Six Sigma
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Lecture No. #17
Measurement System Analysis

Good afternoon, it is Tapan Bagchi again. I resume the talk on six sigma, this is the series of lectures that we have ready for you, and the specific topic that I will be covering today is the analysis of measurement systems.

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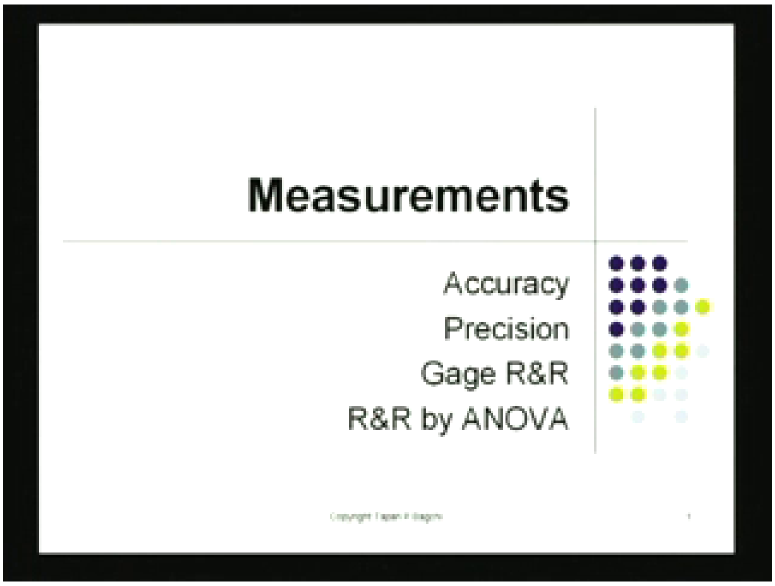

As you all know, whenever we have to take a decision in regard to quality we really make some measurements. And I have a device here, this is a electrical device and it is supposed to be making some measurements that is are relevant to us, and what it would really produces is DATA. Most of our decisions are actually data based, most of the decisions in the assurance of quality or the management of quality and the control of quality; these are all data driven, data based.

So, therefore, we must have really good measurement system same place in order that the data forms the sound bases for us to make our decisions. Unfortunately in situations, most of the situations the data is coming from the process into this system, and then into the form of the data that we observe. Now, this system itself it can introduce errors for us, and the idea of measurement system analysis is to try to take a look at the role played by the gage and also the operator who is using the instrument in producing the data.

And there are some methods that I will be discussing today. Try to make sure you eliminate from the data that you have at least, you have a good estimate of how much errors being committed by the gage. And how much of it is committed by the individual the operator.

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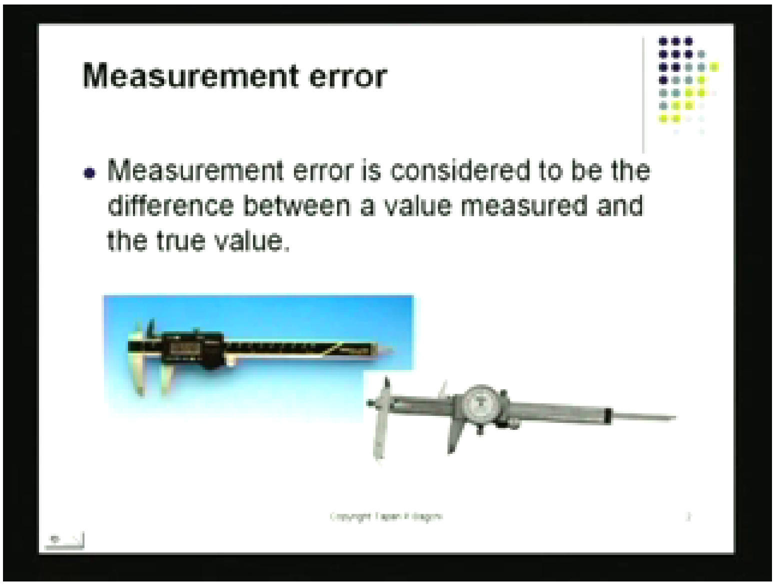

So, we begin on lecture with the point is today would be I will be reviewing, I will be giving you some idea about what is the accuracy? What is precision? What is this phase called gage R and R and we will try to find an idea. We will try to get some idea about how to go about finding that gage R and R? It is a number that we find and we always must have an estimate of this number in order that we can again make our decisions in regard to controlling quality on a sound manner.

Now, measurement error like I just said, these are considered to be the difference between the values that you observe, that you measure and the true value of the item. That is what really is what measurement error is and this would be there. It depends what kind of instrument you are using of course and who that person is who is making the measurement. Therefore, any time we are using an instrument keep your mind open about some possibilities of some error measurement errors being there.

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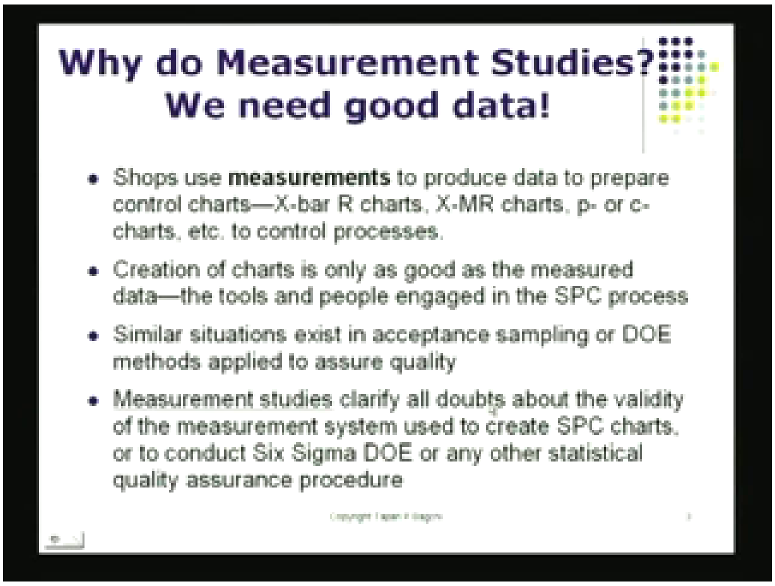

What exactly are we talking about? What we are saying is, we know all this tools in quality assurance. For example, we got the X bar chart, we have got the R chart, P chart. These are using in statistical quality control, then we got the moving range chart or we have the p chart or c chart and so on. These charts are used to control processes, when they are online, when they are basically online. Now, creation of the charts is only as good as the measured data, this is very important. First to realize that unless you have got good data, the charts would give you wrong signals perhaps.

Also many times, we conducting design and experiments particular in this six sigma situation, under that d a m i i c d m a i c framework. In which there is one key element there that is called i the improvement. An improvement is essentially based and it uses designed experiments, as the basis for coming up with factor effects and interactions and so on.

And also it helps you to optimize the process. D O E is totally based on the data that you collect during the experiment. And there again you would not be collecting the data directly, you would be coming through an instrument, you will be coming through an instrument like this. Only then you will be getting your data. It is very important first to realize even if I doing an experiment perfectly.

Well, my instrument must be such that I have an idea, how much error is being introduced by this and also by the person, who is using this instrument? In regard to the production of the data that we that in this case, in this particular case would be the observed data.

