Six Sigma
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Lecture No. # 19
Design of Sampling Plans

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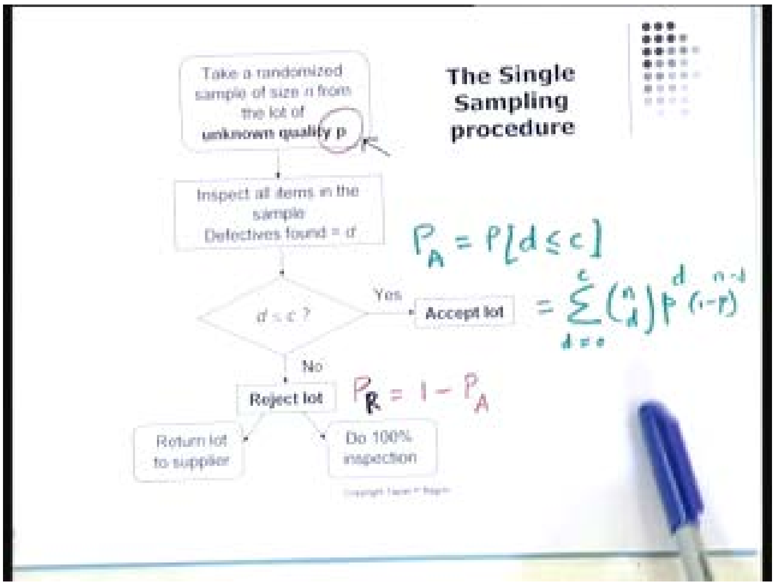

Good afternoon again. We ended the last session with the short view of the signal sample plan. In fact that plan is still showing on the screen there, I take a lot of size big n as sample n. I can sort of it, which is of what quality p, which is unknown quantity. And I inspect all the items there and the number of defects turns out to be d. And then I compare that number of items found defective that little d to be. I compare that to this little control number called c. If d is less than or equal to c, I accept the lot, otherwise I reject the lot. Now in doing that, if you remember I had it on my on my table here. And I utilized what we called a little formula which came from the binomial distribution. Now, I am going to actually give you this formula there, so that you will have a pretty descent idea, how to utilize binomial distribution?

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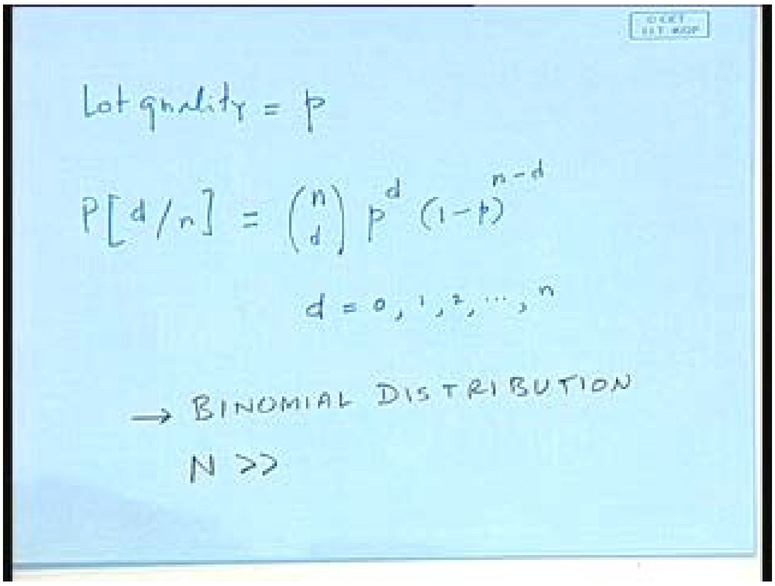

Now, if lot quantity, lot quality, if this is taken to be p that is p is the fraction defective in the lot. Then suppose I picked n items the probability that I will pick, I will find d defectives in n items that I pick out of this. When overall lot quality is p, this turns out to be a binomial distribution. And that goes like this probability is equal to n choose d p to the power d 1 minus p to the power n minus d.

And this formulas tells you that you know this is the exact formula that tells you. If n items were chosen if their inspector and d defectives were found, where p is the overall defect level of the lot Then this is the expression that will give you the quantity p quantity p or P a or the probability of finding d defectives in n items. And here of course, let us write the range of d, d can be as small as 0. It can be 1 2 and so on and it can be at most be equal to n. This is the range of this, and this particular distribution is called the binomial distribution. (No audio from 2:50 to3:03)

Couple of assumptions here, one is that n is very large, that is like one. It is a very big assumption there, and sometimes lots are pretty big, in fact it turns out, if your sample size and lot size if the ratio of if this little n is one tenth of this. You can apply this formula there is no problem. There lots of sampling plans have been designed based on this, and I am going to be showing a you use of this particular formula in designing a single sample plan. I will do that.

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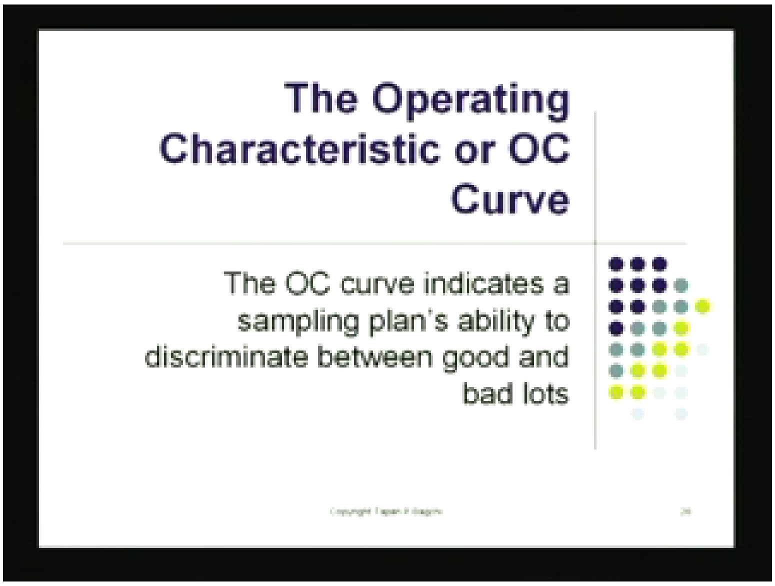

Let us first take a look at something called the OC curve. The operating characteristic curve of the plan. Now notice in the recall that, when I drew the single sampling plan and I had this item there, I had this particular item there.

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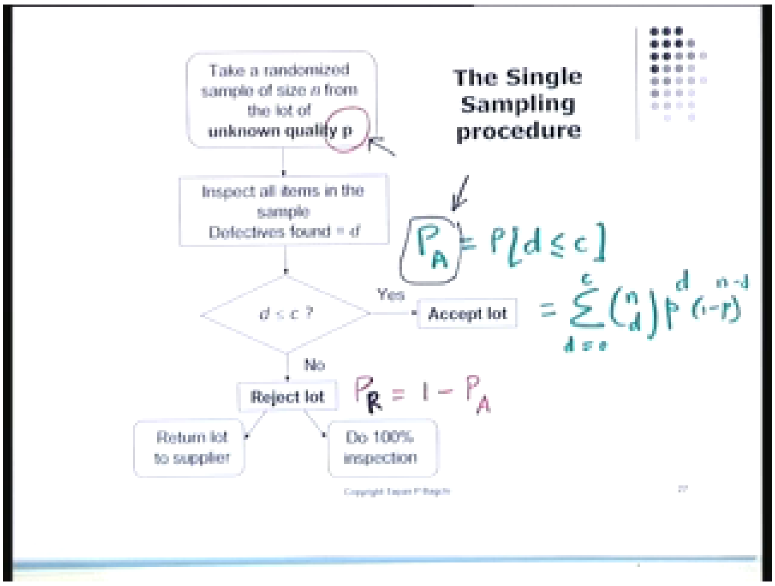

The quantity that I was most interested was this probability. The probability of accepting the lot, that is what I had a lot of interest in this, because this is the probability of accepting the lot which is at quality level p.

So, this is a very important quality. I really need to really have a good field for this. Now, if p changes if little p changes P A also will change, because that will be determined by this formula, there as I change P A is going to change. P A is the chance of accepting the lot, the probability of accepting the lot. Now, there must be A between this little p and P A, that is shown by the OC curve.

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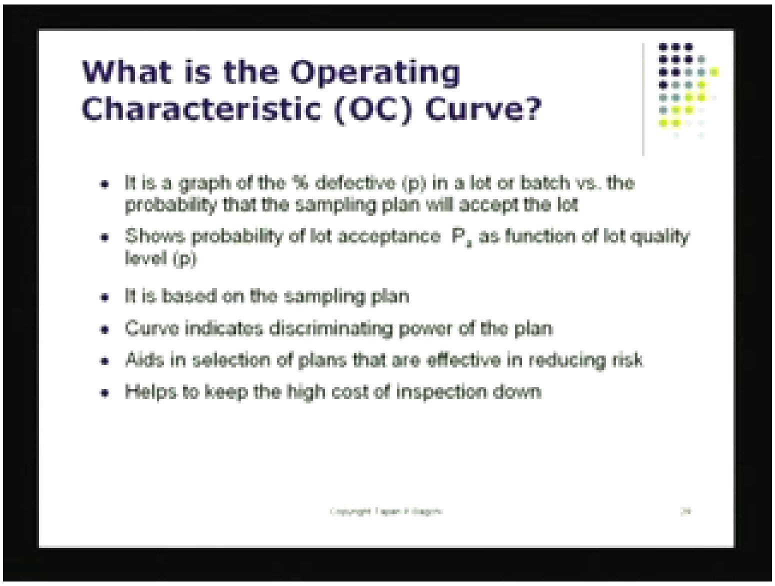

Let us take a look at this curve there. What is this curve called? The OC curve, it is the graph of the percent defective p in a lot or batch verses the probability that the sampling plan will accept the lot, which is P A. So in fact it is a plot between P a big P A and little p. That is what this plot is. That is what the OC curve is. And this is determined on the is all really dependent on the sampling plan. The particular sampling plan you are using that is going to determine the duration ship between little p and P a.

In fact what you would like is, you would like this sampling plan in such a way, that this P a is high. When your lot quality p is very near AQL, and also you would like this P a to be small as small as possible when this lot quality is near or QL. These are the two kind of relation that you must have, in fact the more the plan is able to reject lots at the RQL level to more discriminating it is.

It should accept most of the lots which are at AQL level; it should reject lots which are generally around RQL level .Then I will say it is a good plan, now to get an idea whether the plan is good or bad. We need to look at the OC curve of the plan and I am going to show you, what that OC curve really shows you?

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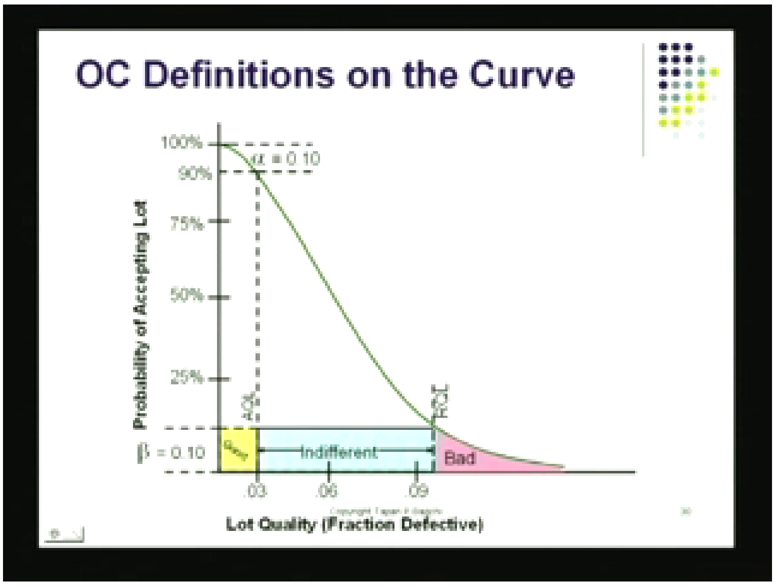

Here is the picture of the OC curve of a typical sampling plan. Let us look at the axis first, on the x axis I have got lot quality which is little p, little p is on the x axis, and on the y axis I have got P a, P a is the probability of accepting the lot that, is big P a big P a goes this way.

Now, notice something I would like I as a user of this sampling plan. I would like to have when my lot quality is near AQL. And let us pretend here that AQL is three percent, if AQL is three percent, I would like to have a high P a at this point. I would like to have a high P a and the curve shows here that this particular some sampling plan from which this plot has been drawn. It will accept ninety percent of the lots which are submitted with your little p at 3 percent level. Now, the same plan will accept a very small number only ten percent of the lots. When your lot quality little p becomes ten percent so like if RQL is 10 percent, then only ten percent of the lots would be accepted by the sampling plan.

Now, what happens in between you see this slopping curve, there is a formula for it. I am going to show you, what that formula is like? That formula gives you for any value of p, it will give you the corresponding value of P A. So what does the OC curve show you?

What does it show you? It shows you the chance of accepting a lot at the RQL level. It is also showing you the chance of rejecting a lot.

