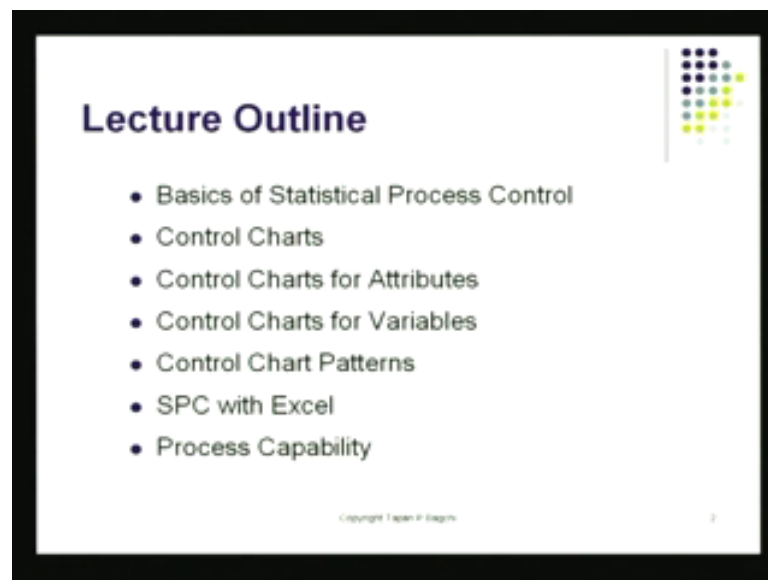


Six Sigma
Prof. Dr. T. P. Bagchi
Department of Management
Indian Institute of Technology, Kharagpur

Lecture No. # 21
Introduction to SPC

Good afternoon we continue with our series on six sigma. The topic today is going to be statistical process control. There will be four lectures; the first three would focus on the use of control charts. The development of control charts which turn out to be a very key tool to try to get consistency in production and also keeping up with quality. Once here where set up a machine and got the process to work. How do you keep it there? SPC turns out to be the technique to be used there. The last the fourth lecture in this sequence of statistical process control we will discuss process capability which I will follow after the first three sessions are over. So, we begin our lecture with our title slide here which says statistical monitoring of processes to detect special causes of variation.

(Refer Slide Time: 01:16)



And as I go into the lecture it will become more and more clear. Why we are talking about different types of causes of variation? You know something that is very, very important for business to understand, for manufacturers to understand, for service providers to understand and also for customers to understand is nothing stays the same when time changes. With time a lot of factors change quality changes, variations occur

lot of factors come and go and they impact the process and therefore, we end up with some variation.

But obviously with any product if there is a lot of variation the user will not be able to live with that. The customer is not going to be able to live with that. So, for that we need some sort of control device and control. The very work control always means there is a feedback mechanism. It looks at the results of the output; it looks at some summary of the output which gives an indication of the nominal level of quality. That uses that feedback to compare to some sort of standard. And if the deviation from the standard is too large then of course, you trigger a control action. This is the approach this is the philosophy of any kind of control. In the case of statistical process control what we do is?

We let this control be activated when the data shows an exceptional behaviour. Then we go out and we sort of apply some fairly simple statistical methods to get a signal out of the collective data and then decide based on that collection. Whether or not we need to adjust something on the machine that producing the widgets or do something about the process or to just leave it alone. Any of those decisions would be impacted by a trace of the output and this trace is actually a statistical trace of the output. This is what we are study when we talk about control charts? Control charts are the devices that make the behaviour of the process visible to us as it is running along as time is flowing by. How is the process changing in terms of its output?

Something that is of interest to us, the manufacturer and also to the users, the consumers. The outline of the lecture we will discuss some basics then we will discuss exactly the you know the various things about the control chart. This key tool that is used in statistical process control, two types of charts are fairly common. One chart that is attributed to some quality that you can basically call good or bad. For example, a pen may be able to write or it may not be able to write. So, it is good or bad or this fit here may be a good fit or it may be a poor fit.

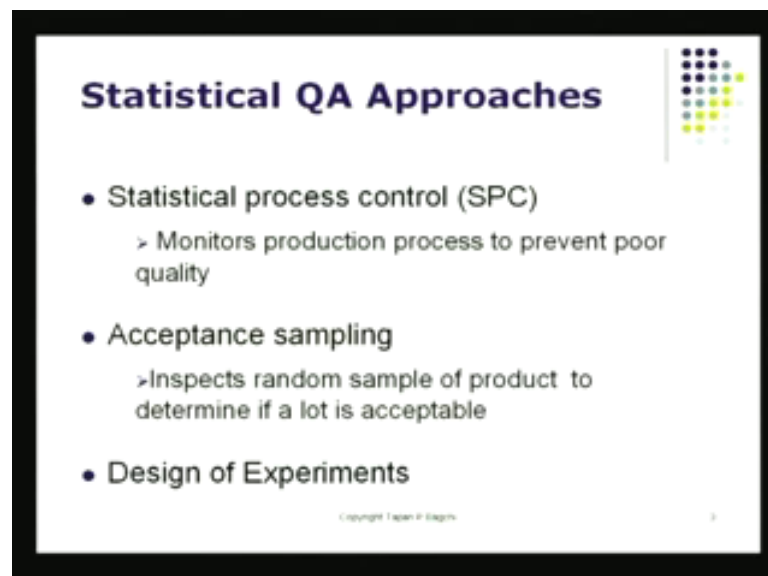
So, this is something where I can really categorize products to be either good or defective. Such quality performance such quality characteristics these are monitored using what we call the attribute control chart. Then we got other types of quality characteristics where I can make a measurement. For example, the dimension, the length

of this pen or some dimension of the roundness of the tip for example. Those can be measured and again those measurements can lead to a set of control charts which are called variables control chart. You might have heard of the charts called X bar chart and range chart, p chart and so on. Those come under the what we call variables control charts and charts that are like the p chart and the c chart and the u chart and so on. Those come under in the category of attribute charts.

Then we shall be looking at what we call some exceptional patterns that basically signal whether or not I need to take some action? So, look at some of these patterns on the common patterns that appear in the chart and these are the triggers. These are the triggers when these exceptional patterns appear these indicate that something has changed in the process and the process change appears to be significant. We will take some control action through a reaction plan. That also I am going to describe to you as we go along.

Then of course, I will give you some hint as to how you can set up this control chart preparation, procedure using this common tool called excel. I will give you some examples there and last of all which should be done in the last session which is the fourth session this sequence. That will be that will cover process capability.

(Refer Slide Time: 05:32)



So, let us get into the lecture and let us try to understand what we mean by statistical process control? Which is also called SPC, what is SPC? It actually monitors the production process online as the widgets or as the output is coming out. You measure

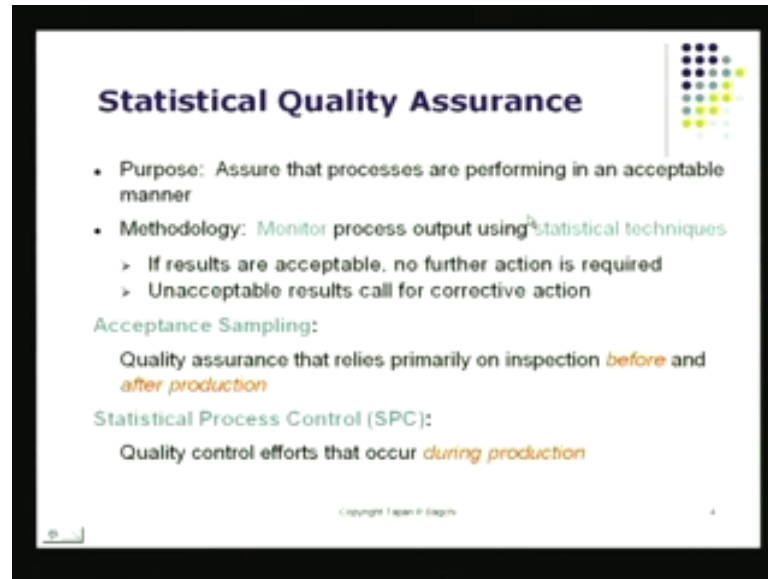
some certain characteristics of interest of the output. Then you collect some data which would be a sample taken out of the output and then you look at the statistic which could be something like \bar{X} or p or r or something like that. And you trace the basically the variation of that statistic.

It could be the \bar{X} , it could be p , it could be c or it could be any of these characteristic. You monitor that as it moves along and that is when you plot a chart and I will show you some examples. You will actually see this trace taking place. As more and more data are collected from the production line you will end up with the chart moving forward. The in the old days the only way to control quality was that by inspection and the statistical process, the statistical procedure which we covered in the earlier sessions.

One of those was called acceptance sample. You basically collect from a handful of items you collect a few samples and based on the on the fate of those samples based on the on the quality of these samples. You will decide whether to accept the lot or to reject the lot. That procedure that particular procedure was called acceptance sampling. That was one way to apply statistical methods in quality assurance and this online control business this is this comes under statistical process control or SPC. This is like the second class of applications of statistical methods in monitoring quality in assuring quality.

