Six Sigma Prof. Dr. T. P. Bagchi Department of Management Indian Institute of Technology, Kharagpur

Module No. # 01 Lecture No. # 22 Control Chart Examples

Good afternoon, we begin again; we continue with our discussion of control charts. The examples that I have here - these are from services.

(Refer Slide Time: 00:29)



As you know, there are two categories of our economy. We got production which produces physical objects like this and we got also services. Like for example, teaching is a service or you know, getting a haircut is a service or getting served in a restaurant is a service. For the majority I will say, most of the things that we have around us, they are a mixture of products and services I mean therefore, need to monitor both. We cannot just say we will only worry about products and we could not be bothered about the service that has been provided. I mean, think of a hotel room that you walked into and you found that the drape had wrinkles and the bed had some disorder there and the flowers were thrown all around and so on and so forth and the soap was not there in the bathroom and the faucet was you know leaking or something. These are service quality defects and these also need to be monitored and to be able to do that we also can use control charts. The moment you collect data and the moment you convert that into a message which is like a graphic message, people start taking action. This is what we do on the production shop and it is very easy you know, get your measurements done, come up with your expert chart, come up with your art chart despite on the wall and of course, it is going to show you if the process is in control. That means, if the variations are within that tolerance or if there are exceptions of variation in that process, that you need to worry about. If you do not do that the customer is going to end up with a lot of off spec stuff; that he is not going to like. This is something that can also happens with services; so, you got some examples.

(Refer Slide Time: 02:06)



Nature of defect of course, is different with services. These are not dimensions; these are qualitative. Many times they (()) differently; you look at some of the examples there for a hospital. It could be timeliness and the quickness of care. So, you came to a place where they basically tried to do you know, they have a facility where they provide treatment and it turns out that the timeliness, quickness is not there. This happens sometimes in health care systems' staff responses to different requests that you all might be making. These also may not be there; lab test may not be accurate; cleanliness may not be there; courtesy may not be there; accuracy of paperwork may not be there and so on. Grocery stores: what kind of things may it turn out? Waiting time is a big problem, it is a big issue and this thing can be monitored. The frequently out of stock item that also is something that 1 can look at quite easily. Quality of food items cleanliness and so on.

these are all going to be going to be aspects of service that you would like to worry about. Then the airlines; if you look at airlines you know, most of you, most of us these days we fly and you might be losing baggages; there may be delays in flights or there may be problems with luggage handling. For example, there may be waiting times in ticket counters and also in check in and there may be lack of courtesy which happens with some of the airlines and I hope very much that they go they quickly realizes or they just get out of that businesses because, my time is not to waste there. And, you know worry about the first (()) somebody there accurate flight information this also is very important and; obviously, the passenger cabin cleanliness that is important and the maintenance of the whole aircraft and so on and so forth.

(Refer Slide Time: 03:44)



Fast food restaurants, lots of things are there; again, waiting time and so on. So, you can actually see as you go along service has also provided a lot of opportunities for control for quality control.

(Refer Slide Time: 03:54)



Those also could be done if you wanted to those also could be done using control chart that could be done using control chart. So, airline they could plot charts, grocery stores hey could also produce charts hospitals many of them they do produce charts quality control charts the same thing could be done by banks and I know some banks that have wherein particular procedures for collecting data these are on the quality of service and they track these charts. These charts are displayed probably by those people who are exceptional performers they show that their air rates are near 0. Their problem in regard to counting money is next to 0. There are curtains, they dressed well and all those things if they are there; obviously, customers are going to be attracted toward that bank. So, this quality will lead to definitely more business. That is why you cannot just say that I will be plotting control charts. Only if manufacturing people, I will be only doing it for people who produce widgets and gadgets. No, it is also true for service and experience; their aspect of it that can still be monitored. So, that is like another area where we could use charts. By the way, I did not give you this name; early charts were invented by a gentlemen; his name is Walter Shewhart.

(Refer Slide Time: 04:57)



Shewhart came up with this idea that there are variations. Almost anything there are variations but, is the exceptional variation that we need to worry about. We need not worry about the natural variation of a process because, to change the natural variation of the process if the application changes. Then, I may have to change new technology and that may cost me a lot of money but, I need not really do that. I need not really you know, spend a lot of money in just trying to keep the process accurate and also precise if I use the right kind of chart. Those charts would be there x bar chart or the range chart; these monitors these sample averages or I could in some cases use the x bar chart and the s chart or I could use the medium chart and the range chart or I could use the x bar chart for individual measurements. I could also use the moving range, ranges these are things that you found out now, find out now that you have understood what control charts are. If you looked at an x bar chart and r chart and we would be showing you some more of those the other charts are all very similar. The spirit is the same; try to distinguish between exceptional variation and natural variation the moment there is exceptional variation. This is when you would like to take some action and leave the process alone if the variations are natural because, every teller knows that there will be some variation signature to signature. Same thing happens almost with anything that you do. When you make an omelet at home, does it always turn out to be exactly the way you would like it to be exactly? Every time if you need that you got to have a omelet making machine and then you are using technology; that is going to cost you a lot more. And, that happens sometimes when you have got to standardize things you might use a lot of automation.

But, most of the times we need not really have perfect standardization; we will tolerate some variation there and for that control charts are just fine because, they are going to catch the exceptional situation they are going to catch that. So, same thing we would apply to attributes data. Also, they will be using the p charts or the n p chart or the c chart or the u chart and these displays show you exactly what kind of quality characteristics are going to be there that are going to be doing this.

(Refer Slide Time: 07:21)



Let us come back to the x bar chart and let us try to see what we could do.