Now, coming back to the slides again, measurement studies clarify all doubts about the validity of the measurement system. Because, we may have some doubts what the validity of the measurement system itself, not only in s p c chart, but also in D O E and any other statistical method. That we would like to use that is data based.

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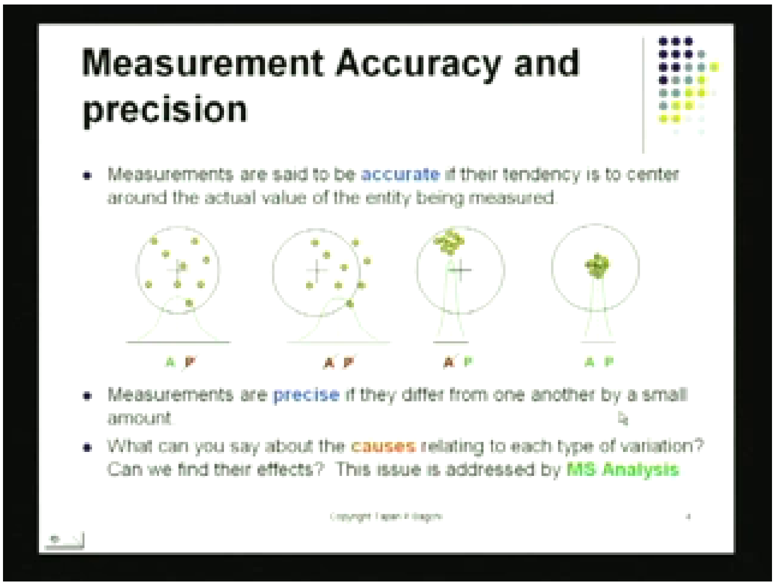

Let us take a look at these phenomena, these two phrases called accuracy and precision. Now, imagine that I have given somebody a gun and this gun is then passed on to other people and I asked them to fire at a target. Now, the both side is right at the middle each of these targets in the both side that is the cross, that is right to the middle and that is why they are supposed to be shooting.

Now, this person look at the shots that he has fired, it split over a wide area and some of the shots are even outside these specification limit which is like it is outside the disk altogether. Now, here look at the situation the average of the shots, that average is shown by this green line it is off target, the target is right there and the average is off target.

This is the situation when I say that the system, the system here lacks accuracy it is not an accurate system. Also look at the wild spread which is shown by this little bell curve here, we showed the distribution of all these shots. And again I say this is a wide distribution here, that actually implies that this system here also lacks precision. Take a look at this system, this system would else given the same gun and firing shots his average tends to be exactly at the target. So, therefore, this system is accurate, but again there is a wide spread in the spread of the wide spread in the positioning of the, of these fires here the shots here. Therefore, precision is poor look at this one particular person who has fired his shots very consistently in a very localized area, but he is off target.

So, here he is lacking accuracy because he is off target. But, his precision is good because most of the shots are pretty close to each other and look at this fourth person. Who fired his shots with pretty decent accuracy the average is right on target and also all the shots are very close to each other. So, again to review this if I do not have good precision I will have wide spread in the data. And if the average is off target I will also be lacking precision I may restore accuracy by cocking the gun slightly to the left. In that case I will be getting my accuracy, but if the spread is wide I will be lacking precision.

I will be lacking precision again if the spreads was wise wide. But, this one particular shot this third picture here has good precision, but it is lacking in accuracy. And this fourth performance here, has both accuracy and also precision. A measurement system also will suffer from inaccuracy and also poor precision. And we got to make sure to understand, how much inaccuracy is there and what we do about it and how much precision is there again what we do about?

It is very important first to realize that, we got to also understand the causes for such variation, wide variation and why this shots are actually off target. And this is done basically; this is done to try to make sure you done what we call measurement system analysis. If we do measurement system analysis, you will have a pretty good idea of how your measurement system is behaving.

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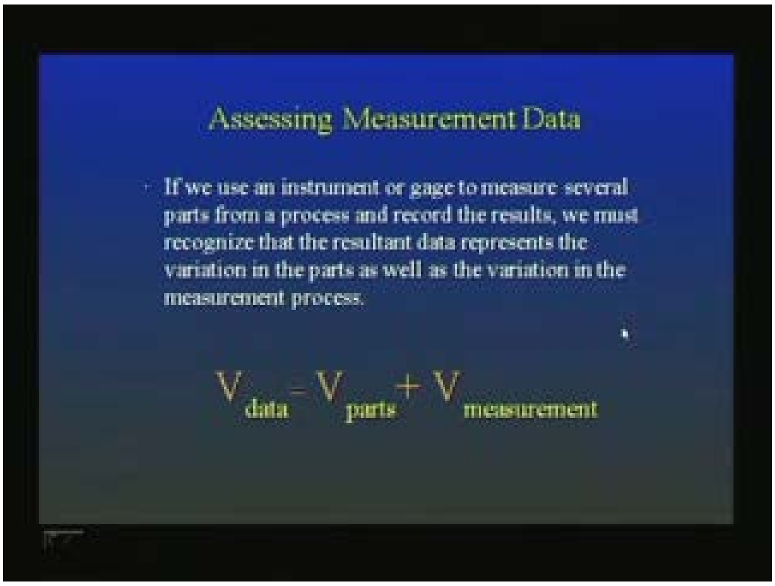

There are a couple of things that actually happen, when you are making measurements. There is of course, what you have really, what you really want to measure? And I show here that by showing that the process or the parts they have their own variability, no two parts are exactly alike. And therefore, we will have a variance that will be based on the variation within the process itself. When I am measuring the data, I introduce additional variation and that is actually the measurement variation.

So, the data that I really observe, data is what I observe. It is the variance of this whole data it can comprise part, the part variation and in part measurement system variation these two add up. And therefore, I really have data that has got inflated variability I would have really liked to get an idea of this, the true variation part to part variation.

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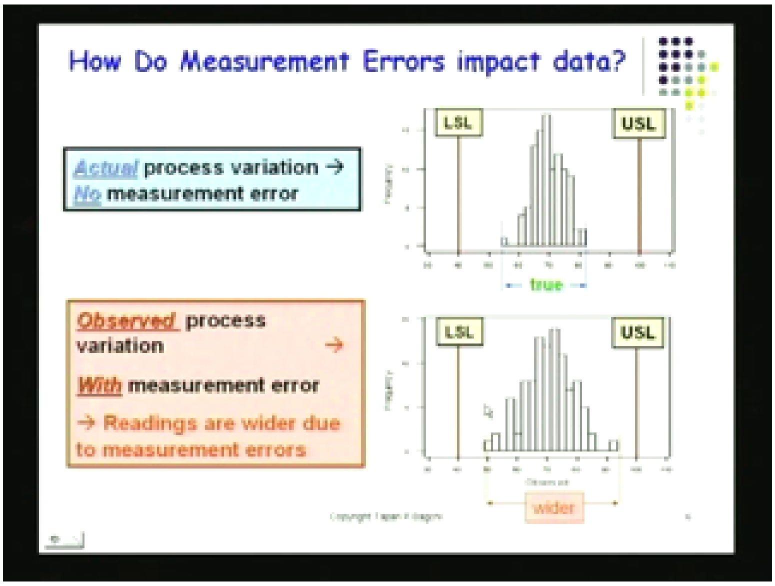

But, because of this measurement system error I end up with inflated data and that is actually something that is really misleading for me. So, I must have an idea of how much this is as compared to the true part to part variation.