When the lot really is at the AQL level and it also shows you the fat of all the lots. Which are at quality levels? Which are between AQL and RQL? That is what the OC curve does? So the OC curve actually it is a pretty useful piece of curve, it is a pretty useful device. And this can be drawn exactly, and I am going to show you the method for drawing this exactly.

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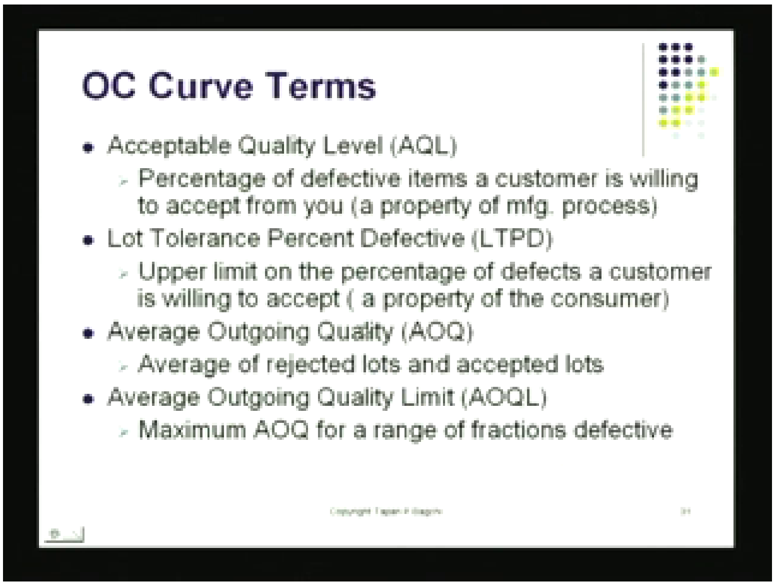

So, again let us come back and look at some of the terms there. Acceptable qualitable level is the percent of defectives that the customer is willing to live with forever. Because that level of defect in the incoming lots is a with him, then there is of course, this other idea called LTPD or RQL. It is the upper limit on the percent of defects that the customer is willing to accept, but only with a very small probability.

Then, there is some other term called AOQ average outgoing quality. I am going to show you what that quantity is and of course, there is a limit on that AOQ also which is called AOQL. And that also, I am going to show you what these are? So, in fact we know what AQL is? We know what LTPD or RQL is? They are the same terms. And I am going to explain to you what AOQ is and what AOQL?

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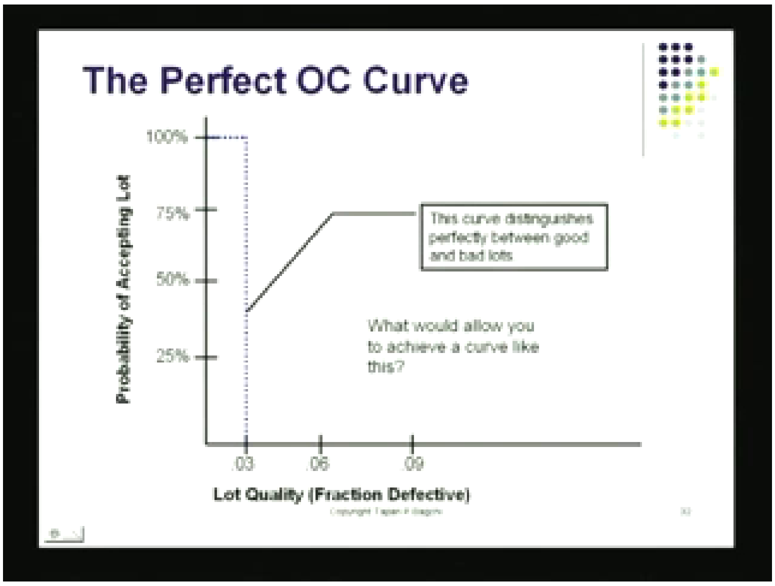

These I am going to show to you now, what about an ideal curve? Ideally speaking of course, you would like to reject as many lots at the near the RQL level as possible. You would like to reject as many lots as possible which are near, which are at quality level. That is near RQL and you would like to accept any lot that is coming at or better than what we call the AQL level. So in fact at AQL level my P A. P A is the probability of accepting it should be very close to one and at RQL level, my probability of accepting should be very close to 0 this is the ideal curve.

Now, think for a minute I could have a sampling plan that will operate actually in this ideal mode, provided I did 100 percent inspection. And I made no inspection errors I committed no inspection errors. If I was able to do that, I will be able to operate pretty comfortably. Because I will be accepting all the lots which are acceptable and I will be rejecting all the lots which are, which should be rejected that I should be able to do.

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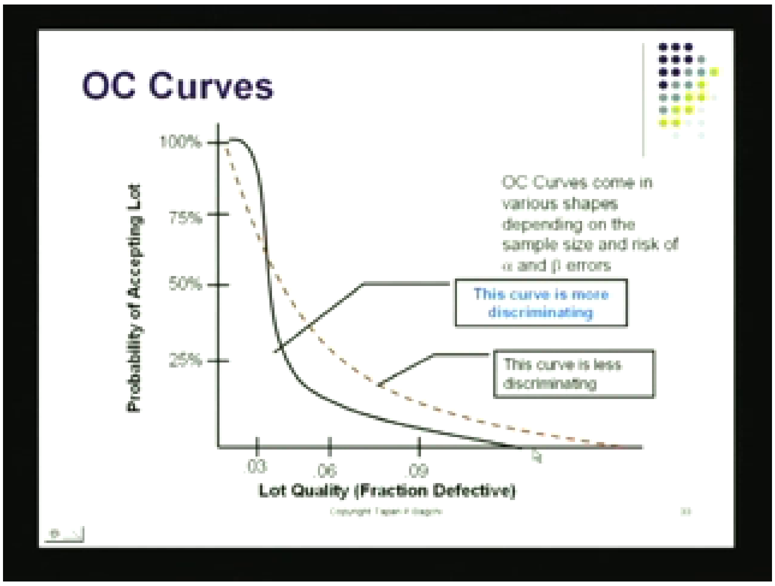

But what happens in real life? In real life I am not doing 100 percent inspection. So my real plan is going to be either like, this is the real OC curve of a plan that I have used or it could be like this, the black curve there. So, either the red curve or the black curve, this is what reality is going to be. Like what is the difference between these two? It turns out that the black curve will throw away, will reject most of the lots that are at poor quality level. It will accept almost all the lots which are at AQL level, AQL being 3 percent and RQL being around here. It will reject most of the lots which are in this area.

Now, look at the red curve, this is another sampling plan. It is got the OC curve of that plan, there the red one. It is not as discriminating; it is not able to separate between good and bad lots as effectively as the black curve. In fact the ideal plan would be like you go down just a little bit, then you come all the way vertically down. Then you go out this way that was the ideal curve. And the black curve approaches it, so the black curve is better than the black sampling plan is better than the red dotted sampling plan.

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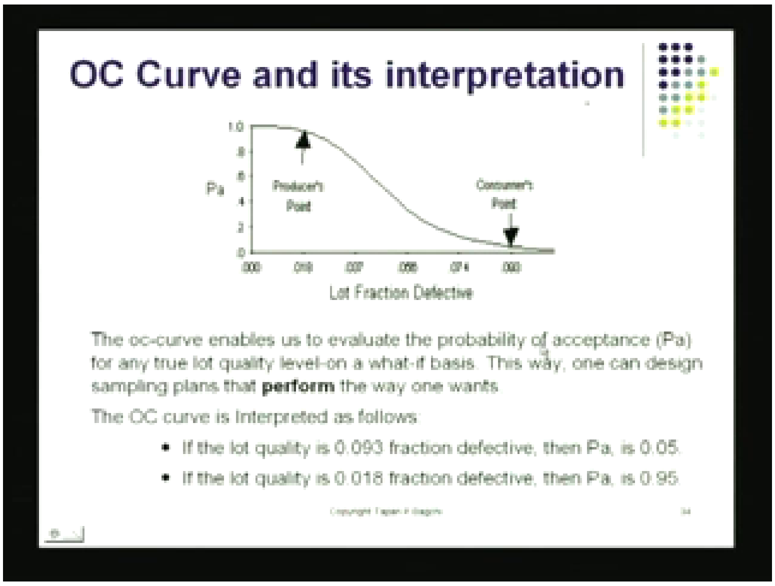

Now, what are the things that are of interested the producer? The producer would like to make sure that the sampling plan accepts as many lots as possible which are submitted at the AQL level. And what is it that the consumers would like to do? The consumer would like to have the sampling plan reject as many lots as possible, which arrive at RQL level. Now, once you set up your sampling procedure, you basically take these rules and you give the gauge. You give the gauge to the inspector, and you tell him now you operate any lots that are arriving by trucks boxes and so on and so forth.

You utilize my sampling rule and my sampling rule is what? Pick little n items and I am also going to give you this control number c. Look up the number of defectives found in those n items, if the number of defectives found exceeds c. Reject the lot, if the number of defectives found in the little n items which were inspected. Because now, this is the sample and you must inspect every item in the sample. If that number d is less than or equal to c accept the lot, this is from my production, this is who you would like to do, you able to do.

Obviously, the producer would like you to accept as many lots as possible. You would like to you, they would like to make sure that your P a at the AQL level should be pretty close to one. And the consumers would like to make sure that the P a at the RQL level is as close to 0 as possible. This is like something that is desired by the two parties which are forming the two ends of the supply chain.

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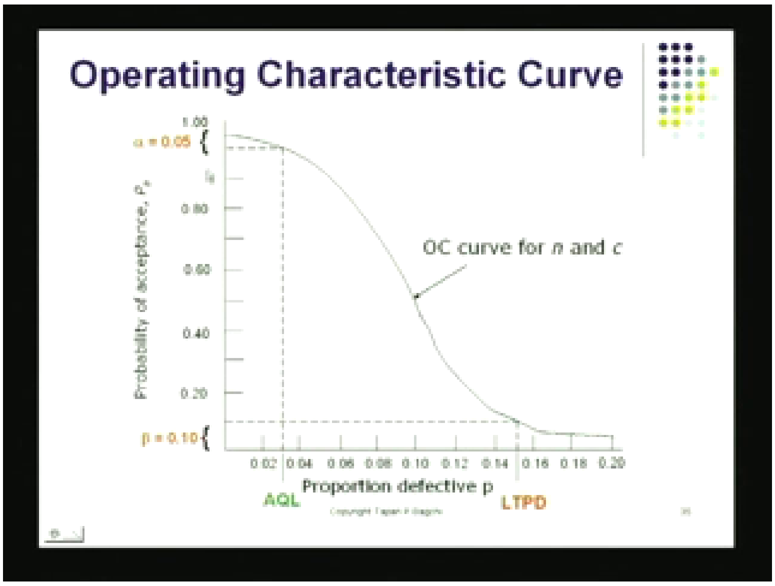

So, in fact in general look at alpha and look at beta now, beta is the probability of my of my accepting a lot which is at LTPD or RQL level. Beta is the chance of the sampling plan accepting a lot which is at RQL or LTPD level. That suppose to be quite small it is kept generally around not more than ten percent. And alpha is the chance of my rejecting a lot which is like 1 minus P a at this point 1 minus P a at this point. That is equal to alpha that is the probability of my rejecting a lot which is otherwise at AQL level.

At this level, ideally I should be accepting all the lots. But I reject alpha fraction of the lots, this alpha and this beta, these are the contention these are actually the ones that we have to balance.

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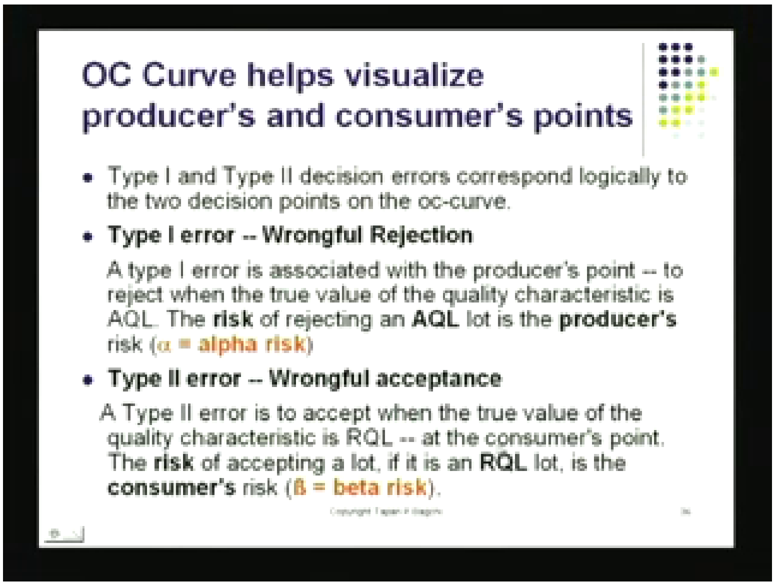

So, there are these two types of risks involved wrongful rejection and the fraction of items fraction of lots that are rejected at the AQL level. That is equal to alpha and wrongful acceptance of poor lots. Poor lots are of quality RQL that quantity is suppose to be beta. I should not exceed alpha in rejecting good lots and I should not exceed beta in accepting what we call bad lots? Which are bad lots are near RQL level.