The third method which is perhaps the most powerful method and that also is basically utilized by six sigma is design of experiments and I am going to be spending some special sessions on that. So, you will get to know what DOE is. What design of experiments is and how we can utilize that to produce better processes and also better designs. That would be doing. So, there are basically three methods for applying, three opportunities for applying statistical methods in assuring quality. The first is of course, acceptance sampling, the second one is SPC and the third one turns out to be DOE or design of experiments.

(Refer Slide Time: 08:01)



Statistical Quality Assurance

- Purpose: Assure that processes are performing in an acceptable manner
- Methodology: Monitor process output using statistical techniques
 - > If results are acceptable, no further action is required
 - > Unacceptable results call for corrective action

Acceptance Sampling:
Quality assurance that relies primarily on inspection *before and after production*

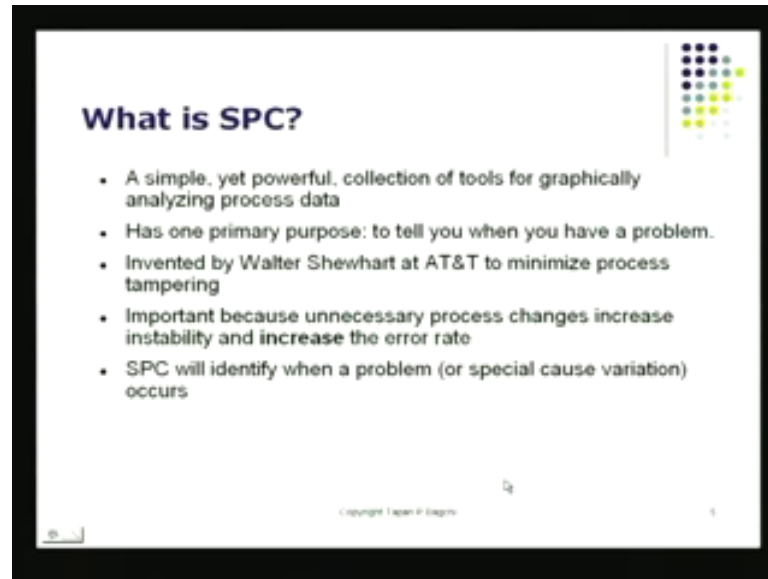
Statistical Process Control (SPC):
Quality control efforts that occur *during production*

Copyright Tapan K. Bagchi

Today we are going to be focussing on SPC, statistical quality assurance. What exactly do we mean by this? We are trying to assure quality. So, trying to be proactive. Basically what we are going to be doing when we do online monitoring? We would be monitoring the data online that means as soon as a widget is produced we make some measurements on it. We access its quality and we take a sample of these. We just do not always go with the item with one item at a time we do not do that. We will collect a few sample and these samples would be collected.

Then from that a statistic would be calculated which is a summary of the data that is coming out of these samples. That summary is what we plot on a chart. That is what we plot on a chart and it leads to what we call statistical process control. The other methods of they have already mentioned to you. One of which is used after production generally, this is the acceptance sampling method. Acceptance sampling method happens to be one of the methods for this and like I said to you earlier. We will be using SPC, we will be studying SPC. An SPC is a method that is applied during production.

(Refer Slide Time: 09:10)



What is SPC?

- A simple, yet powerful, collection of tools for graphically analyzing process data
- Has one primary purpose: to tell you when you have a problem.
- Invented by Walter Shewhart at AT&T to minimize process tampering
- Important because unnecessary process changes increase instability and increase the error rate
- SPC will identify when a problem (or special cause variation) occurs

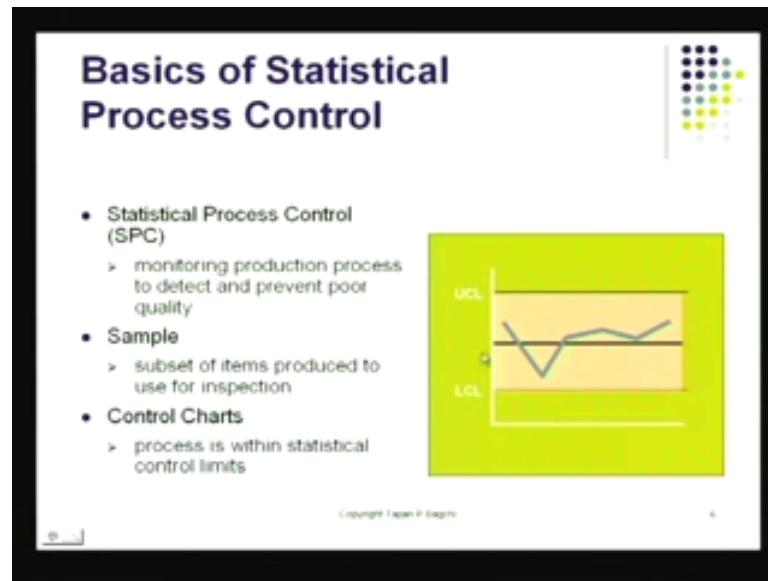
Copyright Tapan K. Bagchi

What exactly is this? It is a simple method; it is a very powerful method at the same time. Once you understand what is going on? It is not very difficult to plot charts and to get signals out of it. And the signals basically tell us whether to adjust the machine or to leave it alone because it is doing alright. There still will be some variations, but those are within the tolerance that the customer is willing to live with. Therefore, we will leave them alone.

But if there are wide variations if there are large variations. Then of course, it is a matter of concern because when there are wide variations in the process. It is very impossible that some of the items that get to the extreme value those may go out of specification. Those may not be fit for use and that is why we want to make sure we monitor the process as it is going on as it is conducting itself. And then look at signals look out for signal.

Once the signal is there that there is something exception that is happening in the process because some tool broke down or something happened or you different procedures was used or something like that or an untrained person came along. And he started running the machine and he producing produced parts that actually need to be basically production of such parts need to be minimized. That is when we would be going into plotting a chart, getting a signal out of it and taking exceptional actions.

(Refer Slide Time: 10:29)



What does a chart look like? I have a little plot here. Notice here there is a trace of the statistic. So, if this was for example and jumping the gun a little bit let us say I plotted \bar{X} and I will show in a minute what \bar{X} is? I plot \bar{X} and time is this way time is on X axis and \bar{X} I have plotted on the y axis. There will be some variation that is what we call the natural variation of the process. That much variation will stay there regardless of what you do unless you tighten up everything. And if you remember the fish pond diagram that we had in one of the earlier lectures. There is almost no process that is not affected by something that is around it.

So, in fact most processes end up with some variation. If these variations stay within some tolerance we have no problem. We probably live with it. For example, when you go to a bank and you put down your signature on a cheque. If the teller knows you probably and if you sign in front of her there is no problem. They should accept and she will give you the payment for that cheque.

But suppose she receives this cheque in the mail and it claims to have your signature. She will compare your signature at a as it appears on the cheque again something that is a standard. That is an electronic of your signature that is already in their data base and she will compare the two if the variation from the standard and your signature on your cheque, if this variation is small. She will say that the two are probably the same and I

need not really worry about anything. This cheque is a genuine in cheque and the signature is genuine.

But suppose, that turns out to be a wide difference between your signature as it exists in their records here and what appears on the cheque. If there is a wide variation between them she was doing sure raise an alarm and in the extreme case she may refuse to make the payment. When she raises an alarm, she could do further investigation. First make you a phone call or something like that and if she thinks it is been a forge signature that is on the cheque there. She will obviously not make the payment and someone up there in the bank upstairs, he would be notified about it and he would be contacted.

If this is not done of course, you know our life will be miserable because lot of exception they would get in as if they are routine and that is not going to be such a good news. We apply the same principle when we do statistical monitoring. So, if you go back here you will notice that the chart that I have. The figures that I have here, I have a chart here and the chart has two quantities. I have got the upper control limit UCL and I have got the lower control limit called LCL. These control limits basically show the limits of natural variation of the process. So, these are the different signature I have done on variety of cheques that I have written. If they are within this tolerance band there is no real problem either for the terror or for the bank to make a payment to me.

But suppose my signature lands up here somewhere or my signature lands up here somewhere. That actually is a wide variation since significant variation from about that normal is the normal signature. Here of course, I am not dealing with signatures I am dealing with product quality. And product quality has been measured and it is been converted into a statistic such as \bar{X} or r or p or u or c and these are the ones that I monitored by plotting them on the chart. And for in all for each of these quantities \bar{X} r p or u or c or any of them these are indications of quality. There are going to be control limits.

