using a different formula which is \bar{x} double bar it is the grand average of all the data values that I have collected at the output \bar{x} double bar like in the old control chart situation. It is the average of all the \bar{x} I values $\bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4$ and so on so you could have $\bar{x}_2 100$. You take all of them make compute one average that is this \bar{x} double bar that is the overall process average; this indicates the midpoint of the distribution of the output that is what I have used here this C pk formula has two parts. One is going to be \bar{x} double bar minus the lower specification limit divided by 3 sigma is still our old process standard deviation comma upper specification limit minus \bar{x} double bar divided by 3 sigma. Notice here C pk is the minimum of this quantity or this quantity whichever turns out to be nearer to the specification limit either because of this too close to the lower specification limit or too close to the upper specification limit. I will have a smaller quantity there and that is going to be reduce the value of C p k.

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Computing C_{pk}

Net weight specification = 9.0 oz \pm 0.5 oz
Process mean = 8.80 oz
Process standard deviation $\sigma = 0.12$ oz = $R\bar{d}_2$

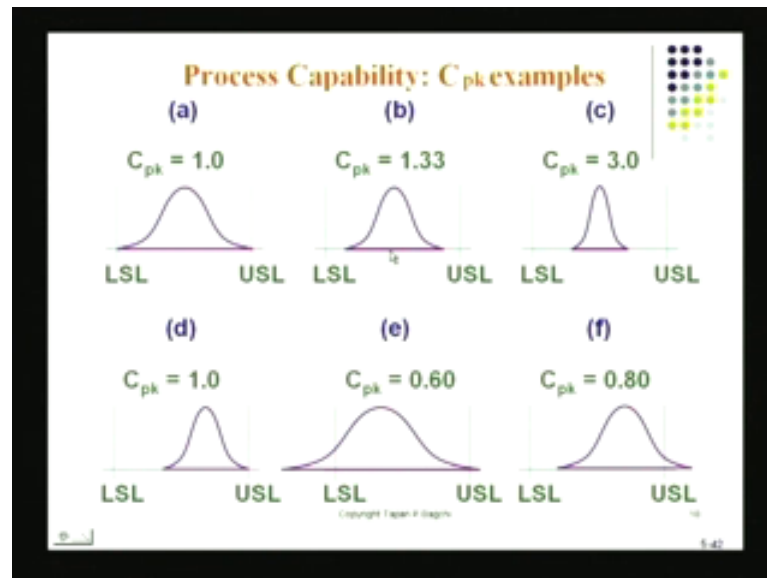
$$C_{pk} = \text{minimum} \left[\frac{\bar{x} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{x}}{3\sigma} \right]$$
$$= \text{minimum} \left[\frac{8.80 - 8.50}{3(0.12)}, \frac{9.50 - 8.80}{3(0.12)} \right] = 0.83$$

And if I show you calculation here is a little calculation I use the same data that we had before my tolerances were given as 9.0 plus or minus 0.5 ounce process mean this the x double bar turned out to be 8.80 by actual measurement process standard deviation turns out to be sigma which is 0.12 ounces and this I formed by calculating R bar by d 2. Remember this; what we also used in one of the earlier calculation when we were doing s p c.

Now, we bring in the C pk formula; this is different from the C p formula. C pk formula is minimum of x double bar minus lower specification limit divided by 3 sigma or the upper specification limit minus x double bar divided by 3 6 sigma and that is exactly what I find here now where is x double bar this quantity is x double bar 8.80 is x double bar so what I do is I come out here and I plug in the **right** numbers I plug in x double bar minus lower specification limit which is 8.50 divide that by 3 times sigma that is like one quantity the other is 9.5 which is actually the upper specification limit minus x double bar which is 8.80 divide by again 3 times sigma the lower of these two is 0.83 and this is definitely less than one which indicates to me that part of this tail of production is perhaps beyond one of the specification limits is probably beyond on the specification limits and that is bad news that means this process again is not capable.