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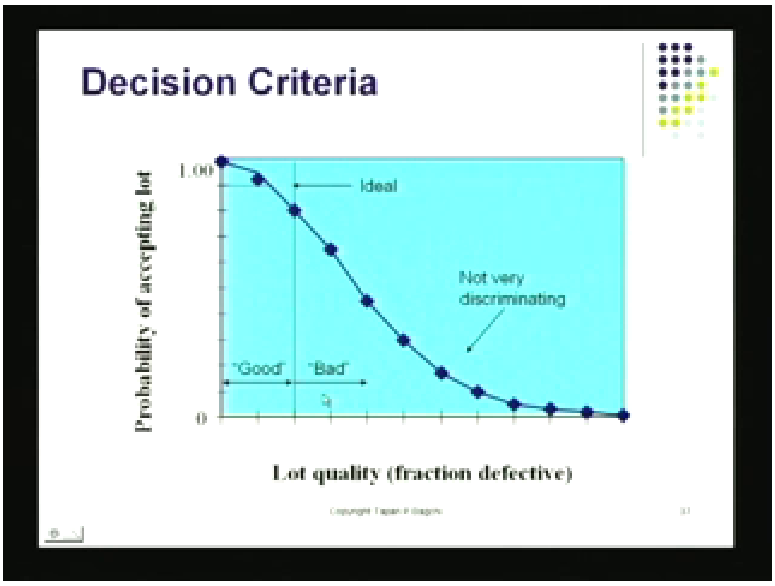

And, a curve like this is not very discriminating; it will probably allow a lot of bad lots. Also to come in to the system, the ideal curve is like this, which you know stays near one near AQL and falls to the ground. And, it rejects every lot that is slightly different from your AQL level. Therefore, there is actually there is some reason for us to take plans which are here. And to try to make sure that these plans are approach this ideal plot, this is what we would like to be able to do so? We would like to design sampling plan, this is like a particular sampling plan.

This is not very discriminating, what we would like to be able to do is? We would like to design a sampling plan in such a way, that this plan is as close to this ideal shape as possible.

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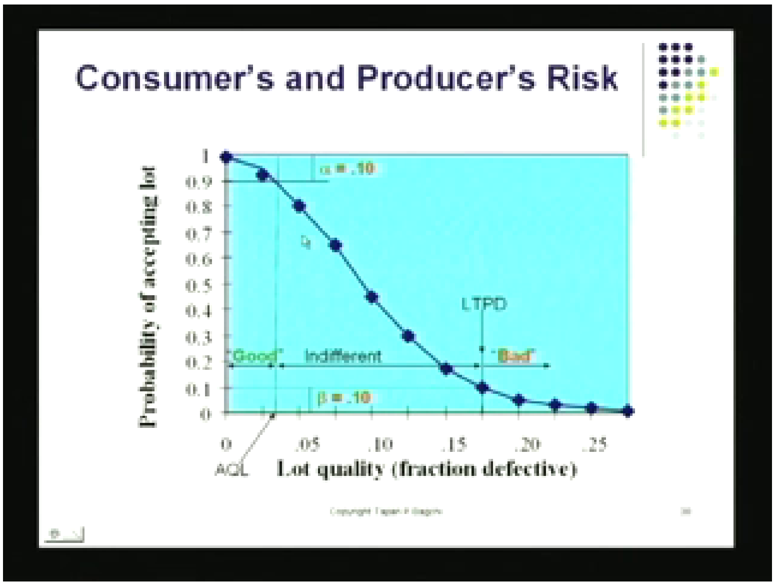

Let us see, how we do that? This is again a continuation of the same thing bad lots differently at this end, and good lots are at this end. That is what they are in between of course. There are lots that will be accepted, because I am not doing 100 percent inspection. And I would like to make sure that this curve approaches the ideal curve as much as possible; this is what I would like to be able to do?

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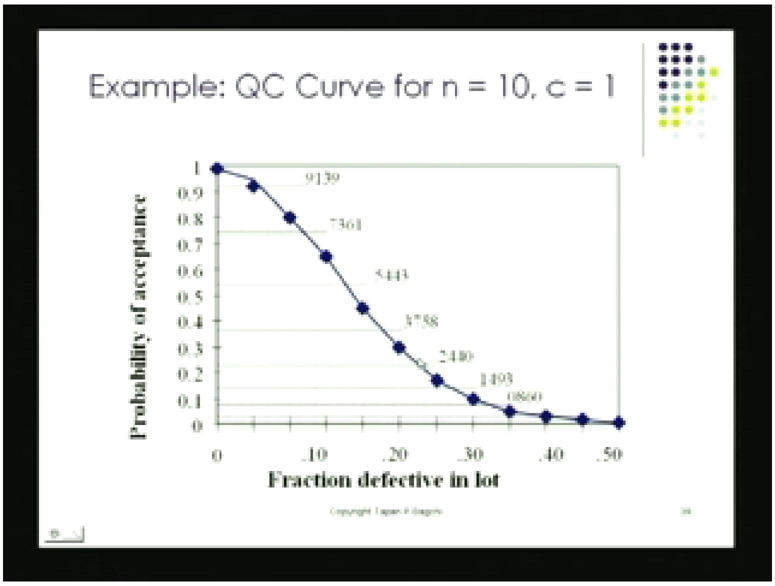

Now, there are ways to calculate these numbers, there are ways to calculate these different numbers. And those would be using the remember the binomial formula that I had there. The binomial formula is shown here, this is the binomial formula, this P a equal to summation d equal to 0 to c n choose d p to the power d 1 minus p to the power n minus d. This is the formula that gives me P a and with the help of this by plugging in different values of p I can determine the values of P a Q. And I can produce a plot; the plot looks like the plot that I have got on the screen here. This plot can be plotted once I have my binomial formula with me.

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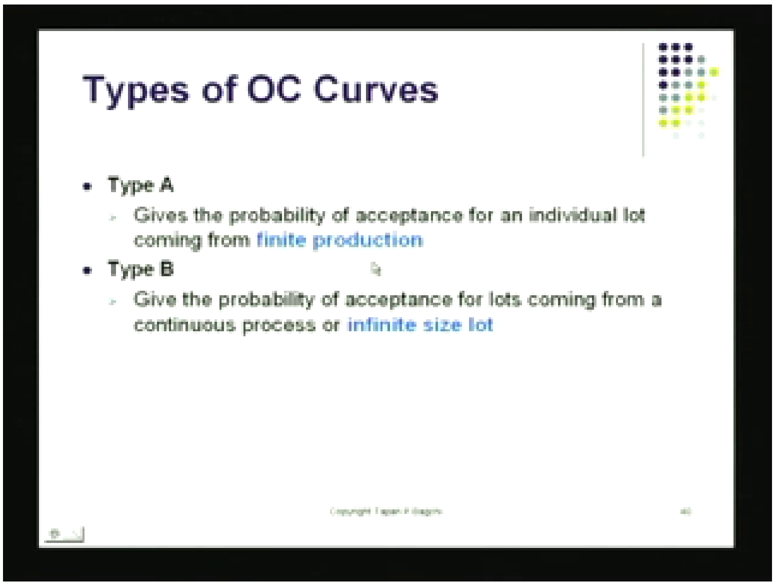

Now, some times what happens? That the lots supplied is of finite size like for example, a truck or a bus. In that case we use a distribution that is the hyper geometric distribution not the binomial distribution. We use the binomial distribution when the lot size is so large, then it can pretend that big n is too large. When I compare that to little n and I can use the binomial distribution, I should be able to use this binomial distribution which is here. When lot size is quiet large, but if lot size itself that big n there, if this big n is not too large, I have to use what we call the hyper geometric distribution? And in that case, I will be using the type A OC curves.

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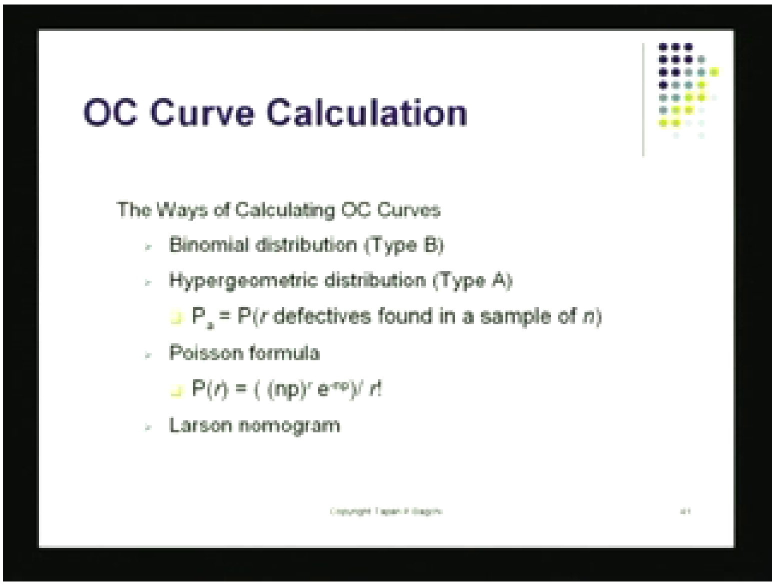

It is also possible for me instead of using the binomial distribution. I may be able to use the Poisson distribution also and the formula is shown here. P r is now the probability.

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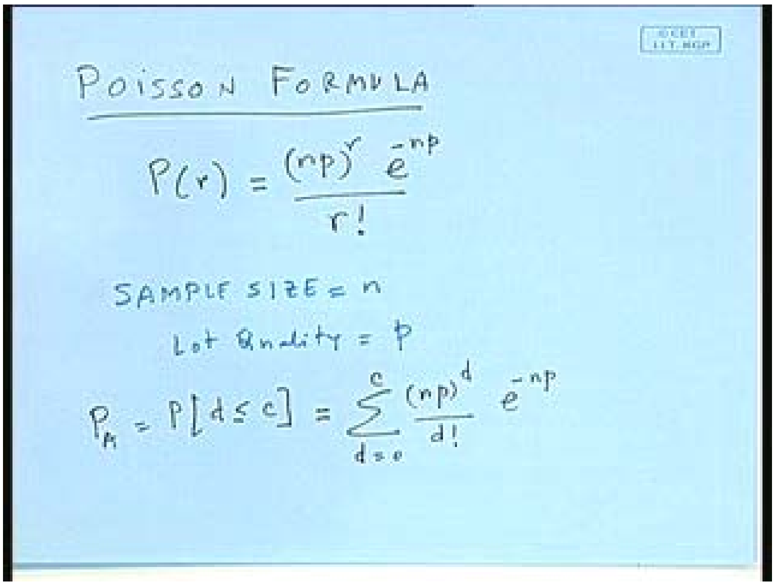

And I am going to be drawing that for you, P r is the probability. And if I draw it correctly using the Poisson distribution, the Poisson formula goes like this. First of all there being r defectives, this is equal to n p raise to the power r e minus n p divide by r factorial. This is the formula for finding exactly r defectives, when I sample from an item which is got n and p is the overall quality of that lot. There n items have been sampled from it in which r defectives have been found, p is the overall quality of the lot that is supplying those parts there.

Now, what have I got there couple of things are happened here. One is I made a decision, Now I made a decision here to select a sample size of n. So sample size is n and lot quality is p, these two are given to me. If these two are given to me then the Poisson formula gives me the chance of finding r defects in n items. Chosen, this is a formula which I can also use in designing a sampling plan or in fact in constructing the P A. The P A number, the P A formula, how will I find P A? Suppose my rule is that you accept the lot. P A is the probability of my finding d less than or equal to c. If I use the Poisson formula this will turn out to be d equal to 0 to c n p to the power d divided by d factorial e minus n p. This is the formula I will be using if I have to construct an OC curve that uses the Poisson formula as the basis.

Now, this is an alternative to the binomial distribution. So, in fact there are several ways I could construct this OC curve. I could use the binomial distribution which is this formula, remember this formula. There, this is the binomial distribution or I could use the Poisson distribution I could use the Poisson distribution whichever is convenient for me.