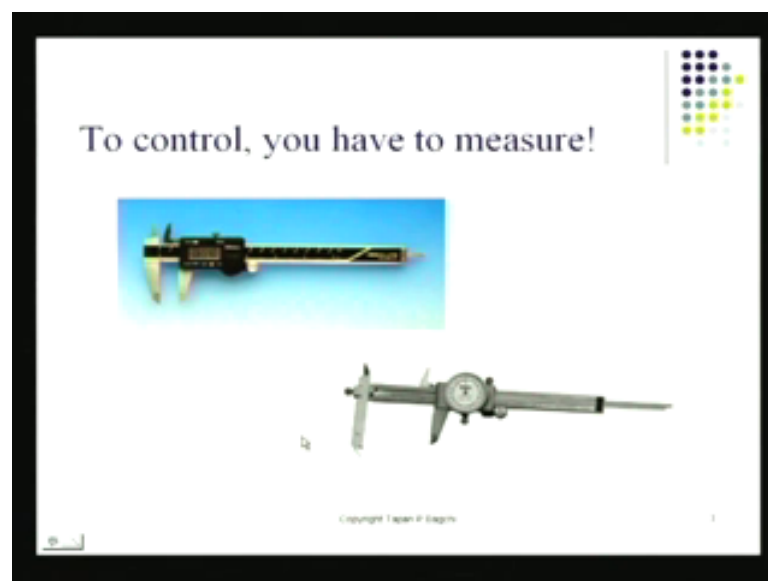
So, one of our jobs is going to be for the selected statistic we must also come up with these correct control limits. That is going to be an important. Now, what all things do I need? I need a couple of things; I need a sample or a production going. I have a full production going smoothly, but once in a while I would like to check the quality of it. Is the average quality being maintained the way the target was set? Is it being maintained or

is it the deviation from it is the process drifting upward or downward or is it going away and then coming back again. Such things are really not very desirable when you are assuring quality, when you are running a production system.

Such variations are sometimes they are not acceptable. So, only they are not going to be acceptable by a customer who sees a lot of inconsistency in the output. So, this is something we would like to be able to avoid. What do I do here? We plot a chart and we compare the data to these control limits. The data you was from this sample and a sample actually basically is a representation of production that is going on. The real production may produce thousands of these pens per day or perhaps even per hour. And perhaps every 15 minutes I walk up to the machine I collect a handful of those pens they become my sample.

These are the ones that I inspect 100 percent. I check out the quality of each of them and I compute this quantity called \bar{X} from this, from these sample data that I have and \bar{X} bar is what I bring down to my chart now and I plot my \bar{X} bar. And by plotting that \bar{X} bar I basically I reach a decision where I need whether I need to adjust the process or can I leave it alone. This device this device is called the control chart. And the goal of this chart is to try to help us keep the process within control limits or within a natural variation.

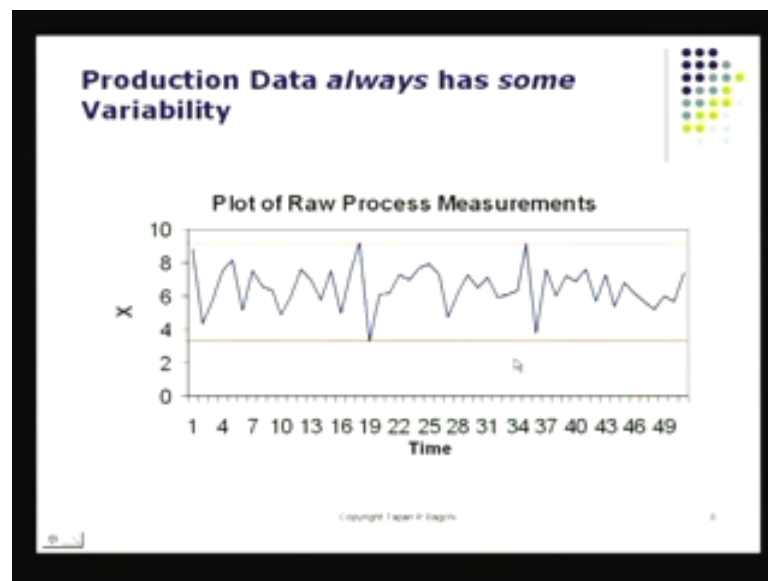
(Refer Slide Time: 16:09)



For doing any of these things you need to do some measurement and it goes without saying that your tools must be good, your instruments must be good. If I am going to be using these for plotting control charts you better make sure that the data collected are correct and of course, the calculations also correct. To make sure that the data collected are correct you have to make sure you have done your gage R and R. Gage reproducibility and repeatability study you have done.

So, you make sure that these instruments which I am going to be using here. These are the instruments that by the way they measure dimension. You got to make sure they have been calibrated and also we know exactly if they are when it drifts and it is something like that. So, gage R and R, gage R and R which is the variability that is introduced by using the gage as a measuring device as something to produce data. That is behaving alright that is that has its own variation, own contribution to variation to the ultimate data kept within a very tight tolerance.

(Refer Slide Time: 17:09)



If I plot raw data my data (()) is going to be looking like this. Look at the variation this is just the plot of raw data. So, if I produced a lot of these, I produced a lot of these pens and I made a measurement of the dimension for example, you know there are certain things that are critical in manufacturing a pen. And it could be the dimension the dimension has to be within some limits within some specification. Suppose, I just take

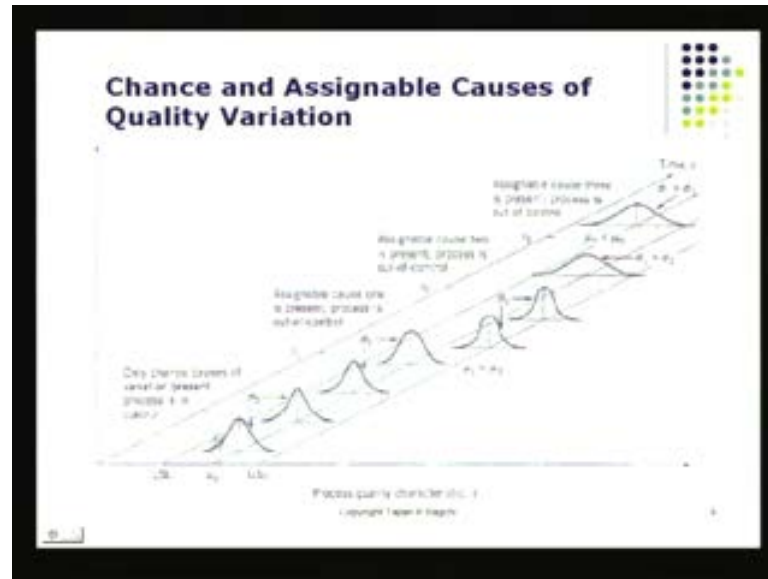
pen after pen and I write down the dimension length is one, then length is L 1, then L 2 then L 3 and so on and so forth.

And I basically take that set of raw data and I plot them. If I plot them I get a trace just like this trace here, I get some variation. Now, these variations are there because if you recall your fish pond diagram. There are many factors that can impact the production of these pens. Now, many of these factors are such that they are own individual contribution to this variation a small, but there are many such factors in place while production is running. I have got many such factors impacting the thing and therefore, there is going to be the combined compound effect. That is why you see a lot of variation when you see a chart like this.

But this is a raw chart. Is this raw chart good enough? The only thing I can do if I have got only one piece of data in my hand which is the raw measurement taken on one item. The only thing I can do is to try to check and see whether it needs specification that is all. There are other things that I would like to know about the process. I would like to know about the behaviour of the process and the two key ones are is the process running accurately. That means is the average production quality pretty close to my target. That is like the indication of accuracy. The other things I would also like the production to be I would like my process to be precise.

That means whatever these variations are they should be within a tight tolerance. Both of those things are important certainly for the user. The final user is going to be using these outputs that I produced out of a process. Those got to be accurate and also it got to be precise. So, the output has to be output has to be precise and also it got to be accurate. I cannot do this I cannot really control these two aspects of quality which is accuracy and precision just by using the raw chart, I cannot do that. It is not possible, I have to convert. I have to convert this raw data into something else and I will show exactly what I do?

(Refer Slide Time: 19:54)



In fact this is the heart of statistical process control. Now, look at a process I have got a process here on the screen and the process started here. Notice here process mean was μ_0 and since specification limits were put down here lower spect limit and upper spect limit. And the natural variation the process kept all the data within this thing and this is what we call the process. When we say that the process is under the influence of chance causes only. Chance causes are exactly those things that let your signature vary a little bit from cheque to cheque to cheque.

Those small variations they are due to chance causes. You will have to hold the pen exactly the same way every time you do your signature. Your speed may change, your strokes may change a little bit, but there all will be within this small tolerance. This small tolerance basically reflects the natural variability of my signature and the same thing here for quality. These bell curves here look at the bell curve on the left the starting one. It shows the natural variation this range is the natural variation of the process, but as time goes along once in a while I find that the process whole process has shifted out.