(Refer Slide Time: 07:32)

Max Xi R = Range =

Remember the formula that I gave you? I gave you a little formula and the formula was right there. If you remember, I have this formula there and that shows you how to calculate your x bar quantity. It shows you exactly how to do that; x bar is sigma x i divided by n that is it and range is even simpler max value of x i minus min value of x i that is that is range x bar is going to help you monitor accuracy of the process and r is going to help you monitor the procedure of the process. So, we need both of them because, we need both accuracy and also we need what we call procedure. We need both of these; a process must have this and also a process must have this. Only then we will have a process that is going to be a satisfying process that is going to be fit to be used in normal use in business; let us try to see what else we do.

(Refer Slide Time: 08:23)



So, we got these formulae here; let us work with some real data I have here. The first thing I have to do is I have to set up the system in place. I set up the x axis which is going to be your sample number and I set up the y axis which is going to be your x bar values but, I need something more I need the upper and lower control limits.

(Refer Slide Time: 08:24)



I need the upper control limit and I need the lower control limit. These are statistical limits. These are ranges that define the natural variation of the process. if a process if an x bar value seems to be either exceeding the upper control limit or going below the lower control limit, the variation, the process is too much. It is not something that I should be living with; I have to do some trouble shooting there; I have to remove this. So, for that I need the formula for u c l and the formula for l c l the upper control limit and the lower control limit. How do I do that? It is not that difficult; all I need is I need a couple of quantities there and the quantity shown here are the average x bar. That means, a long range average of x bar and in our case that turns out to be 82 point 5 kilogram. This being something that probably is looking at the fill quantity in a bag and there are many such bags being filled may be fertilizers or something like that. Or, may be cement the average x bar value is turning out to be 8y 2 point 5 the standard deviation of this when I do my calculation I find it to be 1 point 6. Now, let me come back and little remind you of a couple of things. If you look at the normal distribution; the normal distribution will have if I show it here, it will have some variation and that variation is going to be shown like the bell curve like this.

(Refer Slide Time: 10:06)



This is the bell curve and something you got to remember is that the average is going to be at the midpoint. No problem and that is mu then there is going to be variation that is going to be slowly decreasing. As you go to this side and the frequency has been going to be decreasing on this side, it turns out if you look at the distance of 1 sigma and if you look at 1 sigma this way and 1 sigma this way this area contains 66 percent of all the points in this area. So, this area here this area is 66 percent. If I change this, if I make it another sigma, this side I came up to this point here that is like another sigma from here; so, from the midpoint I am now 2 sigma away. So, from the midpoint I am 2 sigma away, 1 sigma and 1 sigma this distance from here to here. This will contain 95 percent of the area under the distribution and of course, if I go out to be far end and if I come to this area which is like another sigma away, 1 sigma here and 1 sigma here if I go all the way out there then the area that is between these 2 extremes and I am going to be approximately drawing that for you, this distance here that covers 99 point 7 percent of the total area. Now, if there is some random variable call it diameter, I have got an average diameter there and this is my random variable x; these are these 2 ends are rare. These are rare values of x these are rare values of x to find a value for x i have to really go through a thousand samples and perhaps 1 or 2 items will find falling at this end or this end these are rare values of x the more likely values of x are right here. These are the most likely values of x these are going to be the most likely values of x these are going to be rare values of.

So, this is rare, also this is rare; why I say that is that the control limits are set the control limits are set exactly at this rare limit there. So, there is the upper control limit upper control limit is here and the lower control limit is here anything beyond this is rare and anything beyond this is also rare. Therefore, when I plot my control chart when I plot my control chart and I find points coming over here or coming over here, that is no more explainable by the natural variation of the process natural variation stays right here. But, the moment I find 1 or 2 x bar points there 1 or 2 x bar points there and all I have to do to understand this is to turn this sideways and I do just by putting it like this and you can barely see on the screen. You are going to see the lower control limit and the upper control limit all or let me just do it this way. Now, you can see the upper control limit right there and the lower control limit right there. If the data points that I plot, if they turn out to be here or here or here, no problem there and let me try to make them a little dark for you and I am just going to making them dark by filling them in green. So, there is a green point, there is a green point there. These are all within our natural variation; no problem there, but by (()) if it turns out 1 of the points is here or 1 of the points falls here, that is way below the lower control limit is throughout. There is something that requires investigation and also the upper point here it requires and if it is an investigation.

Now, just remember one thing; I drew the upper control limit 3 sigma away. I drew the upper control limit 3 sigma away from the midpoint. So, from midpoint to this point, it is 3 sigma and because this is x bar, if it is x bar it will be 3 sigma x bar and the same thing goes this way. The lower control limit was also 3 sigma x bar below the midpoint this is a midpoint, there is the middle point there. So, my control limits are located just to indicate the beginning of those rare events and this is exactly what I have got on my screen here. What is my screen here? I have got the red line going as upper control limit; I got the lower line going as lower control limit. These turn out to be the limits; within that I have got my natural variation anything beyond that is something is the region that I need to, if a point falls there I need to investigate the (). How do I find them? All I have to do is find the value of sigma x bar multiply by 3 at that to the average just like you did it here; just like I did it in my on my sheet here I on my sheet I have my midpoint right there which is the x double bar value the x double bar value is right there the overall average x bar value I added 3 sigma x bar I got my upper control limit and I subtracted 3 sigma x bar.

(Refer Slide Time: 16:02)



I got my lower control limit there and that is exactly what I have done there. Notice here it says 82 point 5 which is my average grand average plus 3 sigma x bar and I end up with 87 point 3 and I do 82 point 5 minus 3 sigma x bar which turns out to be 77 point 7 this is these are the values of my lower and upper control limits, that is all. The calculation I need to do. Of course, to be able to do this I need to find the value of sigma x bar and there are some shortcuts to do that.

(Refer Slide Time: 16:39)



So, again you know this is the normal curve which I just showed you earlier the region; that is near in the vicinity of the main or the average there the grand average. Those are all events and the ones are rare the ones that are rare are going to be the ones that are going to be beyond the 3 sigma limit and those are the ones that is, those are going to be our rare events and those are ones that need to be investigated.