So notice the difference in going from C_p to C_{pk} C_p basically looks at the overall variability of the process C_{pk} looks at the centrality of the process C_{pk} worries about the displacement of the overall average from the targeted value that is what C_{pk} does so C_{pk} turns out to be a better value for measuring process capability.

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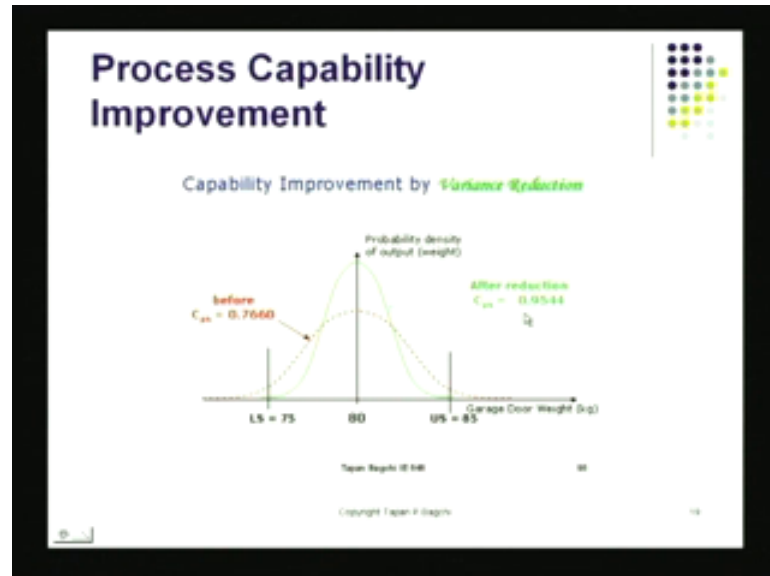


Now here are some scenarios if a process fix in exactly within the specification limit the value of C_{pk} is going to be one if it is well within that my value of C_{pk} could be 1.10 if it is really well **well well** within the specification limits C_{pk} value may rise to 3.0 and this is a really capable process no customer ever will be unhappy with this process in fact they will be pretty happy even there then of course, I have got C_{pk} equal to 1 even if there is slight displacement there is some slight displacement both the curves are the distribution still wholly contain within the spec range and therefore, this C_{pk} turns out to be 1.0.

If there is a displacement of \bar{x} from the targeted value there **unfortun[ately]**- you cannot see them because those have been erased from this picture there but, \bar{x} double bar stands here and your targeted value stands there **there** is a difference between those two so this process lacks accuracy and look at the tail it is fallen out same thing with this even if this process is narrower its tighter than the process here still C_{pk} is poor.

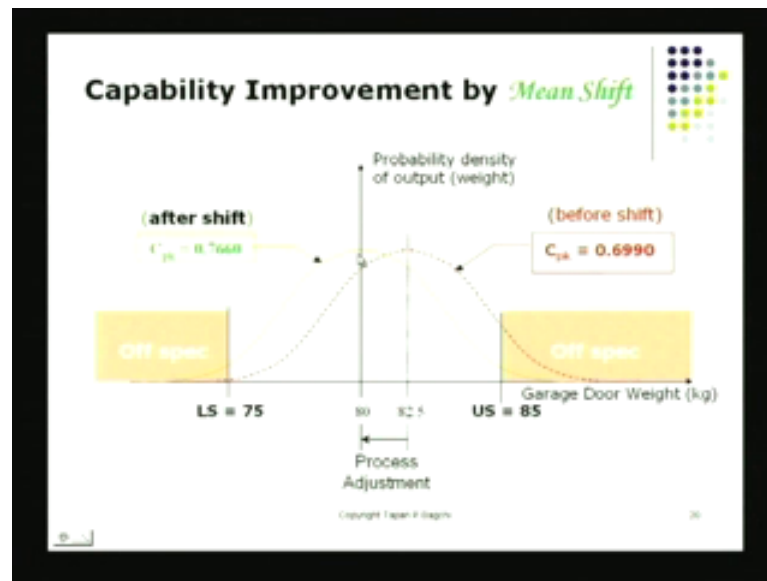
So it turns out to the best this situation to be in is to be well within the tolerance limits of course, number one number two the average has to be pretty close to the target that is there your process is very capable and this could be measured using this C pk process capability index how do we improve it that couple of ways to do it.