What actually happens, when we make you; we have a system or you have an instrument that is not doing a good job, what actually happens there? Suppose, your true variation was as I show here. The true variation is I have got the upper and lower spec limits. And then of course, I have got what we call the variation that is within these specification limits, well within this specification. So, the true variations as you see there, it is well within this specification limit that is shown by this green letter there. And the blue letters they show you the range of the natural variation of the process.

But, when I introduce measurement errors the data actually gets wider, the data will have a wider spread. And therefore, the true value would look like this, but in actual factor smaller than this. Now, if I am using a measurement system I have got to make sure, that I have an idea of how much this in inflation is and that is the variation that has been introduced by the measurement system.

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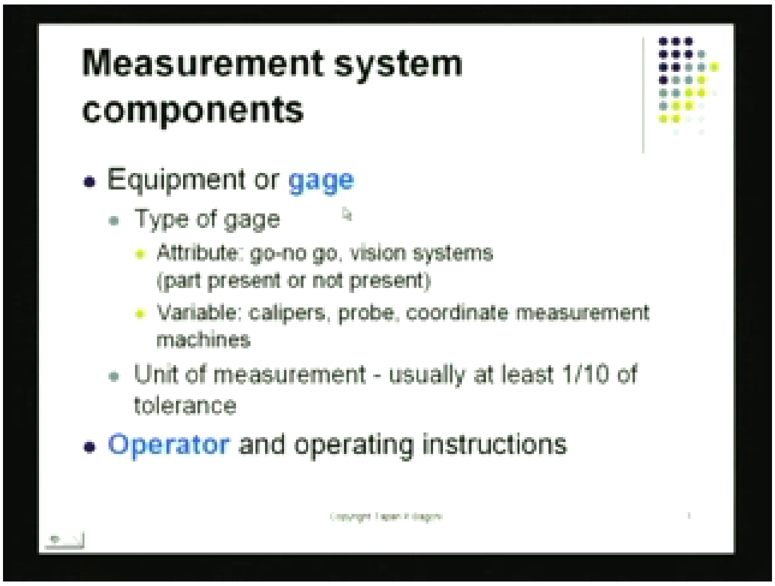

Just take a look at some of the components of what actually introduces variation the additional variation due to the measurement system not being perfect. The first is of course, the source of variation could come from the gage itself or when I apply the same gage I take the same gage. And I use the same part let us say some voltage that is coming out of a battery. And I try to measure that i n the system here, the gage itself may gage and the method of measurement, it may introduce some variation and that is actually what we call gage error or repeatability error.

So, if I repeat my readings I will find there is some variation in it. This is due to the gage itself person making multiple measurements will end up with that variation, which is the measurement variation. But, there is another component that introduces error in the data, it is the person, it is the operator who is using this instrument and that is what we call operator cause variation. So, in fact, if you look at the total data, there will be some part of the data that will be truly caused by the part to part variation. That is the true process variation that you are after, then there is one part that is caused by the gage.

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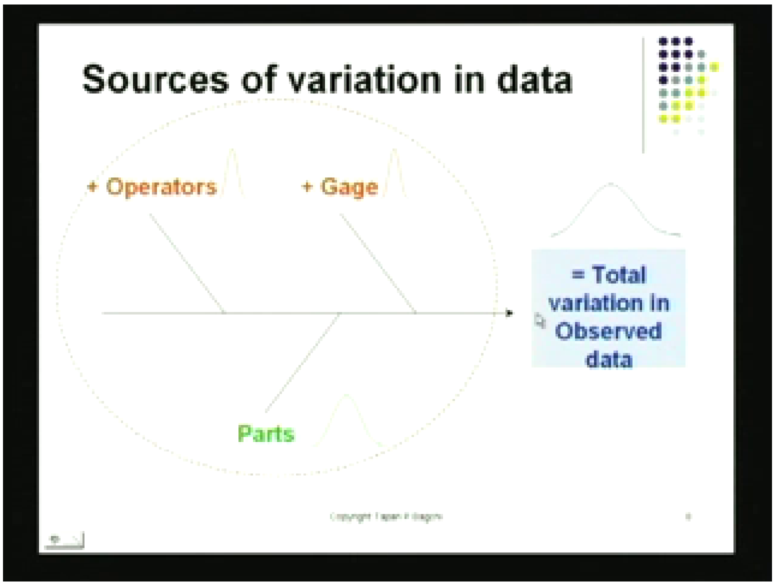

So, the gage introduces some error, because of the repeatability issues there. Then of course, the third component is the human being, the operator who is making using this instrument and making the measurement. He puts the object there and he applies the gage and he gets a reading there, there may be some errors introduced by the individual also this is what we call reproducibility. And that changes person to person, if we keep moving here and if we look at this total picture, the overall variation is over here and this overall variation is caused partly by the true variation the part.

And you see the little green well curve there that shows the variation the part itself; this is the true variation in the process. But, the gage itself will introduce some additional variation which is the gage variation; this is what we call repeatability. And then of course, the operator who is using the gage, he also introduces a additional variation and that is called reproducibility. So, in fact, the total variation would be part this which is the part to part variation or the error that is introduced by the gage. And part error that is introduced due to the operator. These are the different sources of variation in the data that we measured.

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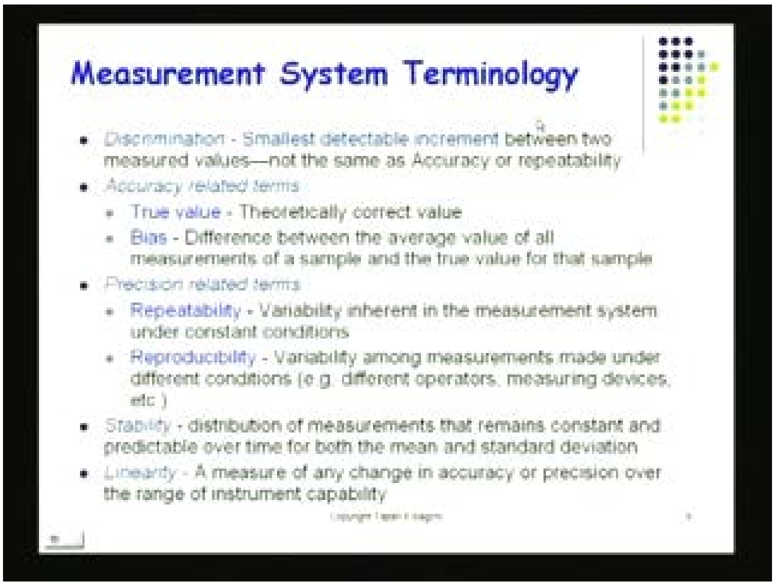

If you look at some of the terminology now, this has become a science measurement system analysis it is a science. And we have done, we have tried to understand what are the different terms that, we have to study what are the different aspects we have to study. And we have to coined our own terminology here, the measurement systems people these expert they have coined their own terminology. First of all there is something called discrimination, whenever you are using a gage.

For example, a gage that I have here there is a least count there is a least count there is a minimum value, there is a minimum value that you can measure with this gage. Of course, this is a digital gage, this is much better than the analogy gage that we had in the older times. But, this itself would come a with different amounts of different extends of least count, least count is the discrimination. That is the amount of voltage or there is the amount of say resistance or current strength that you would like to measure that you can measure with this gage there.