(Refer Slide Time: 16:59)



So, if a process we say that the process is in control if there are no sample points sample point are like x bar x bar values or r values beyond the upper and lower control limits most of the point are near the process average. About equal number of point are above and below the centerline and points appear to be randomly distributed when I look at the chart.

(Refer Slide Time: 17:32)



If I do that, my distribution is going to be mostly in this green area and that is going to be the process in good control there. Now, there is something written up there notice this central limit theorem which basically says this no matter what kind of raw data you have. The moment you calculate your x bar value it will have a normal distribution and because it will have a normal distribution I can use the 3 sigma x bar. So, the average plus 3 sigma x bar and the average and 3 sigma x bar below it lows as control limits because the distribution x bar is normal and that is the reason for us actually even constructing a subgroup that is of some size subgroup is basically this sample I have individual data points and what I really do is I collect a few of these data points. I look at their average; that is how I calculate my x bar value and that is something that we will be doing again as we go into our example.

(Refer Slide Time: 18:33)



So, my raw data has this distribution and my x bar value will have a shrunken distribution it will always be normal in some cases of course, even the raw data is normal. Then of course, this definitely is going to be normal, but even if the raw data is not normal when I construct my x bar value which is x bar is now the average of you know, n number of x measurements and we will do some examples and you will see how that is being done.

(Refer Slide Time: 19:05)



So, the control limits actually turn out to be narrower than specification limit the process variation is going to be much wider by itself raw process variation, but the variation which we got interest is the variation of the x bar quantity or the range quantity and that is got to stay within these control limits. So, I must use the formula for u c l and I must use the formula for l c l to know exactly what this range is where if x bar value is lie within this limit of course, then of course, I have got no anxiety at all.

(Refer Slide Time: 19:36)



After I have plotted the chart and these are points that I have actually come about as I started running the process I collect the data and I calculated this is the first x bar quantity that I found second x bar quantity I found third and so and so forth. When I get to this point, I find that the point has now gone beyond the control limit I should stop the process because the process is now out of control. It is not under the influence of those small chance factors only there are some unusual factors that have impacted the process and the chart is telling me please stop the process take a look do an investigation and then do it.

(Refer Slide Time: 20:15)

toutine use of	the Process Control Chart		
ω.,	Out of control samples		
feasur	· · · · · · · · · · · · · · · · · · ·	Upper control limit	
- 8	1 2 2 4		
Proce		Lower control limit	
	Samples		
DataInformat	on: Monitor process variability over time		
Control Limit	Average + z - Normal Variabi	lity	
Decision Rule	 Ignore variability when points are w Investigate variation when outside 	thin limits as 'abnormal'	
• Errors Type	- False alarm (unnecessary investigation)		
Type	II - Missed signar (to think ") and correct)		

Then, move along with this and of course, this is the repeat of the same thing. I am again showing the variation and I am showing you a couple of data points that are out of control.

(Refer Slide Time: 20:26)

Control Charts	
 Basic Types 	
Most typical three	
X-Bar and R	
p chart	
c chart	
> Depend Upon Data Type	
Variables	
Attribute	
 All are Applications of Hypothesis Testing: 	
H _o : Process is in statistical control; H _i : Process is out	of control
Converted Tastan P Tastania	
P	

(Refer Slide Time: 20:39)



The same principle basically applies to the x bar r chart the p chart the c chart and so on. All these charts they are due to this doctor Walter Shewhart. He is the one who actually conceive this idea and he came up with this idea of a separating between assignable causes or the special causes of variation from what we call the chance of the normal causes of variation. This is something we got to figure out and how will we figure them out we figure that out by just plotting the chart and finding out if there are any point that are beyond the control limits. The moment a chart shows points which are beyond the control limits either above the upper control limits or below the lower control limit stop the process. Do not continue with your production go and take a look at the tools and a variety of things and I will show you how to do that. Once you have done that you basically remove the factors that could have cause the process to actually go out of control.

(Refer Slide Time: 21:22)



Special causes of variation these are the ones that actually are the assignable causes of variation and there are many examples and as experience grows we find that we are able to tell between this special causes and the common causes of variation that is not going to be that difficult.

(Refer Slide Time: 21:30)



(Refer Slide Time: 21:39)



(Refer Slide Time: 21:48)



Now, something more we have to actually tell you. Notice here I constructed my charts using the x bar quantity x bar quantity and this I use to try to see if my accuracy was then I could also look at the variation the process and for which I could really construct an s chart. And, this could give me an idea whether precision is in control, but you remember the formula for s which is really complicated a square root and then we do sum x i minus x bar and then I do I equal to 1 2 n and I divide this whole quantity by n. I take that square root that is my s quantity; this is rather complicated formula. If you are doing this in production, even if you had a calculator, even if you had a calculator with you it

would not be easy to execute this. So, what have we done we said we will probably use the calculator to find my x bar quantity only, but we use in place of s we use range and range is just max x i minus n x i and this will do the same job. it will also let you control precision just as s does this will be the job just as good. So, if at I can control accuracies in x bar chart. I can control precision I can monitor precision by looking at the r chart what is the link between these 2. You might just say then is there a correlation between these 2 let us look at some real data I have on the screen here the top chart is the s chart the standard deviation chart. I have got my samples here I have got the values there for the same data I calculated range and they are like this and notice how close these 2 are. In fact, if you do a correlation plot between if you do a scatter diagram that correlation turns out to be like point 99 or something like that. That is really good which really means, in place of using this complicated formula to find my standard deviation I could just find r and use r to monitor precision of the process that I could do quite easily.

(Refer Slide Time: 23:59)



(Refer Slide Time: 24:07)



In fact, the common practice says plot the x bar chart and then plot the range chart and you carry along with this. How do I construct these charts?