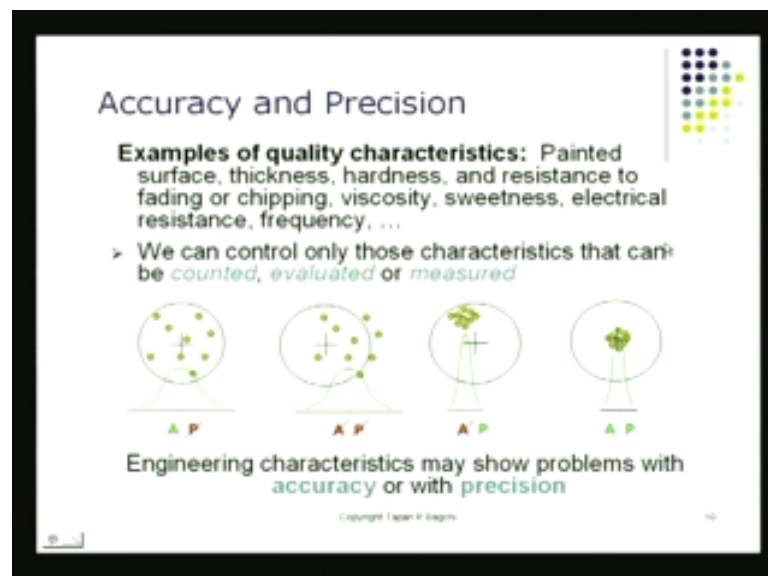
It ran alright up to this point then I find that my process shifted to the right. That is the new mean μ_1 is greater than the original mean which is μ_0 is shifted out to the right. Now, this may cause this actually causes this impacts the accuracy of the process because the mean has changed. So, it will impact the accuracy of the process. It is also possible that your accuracy is fine like for example, look at this one, this third curve here

accuracy is fine because the average is still at μ_0 , but the problem is the process has become wider. It is got it is taken over a wider **wider** distribution.

This is when precision has got affected it is become wider and that would be there is a chance of course, that throughout the tail end of these this output. This population output I may have some parts which are not going to be fit for some sequent use. So, we have two problems here as you noticed. One is the issue of accuracy that means is the process I have been staying on target and the second thing that we have is the process precise it is holding on to consistency. That means whatever sigma was there in the process to begin with.

When I started out I set up the process and I turned on system on and started to produce my widgets. Is it staying with the same precision or is precision getting worst? Generally speaking we get concerned when precision becomes worst. You see that when you see this path here. There is a accuracy changing once in a while and also there is precision changing once in a while. Both of these could be problems, both of these situations which is a shift in accuracy could be a problem and also a shift in precision could be a problem. So, we got to do something about them.

(Refer Slide Time: 22:52)



Let us try to again recall this diagram which some of you might have seen earlier. I have four people who are trying to shoot their gun at the targets which are here. Now, there is one person is totally untrained, he is been given a gun and he is been asked to fire and he

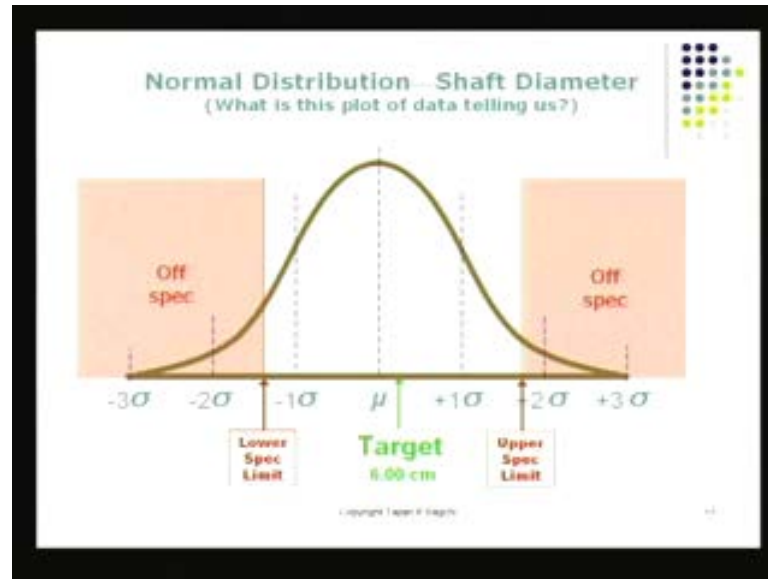
fires a few shots. The distribution of those shots they are shown here. The target is right there bulls eye is right there and these are these indicate the positions of where the bullets went by. So, you have this variation here. There are two problems with this sort of performance. One is look at the average of the distribution of these little holes there, bullet holes. The average itself is of the what we call the bulls eye.

So, there is a problem with accuracy. Also notice this spread is rather wide so, I will say precision is also poor. There is poor accuracy and also there is poor performance in terms of precision as far as this shooter is concerned. Look at this shooter now this shooter has his average right on the bulls eye. Therefore, accuracy is not a problem for him and I show that in green letter A, which indicates accuracy it is green. That means we have accuracy in control there, but precision is poor again. Look at the wide spread, look at the third shooter here who has got his distribution of different shots pretty tight. So, precision is there, but unfortunately the average is off target. So, it lacks accuracy.

And look at this fourth guy he shot all his shots very close to each other and the average is right on target. This is how we would like to be, we would like to be not here, not here, not here, but right there. This is what our process should be like. If you have a process going like this, the parts that come out of a machine that is doing the performance poor type of performance. These parts are all going to be fitting well. So, no matter what cap you pick and what body you would you pick? If these are both accurate and precise they are all going to fit. This is something that we would really like to have any time. You got mass production we would like to have accuracy and also precision in the manufacture of all parts.

Let us see now, what are tasks then becomes. We have a running process and the running process as I show you showed you. The raw process may have all kinds of variations, but what we should be concerned about is? Is the process accurate? Is it producing on the average it is it producing output that is staying right on target? Number 1. Number 2 is the process also performing in a precise manner. That means are the different pieces output at the close to each other. That is also something that we would like to be able to do.

(Refer Slide Time: 25:46)



If I do not control these things and if I just look at the output of a process which is not being controlled either for accuracy or for precision. We will end up with performance that looks like this. Now, before I show exactly what is going on here? Let me show you the quality characteristic is plotted on the X axis. The quality characteristic is plotted on the X axis. Now, there is a limit here which is called the upper spec limit for the parts. This the upper spec limit which actually says if I produce parts which have dimensions, which are beyond this upper spec limit. Those parts are going to be useless for application.

So, in fact if I was producing these caps and these caps will produce with two wide holes again those would be useless to be used in the assembly of pens. In fact it might turn out that those caps those would end up being in the off spec area. Either to the left which is like the type the caps are too tight or they might be too loose which is to the side. Both of these are harmful. One reason is this it costs me the same whether I produce a part that is within specification or outside specification. The machine process itself does not change the cost of production from item to item to item.

It does more or less the same thing as the as the different pieces are floating through the thing, but if I end up with a lot of products which turned out in the lot of parts, which turned out in the red area either off spec because it is too tight or it is too loose. Therefore, it is again off spec. It tends out both of these are large share of our production

we have lost that much money because we cannot sell these parts, we cannot use those parts. The only parts that we can use are those that are within specification. So, what would you would like to do is?

First of all we would like to have the output average exactly on target not neither to the left nor to the right. So, I show the target here with a green arrow there. The process average should ideally be exactly right there. That is like something we would like to be able to do. Now, that is the midpoint of the specification range. The second thing that we would also like to do is we would not like to see as much variation as its shown on this graph. Rather we should have a sigma that is fairly tight and stays pretty close to the that midpoint there.

So, that way we would getting accurate production and also you will be getting precise production. The role of control charts is to try to help us remove all those factors that might either impact accuracy or it might impact the precision of the process. To make sure what we produce can be sold without any problem at all or those could be used in your consequent your subsequent processes.

(Refer Slide Time: 28:40)

Variability

- Random
 - > common causes
 - > inherent in a process
 - > can be eliminated only through improvements in the system
- Non-Random
 - > special causes
 - > due to identifiable factors
 - > can be modified through operator or management action

Copyright Tapan K. Sengupta 12

Where do these variations come from? They come from a variety of different causes. Causes which are large in number, but the individual will produce very little impact. It is like the manner in which you hold your pen for example, when I doing a signature. Every time I pick up the pen and I hold it there may be some slight variation in my

holding the thing. That might affect the quality of the signature; it might affect the quality of the signature. Those are called chance causes and those are also called random causes.

These are these causes are inherent in the process that is part of my biology. The way I hold the pen is part of my biology and these may be eliminated only through improvement of the system. Say if I take some training for example, children when they are small when they are just turning trying to write. They are learning to write. They are told how to hold the pen? You know have to put it like this, you have to rest the thumb against it. You have got to rest the this index finger there and the mid finger should be somewhere else. They got to hold it like this; the angle should be like that and so on. If you do not teach it many times children they tend to hold a pen like this or they may hold it like this and they may hold it in various different ways.

If you hold it in any manner that is not the standard writing manner. There be more variation in writing and your handwriting is not going to be in your control. There will be lot of variation. Those are you know holding the pen like this or like in some manner that is not the standard method. For example, those are what we call non random causes of a variation in your signature. The same thing we have in production also when we produce these widgets these items or these parts. There are these special factors and I will give you many examples of these as we go along. These may cause large variations in the system, large variations in the output and these factors these outputs are generally the these factors can generally be identified.