So, that one particular input the very first input is of course, what we call the discriminating capability of the gage itself, then you got some term that is called the true value. True value is the theoretically correct value that is actually the part to part variation that is the true value of the data that you are trying to measure.

So, if I get the cursor here, you would be able to see what that thing is the true value is the theoretically correct value, then there is something called Bias. Bias is the difference between the average value of all the measurements that we have done and the true value for that sample. So, there is a difference now between the average that you produce as the result of multiple measurements and the true value. There is a difference between these two and this is called the bias in the system, in the measurement system. Then of course, we have got something called repeatability.

And repeatability is actually is the issue of using the same gage and basically applying the same gage again and again. This would be done by the same individual. It would be done by the same operator and basically what we will end up doing is here we apply the same gage and we will end up with different readings and those readings would be caused about by the gage itself. And this is the issue of repeatability. Then there is the issue of reproducibility and that is actually the issue.

When we have different people using the same gage and coming up with different measurements. That also is something that ideally we should really reduce. We should try to make that next to 0 and that is also something that we would like to make sure that we have an idea about. Then as you are applying the gage the reading today may be slightly different from reading next week or the reading a month later and so on. That is because there is some drift in the system and that happens because of a variety of reasons.

So, in fact again measurement, measurements that you produce the data that you produce may have a drift and the measurement system if it has a lot of drift then it is not a stable system. This is also a problem then of course, we have got the issue of linearity. Many times what happens is when we got the process in place the accuracy or the precision it may change over the range over which you are trying to make the measurement. So, for example, linearity can also be an issue with a measurement system that you using. It is very possible that is not linear in the full range of the measurements that are possible to be made with this gage. That is also is an issue that you would like to take care of.

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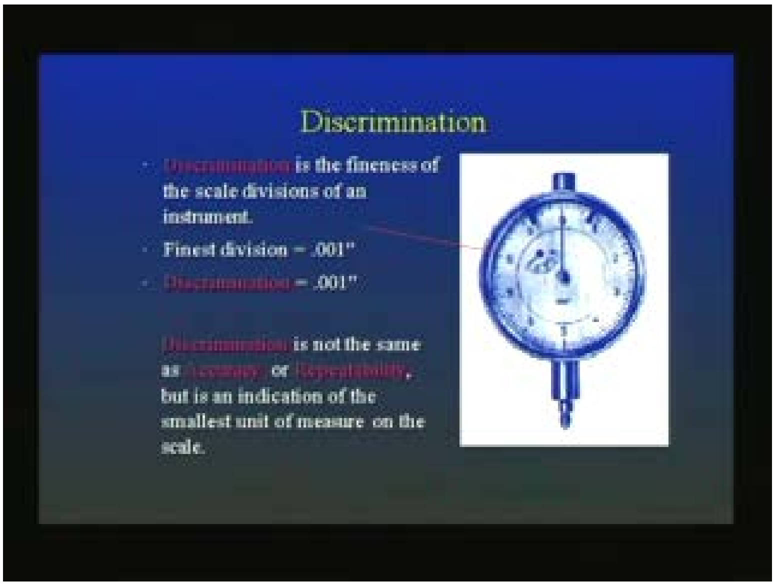

So, discrimination. What is discrimination? You take any gage like a gage that I am showing here in the screen. There it is the least count that will tell you. Can I discriminate between two readings which are at least it will be the least count of the thing? That would turn on to be the least count or the discrimination of the object itself and that would give an indication of what the discriminating capabilities of this measurement system.

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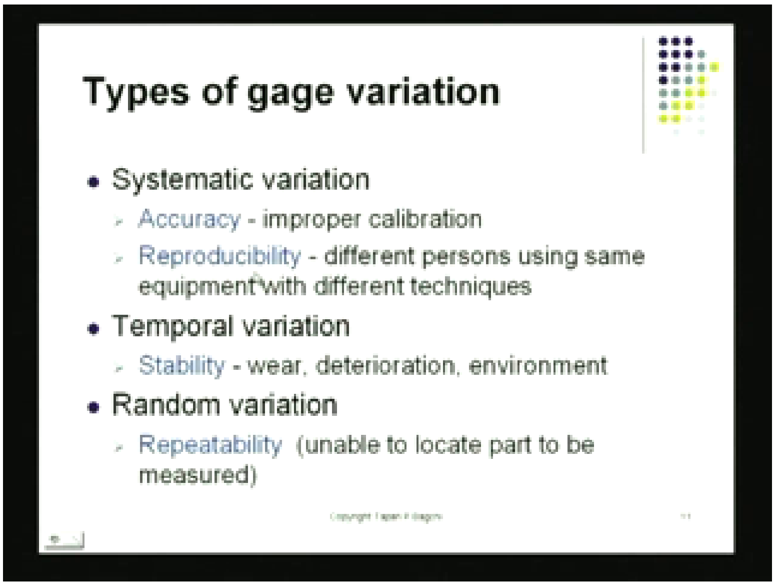

There are certain things that are called systematic variation and things that fall under are those would be accuracy and also those would be reproducibility. Accuracy I have already discussed and the way to fix accuracy is when you do calibration. When you do calibration you try to adjust for accuracy that is like something that you do. Then of course, is the issue of reproducibility which would change from person to person.

So, there again I got to have some idea of what is the extent of reproducers portability in the measurement system? Then there is the issue of this temporal change which would really give some indication about the wear and tear that might be taking place in the total measurement system. And that would give an indication of how stable the system is? Then of course, we have got some factor called the random variation this is captured under repeatability.

Now, when we look at a measurement system we do try to get an idea of is it properly calibrated? So, you try to get an idea that the system itself is accurate. It can provide if I look at the average it will provide me current readings. Also I would like to make sure I have an idea of the reproducibility, this is a variance measurement, this is not an average measurement it is a variance measurement. I also make variance measurement when it comes down to repeatability which is like repeat readings made with the same instrument. And of course, the issue of stability is also there when the reading itself may change a little bit as time goes by.

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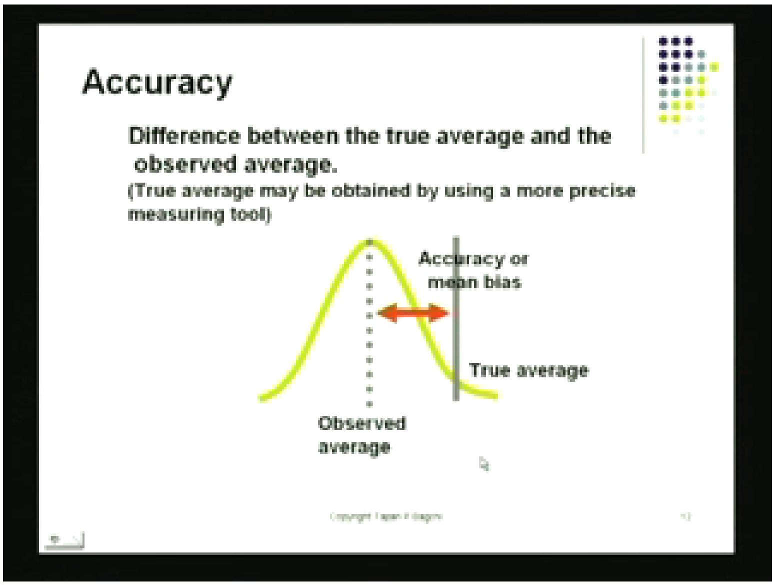

Let us try to get a good picture of what we were talking about. The observed value I have a true value there which is the true average of the process. Then I have got this average value. This average value is the observed average value. The moment the average value differs from the true average. I say there is a bias in the system or the system is not accurate. So, that is an indication of inaccuracy of the system.