(Refer Slide Time: 24:12)



Well, you have the formula which I have shown here and the finding the grand average of x bar is not that difficult the overall average which is I am showing. Here I am showing that by showing the average of sample means our little x bar values and I found x double bar which is the average of these sample means collected over some history. Maybe, last 1 week production or something like that. So, I have got an idea of x double

bar which I use to find the upper control limit. I should really be doing x double bar plus 3 sigma x bar that is what I should be doing very fortunately other people have worked on this. Other people have worked; they said in place doing the 3 sigma 3 times sigma x bar. I can use the quantity called a 2 which is a multiplier a constant that depends on sample size and r bar which is the average just like a found a grand average of average is sample average. I find the grand average of what we called ranges use that as r bar. I can use this formula to find my upper control limit I can use this formula to find my lower control limit. So, what are the new constants I have used here new quantities k is of course, sample size number of samples I have got number of samples. I have got to calculate my x double bar r bar is again the average of the different r's that I calculated for the k sample that I draw that. I took a 2 is a quantity that will depend on sample size and I am going to show you how to do that. Once I know those I have got my u c 1 known and I have got my 1 c 1 known and these can be clamped on right on the control chart.

(Refer Slide Time: 26:03)

							_
	OBS	ERVAT	IONS (S	SLIP-R	NG DU	AMETER	t, CM
1	5.02	5.01	4.94	4.99	4.96	4.98	0.05
2	5.01	5.03	5.07	4.95	4.96	5.00	0.17
3	4.99	5.00	4.93	4.92	4.99	4.97	0.01
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5,10	4.96	4.99	5.02	0.14
8	5,09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	4.44

Let us try to do some of these calculations. Let us try to do this let us try to see if you could come up with the quantities to be able to do our calculations here. Now, let us first become familiar with the table itself samples are 10 samples have been collected. So, sample number 1 it has got 5 individual data pieces. So, these are the x values these are the x values x 1 is 5 point 0 2 x 2 is 5 point 0 1 x 3 is 4 point 94 x 4 is 4 point 99x 5 is 4 point 96; let us write them now. In fact, what I could do is I could go straight to my

calculator I could go straight to my calculator and I am now using my calculator and I just read off I just look at this table and I will use my calculator which I am showing on the table here and just watch my fingers, I am going to be using this. So, the first quantity is 5 point 02 plus 5 point 01 plus 4 point 94 plus 4 point 99 plus 4 point 96 and this is going to be the average. So, I got the average there I have got 24 point 92. I divide now that by 5. So, I divide this by 5 to get my first x bar value. What is the quantity there? 4 point 984; I am a little more accurate than the table here. The table shows the value of x bar to be 4 point 98. Let us now try to find lets now try to find the value of r. How do I do that? I locate the maximum value of x in that sample there. What is the maximum value 5 point 025 point 025 point 0 1 4 point 94 and so on and so forth. So, this guy is maximum. I enter that quantity there 5 point 0 2 now this is the max x value. What is the main x value in this? It is 4 point 94. So, I do minus 4 point 94 and I do an equal and I get 0 point 0 8 that is the value of r. So, what have I got here? I have got 2 quantities there.

(Refer Slide Time: 28:38)



I have got my x bar 1 equal to 4 point 98 and I have got r 1 this is equal to 0 point 0 8 these quantities are for the first sample. I will do the same thing for second sample third sample and so on and so forth and you can notice here on the screen. I show here the different values of x bar that have been calculated by doing there this is x bar 1 x bar 2 x bar 3 and so on and so forth. If I take the grand sum of all these that is this 50 point 0 9 if I take the grand average of all these I have got 1 point 1 5.

So, let us try to find now x double bar which is the grand average of all. So, I again come back to my calculator and what I do here I use my calculator in a very special way. Now, I do 50 point 0 9 and I divide that guy. How many samples are there 10 samples divide that by 10 and that turns out to be 5 point 0095 point 009. So, my x double bar is going to be 5 point 009; what about r bar. How do I find r bar all I do is I look at the sum of r's and that is 1 point 1 5. So, I put in 1 point 1 5; I divide that by 10. So, my r bar value turns out to be 0 point 115. I have these 2 key quantities that I will be using to construct my u c l and l c l. The only things that I need; now I need some constants and I will find out what those constants are and I need to remember something called sample size. What was my sample size for this problem? My sample size was n equal to 5 n equal to 5 was my sample size. I will be needing this quantity to be able to find those you know the constants a 2 and there will be some other constants also in trying to find the control limit for the range chart that I will be doing that.

(Refer Slide Time: 30:55)



So, let us turn the page over and let us see how they find their quantities there and look at this x double bar he had 50 point 0 9. He divided by 10 and he got in his own approximate way he found 50 point 5 point 0 1 and I found of course, 5 point 0 $\frac{0}{9}$. That is the only difference there then with this he also has calculated his r bar and r bar of course, luckily coincide for this r bar in his case is point 115 which I see there. I see point 115; no real problem there and I have to multiply that now by this quantity called a 2 and we will try to find the value of a 2. We will try to find the value of a 2; a 2 can be

found once I know n. So, remember our n our n is equal to 5 from this I can find my value of a 2 and for that I just have to go to a particular table a particular statistical table that will give me the value of a 2 and I will get to the table in a minute. But, the anchor is going to be my sample size n once n is equal to 5. I will be able to find my value of a 2 the moment I find the value of a 2 I can come back and plot it and I can just plug it in my thing there and notice he is found the value of a 2 and I am going to be find that also for you. I end up with my value of upper control limit for the x bar chart I do the same thing and I just flip the sign here; I find my quantity for my value for the lower control limit.

(Refer Slide Time: 32:27)



Of course, then I have got my lower and upper control limit and I have got all those x bar values remember I have the x bar values here I have got my x bar values there I have also got range values there now because I have got my control limit there it is very easy for me to plot this chart what have I not found. So, far for you I have not found the value of a 2 a 2 is something that I will have to find out from somewhere and a 2 is found in books of quality control. They provide a special table there and they tell you please tell please indicate what is your value of n the samples size. Once you tell it the value of n they will tell you what this value of a 2 is going to be a 2; depends only on n the sample size. So, we will be able to do that once you go to the right table then we will be able to value find the value of a 2, but returning to the chart again notice here the chart has all the things there he is got the upper control limit the lower control limit the midpoint there sample number share and so on. Now, this chart is ready; now to monitor

production beyond this, beyond sample number 10, I could continue to have the same control limits. I will keep plotting the new point there which will probably fall there and so on and so forth and let us hope that they will not cross the limits.