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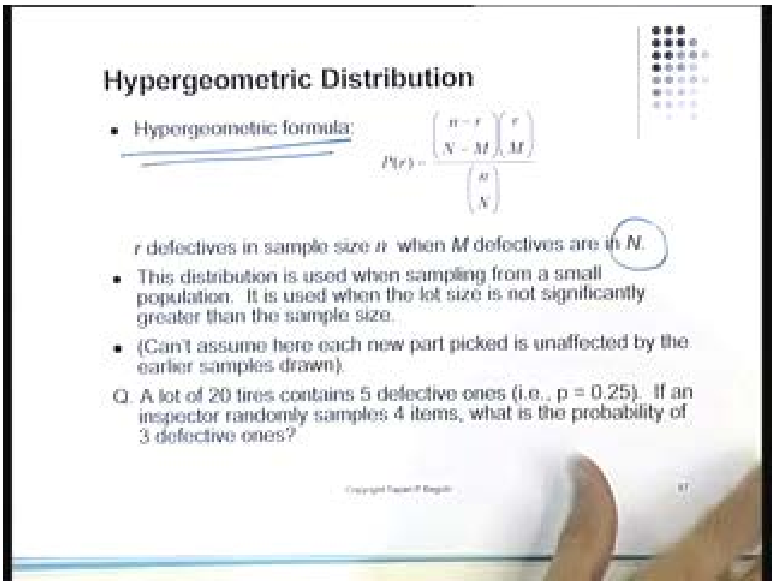

If I doing the calculation, I could use that and by chance if it turns out that my sample size is, what it is suppose to be, which is little n but my lot size is finite, my lot size is my n, n is my lot size. And this n is not too large in that case I will be using the hyper geometric distribution. And there, the formula is given by this and again I can work out the exact formula for P a. The probability of accepting the lot, now this again is given in your slides. As we go down you will be able to see this one and we will come back and we will use this in a couple of minutes, we will be using this.

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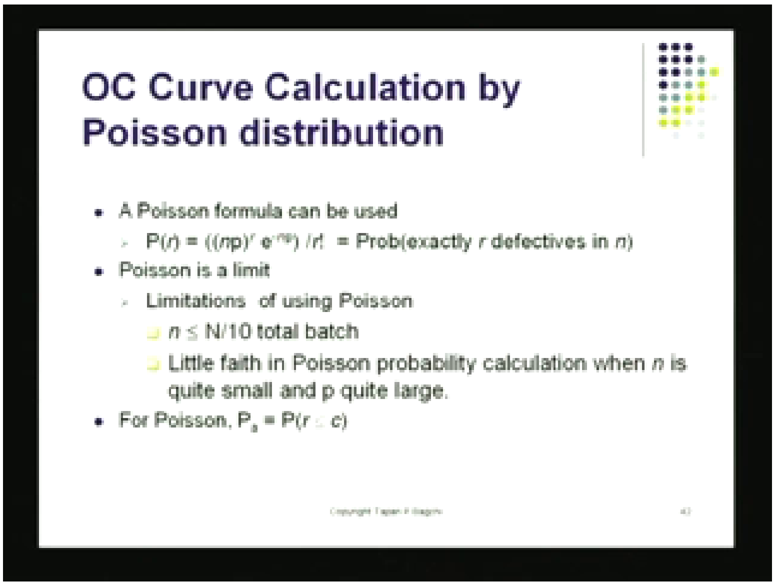

Let us carry on with our distribution. There, we were using the Poisson distribution, so we got the Poisson formula there and with Poisson formula I can actually work out my P a.

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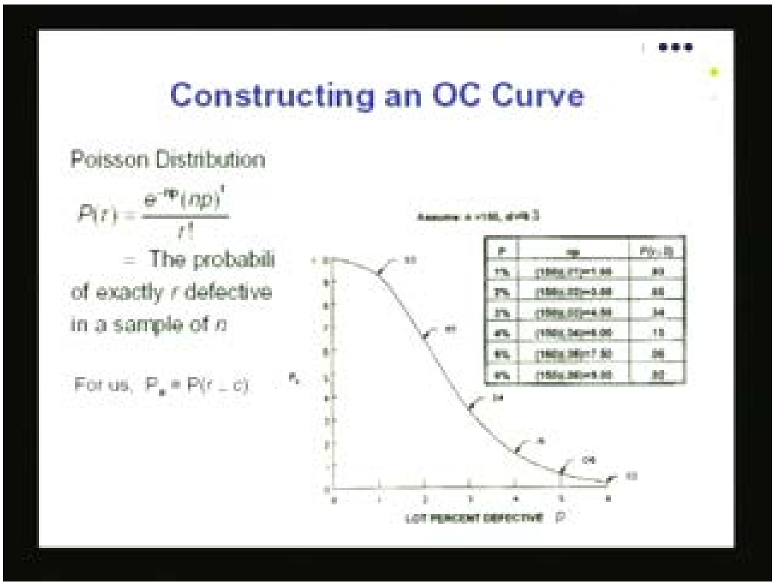

P a, remember P a is the y axis .P a turns out to y axis in the OC curve and the x axis in that case turns out to be a little lot quality p lot. Quality p is here and P a is there and when they end up with for a given n and a given value of p. I end up with the OC curve there, what does the OC curve tell us? It tells us the risk of accepting a bad lot or the risk of rejecting a good lot. Rejecting as a good lot is something that the supplier is going to be upset with and accepting a bad lot is something that the user is going to be upset with. And both of these we have to balance both of these, we have to minimize and generally speaking the considerations are economic.

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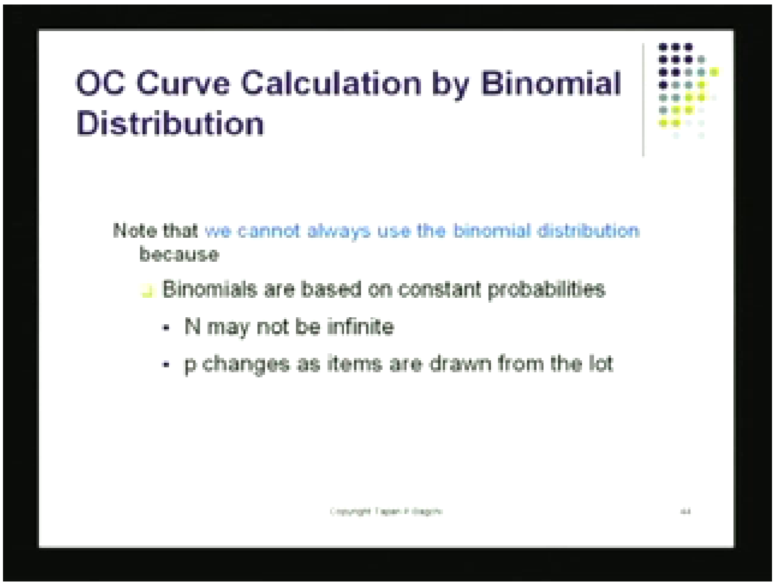

Like, I said to you earlier, we need not always worry about the binomial distribution. As the only way we may be able to use the hyper geometric formula.

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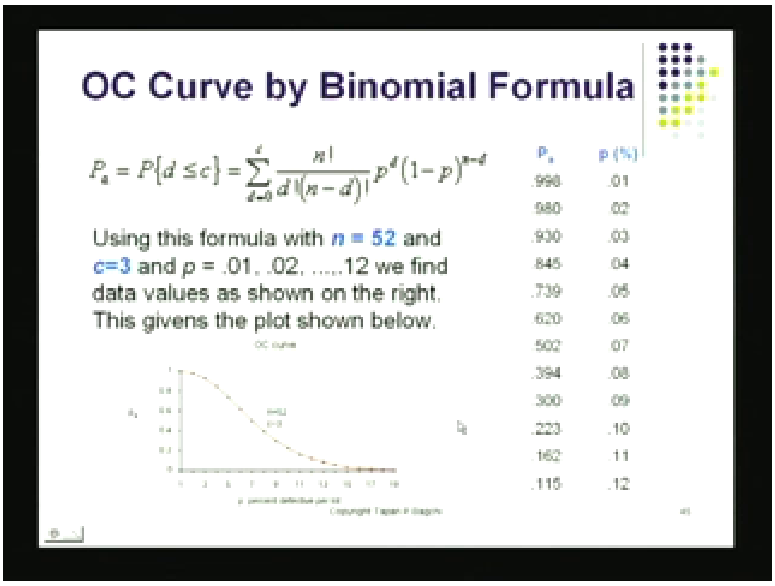

Or we may be able to utilize the binomial formula also. Hyper geometric Poisson or binomial, these are three popular formulas which are utilized, in trying to draw the OC curve in most cases.

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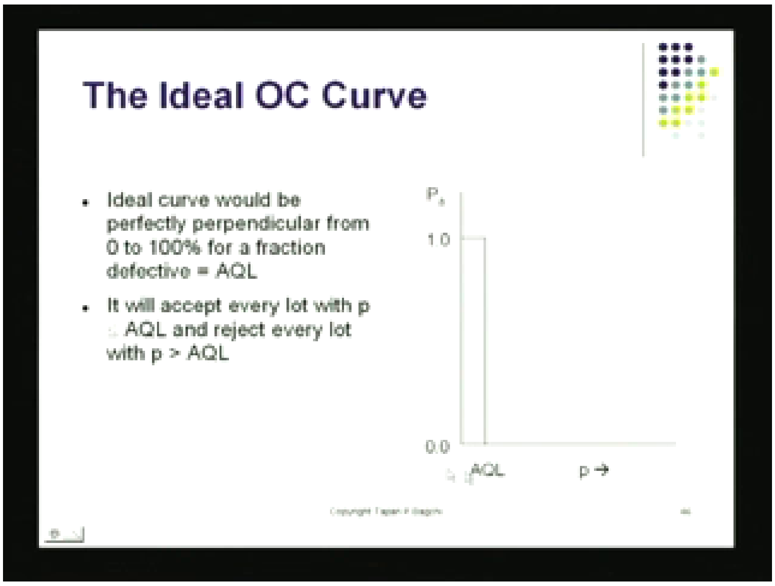

The ideal OC curve of course, is like this. It just starts out; it accepts everything that is near AQL; and it rejects everything that is beyond AQL; and this would be this situation; I can guarantee this, provided I take the full lot. And inspect each and every item, I take the full lot and I inspect every item. So, that I can pull out any defectives that might be there, I pull out all the defectives and I end up with only good parts there. If I would do that, I could operate in a way that would be pretty close to what we call the ideal curve that we could do.

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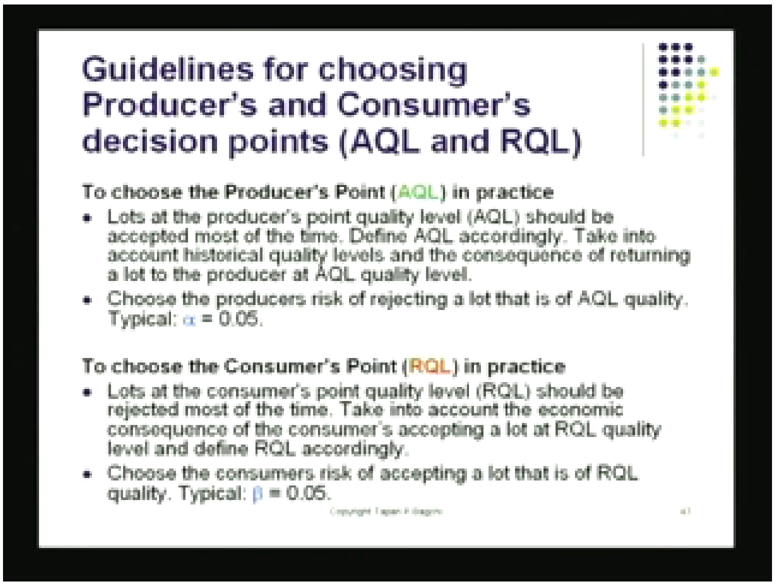

Now, the let us see how we balance the two risks. Now AQL is something that the supplier knows, AQL is something that the consumer is willing to live with forever and ever. And, therefore, the consumer is interested in making sure that he gets generally items which are near AQL level. Occasionally, if he does get something that is near the RQL level, the consumer would like to make sure that lots coming at near the RQL level. Those are rejected, the producer on the other hand, he also knows that AQL is a quality level that is acceptable to the consumer, to the user or the consumer. Therefore, what the producer would like to make sure is that any lot that is submitted at AQL level of overall quality, it is accepted by the customer.

Now, the decision to accept or reject the lot; that is based on the n c sampling plan that I have in the single sampling case; therefore, we got to make sure when we design a sampling plan, we do it in such a way that we take care of the interest of the of the user and also the interest of the supplier. Both of those interests they have to be brought together and there is a easy way to do this. Say, it is surprisingly easy they might appear to be conflicting. But I will show you a process by which you will really see, it is not that difficult to work this out.

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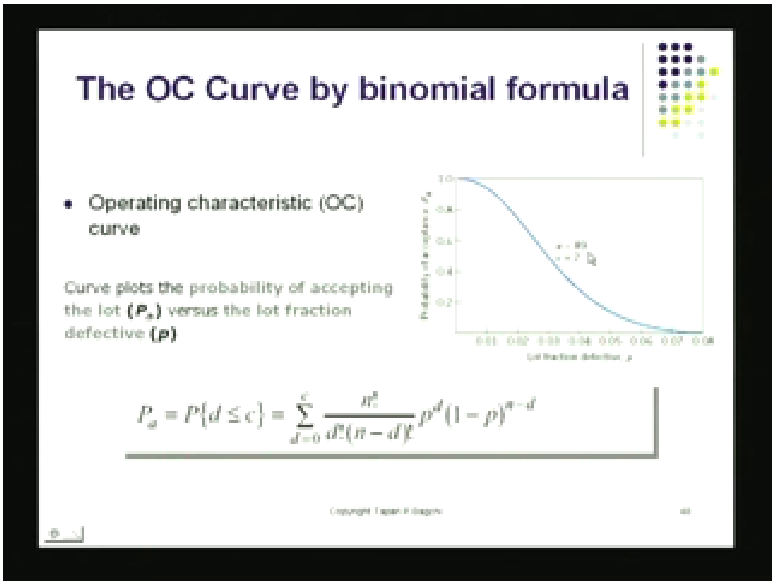

This is of course, the binomial formula and it is just showing you that if you use the binomial formula. You can construct the OC curve by changing the value of p, given a value of n and given a value of c. If you change the little value of this, little p there you can determine different values of P a. That will be generated from this and then you can plot p verses P a, which is actually the trace of the OC curve itself. For any n for any sample size and any control number c that is given to you.