For example, if we see a crazy signature. You might actually ask the person to do a signature again and if you find for some reason may be he is got a cut in his finger and he is got a band aid on it and it is paining. That finger is paining he is not going to be able to rest it the way he would normally do. He will probably make some exception he might buckle his finger or something like that and he may just put his signature like that. That is an exception and this would impact the output. This would impact the quality of the signature itself and this is something naturally if it is going to be used if it is a signature for a cheque, this is an exceptional situation.

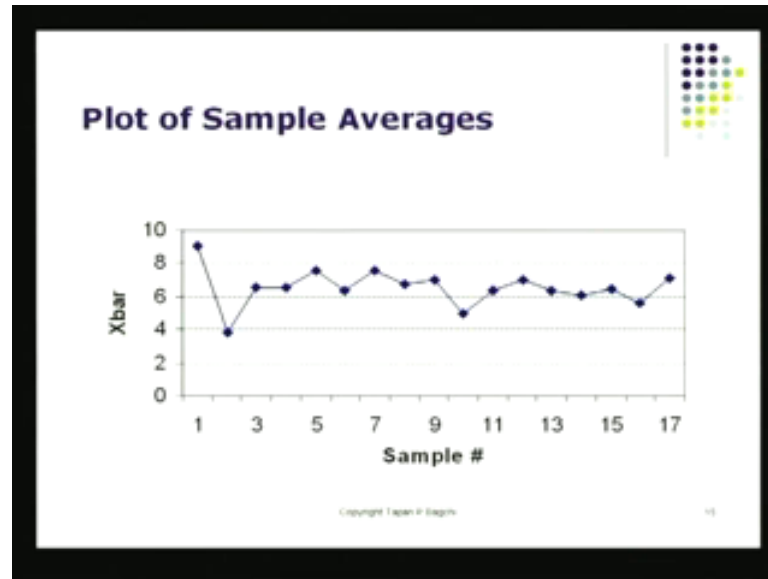
And people have to be notified about it and in general if this happens in production you do not want to continue doing production when you got a situation like this. You have

got one of these one or two of these exceptional factors impacting production. That you should not be able to should not be forced to do. And these special causes the variations are the ones that we detect by using the control chart which we will be seeing in a few a minutes. So, again to summarize there are these common causes. Those cause small vibrations, small variations and they will many number, they are large in number, but they do not really, really have a big impact on quality, but then there are the assignable causes.

These are the special factors such as I have a cut on my finger and I am trying to write the with the pen I am not able to get a good signature. That is going to be the exceptional case and in production also if the tool has broken or if the cooling fluid has stopped for some reason or there is some other problem with the machine itself. It may not be something that I would be like to carry on with and this is something would like to able to avoid and again the control chart is the way to do it. How do we actually use control charts? We take periodic samples; we plot the sample points on the control chart. We determine if the plotted points are within what we call control limits.

These are different from the specification limit. Specification limits are used to evaluate each single item where as control limits are used to look at the statistic, the summary of the data, which could be the some total averaged out of measurements that I have made on a number of different parts. For example, in this case it could be the X bar and it could be the range, the R which is what I calculate using the raw data there and that is the one that I plot. Those are the ones that I plot on a chart and I try to see if the plotted X bar value or the R value is within what we call control limits.

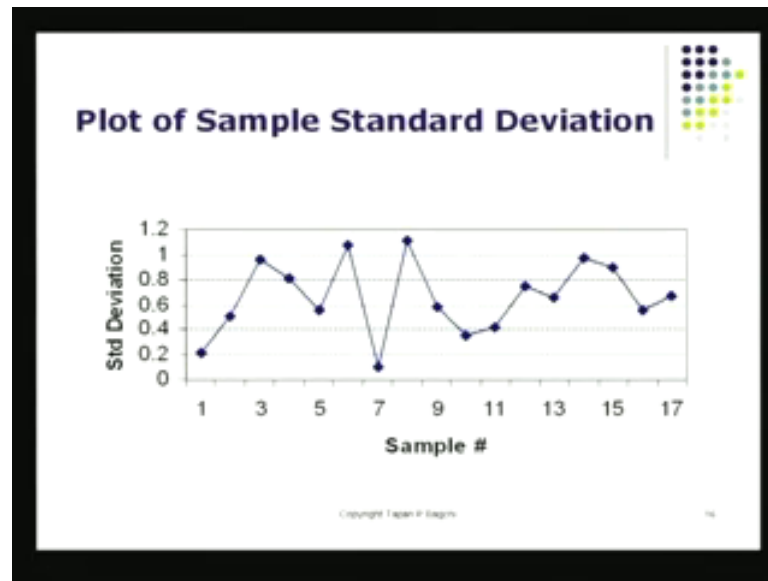
(Refer Slide Time: 33:35)



If we do that we would be preventing a lot of quality problems. So, again let us take a look at it. We saw the plot of the raw data, the same data was converted into \bar{X} and I am going to be showing you how to do that? You have a collection of raw data which are like individual measurements on these pens, dimension of these pens and you do a simple calculation. You calculate the quantity \bar{X} from the X values. So, you have got X_1, X_2, X_3, X_4, X_5 ; these are 5 individual data points. You will calculate \bar{X} and as the name implies \bar{X} is the average of those X_i values. This is like one statistic.

I calculate a similar statistic called the range which looks at the highest value in the data collected and the smallest value in the data collected within that sample. The difference between the highest value and the lowest value is what we call the range of the data the variation. This range actually is the indication of the consistency of the process. If the process is very consistent the range is going to shrink down to pretty close to 0, but if the process has a lot of variation. Range is going to expand. In some sense, this range is similar to what we call standard deviation and we will see that as we go into this theory here.

(Refer Slide Time: 34:50)



This is now a plot of the standard deviation the same data that we saw earlier. The same raw data first we constructed the \bar{X} statistic. Then we also calculated the standard deviation and statistic. Let us just recall for a minute what these formulas are? And I will be using this little piece of paper here to be able to do that.

(Refer Slide Time: 35:14)

$$\bar{X} = \bar{X}_{bar} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{\sum x_i}{n}$$
$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1}}$$
$$R = \text{Range} = \text{Max } x_i - \text{Min } x_i$$

Let us say I collected some raw data and I looked at 5 pieces or parts that were produced. These are the five pieces of individual data points that I collected and these are what they are measurements, they are measurements done on these individual part. So, I had one

measurement done for the first one, second one, third one, fourth one and fifth one. I have got 5 pieces of raw data what is \bar{X} ? Let us try to write the quantity for \bar{X} . \bar{X} is a very simple formula, \bar{X} which is also written sometimes as \bar{X} is basically the average of this. So, this goes as X_1 plus X_2 plus X_3 plus X_4 plus X_5 divided by sample size which is 5. This can also be written as $\sum X_i$ divided by n when n is your group size sub group size.

This is one statistic which I can plot and I assured you that plot of \bar{X} . That will actually let me just show you the \bar{X} chart. This is the \bar{X} chart and notice here on the y axis I have got something calling itself \bar{X} and on the horizontal side I have got sample number. So, sample number could be sample 1 then sample 2, sample 3 and so on. For each sample I am calculating an \bar{X} value. Then the other quantity that I can also calculate is the standard deviation and remember the formula for that I will just write it here as S equal to and for this I need to do a little bit of calculation not too bad.

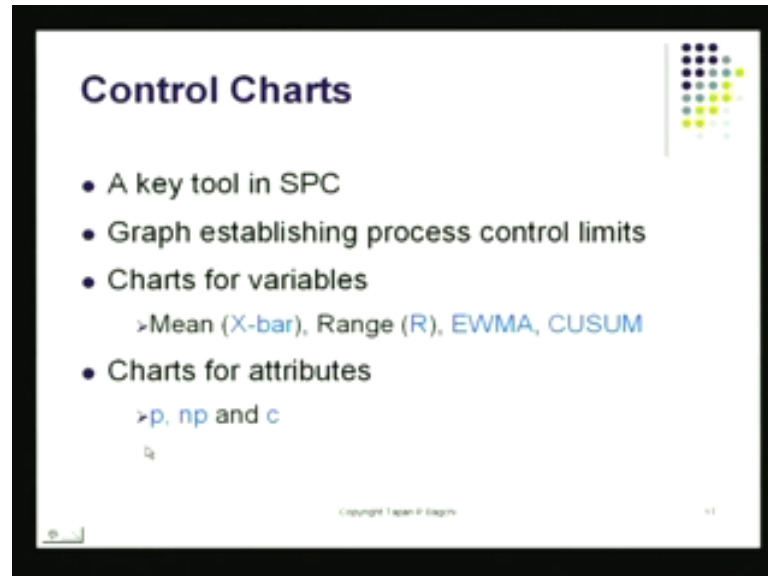
I will do a square root first then I will do $\sum (X_i - \bar{X})^2$ and this is to be summed over the way the full sample, i equal to 1 to n . That is all fine, divide this by $n - 1$ and take this square root of the whole thing. This is my standard deviation. This is my sample standard deviation. \bar{X} would be utilized to get some sense of accuracy of the process. It basically indicates the central tendency of the process. Then we got something called standard deviation which indicates basically the spread of the process.