(Refer Slide Time: 33:42)



Now, you know there are some other issues which are brought up once in a while. What is the chance that the chart is going to give you a false signal? That is done by using this concept called average run length of the chart. When you got the alpha, alpha is the chance of what we called type 1 error used in a control chart which is like getting a false signal out of the chart. A false signal will come when you get a signal out of the chart which means, you find 1 point going out of control even if no sign of causes have really disturbed the process. So, the process is still fully under the influence of random causes only but, because of this statistical acceptance. Because, you remember the normal curve had tails that went beyond the 3 signal limit is those teeny-weeny tails that are there. They also cause sometimes this natural variation to push out a few points beyond the control limit and that will happen at this rate alpha by 2 on the high side and alpha by 2 on the low side. Therefore, there is a little formula there which tells you average run length that is the average number of sample data point that you need to plot that you should plot before you should expect to see a point that is out of control even if the process is in control. So, that is the chance of my making a type 1 error type 1 basically means, a false which hunt you think that the process is out of control. Actually, it is in control its under the influence of random causes a chance causes only and just under the

influence of these chance causes it has pushed out a few data point that are beyond the control limit.

That is all; nothing else, no nothing else is gone wrong with this but obviously, if the normal chart shows you that it has gone beyond control limits and we got many examples of that and we will see some more examples as we go into this. For example, that we just bring up 1 or 2 examples; we had seen them earlier and example is going to be this one. For example, control limit exceeded control limit exceeded these are going to be instances when you got to stop the process regardless do not second guess the moment the chart has gone beyond control limits. Stop the process and go to something that is called the reaction plan which is what to construct using the cause and effect diagram and so on. But, then go through your full checklist full list of checklist and check everything about that machine just to make sure that you will not really end up with that problem being there and you think everything is in control and you will end up producing products (()). Let us take a look at 1 or 2 other items which are related to this.

(Refer Slide Time: 36:31)



There are certain things called warning limits which are put 2 standard deviations away you know the control limits which are like the action point those who put 3 sigma limits away, but sometimes people also put a limit at 2 symbol limits away 2 symbol limit from the base side these are warning limits they actually tell you where we are finding too many points in the vicinity of the between beyond the 2 sigma limit which actually is

saying perhaps, something is creeping in some problem is creeping in. and, it is just a warning limit it is like the amber light when you are driving the amber light appears you will become cautious. So, if your point goes beyond the 2 sigma limit you are in that amber zone there. Just be cautious; look out for any tools and make a few checks just to make sure nothing really is going to breakdown again in a little while. Of course, the bad thing is when you get you know no signal out of it the process is actually change, but you get no signal out of it.

(Refer Slide Time: 37:25)

Calculation of Xbar Chart's Control Limits	
Def : Xbar = $\overline{x} = (x_1 + x_2 + x_3 + x_4 + x_5)/5$ Range $R = Max - Max$.	
A quick method for finding control limits is to use	
average sample range \overline{R} as a measure of process variability. Upper control limit, UCL = $\overline{x} + z\sigma$, = $\overline{x} + A, \overline{R}$	
Lower control limit, LCL = $\overline{x} - z\sigma_{zzar} = \overline{x} - A_2\overline{R}$	
where \overline{R} = Average of sample ranges and A_i is found from a ta	ble.
Copyright Tapen P Elegini	54

I have exactly the same formula again here. I show here by showing you that how to calculate x bar, how to calculate r and how to control the get the control limits. and of course, again the issues that of knowing what a 2 values are and a 2 values are now found by this table here. If it is got a few constants also which are used for other purposes in drawing control charts, but notice here I have got sample size here in column 1 and column 5 column. 1 row 5 gives you the value of a 2 in column 2 which those guys took to be a point 5 8. The true value of a 2 is going to be point 5 7 7.

(Refer Slide Time: 37:46)

1	Process Co	ntrol Cha	rt Factors	
Sample Subgroup) Size (n)	Control Limit Factor for Averages (Mean Charts) (A ₂)	UCL Factor for Ranges (Range Charts) (D ₄)	LCL Factor for Ranges (Range Charts) (D ₃)	Factor for Estimating Sigma ($\sigma = R/d_2$) (d_2)
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059
5	0.577	2.115	0	2.326
6	0.483	2.004	0	2.534
7	0.419	1.924	0.076	2.704
8	0.373	1.864	0.136	2.847
9	0.337	1.816	0.184	2.970
10	0.308	1.777	0.223	3.078
			La .	
		Copyright Taplan P Engl	n-	
1				

So, if since our sample size is 5 I can use this quantity is the value of a 2 and go back into my formula and plug that in there and I will end up with my control limits here this is the way you calculate your control limits for the x bar chart. I have a very similar procedure which is used for controlling my for plotting the r charts I do that and here is an example again just like the example that we saw there.

(Refer Slide Time: 38:34)

Select 25 small samples (in this case, n = 4) % Find X and R of each sample. % The X chart is used to control the process mean. Sum X The R chart is used to control process variation. R	Sample Number 1 1 4 7 6 5 8 8 5 6 9 20 24 28 5 6 8 20 24 28 5 6 8 2 5 3 2 75 3 2 3
Copyright Tapen	P Engine 51

Here is an example again and we can verify that the sums of these column. These are this sample number 1 sample number 2 and so on. They are 25 samples corrected; these are

the raw values and here is the value of x bar right there and here is the value of r and you can verify. For example, 6 minus 4 is equal to 2 which is the range value first one. Similarly, range for second 1 third 1 4th and so on and so forth. Once I get all these things I can calculate the totals I can calculate my x double bar and r bar and I can use the same formula.