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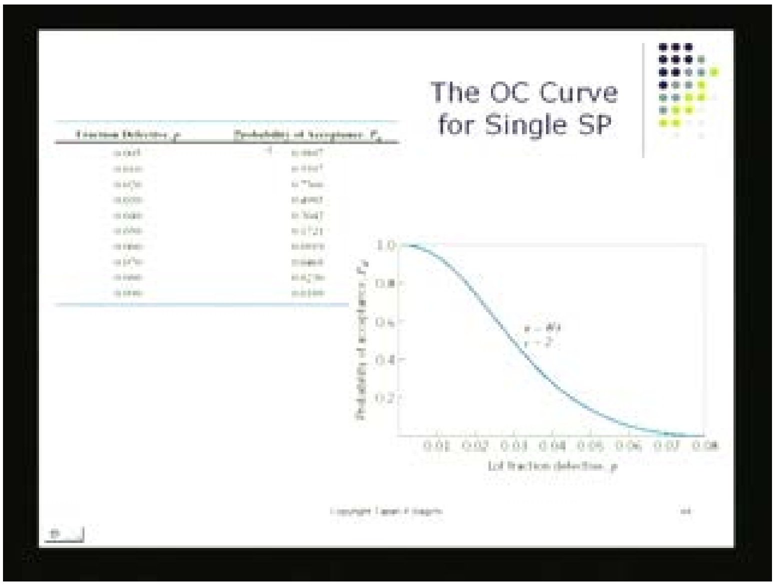

So, in fact it is the same manner in which this OC curve has been generated. It is not very difficult to do.

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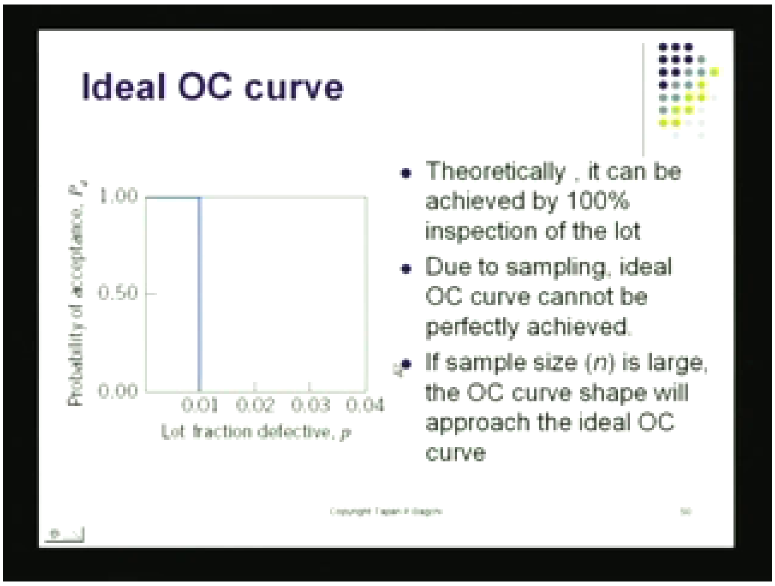

And of course, the ideal curve also I showed you the ideal curve is valid only when you are doing a 100 percent inspection. It is not valid, otherwise that is something we would like to be able to do.

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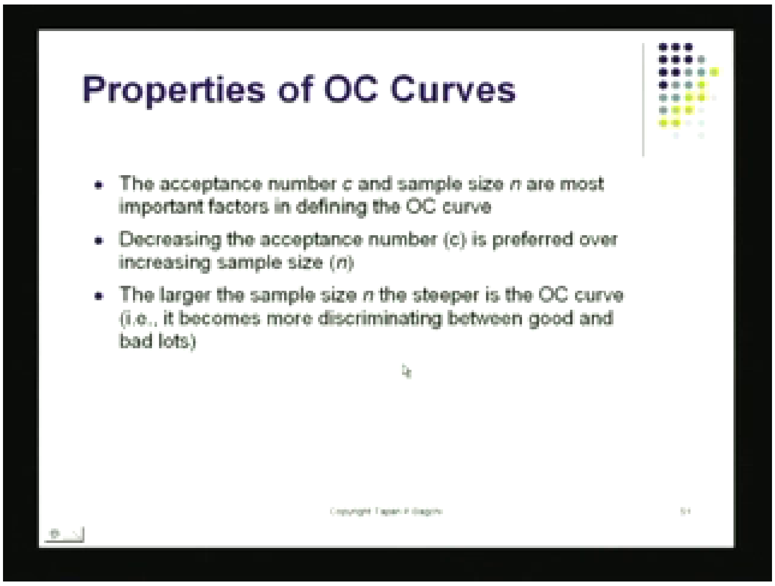

Now, you know there are curves; there are various types of curve possible. Let me draw a few curves here for you. I am going to be using this paper here.

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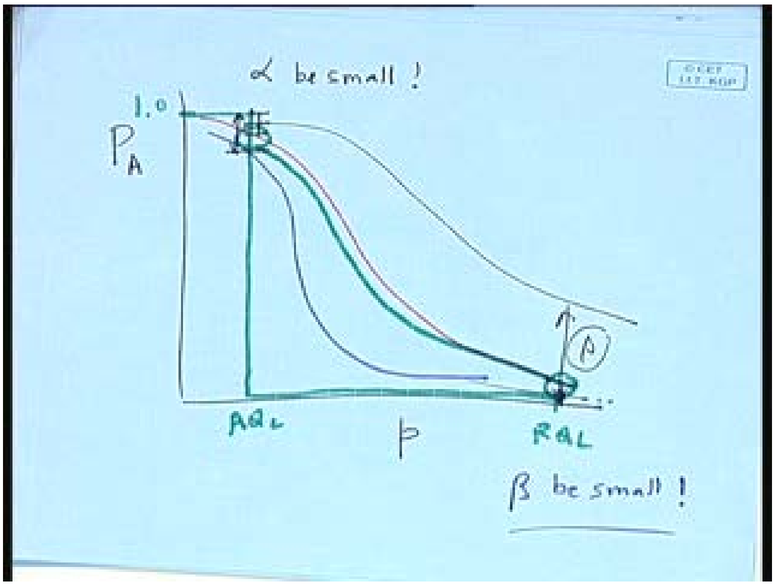

And I am going to be drawing the two axis there, one axis is p is the overall quality and this side I have got P A. P is the chance of accepting the lot the ideal curve of course, is it is going to start at 1.0 and it is going to come down to RQL, AQL level and from that point on it is going to stay right there near 0. And all the way beyond this RQL is here this is like a an ideal OC curve but generally what happens here? OC curve. They behave like this in other curve they may, it may behave like this a third curve, it may behave like this.

Now, obviously look at these curves a little carefully, look at the value of beta. Let us say RQL is here, beta here is quiet small. But beta here is somewhat larger and the beta for this curve is quiet large now. What is beta provides the user or the consumer of the protection? Say if large beta is there, the consumer is not going to be protected very well. And you look at alpha, now alpha is small here. Alpha is protection provided to the supplier, then I have got alpha here, which is slightly larger. And this one has got the largest alpha, the larger is the alpha, the larger is the suppliers risk or the producers risk.

So in fact what the producer would like to do is? He would like to have as small as far as possible alpha be small. This is what the producer would like to be able to do? And what is it that the consumer would like? The consumer would like beta to be small, this is what the user would like to be able to do or the producer would like to be able to do there is the consumer would like to do.

So, the consumer looks at this end and the producer looks at this end. What we would like to be able to do is? We would like set some target. Here I would like to set some target here, and I would like to see can we construct a curve that goes through these two points? This point and this point, can we do that this is something that we would like to attempt.

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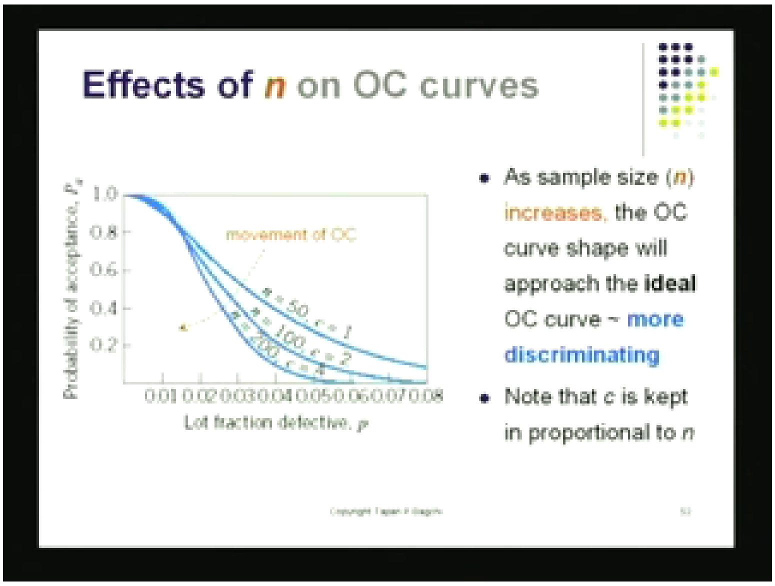

Now, let us go to the screen, there notice here. What happens if I vary this curves, if I change n, if I change n little n is the sample size. As sample size is increased, the OC curve becomes more and more discriminating. So, it moves this way which is like it, becomes better and better from the perspective of here. We have got RQL and the guy who is most concerned with this is the consumer. So, it is better for the consumer to take a large sample, remember this now it is better for the consumer to use large sample in sampling.

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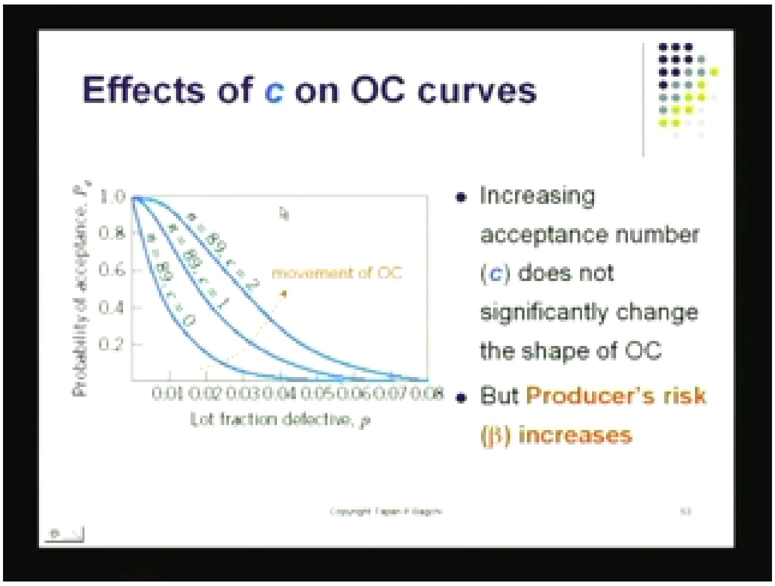

And, now what about the other story? There, if I increase my c, the curve turns to rise and when the curve rises, what happens to alpha? Look at what happens to alpha, becomes smaller and smaller. When alpha become smaller, the producers risk goes down the producers risk was high here. It became quiet small; there on the other hand the consumers risk was low. Here low here and it got to a point which is like more than 0 definitely.

So, in fact, what we have to do is? We have to find a compromise between sample size and this control number c in such a way that I am able to provide a protection to here. The supplier or the producer and the consumer, who is the user of these items which are arriving by lots.

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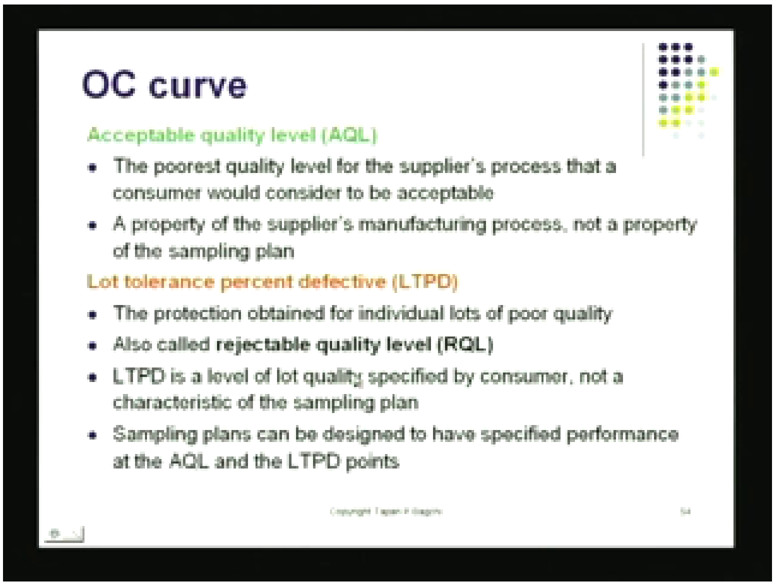

Let us see, how we do that? We have some general definitions here; in fact these again define AQL and RQL for us? AQL I have put in green and RQL or LTPD I have put in red here.