There is another quantity which is not as complicated to calculate and that is called the range R . R is the same as range and what is this? This is equal to $\max X_i - \min X_i$. The largest value in the collection of the data there and the smallest value in the collection of the data there. So, the difference between the largest quantity and the smallest quantity this difference here is R is the range and that is something that we would like to get an idea of. Because range indicates again like standard deviation it indicates precision of the process and we will be looking at some real data.

We will actually see how these calculations are done and to be able to do those calculations of course, I have brought my you know the real pressures computer my little calculator. I will be using the calculator every now and then to calculate the data points various data points whereas, the \bar{X} quantity and also for the range quantity that we will be doing. Most of the time I will not have to use the standard deviation formula I,

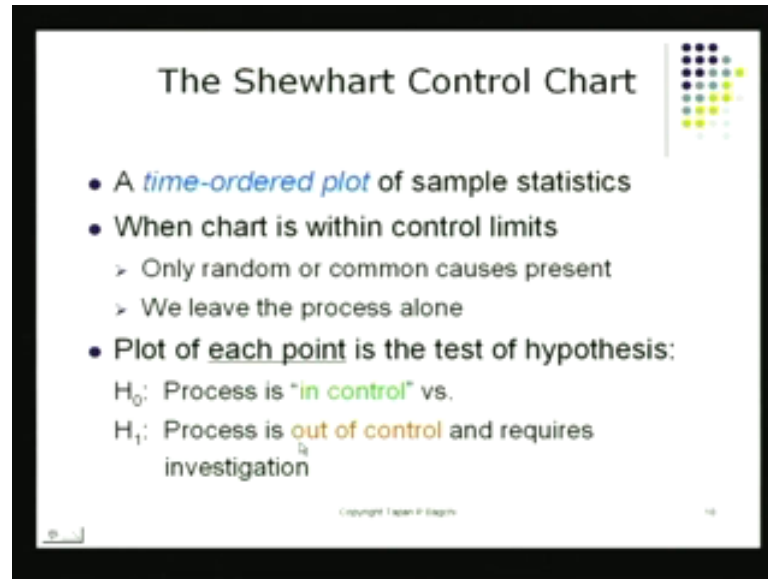
one could do this, but slot is here; if I getting the same use as this or this, when I am trying to do process control. Then why do not you use the simpler one which is the range chart.

(Refer Slide Time: 39:13)



So, I have here if you see the screen there I have the standard deviation chart and the very next chart is going to be the range chart. That also is shown control charts are there for a key tool in SPC and I can with this I can monitor the process graphically. I can do that by plotting the X-bar chart or the range chart. Then there are some other charts also for example, the weighted moving average chart that also can be used or the CUSUM chart this also can be used. These are other charts that have some special benefits but they are more complicated. We will focussing today on the we will focussing on the X-bar chart and the range chart. Then of course, there are other charts called the p chart and the np chart and the c chart which we shall see as we go along.

(Refer Slide Time: 39:54)



The Shewhart Control Chart

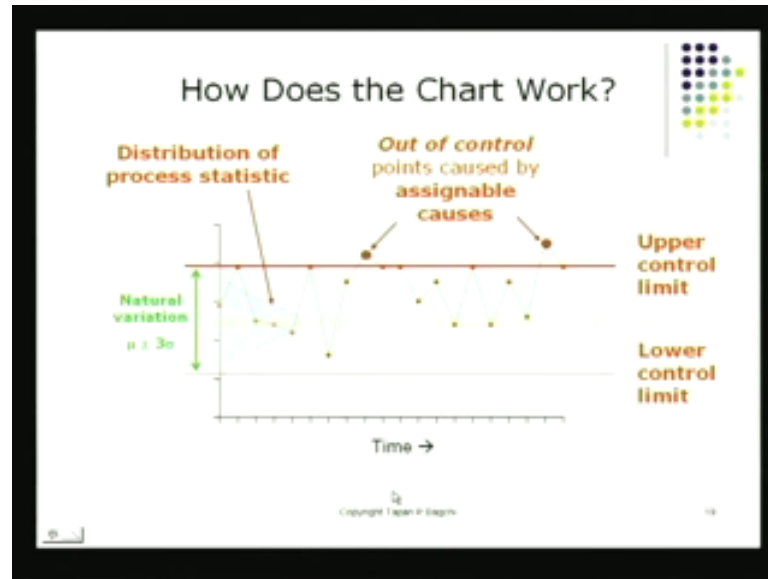
- A *time-ordered plot* of sample statistics
- When chart is within control limits
 - Only random or common causes present
 - We leave the process alone
- Plot of each point is the test of hypothesis:
 - H_0 : Process is "in control" vs.
 - H_1 : Process is out of control and requires investigation

Copyright Japan P. Page 18

So, what is a control chart? It is a time-ordered plot. That is what this is and it provides you with a signal. When there is no special cause impacting the process nothing has gone wrong, nothing perform has happened to their process and the process remains within what we call your in control is a in control state. And of course, if some wild things happen in the process like a tool breaks down or the raw material is changed or a operator comes along and he starts operating the machine.

Those are lightly to cause some exceptional variations in the process and the process the control chart may go then out of control. So, I have got these two situations. I have got something called the process is in control which is the base case and the exceptional cases the process is out of control. These two I figure out by looking at the actual chart itself.

(Refer Slide Time: 40:46)



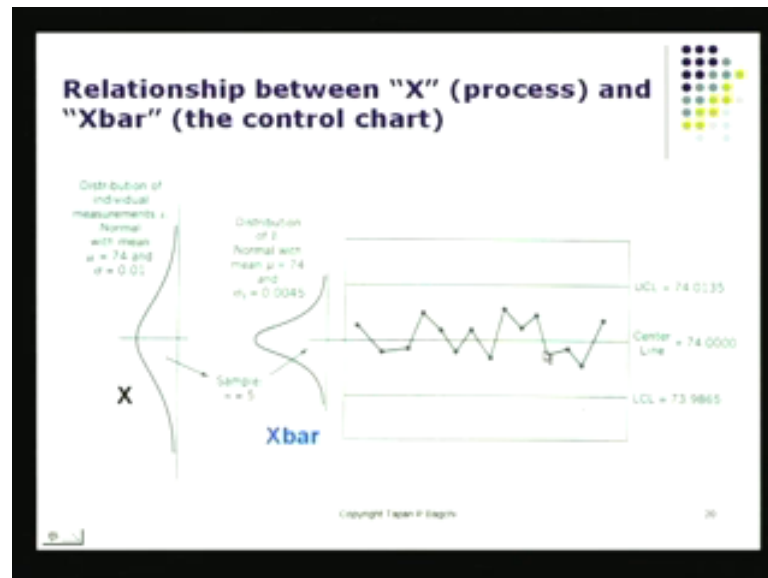
So, here is an example I have plotted the chart. I have plotted the charts of plotted here for example, I might have plotted the X bar value. When I plotted the X bar value and I have the plotting done for various samples that I have come along. I notice the upper lower control limits. The control limits have been marked and I will show you the formulas for these. There are some special formulas that you utilize to control the to calculate the upper limit and lower control limit. Once these are known I can plot the chart, I can plot the data. These are going to X bar values and I can check to see whether any point plotted has gone beyond the control limit either beyond the upper control limit or below the lower control limit.

If that happens that is an indication. That perhaps this variation has been caused by something exceptional. Perhaps the turner has to the look at the person and figure out, Oh he is got a cut finger that is why probably his signature is bad. If not she might just say I have a suspicion that somebody has tried to forge your signature. Would you please verify this? Because suppose she treats this as normal variation. It is possible that some fraud had come along and he forged the signature. It is very possible. You want to prevent such things; you want to do the same thing in a manufacturing situation.

When there is production going on to produce hundreds and thousands of these parts. You do not want these parts to be produced such that they actually violate the specification limits or they are unfit for the users use. These are not going to be good

news for any one. So, you plot the chart and you try to see are there any points that have gone beyond the control limit. If they have we have to stop the process, do some investigation.

(Refer Slide Time: 42:37)



Now, notice here I did not plot X , when I plotted the control chart. I plotted something called \bar{X} and the distributions of \bar{X} turns out to be narrower. Why is the distribution of the \bar{X} narrower? Let me just show you just by showing you what happens when I take I go back to the formula that I wrote here. Notice, the \bar{X} the formula for \bar{X} is I have $\sigma \sum X_i$ divided by n .