(Refer Slide Time: 39:16)



I can calculate the thing and here the calculations and if you want to see the chart the chart is right there. So, the same data that I showed you this time, those data have been used to set up the control limits and now of course, the 2 charts are there to remind you again we need both the mean chart which is the x bar chart and the range chart because we are concerned above 2 aspects of the process. One is accuracy the other is precision both of these have to be certainly when the data quality is measurable I need to know if the process is producing items which have their values pretty close to the target value and this is the indication of accuracy and precision is when the production is such that most of on my production is consistent; that means, regardless of which cap I pick they will fit in. Actually, simply, they will fit because they are they are all pretty tied to each other they are with processes precise.

(Refer Slide Time: 40:18)



Here is another example and you can try that example you can look at the numbers there you can freeze that screen there and you can check it out and this again will give your values there no real problem there in some places.

(Refer Slide Time: 40:27)

Example of Control Limits Calculations using σ_{sbar}	
$\frac{\pi}{\pi} = \frac{12.10 - 12.12 - 12.11 + 12.10 - 12.12}{\pi} = 12.11$	1
and $\sigma = 0.02$ (gaven). Note that sample size $n = 4$	
Hence $\sigma_3 = \frac{\sigma}{\sqrt{\pi}} = \frac{0.02}{\sqrt{4}} = 0.01$	
Upper control limit	
UCL = $\overline{3} + z \sigma_7 = 12.11 + 3 + 0.01 = 12.14$	
Lower control lumit :	
LCL = $\sqrt{3} - 2\sigma_{1} = 12.11 - 3 + 0.04 = 12.08$	
where $\sigma_2 = \frac{\sigma}{\sqrt{n}}$	
: Standard deviation of distributi on of sample means τ	
σ = Process standard deviation and u = Sample size.	
Copyright Tapan P Bagch	62

(Refer Slide Time: 40:32)

Saint Szu-	Factor for 3bar limits	Factor for R L CL	Factority R U.C.
n	Α.,	D,	D.,
2	1.00	0	3.27
3	1.02		2.57
4	0.73		2.29
5	0.58	0	2.11
	0.48	0	2.00
7	0.42	0.08	1.92
	0.57	0.54	1.06
	0.43	0.18	1.82
10	0.51	0.22	1.70
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.81	1.69
14	0.24	0.33	1.67
15	0.22	0.95	1.05
16	0.21	0.56	1.64
17	0.20	0.30	1.62
10	0.19	0.96	161
19	0.19	0.40	160
20	0.18	0.41	1.99

I have shown you how to do your x bar chart and how to (()) r chart and in some cases I have also I have also shown you how to do the s chart and they turn out to be very similar. Here is the range chart the formula is right there and I followed the exact same procedure I ended ended up with the value of r I.

(Refer Slide Time: 40:40)



(Refer Slide Time: 40:44)

R-Ch	nart I	Exa	mp	le			
	OBS	ERVAT	IONS (LIP-RI	NG DIA	METER	. CM
SAMPLE &	1	2	J	4	5	x	R
1	5.02	5.01	4,94	4.99	4,96	4.98	0.0
2	5.01	5.03	5.07	4.95	4.96	5.00	0.1
3	4.99	5.00	4.93	4.92	4.99	4.97	0.0
4	5.03	4.91	5.01	4,98	4,89	4,96	0.1
5	4.95	4.92	5.03	5.05	5.01	4.99	0.1
6	4.97	5.06	5.06	4.96	5.03	5.01	0.1
7	5.05	5.01	5,10	4.96	4.99	5.02	0.1
8	5.09	5.10	5.00	4.99	5.08	5.05	0.1
9	5.14	5.10	4.99	5,08	5.09	5.08	0.1
10	5.01	4.98	5.08	5.07	4.99	5.03	0,10
						50.09	1.1

(Refer Slide Time: 40:50)

R-Chart Example (cont.)	
$\overline{R} = \frac{\Sigma R}{k} = \frac{1.15}{10} = 0.115$ UCL = $D_4 \overline{R} = 2.11(0.115) = 0$ LCL = $D_3 \overline{R} = 0(0.115) = 0$	0.243
Retrieve Factor Values D ₃ and D ₄	
Copyright Tapan P Bagini	

Find r bar from there and of course, I calculate this now something I wanted. I want to point out here remember we used a 2 we remembered we use the quantity a 2 to a 2.

(Refer Slide Time: 41:05)

X A=5 -> A== 0.55 = De LCLL = D, R

In our case turn out to be point 5 8 that I that is what I use for my x bar chart and for my range chart I use something else I use some other constants and these constants are going to be d 3 and d 4 and what is the formula for the range chart u c l range its d 4 times r bar and l c l r is going to be d 3 times r bar these are the formulas these are the formulas that I utilized and constructing a range chart and putting control limits on it and that is what they have done on the screen there notice again where do I find my d 3 and d 4 I go to the same table I go to the same table and I have there d 3 value and d 4 value what are these dependent on again they are dependent on sample size n. So, if I know my sample size n which in our case was 5 I find that the value of d 3 is 0 and the value of d 4 is 2 point 1 1.

Let us go and see what they have done they use the value of 0 to find the lower control limit. So, the power control limit for the r chart or something will be 0 and for the upper control limit they have used d 4 d 4 times r bar and they use 2 point 1; one for d 4 and they end up with a upper control limit there. So, these are the ones that then become our control limits there is a chart this is the chart that actually was constructed using the same data there.