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Suppose you are suppose your a process had the distribution that was like the red dotted line this was the distribution of the output that we had coming out of here system your machine or whatever it was it was producing output that had this kind of distribution the spec limits are shown here and notice carefully that some tail of the old process is below these specification limit and also above the specification limit this process is clearly not capable and the C pk measure here turns out to be some 0.76 x which is less than 1. I can improve this process and here I can do that because centrality is not an issue the whole process; also, is really right on target. The average double x double bar is standing right at the target; no issue there. But, if I make the process tighter that means if I reduce sigma I can go from that green line to this red line which also is not quite perfect yet because it is still got a little bit of tail there but, certainly C pk has improved from 0.766 to now 0.9544 this is the better process clearly it will have very few rejects.

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The other way to try to improve a process that has got poor capability is to shift the mean, if the mean was of some of target if we just move, if you adjust your factors in such a way that the average of the process shifts and becomes coincident with the what we call the target process, which is what has been set by doing some market research, it turns out you will have a larger area of your process that will come within the tolerance limits. And, again process capability will improve the old process capability with the off centered process was 0.699. The new process with your \bar{x} double bar shifted to coincide with your what we call the targeted value that C_{pk} turns out to be 0.766 which is larger than 0.699. Therefore, this gone a improvement in the process and this improvement we have achieved by shifting the mean of the process. The red line, the red curve was shifted to this yellowish curve there and that is how that is how you end up with improving the process.

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Another Capability measure Process Performance Capability P_{pk}

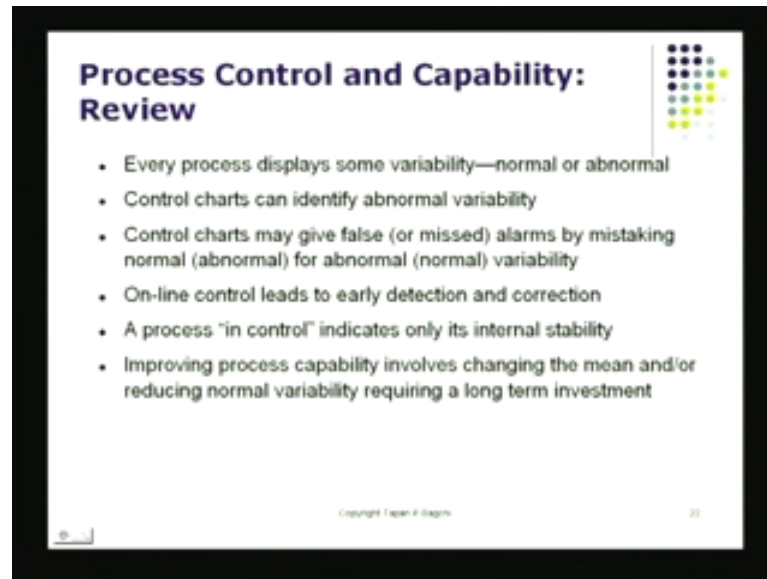
Net weight specification = 9.0 oz \pm 0.5 oz
Process mean = 8.80 oz = \bar{x}
Sample standard deviation s = 0.12 oz

$$P_{pk} = \min \left(\frac{\bar{x} - \text{lower specification limit}}{3s}, \frac{\text{upper specification limit} - \bar{x}}{3s} \right)$$
$$= \min \left(\frac{8.80 - 8.50}{3(0.12)}, \frac{9.50 - 8.80}{3(0.12)} \right) = 0.83$$

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Now, there is a process capability index that is that goes even beyond this and this one is called P_{pk} there is no real difference between what we done before with our C_{pk} and P_{pk} . P_{pk} basically looks at process performance it looks like process performance the only difference in the formula between P_{pk} and C_{pk} is look at the denominator the denominator of P_{pk} is $3s$ sample standard deviation the denominator in the other case I'm just going to go back there C_{pk} the denominator of C_{pk} was 3σ and this sigma was found by this R/\bar{d}_2 which is really the process standard deviation there is one thing called process standard deviation then there is something called sample standard deviation which is the local which is a local standard deviation. If we do that we end up producing use the same formula we end up producing a process capability index that is called P_{pk} turns out to be another measure, another process capability measure which is also utilized.