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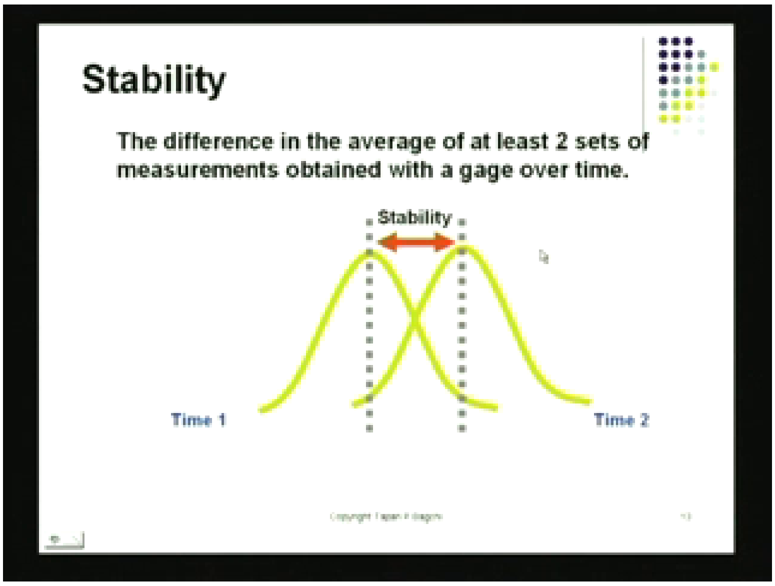

Then we got something called stability. If I make measurements at time 1. I get one distribution of data and the average is the average is average reading is at a certain point. When I make the same measurement, I measure the same object at some other point in time again I get some variability due to gage variation and due to the issue of reproducibility. When it turns out that the average also is shifted and this shift is taken place over time. This is an issue of stability or instability. This is also something that we would like to get an idea of when I am using a measurement system.

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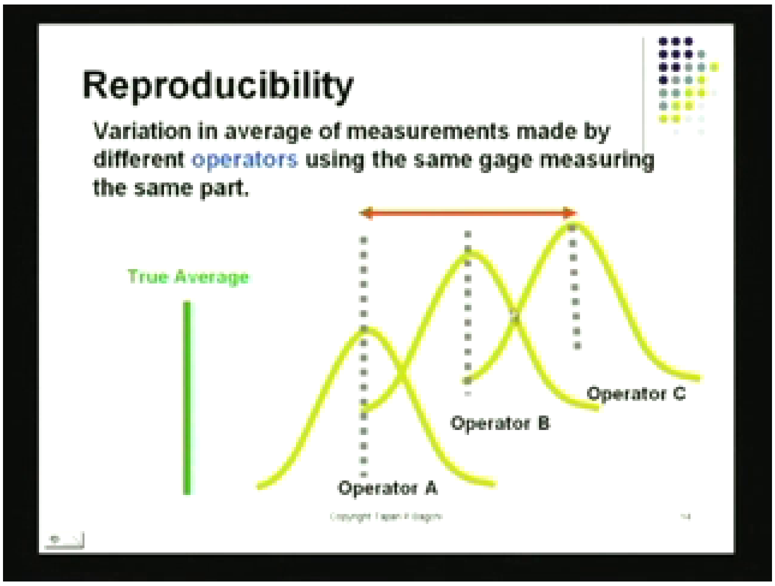

Then of course, this issue of reproducibility. Alike I told you reproducibility is basically it hinges on operators. So, operator A has given me these readings and the average is here. The true value is here of course, operator B has given me these values and operator C has given me these values. And look at the difference of the averages produced by these three different people and this difference here is attributed to what we call poor reproducibility of the system? People have a large effect in the actual data that is produced.

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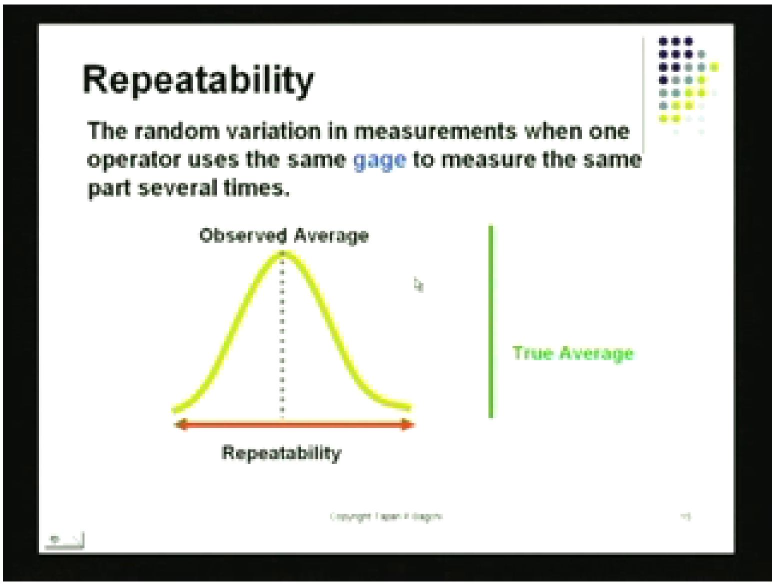

Then of course, this issue of repeatability. This is actually caused by all kinds of random factors that might be affecting the gage it is to be some slight variation temperature gagging positioning. So, many things are there because of which the gage may produce slightly varying readings even if I measure the same current or the same voltage or the same resistance. That is also something that we would like to get an idea of.

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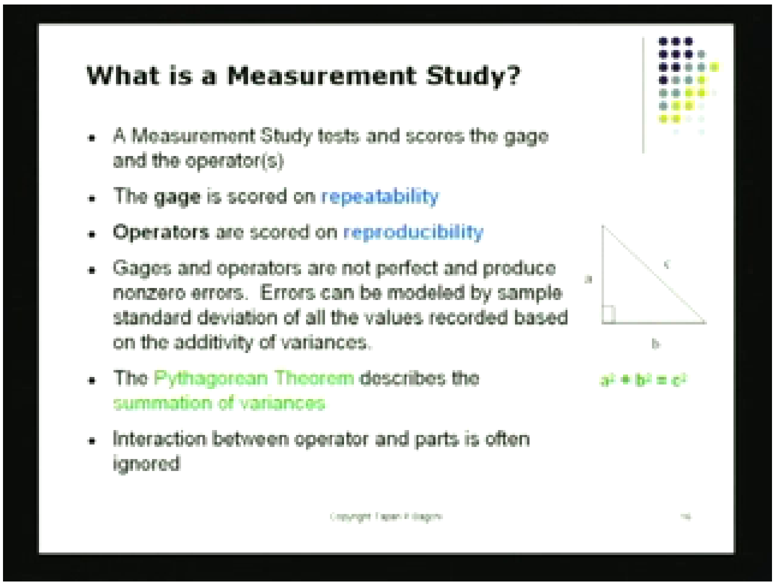

Now, there is something very interesting of course. In most cases these different factors one of which was reproducibility, the other was repeatability. Many times these different factors they do not their effects, do not interact and that is a really really good thing for us because what it does is? It apply it helps us it allows us to utilize principles that we learnt in back in high school.