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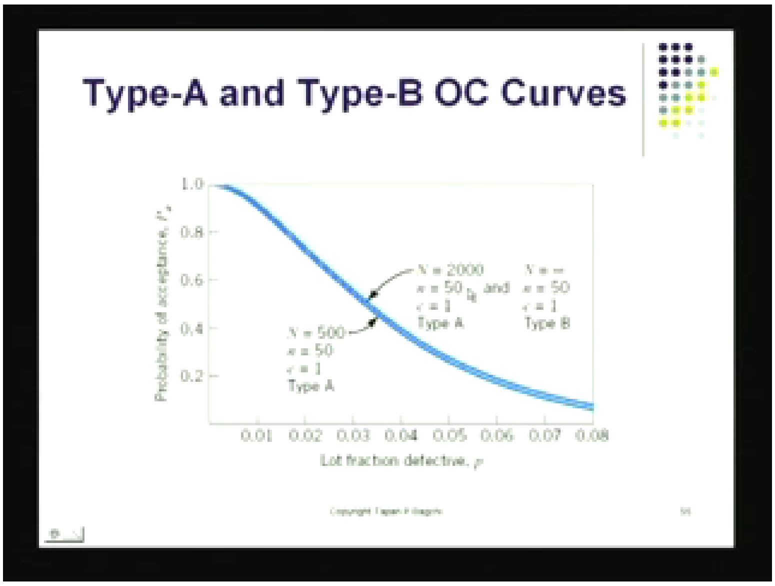

And, these are two curves, now they are both actually for finite size lots. They are both for finite size lots, if I look at infinite size lots, which happen sometimes when the supply is such that the truck size is very large. And I have got continuous production feeding, my factory continuous production by the supplier feeding my factory. Then of course, the suppliers lot size is very large and I could be using the type B curve there. Now, these are only to indicate to you that as far as large lots are concerned, there is not that much difference in the OC curve.

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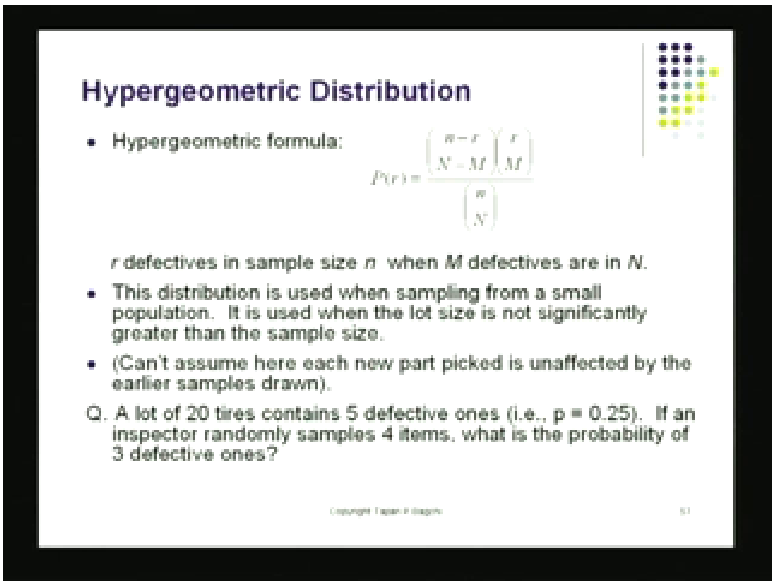

Let us, take a look at, whether we use the Poisson distribution or some other distribution. If lot size is large use the binomial distribution or the Poisson distribution. If lot size is small use the binomial distribution, use the hyper geometric distribution. And let us try to do a small problem; let us try to work out a very, very small problem, pretty simple problem there. And what we would like to do is you see the problem that is indicated right at the bottom. A lot of 20 tires containing 5 defective items, 5 defective would lead to basically p being 25 percent defective, those are supplied to you.

If an inspector, randomly samples 4 items out of the 20 tires. What is the probability of finding three defective ones? This exercise basically is a direct application of this formula, there the hyper geometric formula. Let us see, how we do that?

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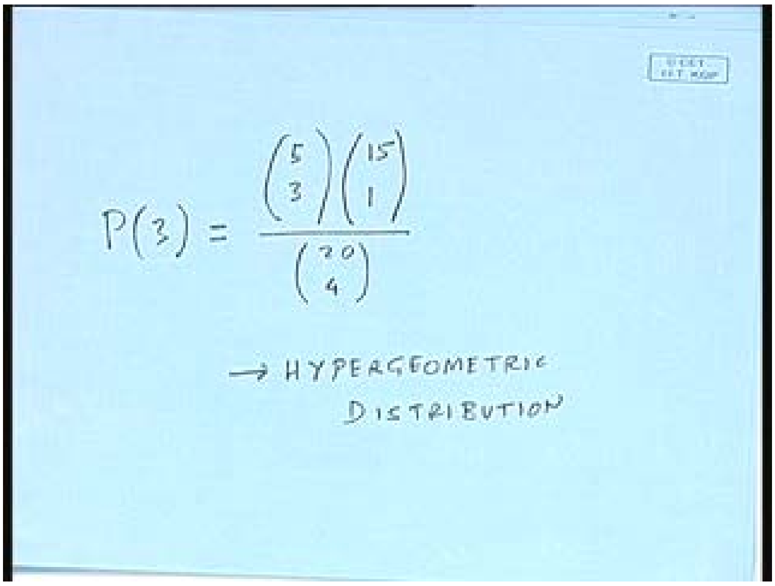

I am going to be doing this; I am looking for the probability of three defectives. So I will write this p three defectives and these three defectives are in four items. This should be equal to now, look at the formula, out look at the formula given there, n minus r divided by n minus r. Choose n minus m, let us try to write that. Here the first thing I would like to do is, I would like to make sure that, I account for the number of items which are picked out of the lot.

The lot size here is 20, so I put 20 there and sample size is 5 sample size in this case is 4 items only. So 4 is there, now this is the total number of combinations. Different number of ways I could construct samples of size four picking out of 20 items, 20 items are the basically the size of the lot.

Now, let us take a look at the number of defectives in the original lot which consist of 20 tires. There are 5 defective tires and I would like to have 3 defective tires appear in the hands of the inspector. So 5, choose 3 is the number of different ways 3 defectives could be picked out of 5 defectives, which are present in the lot itself.

Then, what about the items which are not defective? Those are 20 minus 5 and that is equal to fifteen. And then what is the remaining number, it turns out that if I pick 4 items if I pick 4 tires and 3 are defective. There is only going to be one that is going to be one good one. This is the probability, now of my finding 3 defective tires, when I choose 4 tires out of lot of 20 which is got a total of 5 defective items in that. This has been done using the hyper geometric distribution and this is a very important formula which we use. Whenever we have got finite size lots, would like to use the hyper geometric distribution.

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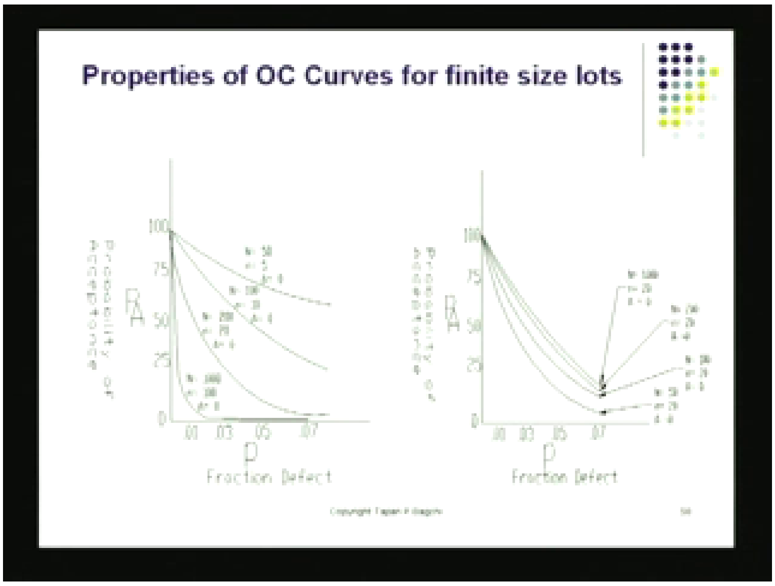

Let us, get back to the curves here, again these are couple of other curves that I show here, to give you an idea that OC curves are never the same. OC curves depend a lot on whether the lot is finite or the lot is infinite size. And, also if there are items which are like the number of defectives that I allow to be in the lot. In the sample, which is the control number c, if c changes the OC curve will change. If lot size changes big n changes the curves will change the OC curve will change. And of course, if sample size n changes little n changes then again the OC curves will change.

So, the OC curves dependent on all of these properties and of course, the OC curves tell us whether a plan that I make my plan is always going to be an n c plan. Remember, basically any kind of plan that I workout, whether it is a single sampling plan or a double sampling plan. Anytime I have got a plan worked out that plan is going to be searched, that I end up with protection provided to the user. And also it protect the supplier, the user should not end up with a lot of defectives items. And supplier should not end up with a lot of lots returned to him, which are otherwise at AQL quality level or near AQL quality level. To do that we obviously have to optimally design the sampling plan.

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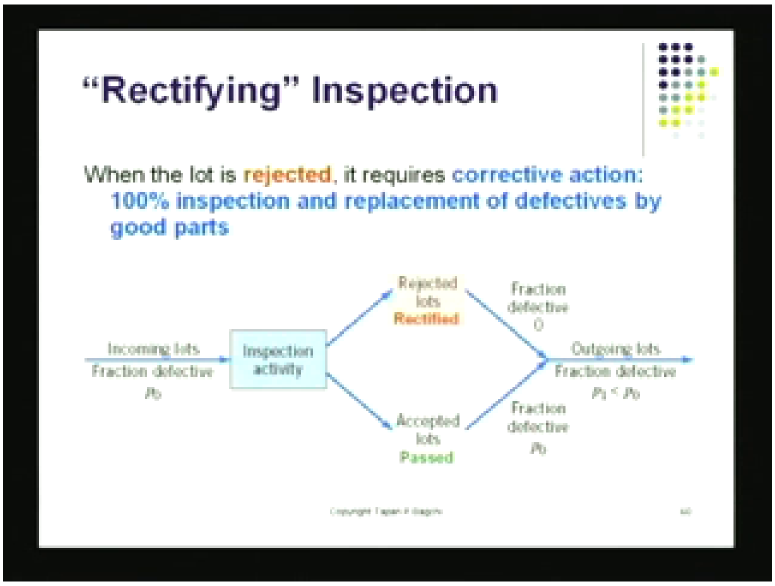

This is like one other item that I should mention to you, might reject the lot. What is very possible when you reject the lot? You would not want to return it to the supplier, what you would like to do is? You would like to probably charge in for the defective items, what you would like to sort the lot by doing 100 percent inspection. Remove the defective items and make sure you can carry on with your production, because your stoppage of production can be more expensive than returning the lot, and getting replacement and so on and so forth. So what you would like to do most of the time is? you would like to do rectification.

You look at the defective lots, once that are been rejected by your inspection procedure, by your acceptance sampling procedure. And sort out the good items from the bad items. Take the bad items; replace them with good items, now you got supplies now that can carry through your production. So actually now you have got parts that you can carry, that you can utilize in your production process. Mean while of course, you will negotiate the deal with your supplier, as to what to do with the defective parts. Should he get more replacement or should he charge him for some inspection cost and so on and so forth. Those things you sort out separately, that you do when you are doing this? You would be doing either, you would be doing rectified inspection or you would be doing just standard rejection of the lot and returning the lot to the supplier.

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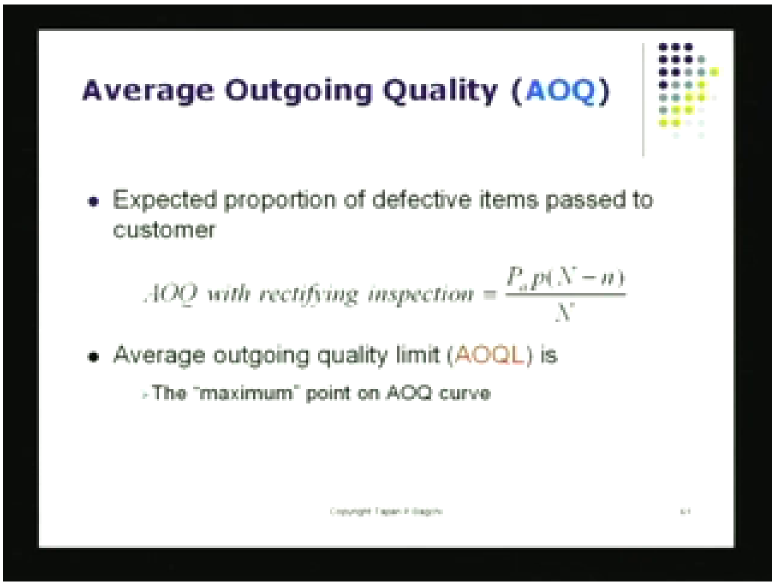

Now, there is something that is also of interest to us. This is something that we would like to call the AQL. AQL leading up to what we call AOQ? Now, suppose there is an inspection plan in effect. What I have here, I have got an inspection plan that is in effect.