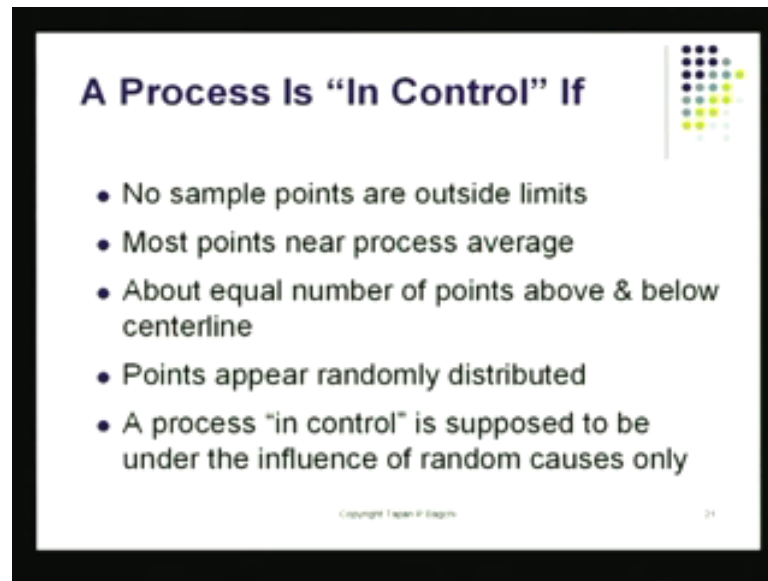
(Refer Slide Time: 43:15)

$$x_1, x_2, x_3, x_4, x_5$$
$$\bar{X} = \bar{X} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{\sum x_i}{n}$$
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$
$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$
$$R = \text{Range} = \text{Max } x_i - \text{Min } x_i$$

If I try to write down these quantities sigma X bar. Sigma X bar is actually the standard deviation of this quantity X bar. Now, these X are random variable therefore, this is also a random variable and this will also have it is own mean, it is own standard deviation. It turns out the standard deviation of X bar, it turns out to be this standard deviation of X divided by square root n when n is sample size. So, the larger is n the narrower is going to be these the title is going to be my sigma X bar and that is why you notice here when I go back to my screen there.

The curve on the left, the bell curve on the left, this wide one has a larger sigma and this guy has a smaller sigma because this sigma is going to be sigma X divided by square root n. That is what this thing is. That is why control limits are always narrower. Control limits actually have nothing to do with specification limits. Control limits are calculated based on the data. The data that you have and some constants are there, some multipliers are there, but basically control limits are calculated using the data that you collected when the process is in control when there are no wide variations there. That we will see again when we get into calculation of control charts, the control limits.

(Refer Slide Time: 44:38)



A Process Is "In Control" If

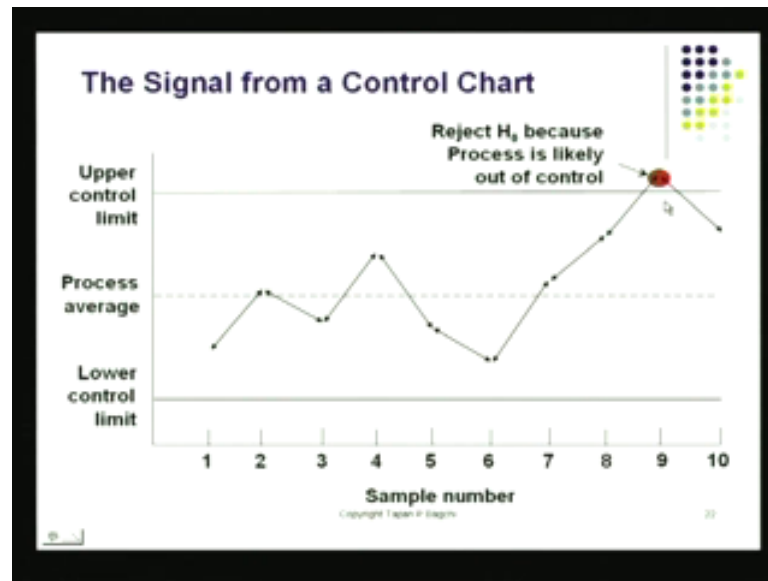
- No sample points are outside limits
- Most points near process average
- About equal number of points above & below centerline
- Points appear randomly distributed
- A process "in control" is supposed to be under the influence of random causes only

Copyright 2009 Pearson Education, Inc. 24

So, the process basically is in control if there are no sample points that are outside the control limit. Most points are near the process average. This is also a good indication that the process is in control. That means its somewhere the influence of random causes are chance causes only and none of these exceptional assignable causes. The ones that really create trouble. About equal number of points should be above or below the chart. If that is there above below the central line of the chart that actually says of the process is stable and it is staying pretty close to where it should be.

Points appear to be randomly distributed there is no like biasing toward the upper side of the chart there or the lower side of the chart there. That sort of pattern is not there. In fact it does appear to be completely random. Then of course, the process is likely to be in control and of course, we say that the a process is in control, when it is under the influence of random causes only.

(Refer Slide Time: 45:39)

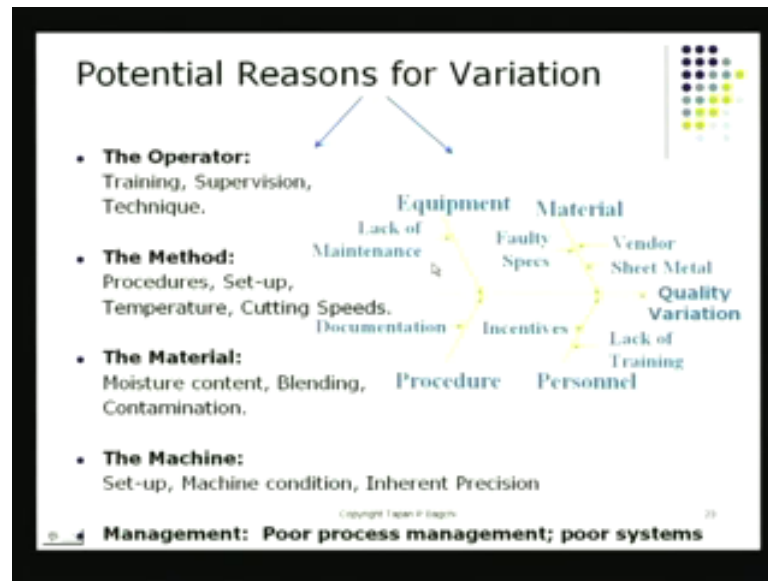


Look at one chart NOVAN chart that is shown here and it is very possible that this chart till this point the chart was in good process control. And something exceptional happened that bomb this thing. That bomb the chart, bomb the \bar{X} value and it went beyond the control limit. It is very, very possible when you investigate this chart. When you go back and investigate this process you will find a broken tool or a stopped hydraulic line or something or may be a new person came along. He started fiddling with the machine or raw material changed or voltages fluctuate, fluctuated.

Therefore, the cutting speed changed or something like that. When those things happen of course, you see charts going beyond the control limits and you end up with control, you end up with exceptional situation like this. At this stage you are suppose to investigate the process and you are suppose to remove that cause of variation, the cause of extra variation and you are supposed to then resume your normal production. What are the various things that can lead to process variation? There are number of them and these have been broadly classified through history as factors that can be trace down to operators.

For example, their training, the quality of their training, the kind of supervision they are receiving, the technique that they have been taught that they have learnt. If there are variations there this would end up showing up on the control chart because operators are a key part in that total production system.

(Refer Slide Time: 47:11)



Then of course, the method itself, the method itself what are the procedures are there following the standard operating procedures? What is the set up like? Does it change from set up to set up? What about temperature cutting speeds and so on? These are all parts and partial of the method itself. These may also lead to like variations in the process. Then of course, raw material I do not have to tell you that when that thing changes the again the quality will change and the machine that also can be rare and the management process.

Poor process management that can also lead to a lot of variation and here is like another way to present the same thing. I have got the same thing if you recognize the tool it is the fish pond or the cause and effect diagram. And quality variations would be there when there is lack of maintenance, faulty specifications, the vendor has some problems there or the sheet metal that is being used. It is got some problem there personnel also might be causing variation because the incentives are not good enough lack of training is there and the procedure is there, but there is no documentation. So, each person he uses the procedure in accordance to his own understanding. These are the variety of different factors that can lead to the chart fluctuating.

If you had a lot of money you would remove all of these factors, but generally speaking we do not have that. So, what we have to do is the slight if you just wanted to get a taxi and you wanted the president to ride in that taxi. Of course, the president were ride in a

limo. The driver that you are going to pick for that limo that driver is got to be an exceptional driver because he should not have any kind of problems in regard to his performance as a driver. But if it is like an ordinary person you know who does his probably own driving or probably you know his demands are minimal.

Then your moderate training perhaps he drives only a tractor. He could also drive a car and he is his performance is going to be highly variable. So, with training we can improve with a lot of other interventions you can improve a process; and slowly and slowly as you spend money the variation in the output. It is going to get narrowed down and we will see that as we go into some of these examples. We will see how it started with a wide variation? Then it became somewhat tighter then it became somewhat tighter and then it converged eventually to exactly what the customer wanted the output to be?