The Pythagorean theorem that actually sees this that the total variation and I am going to be repeating this several times. The total variation I am just going to call it c square is the sum of component variation. One of which is a square plus b square is equal to c square. So, this could be repeatability and this could be reproducibility. This is the total variation produced by the measurement system. Ideally of course, what we would like is would like to have this to be reduced to 0 and also this to be reduced to 0. So, that the measurement system itself contributes no error in the system. This is what we would like to be able to see.

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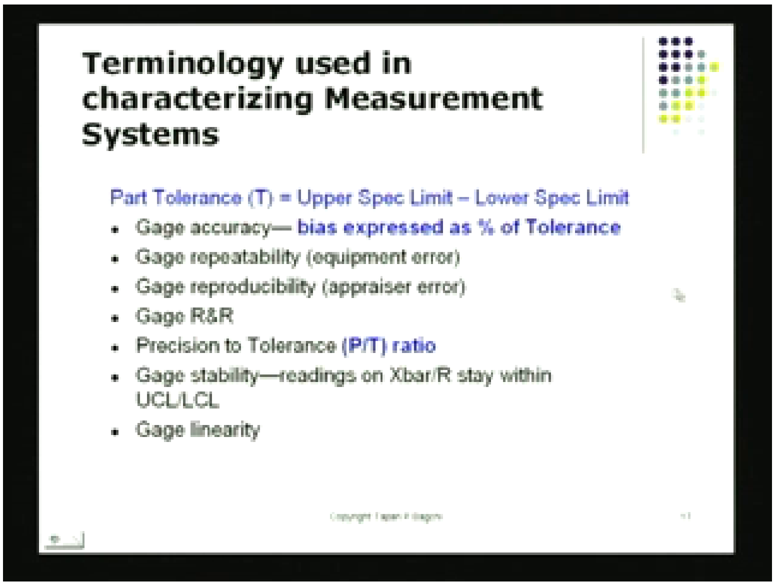

Now, can you really find a perfect gage you know it may cost you a lot of money. In many cases when I am doing regular production and manufacturing some widgets and objects and so on. I make some half decent profit, but probably I cannot really afford to have top quality instruments. That we just cannot afford to have that. So, many times what I do is? Industry in fact, I accept this that they will allow a certain amount of precision to tolerance. What is tolerance now? Tolerance is the difference between if you see the slide, tolerance is the difference between the upper spec limit of the part that I am trying to produce and the lower spec limit of the part.

This basically this difference is what we call the tolerance. The bias itself is sometimes measured as a percent of the tolerance. If this bias is high this percent will be high and of course, your data would be biased. And the data would not be reflecting of the true variation or the true size of the parts of the true dimensions of the parts, that is why we would like to have this bias, as small as possible would like it to be a very small percent of the total tolerance. That is the difference between the upper and the lower spec limits, then there is something called precision to tolerance ratio precision.

Precision believe me, the first part was just the difference between the averages that is what I that is how I measure my bias. But, when I measure my precision I look at gage repeatability that is like one variance component and also I look at what we call gage reproducibility. This is another component of variation and these two together, when I got gage repeatability I have got gage reproducibility I have got two components, two contributors of variance.

This is what leads to imprecision and that is measured by what we call P and this ratio of precision to tolerance this also should be quiet small, we will see later that industry has set some norms there. Then of course, if am using a system, that can shift it is performance over time I may produce some control charts and that is to be done with some standard objects. I may take some standard objects and I may measure this the dimensions of the standard objects at various points in time.

And I may produce a control chart with this and the control chart can tell me, if the measurement system itself is drifting or if it is staying stable. That is something I can also do quiet often by plotting various types of control chart that is a pretty decent approach. In fact, to try to make sure that the system stays the same way as time goes by.

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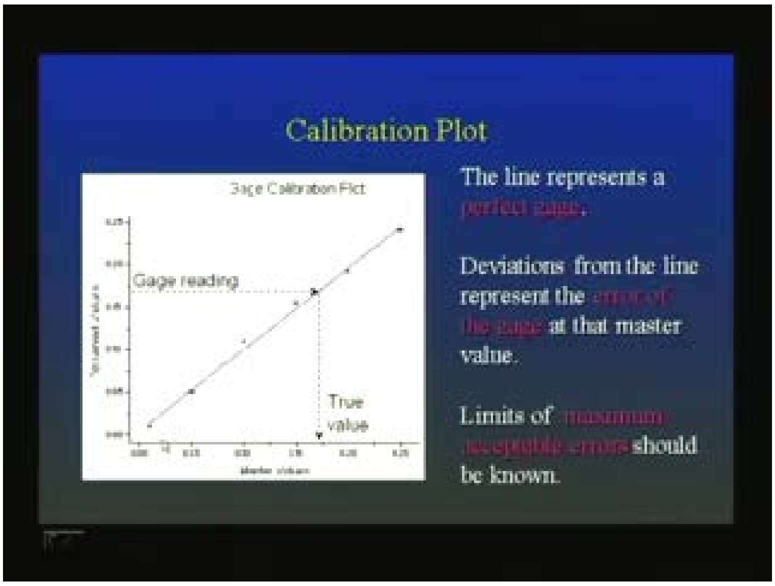

Calibration of course, is a way like I mentioned, it is a way to make sure you have some idea of the inaccuracy in the system. Inaccuracy is what it is the difference between your observed average and the true dimension. The difference between the observed average and the true dimension this is the inaccuracy of the system. And this is something, that you would like to eliminate and for this what we do is periodically, we will take a gage I would like to get it calibrated. So, would submit to a standard lab and those standards people, they will you know make the settings the way, they are supposed to be and they will apply some standard voltage there.

And they will take a look at the gage, at various settings, at various readings they will be checking with different amounts of voltage applied. They would take a look at the readings there, in the process they produce what we end up calling the calibration chart. Which is what you see on the screen there? That is the calibration chart. I have master values or the true values of let us say, voltage that I applied and the readings, the observed readings is what I apply there. Now, obviously ideally of course, this line should really at a 45 degree angle, but most of you know systems have really not in proper calibration all the time.

There is some inaccuracy there and, because of this there might be some variation there, also you see an issue of nonlinearity also. To try to take care of that, we do calibration that is like something there that is invariably done, when you have got a good gage. And a good gage that is being used for research purposes, it must have a calibration chart with you. Otherwise the data that you produce using this, it may not really end up being a good piece of data there. And there may be questions, about it is validity. That is like something, we would like to minimize to the extent possible.