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Process Control and Capability: Review

- Every process displays some variability—normal or abnormal
- Control charts can identify abnormal variability
- Control charts may give false (or missed) alarms by mistaking normal (abnormal) for abnormal (normal) variability
- On-line control leads to early detection and correction
- A process “in control” indicates only its internal stability
- Improving process capability involves changing the mean and/or reducing normal variability requiring a long term investment

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Let us now do a little broad review of what we have done so far. Every process display some variability, some may be normal, some may be abnormal. Notice here I have a process here which is controlled by some control factors. These are process control factor it is also influenced by these noise factors the result is this variation this is something that will happen in every process and it turns out some of these variation would be normal and some of it would be abnormal. In order for us to what we call get an assessment of the capability of the process, that is how good the process is in meeting customer requirements, we have to come up with an index that is called process capability.

There are a couple of ways to measure that one is of course, you worry just about the spread of the process. You do not worry about the centrality of the process in that case you would be using this quantity called C_p . If you use C_p you end up with a simple formula; this just basically gives you an idea what is the chance of by producing something that is going to be within these specification limits because my process is pretty tight. This does not C_p by itself does not worry about the centrality of the process.

If you are worried about is there some way I could get an idea of how good my process is, how poor my process is, look at your control chart. If there are lot of points is there lot

of statistical points, these are the expired values of the R value. If they are beyond the control limits clearly it is not very lightly that your process is capable this is something you got to worry about so if you if you are starting out trying to control your process of course, you could produce the histogram of the output and I could do that but, it will be better idea to try to run a control chart and take a look at r there take a look at whether there are many assignable causes that are also impact in the process control chart.

The process if the process is being influenced by lot of these what we call abnormal causes or assignable causes spend some time to try to remove them once you remove them what is left is just a chance causes; these are the random factors that will be there no matter what unless you improve the technology of the process. if you improve the technology you can of course, tighten of the process just like one great way to try to improve your process.

So, control charts would be a pretty decent place to begin your study and take a look at just in case this chart show some sort of normal variability it is possible that control charts would give you false signals. It is very possible that the variation that you see is normal but, remember alpha error; alpha error is the type of an error control committed with a control chart which says I get a signal that means I see a point beyond what we call control limit upper and lower control limits on a control chart. And, I go which hunting I tried to sort of diagnose the process I go back to my process and I start taking a look at all these different factors to try to see is there something wrong with A? Is there something wrong with B? Is there something with C? This investigated little bit more to try to sort out so it can be removed something there.

Suppose the process was not disturbed, you would have adjusted statistical variation that lead to that condition. There you will end up with the process you will end up needless to troubleshoot in the process. So, that can happen because the control chart you know there is like a tiny chance 3 or 4 parts per 1000 when the chart itself may give you false signal that is there and this is the type 1 error. If we do, now what we call in control? If we remove assignable causes and what we are left with is basically essentially the process under the influence of chance causes only which is the natural variation of the process it is at that point. You can try to, you should to make an assessment of what we call process

capability. Process capability is the capability of the process to be able to just basically produce any output. It will meet customer requirements it is wholly within the tolerance that is tolerated by the customer if it is if there is a process like that you have nothing to worry about it. I should of course, caution you that many times the range of the process that means the difference between the maximum value of the output produced on minimum value it may be pretty tight but, the overall process itself may be aspect and that is because sometimes the average which is \bar{x} may not () inside with the target μ .