But it would cost you money and time to come down from this end to this end to this end to end. It would cost you time and money. If we do it the right way using for example, design of experiments this march can be pretty quick and if we just count on trial and error it will you will probably get there it would take a long time. If I were good trainer if I am someone who is knowledgeable about the process he can probably let you reach that end point pretty, pretty quickly.

(Refer Slide Time: 50:03)

Charts may signal incorrectly!

Charts repeatedly apply hypothesis testing!

Type I error with charts:
Concluding that a process is not in control when it actually is

Type II error with charts:
Concluding that a process is in control when it is not

Copyright 2009 by Pearson Education, Inc.

Charts may also of course, signal incorrectly. It is very possible because these are all random variables. So, it is very possible when you see a point, which is beyond the

control limits. There really were no assignable causes that lifted it up there. It could be a few random causes that combine together then they produce that big big jump there. So, it is very possible that charts once in a while may give you a wrong signal. We should be ready for it, we should be able to tolerate. If you are expecting to produce output that is going to be satisfactory for the user we might be a little conservative there is no harm there. The result is going to be always better; it is always going to be good.

Let us take our time, let us make sure we understand the process. Let us make sure we install the control chart system there and we will do some design work there. That it will like will calculate the control limits and so on. We should not make any mistakes there if you make a mistake there of course, the chart will give wrong signal. We should not do that, but once you have things in place then there is going to be no real problem at all monitoring this process day in and day out. You will get the same performance same output.

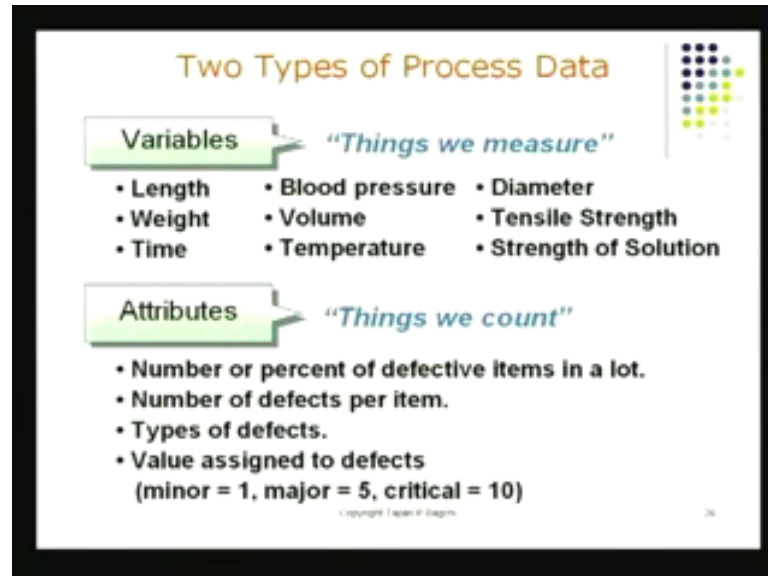
Do stay open to some of these errors with the control chart that means the chart shows that it has gone out of control and you end up doing some unnecessary trouble shooting as a result of that. Please tolerate that because you are trying to be conservative. Even if there is this slightest chance of an assignable cause that might have come there. You do not want to miss that chance catching that chance sort of. So, you better make sure that you got lots and lots of extra time to do your investigation. You might be doing a bit of which for hunting once in a while. That means the process is fine, but the chart gave a wrong signal because of the random combination of things. It is also possible of course, that the process itself has become worst.

It has become (()), but even if it is bad it turns out that the chart is within your control limits. That is also wrong signal in fact you are not getting the signal when you should be getting a signal. So, let us try to see what we could do? We have attributes, attributes as you understand they are quality characteristics that I can categorise between good and bad. And I can draw control charts based on these and those who be the p chart and the n p chart and the c chart and the u chart for attribute characteristics.

Then I could have something that we call variables characteristics such as dimension. Dimensions for example, those are measured and then you convert those measurements

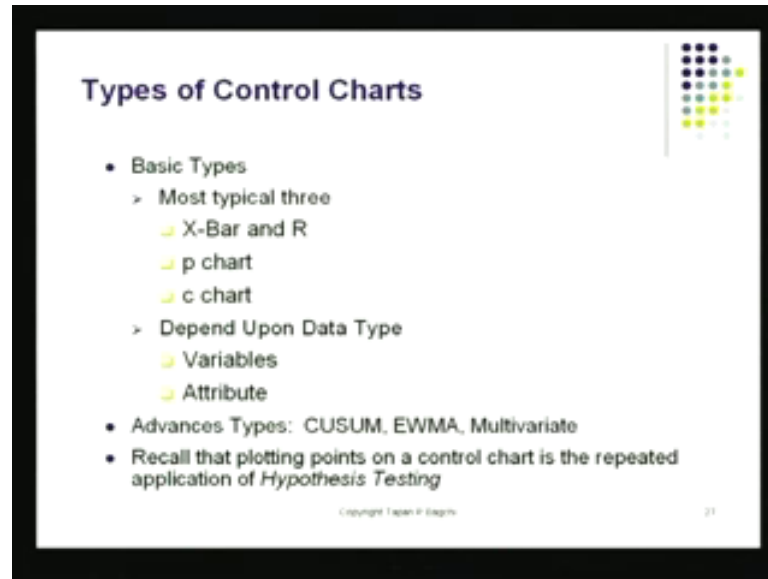
into either the X bar or the R chart in fact, you do both and then you plot the X bar chart or the R chart on a control chart paper.

(Refer Slide Time: 52:55)



And you try to see are there signals there, Variables; examples of various characteristics and lot of examples given here length, width, time, volume, temperature and so on, these are all quantities that I can measure. Attributes on the other hand are good and bad that is like is the pen able to write or is it not able to write. It just sorted between good and bad and so on. So, that is an attribute that can only be monitored using what we call the p chart or the n p chart or the c chart or the u chart.

(Refer Slide Time: 53:25)



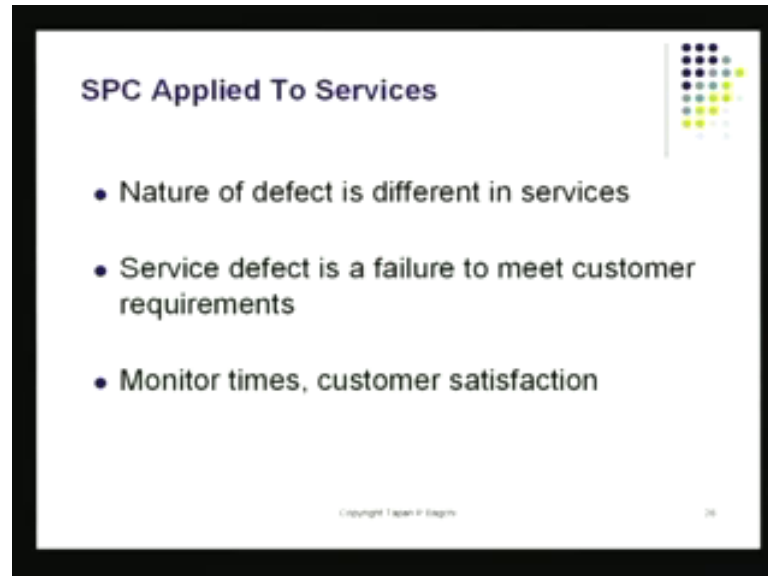
So, these are the ones that lead to these various types of chart that we see. We see the X bar chart and the R chart these are the most popular chart. Then you got the p chart which is also pretty close to the X-Bar and R chart in terms of popularity. Then you got the c chart and we will see that we got we make this choice based on whether the characteristic is variable or it is an attribute. And of course, in some exceptional cases you might like to use the CUSUM chart, which has got some extra power or the weighted moving average chart. That also explanation weighted moving average chart, those also you we may use once in a while.

And of course, I was only talking about dimension which is only one characteristic. It is also very possible I may multiple characteristics that I start plotting on a chart. Not on a single chart, but there are special charts which are like multi attribute charts and these are multivariate charts. They also give signal just like this and they also have something called control limits and so on and so forth. We will see them we will see a couple of examples of these as we go along.

What does a chart really do? It let you it test this hypothesis. What your hypothesis? The first hypothesis is the process is in control. The test of the hypothesis plot that point. The moment you plotted that point and it has turned out to be it has gone beyond the control limit. You say to yourself I cannot hold on to the hypothesis that the process is in control

the process. I must abandon this and I must investigate the process to say that process is out of control. Can I find the root cause? Can I find the reason why it is out of control?

(Refer Slide Time: 55:00)



SPC Applied To Services

- Nature of defect is different in services
- Service defect is a failure to meet customer requirements
- Monitor times, customer satisfaction

Copyright Tapan K. Bagchi 26

And the same thing can be done for services also. We talked about manufacturing; we talked about pen production and so on and so forth. We can do the same thing with services also and I will continue with this as we go into this session. Till the next session we will be looking at some of the service examples. Thank you.