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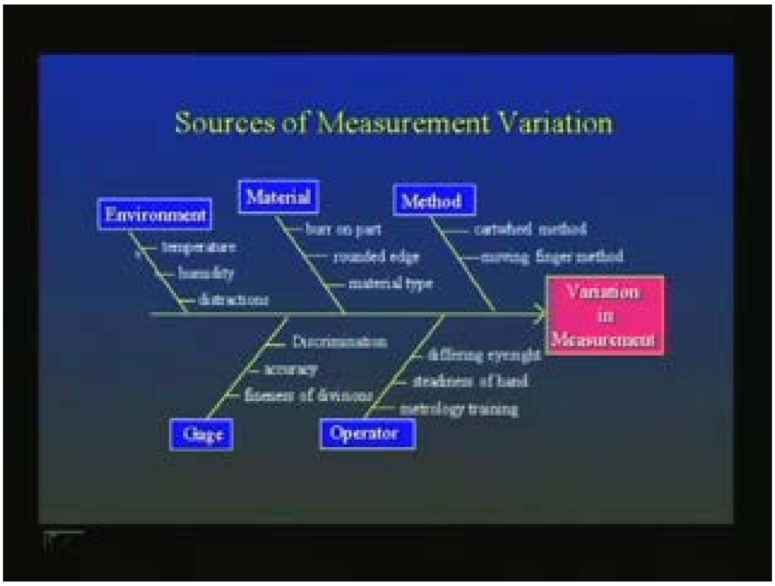

Now, there are many different things that can lead to measurement variation. I have got a few things there, I have got the environment, I have got materials, I have methods.

Methods of priority and doing the measurement itself, then the gage itself may introduce variation and that would be the issue of repeatability. And of course, then I have got operator who introduces the issue of reproducibility. So, you see there I have listed the various factors that could lead to variation in the measured data.

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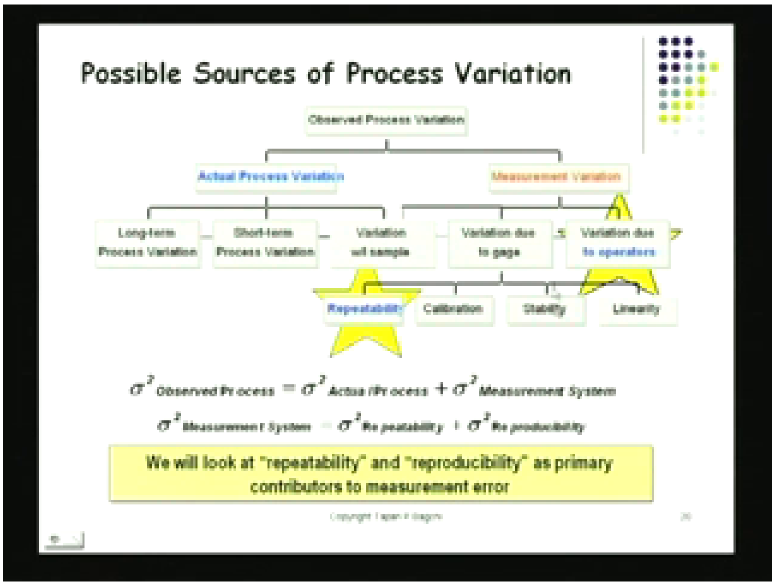

And I have got this is another picture of it, but what I would like to point out to you is that the concerned, that industry has most of the time is the issue of repeatability which means. If I apply the same gage on the same object again and again will I get the same reading. The other issue that industry worries about the issue of reproducibility, that means, if I use different operators to do my gagging.

Will they produce numbers which are all pretty close to each other? If there is a wide variation between going to operator, a to operator b to operator c that measurement system is actually that measurement practise is not probably a good one. Therefore, what we would like to do is, would like to get an idea of the total variation in the observed process, which is part due to the actual variation in the process. And part of the variance that is introduced by the measurement system.

And the measurement system variance itself can be split now, between repeatability and reproducibility what we would like to do is? Would like to take a look at; Now, a closer look at repeatability which is caused by the gage and reproducibility which is caused by the operator in regard to the total variation. That is there now in the observed data.

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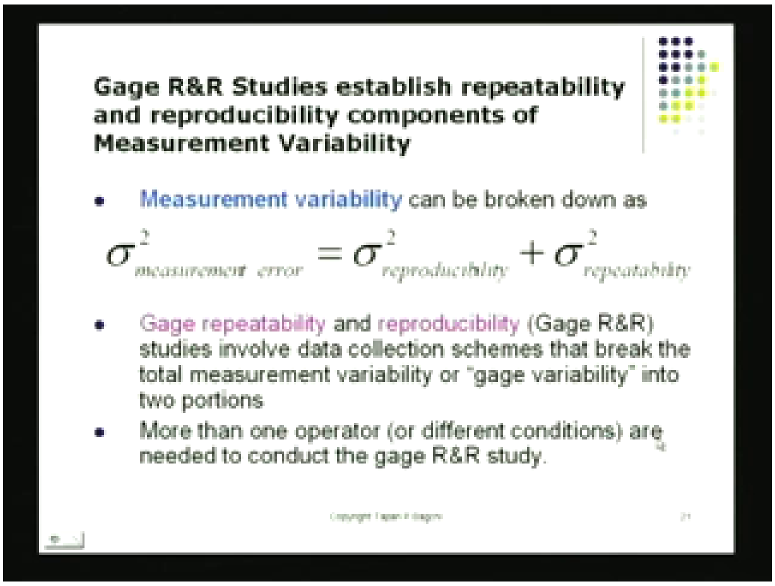

Now, this is another way to state that Pythagoras theorem, that we that I mentioned to you earlier the total variance of measurements. This is the total error, the measurement error that I see the total error, that is contributed part by reproducibility the variance due to reproducibility and also the variance due to repeatability. So, I have got reproducibility, which is stress able to people the operators and repeatability which is stress able to gages.

So, I have got really, if you look at the components of the measurement system. I have the true process, I have got the standard objects, I have got production objects, which are being produced and I have got to make some measurements on them. I have got to make sure I got get the understanding of the of the real variation itself.

So, I have these objects that are being produced by the system. Now, how do I get the knowledge of what exactly are the dimensions of this or some quality characteristic? I apply some measurement system. What does the measurement system comprise? It consists of the gage and it consists of the operator. Now, there is some variance that is contributed by the gage and this is captured under repeatability. And there is some variance that is contributed by the operator and this is captured under reproducibility.

And I have got to quantify both of them, I am going to show you calculation procedure by which you can find the contribution, that is the variance contribution that is due to the gage and the variance contribution that is due to the operator. I am going to show that show that to you in a couple of minutes.

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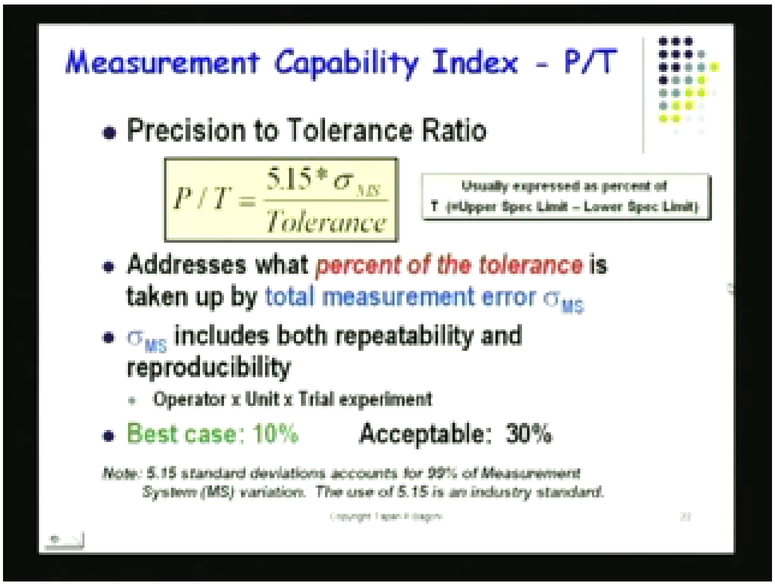

Again here is an idea of the kind of industry accepted norm for precision to tolerance ratio. Tolerance as I told you earlier, it is the difference between the upper and lower spec limits of the part that you are trying to measure. The values that is introduced by the measurement system that multiplied by 5. 15 sigma multiplied by 5.15. That actually captures a pretty large span, almost like 90, 95 percent or perhaps. Even more of the variance, that is there in a measurement system that is totally captured. If you do 5.15 multiple by sigma M S, M S is the measurement system.