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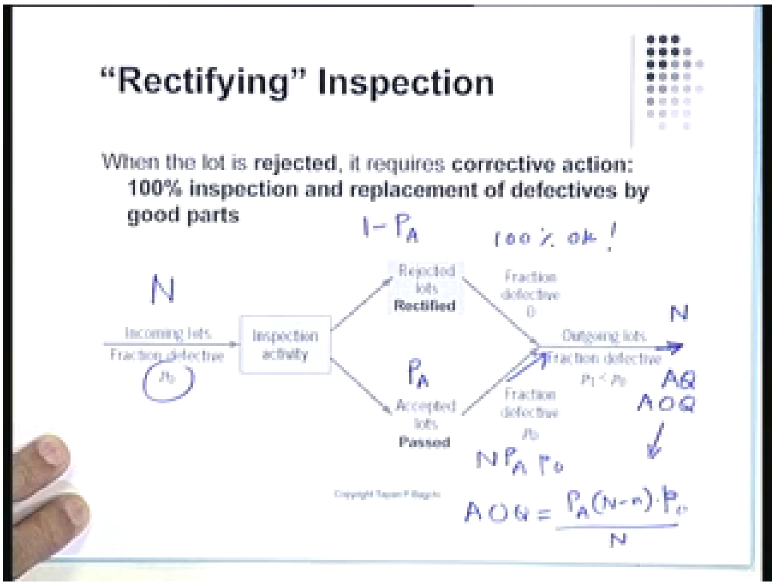

And, I have here, I am going to be doing that using a piece of paper, there my incoming parts, those are coming in lot size of N is the lot size of an incoming parts. That is the lot size and it is coming in at a quality level p 0. There is a certain probability of accepting this lot and that probability is P A P A at p 0 level. And there is a certain chance of my rejecting that lot and that is 1 minus P A is the probability of my rejecting the lot, how many items are here?

Now, let us take a look accepted lots, those are accepted at probability P A. And, they are of quality p 0, and the size is n. If you look at the number of items that comedown this way, and if I actually workout the fraction of items the number of items which are defective items in this strain. Let us see how we work it out on this side, on the average if n items came in and I inspected them, I have N times P A. That is a chance of my going this way and that is got a lot quality of p 0.

Now, look at this side, I have here lots coming in, they are getting rejected. And, they are of course, all of them, they have been rectified. So, any items coming this way must all be 100 percent on this side. Unfortunately I have drawn a sample, I have drawn a sample and I have looked at the sample, and I have replaced any defective item that I have found in the sample. Therefore, what I have here? If you look at the quantity here, I have the average quality that is going out this way that if I indicate by this quantity called AOQ.

Average outgoing quality this way, that is going to be P A multiplied by big N minus n multiplied by p 0 divided by N. Let me explain to you, what is going on here? N is the total number of parts going through the system, once I have done rectification. I am not returning anything to the supplier .So, the rejected lots they got rectified, they came along I end up with some items here. And I end up with some items here. But the total number of items coming this way is going to be N items.

Now, the once that have been accepted those are not been rectified on this strain .On this side I have removed all the defective items but on this side I did my sampling. And, I accepted them. Now, if there were any defectives in the little n samples, I remove those defective items .And, I therefore, those n times p 0 those quantities have been removed.

So, the actual number of defectives going this way is going to be big N times p 0 multiplied by P A minus N times p 0 multiplied by P A. This is the number of defectives that I let go this way. Therefore, the fraction that actually comes out as the result of doing all this AOQ. AOQ at the outgoing level, this is at the average outgoing quality living the inspection, both this area. This quantity turns out to be this quantity there.

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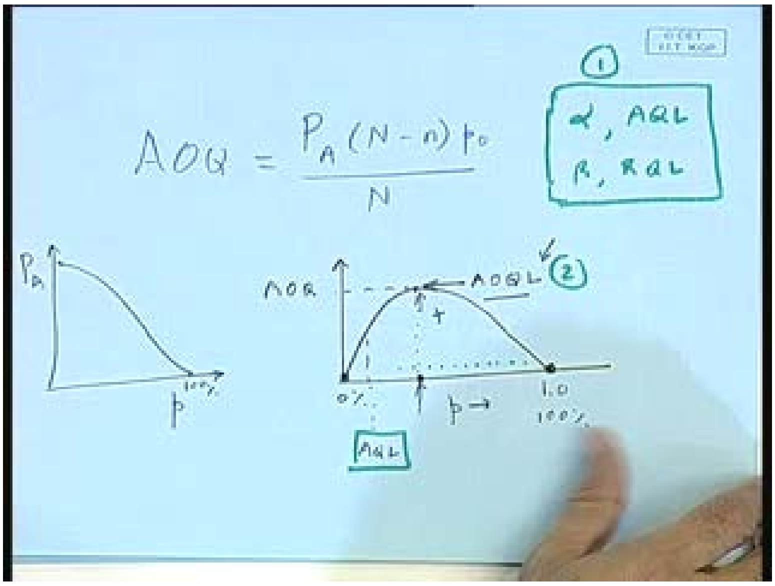

Now, couple of things I would like to show, now let us say I write down my AOQ figure. As this AOQ is equal to P A times N minus n multiplied by p 0 divided by N. Now look at this quantity and let us use a black pen, look at this quantity now. And remember alongside I have got my OC curve also, my OC curve goes like this. I have got p here I have got P A there, and the OC curve goes like this. It turns out when my little p is like 100 percent that means everything is defective, when this is equal to 1.

P A becomes 0. Therefore, the product of this and this product of this is, it would be 0. So, this AOQ quantity if I plotted the AOQ figures here. Now AOQ I have little p there I have this little p equal to 1. That means everything is 100 percent defective, that is the worst possible thing and here it is 0 percent defective, this p rises this way. So I get more and more defective part as I go this way, that is lot quality, it turns out when little p is 0.

When little p is 0, that means this quantity if you look at it, when this is 0 no matter of what this value is? AOQ is going to be 0. So AOQ will have a 0 value there and AOQ will have a value again 0, because P A becomes 0 at this point in time. So even if this becomes 1, this is 0. Therefore, this is also a point that is for the AOQ quantity.

Now, you look at all the other quantities here, this is a positive quantity, this is a positive quantity, this is a positive quantity, this subtraction, this difference is positive, this is also positive. Therefore, I have got only positive quantities there; this is of course a positive quantity. So, this curve between these two extremes must have some positive value, there this is positive which actually says that if I am using the rectification scheme.

If I am using the rectification scheme at some point, I will have the worst level of defects being passed. And this quantity, it can be this quantity, it can be determined. And also this quantity can be determined. Now, sometimes what happens? The consumers would like to tell you that there is a limit beyond which I cannot accept any kind of defects in the lots, there is a limit there and this limit here is called AOQL, Average outgoing quality limit. So, no matter of what inspection scheme? I am using AOQL actually tells you the worst performance.

The worst performance of that sampling plan and many times consumers would like to specify this to you, instead of giving you the, what we call the AQL? Remember AQL was there, AQL you might think, that the consumer would be quite happy saying this to you. But the problem is sometimes, we are also accepting lots which are at these quality levels. And because of that the consumer comes along, he says I am going to give you a quantity called AOQL. And please do not exceed that, no matter of what you do.

Therefore, this is like another criterion by which I could design my sampling plans. I could of course, use the, I could use these things. I could use alpha AQL beta RQL, I could use these constraints, I could use these constraints to design my sampling plan. And, the other so this should be like one way. The other way would be to specify AOQL and then come up with the plan.

I am going to show you a method that uses this technique. I am going to show you that, these are the same calculations which are just shown here in different schemes.

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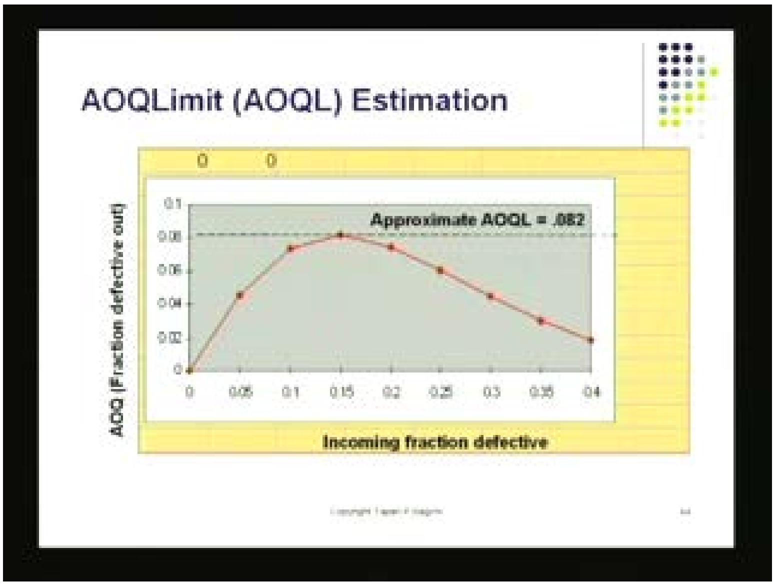

And, here is an example of AOQL and this AOQL was specified by the consumer. And therefore, he gets now 8.2 percent defectives in his items. That is the worst quality level, he is going to experience if the same sampling plan is applied again and again .And here again is the display of AOQL.

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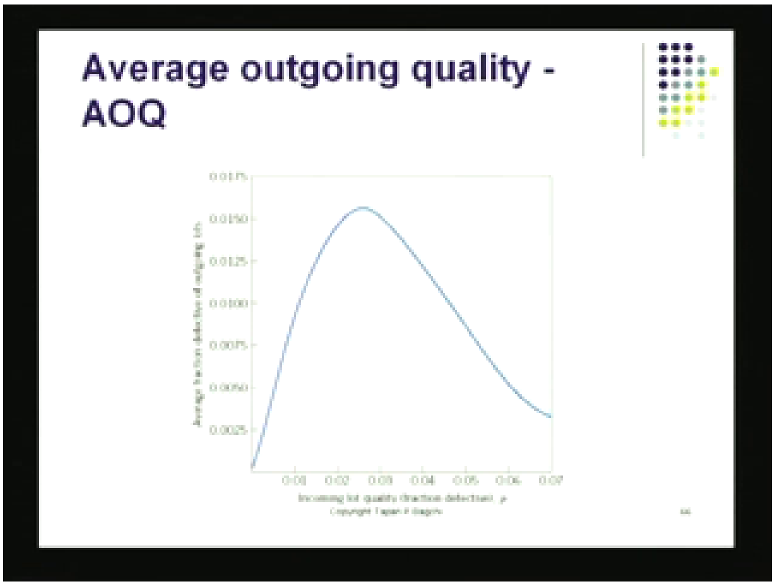

And, these curves can be drawn quiet easily. Because I remember, I have the formula; I have this formula here with me. And I have this OC curve with me and with that I can calculate the different point on this. And I can determine what that AOQL quantity is? That is quite easy to determine.

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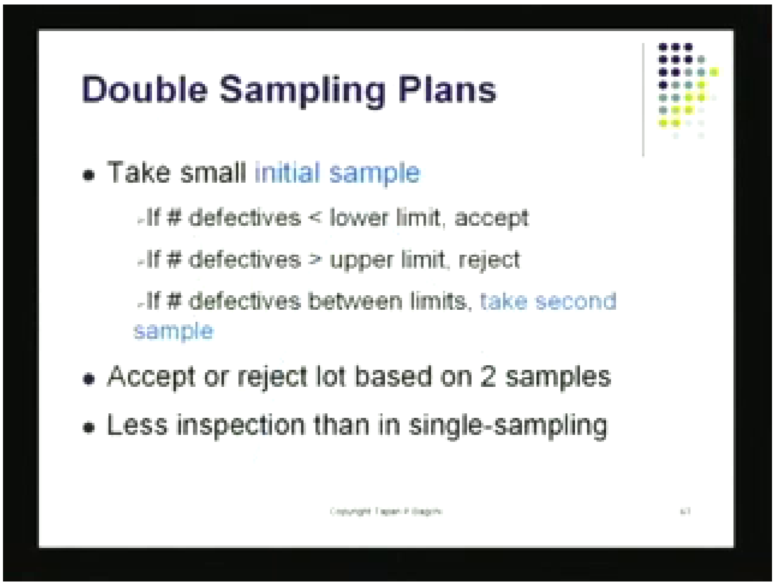

And of course, double sampling plan would be an extension of what we have done so far. I take an initial sample and I use the initial sample to come up with one kind of decision. I may be able to accept the lot, I may be able to reject the lot .And I may not be able to do that when I do double sampling. What I do is take a second sample, so I have taken a first sample. But it did not lead to a decision; I was stuck between these two decision points. Then I take a second sample and based on the result of the first sample plus the second sample I may decide to accept the lot or to reject the lot.