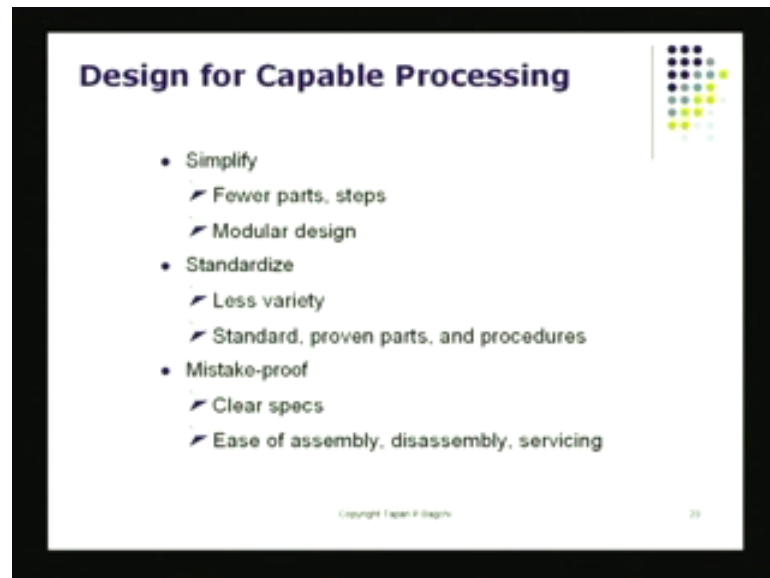
If \bar{x} are μ if they are different from each other this will cause this will cause some part of the process to go beyond the control limits even in the process has descent distribution as we saw in one of the earlier curve curves so what we have to do is we have to do two things now the first thing is get an idea of process variability which you could do just using C p plot some charts try to make sure that there are no assignable causes present in process that disturb the process. This is something you have to do. The third thing you have to do is compute C pk and C pk will tell you if there is a problem of basically positioning the process which means is it being is it running **right**.

Now, the average quantity is average quality characteristic being produced there are beyond what we call either below or above the target that is desired by the customer. If it is off target that means, there is a problem with the process there to do some adjustment to try to make sure the average returns to the target value. Then, I have got the best process then of course, I am going to tighten of the process.

The methods for doing this, some of this can be done through experience a better way to do this is to apply a design of experiments, use factors and those factors eventually will end up tackling your process there and they will come up with those factors that really cause either a shift in mean or a shift in sigma that can be done by doing by picking the right response making the right measurements and conducting experiments in the frame work of what we call a matrix. If you do those matrix experiments you are very lightly to discover the main effect of the different factors that are there the process control factors and also their interaction. Once that is there, once these variations have been located you got not much to worry about. All you have to do is adjust the process, adjust those these

settings on those factors, the factors that turn out to be the culprits. You adjust them in the right direction, the process will be stored. The average will come back \bar{x} will come back to target and also sigma will reduce sigma, will reduce the overall variance of the process will reduce this is something that we need to do.

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This is a very interesting area in fact designed very often aims at simplifying it, looks at simpler parts, pure parts and so on and so forth. Always the idea is to try to come up with the process which has got easy to operate which is easy to operate, easy to assemble, easy to produce and any chance of making human error those also have been reduced those have been minimized. You look at the causes of variation look at the causes of variation; you monitor the process using a control chart you look at the output. If it turns out that the you got a lot of assignable factors there then of course, you go back there do investigation there.

If you are taking care of the assignable factors then, you look at the spread of the process. You look at the spread of the process because that is now the natural variation of the process if that goes either above or below what we call the tolerance you have to then adjust, you have to then play with the sigma of the process and this again like I said it that can be done. This is the overall variability of the process and one great way to tackle

that is to apply design of experiments.

So, we wrap up this session by saying if there is a process that is influenced by many factors I can monitor using process control charts. And, then of course, I have got this tool called C_p or C_{pk} measurement. These give you a pretty decent idea see the process control charts themselves they do not talk about specification but, the moment you bring in C_p or C_{pk} you talking specification and now you are talking about what is of interest to the customer, to the final user. If you do this, your process will stay in control. Also, it will consistently produce output that is well within the tolerance that is tolerated by the customer; we will continue our series with the next lecture; thank you very much.