(Refer Slide Time: 42:42)

- Control	t i mite	hu Dha				
Chanadar		by Roa	53	54	64	
1	12.11	12.16	12.00	12.12	12.00	
2	12.10	12.12	12.09	12.10	12.14	
3	12.11	12.10	12.11	12.08	12.13	
4	12.08	12.11	12.15	12.10	12.12	
Range R	0.03	0.05	0.06	0.04	0.05	
R	= 0 03 + 0 0	$\frac{15 + 0.06 + 0}{5}$ = 4, theref	0.04 + 0.05 ore $A_2 = 0$	- 0 046 73 firom tabl	e	
H	ence, Upp	er Lower C	ontrol Limit	5 382		
U	$CL = \overline{x} + A_{f}$	R = 12.11+	0.73 - 0.046	=12.14		
D	$TL = \overline{x} - A_{\mu} I$	R = 12.11-0 Crevent Ta	0.73 - 0.046 per P Septe	=12.08		

Again, there is an example and you can again verify your construction there and get the things. So, it is like range chart done another example of.

(Refer Slide Time: 42:46)



(Refer Slide Time: 42:51)

Range Control Chart Control Limits	
The control chart used to monitor process	
dispersion or precision is the R - chart.	
Upper control limit, UCL $_{R} = D_{4}\overline{R}$	
Lower control limit, LCL $_{R} = D_{1}\overline{R}$	
where D ₃ and D ₄ are obtained from the	
Control Limit Factors table.	
Costyright Tapan # Bagote	26

What we did before these are all going to be examples and you can flip through them back and forth and you can actually verify that the numbers are right.

(Refer Slide Time: 42:58)



What we have to do is we have to now move toward what we call getting signals out of the chart how do we get signals out of the chart how do we find the normal pattern

(Refer Slide Time: 43:08)



They are going to be found like this both process average and process variability must be in control process average is your x bar process variability is your r or range or standard deviation. Is very possible to have a very tight process a very good procession but, the average may be off this is no good news or it is also possible that the average is right there. But, you got wide variation both of these are problem situation. So, I really got to I cannot just say I will be only using x bare chart in my plant or you go to a manager and all you see a range chart plotted no r chart no x bar chart. These are not going to be good situations because you will have control only one aspect of quality. You will have either control of the accuracy of the process or the procession of the process, but not both but, real life we need control on both you need to have accuracy there and also you need to have precision in the process. So, therefore, you got to have both of these charts.

(Refer Slide Time: 44:15)



Now, what happens after you plot the chart you find some unusual patterns 1 very simple you know signal that you get out of a process is that the 1 of those data points went beyond the control limit that is like 1 signal out of control chart it can happen on a x bar chart or it can also happen on a r chart both of those chart they can actually give a signal. So, I can either get an x bar chart giving a signal there or I can get a get a range chart giving the signal there both of those are possible what we have to do is we have to see can I get more signals out of the same chart I need not really wait till I get a point beyond control limits. There are a number of unusual statistical patterns that are also visible on the control charts which are under the influence of assignable causes or these exceptional causes. What are those examples? What are those pattern examples? Of those patterns and unstable process will look like this. a process with trend will look like this; the blue lines are the control limits and look at the way the process is moving. It is slowly rising and rising and rising and suddenly it will go beyond; perhaps, probably what acceptable to us or there may be cycles. These may be 2 guys; 1 guy works dayshift the other guy works the evening shift or something and their settings are different. And, therefore, they produce the cyclical behavior in the control chart or there may be a genuine shift in the process. Now, this is like you know if you have been in a truck for example, you will find the same truck is used or the same bus is used by various different drivers and every time the driver changes the new driver comes in and he adjusts the seat. In fact, I did that when I was using this chair because, the instructor who came before me they had a class before me. He had made this pretty low because he is like a big tall guy not very fat but,

a very tall person. So, it did not really need that much height in the chair itself, but when I came in, I am like about 7 foot 2. So, I need some lift; no, I am actually 5 foot 4. So, I needed some lift. So, I adjusted that chair. Now, this if it is done if you watch the height of the chair at different hours in this studio, here you will find that the height is changing like this. It is going up and down like this and that is exactly shift in the average that is a lowest pattern this is something also something we would like to see.



(Refer Slide Time: 46:47)

So, the same kind for abnormal behavior is there. You see the chart going from one end one side of the midpoint to the other side of the midpoint. this is also a signal that something is happening with the process or a train going up or a train going down these are also again abnormal patterns. Or, too many points on one side of my chart which is one side of the midpoint; these are also exceptional points. (Refer Slide Time: 46:58)



(Refer Slide Time: 47:06)



So, even if the control point even if the point that you are trying to plot has not gone beyond control limit but, you notice one of these exceptional situations you need one of these things. Stop the process because, it may be too late if you wait for a real point to go beyond the control. It may be too late; you might have produced a lot of useless production. You may produce a lot of widgets that have no real use. They are not going to be fit properly because, the pattern has already shown that there are some unusual variations and even that point going beyond control limit. It is a unusual variation and these are the other patterns that actually also gives you unusual variations.

(Refer Slide Time: 47:53)

Zones Fo Tests	r Non-Ra	ndom Pattern	
UCL	Zone A	3 sigma - X + A ₂ R	
	Zone B	$2 \operatorname{sigma} = \overline{x} + \frac{2}{3} (A_2 R)$	
	Zone C	$1 \text{ sigma} = \overline{X} + \frac{1}{3} (A_2 \overline{R})$	
	Zone C	X	
	Zone B	Sigma I - 3(A2R)	
100	Zone A	$2 \operatorname{sigma} = x - \frac{1}{3} (A_2 R)$	
L'UL		3 sigma - X - A ₂ R	

Sometimes, people set up zones on their charts and they say you know 1 point beyond zone A. bad news; 2 out of 3 points in zone A for example, that is also bad news and so many points in zone B and so many points in zone C. if these things started happening these are also unusual pattern and these are patterns that should be... one should look out for them and we should try to see what could be done to try to prevent them.