This should be the way to do your double sampling plan. What the double sampling plan does beyond the single sampling plan is? The double sampling plan generally gives you a total lower on the average, it gives you fewer items to sample, that is one of the advantages I am using the double sampling plan.

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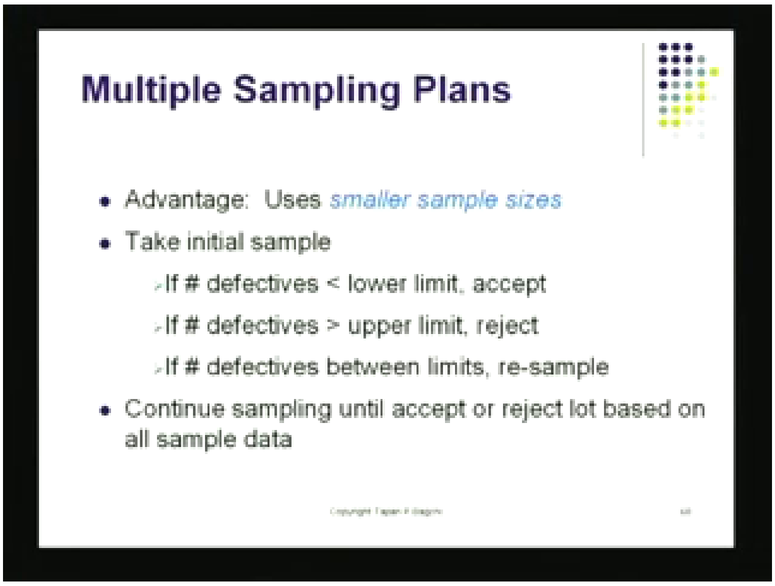

So, it gives you generally speaking a total, a smaller total number of items. I could generalize that I could go to multiple sampling plans, where the rules become slightly more complicated. But the follow essential the same procedure take one sample, check it out. If the number of defectives found in that sample is between the limits where you cannot really decide, you have to take on to another sample so on and so forth. You keep doing this, you take the first sample, second sample, third sample and so on at some point in time, you are going to make a decision either to accept it or to reject it.

Now, this may seem to be a little complicated to you, it turns out. To implement these plans is somewhat of a complexity, because many times these procedures, they are not clearly understood. By they can understand and they can many of them they follow the single sampling plan without any trouble at all. But the movement you bring in more stages of sampling, you go to the double sampling plan, the multiple sampling plan and so on. Then it does become a bit complicated to implement on the floor, but what you gain is? You have to do much fewer. You have got to do far fewer inspections, that is what you should be able to do?

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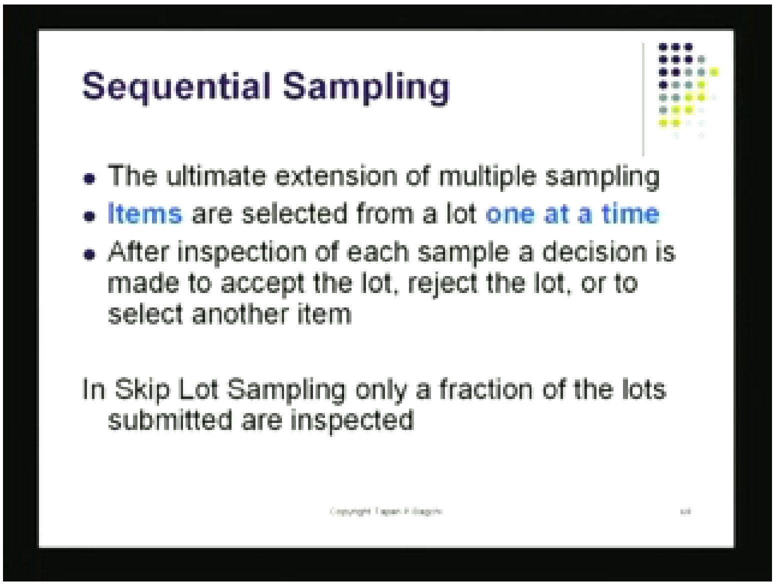

Then, there is something beyond this which is called the sequential sampling plan. And, I am going to kind of just give you a hint of it there, what we do is? Instead of taking like one lot of sample, which is like maybe of first sample will consist of these items? And then the second sample will consist of some more items. There you put them all together, that is your second item, second sample. I have got my first sample, and then I have got my second sample and so on.

Then I tried to decide something on the basis of that the sequential sampling plans does. Then something different, it again works with the same lot; the lot has been supplied to you. What you do is you pick one item out of the lot and you take a look at it, do the inspection and record it as good or bad. Then you take a second item; make sure the record is kept there. Take the second item out and inspect it, to inspect, to see, if it is good or bad. You do that.

Then, you take the third item again, you are, so what you are doing is? You are doing one item at a time. You looked at the first item, you looked at the second item, you looked at the third item, and you looked at the fourth item and so on. You keep going like this, till at some point, you have to make a decision you end up with a decision and that goes like this.

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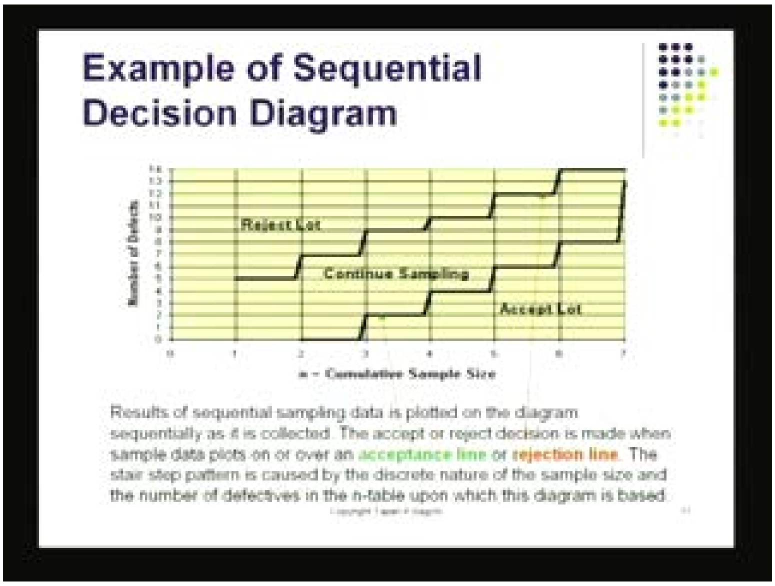

If I show you a curve there it will be not easier to see. Notice, here on the x axis I have got the number of items sampled on the y axis. I have plotted here the number of defectives found in this sequential sampling plan. You start with one item and you basically, you got two control limits here .One is the acceptance control limit and this is like a straight line, which is been drawn here, as a curvy stepwise kind of progress there. I am going to show you, the other one also in a minute.

And, then I am going to get a rejection line also, if the trace of your sampling and results of the sampling, if this stay within these two limits. You continue to sample, the moment it goes either this way or this way, you decide to either accept the lot or to reject the lot.

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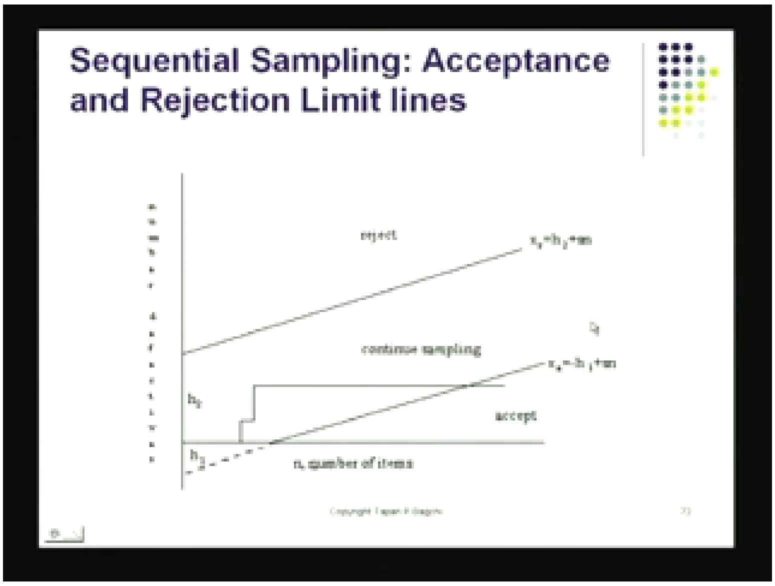

Let me show you, what it looks like in real world? I have got a continuous line here. And this is the acceptance region, see by doing sampling. I end up going here, I accept the lot if by doing sequential sampling, I accept going somewhere here at some point in time. I reject the lot, but if I keep going like this inside it is always going to be rising. I keep doing sampling.

But there is a theorem by Wald, which actually tells us Abraham Wald, he told us you will never end up going forever in this direction. You will either end up going that way or you will end up going this way. This is a very powerful method and the formula is here, these lines are given as the straight equations, straight line equations.

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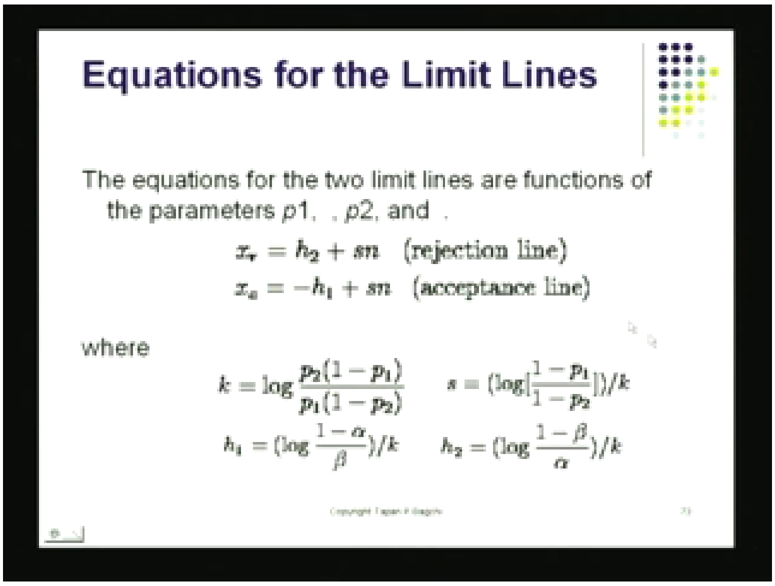

And there parameters are given by this expression. Here, you got a rejection line which is the lower line in the plot. And you got an acceptance line which is the upper line in the plot. So, if we go back to the line. There this is the acceptance line on this region. In this area you will accept, you end up coming here like this point. Here you will accept the lot and if you end up going there, if you go end up going here. You will end up rejecting the lot those are determined by this. And of course, the parameters to go into this equation, those are given here by k 1 h 1 s 1 and h 2, what is the advantage in doing?

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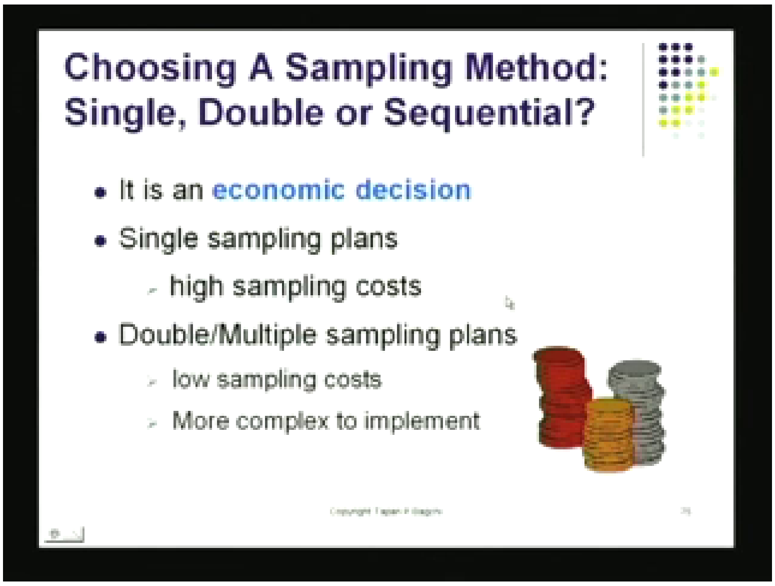

This is the first thing is the OC curves turn out to be very similar to what we done before. But the beauty is that you end up with for fewer items to sample. When you do one item at a time, when you keep picking one at a time. Your decision is reached much earlier, because this is a sequential likelihood, this is a sequential likelihood of finding so many defectives in a sequence. And that is a something that is got much more power, something like that you could quiet easily do and we will follow through. Now, with the design of the actual sampling plans and I will be showing you a couple of methods there. Thank you very much.