(Refer Slide Time: 48:25)



These are all going to be abnormal patterns like some examples are given here. 8 consecutive points falling on one side of the centerline or 8 points running up or running

down, these are also unusual patterns. 14 points alternating up and down; these are all being found by in this case, the AT&T company and they have setup a lot of rules besides the point going beyond the control limit; these are additional rules. So, they give you additional power; truly additional power to stop the process when the process is not gone out of control yet. As far as you know, looking for a point to go beyond the control limit is concerned, 4 out of 5 points in zone A or zone B - remember these zones. So, there is s rule based on the construction of these rules and that say the 4 out of 5 points are in zone A or zone B; that means, there is an exceptional situation.

(Refer Slide Time: 49:26)

Reaction Plan to use when process is out of control	ner en	An event reader to a second to	A static di la segura di la seg

You should stop process and do something. What do you do? You now found out that yes, I need to take some action; I have done my calculations; I have plotted my chart; I have got all the control limits and everything done there. I have got my chart going. I have got absolutely nothing I have done now and I now find there is some abnormal behavior - abnormal point.

Now, I have given you a sheet here which says, the reaction point, reaction plan to use when the process is out of control. How do you know you that you got the... process has gone out of control? Well, those exceptional patterns or a point beyond control limit then what you do is well, before that you got to do some work. You have to do got to prepare this reaction plan. What you do is, you sit with people those who are actually familiar with the process and you construct; look at the top half of the program of the page here.

You recognize this tool here; this is your simple fishbone and the fishbone says there are certain conditions that will drive your process beyond specification limit and many times these are the factors that also make the make the control charts go beyond control limits or show some exceptional patterns and so on.

Now, say you have found a point beyond control limits. Stop the process; we will have to do your checklist and your checklist is this. This is your checklist and very systematically go over the items I have listed them out here. So, you notice the factors here; the causes that might have caused in this case, imparity to go beyond specification imparity is within some specification limits the first factor is this error in calculations. So, probably some calculations were done using a calculator to adjust maybe the quantity of catalyst or something. when this process was running and perhaps somebody pushed the wrong button here and he created a number that turned out to be different. If that thing happens that is actually not so good news. If you got a wrong data there that is now gone into your process and its cause, this variation there what you do is, you recheck your calculations. I have listed that because, it was in a cause and effect diagram. I have listed that and I do the calculations there; redo the calculation; turned out to be to make sure I in fact, turn to the process and make sure they have done all the rechecking and so on.

I marked right here; have I found the problem calculations correct? Yes; that means, I need not really worry about this factor. Then, go down to the next one. New batch of catalyst started is that true or false that could have caused the problem; this is also not a problem. That means, no it is not a new batch; too much quantity added or too little added. Check the batch records; check the sheet; look at the scale and so on. Make sure the scale was calibrated, all those things. If that is happening empirically then of course, you probably are missing on gauge r and r that is something you probably should not do, but even if I if you are trying to be a 6 sigma production process, you should not really do that. So, you go over these things; low purity catalyst - that also could be a reason for this incorrect temperature, incorrect pressure, error in testing or bad lab chemicals. So, you actually are getting a bad test coming out of the laboratory. These are this is our this is our checklist. So, whenever we have got an exceptional situation pull out this checklist and you start very systematically going over 1, 2, 3, 4 these items and just see the results of your investigation. If you have done it properly, if you have done it intelligently, you

are very like to catch the problem. If the cause and effect diagram has been constructed with a lot of honesty and by looking at previous history previous problems and so on and so forth, not many other things are there that are going to be beyond this. Most likely you problem is going to be one of that - of calculations wrong or you added something differently or added a wrong quantity or something like that. or, maybe a new person came along or something like that. But, something that you can put your finger on and this is called assignable. The very fact we use this word assignable means that the chance of a point and now, I go back to my, you know, this fancy diagram that the chance of a point to be found beyond the control limit either here or here. I can go back to the root of it; I can go back to the, what we call the checklist there and I can put my finger on the source of it.

(Refer Slide Time: 53:58)



Once I have done that the battle is half over because, I have found the culprit and the next thing I do is I basically take care of that. Something that is important for us to also remember when we are doing control charts is the sample size. Normally speaking when you are calculating these quantities the x w bar and the r bar and so on. Take 50 to a 100 data points 50 to 100 samples and as far as the these are going to be for attribute charts for the p chart and the m p chart and the c chart and so on.

(Refer Slide Time: 54:22)



If you are doing basically if you are looking at the variables chart what should be the size of your n which is the subgroup of items that you decide to pick every time. So, I collect. So, many data point and I average them out I calculate 1 value of x bar and 1 value of r how many data points do I need it should be between 2 and 10. if you do that it will be very easy to go to that table there and find your quantities.

(Refer Slide Time: 55:09)



You could start from the process that is like this; I started a process that was like this in Singapore and we did not even have the cause effect diagram it took as an effort for us to go down to a stable process. This was not stable there; look at the point out of control and this was more like the general story. Almost every other day we had a problem like this. Then, we understood that we did a lot of studies and so on; then we got to this. What is the problem with this? I notice here the target is off this is still target is off. So, we have to restore the process to be correct target and this is done using design of experiment. That is a very powerful method to try to adjust the process and that is also a key tool used in 6 sigma; especially if you remember 6 sigma D M A I C. The I or the improvement step uses design of experiments and here is also an improvement and here what we got is we got the process there which is filled in white. I have got to first of all control the precision who also control the, you got to basically restore the accuracy of the process.

So, I have started with the () process. I bring it in control; then I tighten it and I restore the accuracy of the process. This is the improved process. In fact, this is the way the 6 sigma projects move. They start with a very you know, wide sort of variation and they slowly and slowly as they understand the process they apply the right tools. One of which for your D O E; but for monitoring, process control charts are the way because without control charts we would not know how we are doing. There are some other things that we would be doing.

(Refer Slide Time: 56:47)



We have other types of charts. They are the p chart and the c chart and we are looking at what those charts are in a couple of minutes. Thank you very much; we will continue in a just a short while with the next session; thank you.