So, that gives you actually an idea of the variance that is introduced by the measurement system itself. Divide that by the tolerance and this ratio should be of the order of 10 percent. The best case picture for this is 10 percent, unacceptable limit in some industries 30 percent not in all industries. But, some industries will accept this precision to tolerance ratio as high as 30 percent ,what does the precision or imprecision comprise? It comprise partly the measurement error which consists of repeatability and reproducibility and into also be due to stability and so on and so forth. We shall focus here between two things, on two. In fact, we will focus on two things, one is going to reproducibility the other is going to be repeatability

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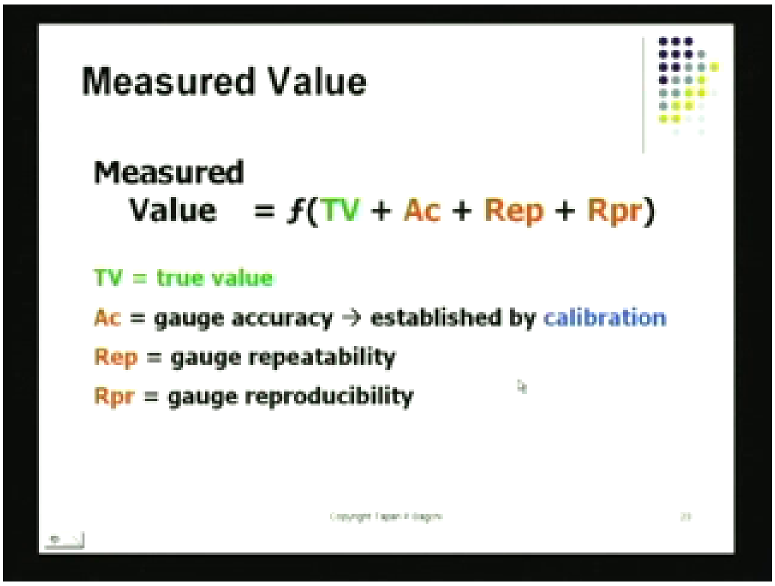

If you look at the measurement value, if you look at measured data that measured data consist of the true value. It consists of the inaccuracy which is really fix by calibration and then I have got repeatability and I have got reproducibility, these are the different factors that impact the measured value. This is the observed measured value and the true value is here. Now, unless I have a decent idea what these are I will never know what the true value is? Because, all the time I am observing is this.

If these values are unknown and if they are large, then of course, I might just think that I have got an idea of the true value, what I am an actual fact I do not have that because of these hidden things, which are also there inside the measured value.

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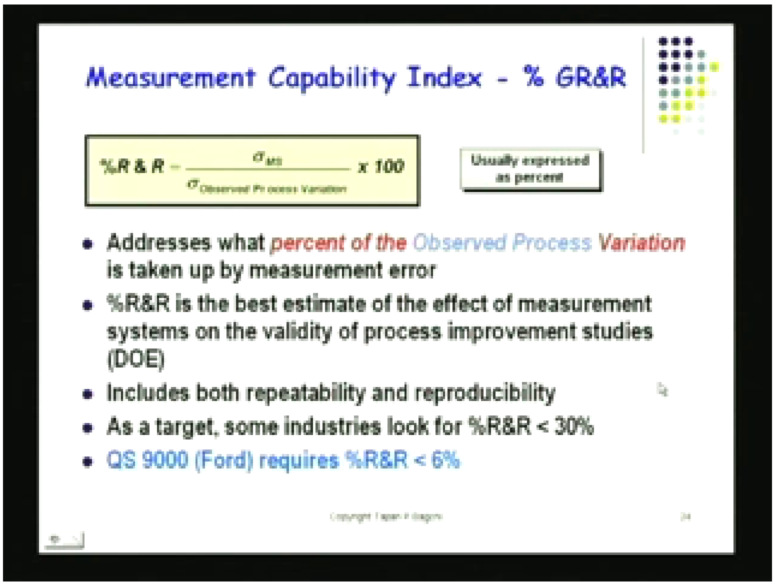

Again let us take a look at capability this is like another measure of gage R and R or the measurement system, which are there. And one of the way is to that you take the standard deviation of the measurement system, multiple that by 100 and divide that by the observed process variation. Observed process variation is now part to part variation as observed by through a gage. If this turns out to be small, it actually means that the measurement system itself is actually contributing very little of variation. What you seen in the observed data is mostly is it true part to part variation, this is what we would like to see.

In fact, it turns out, if you look at the Q S 9000 system some of you might remember, that the automotive industry, automotive parts supplier. They must be using, they are required to use the Q S 9000 system, to be able to supply to ford g m and Chrysler and also to other companies large truck manufacturers and so on. It is it is actually held by the automotive industry, those who really get Q S 9000 type part supply. That this ratio, this capability should be 6 percent or less, that is the extent to which I can allow the measurement system to be slight layoff otherwise, I really will not I be should not be using that measurement system there.

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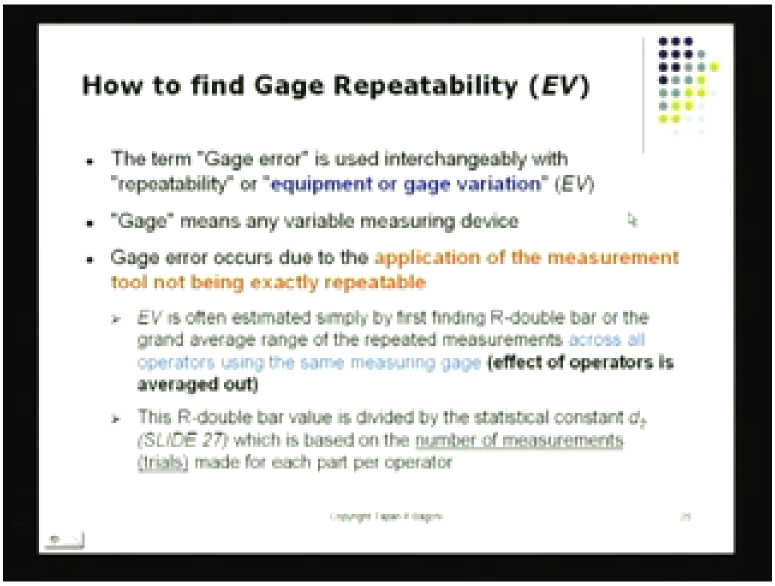

Now, how do I find the repeatability as an example? Let us, take a look at how we find repeatability, then we will also take a look at how we find reproducibility? And you can see the slide here, which actually is using now E V is the notation for what we call the variance due to repeatability. This is, these are the different readings are obtain by the same gage by the same operator, there are some variation there. And that whole thing is quantified as variance and this is called the equipment or gage variation.

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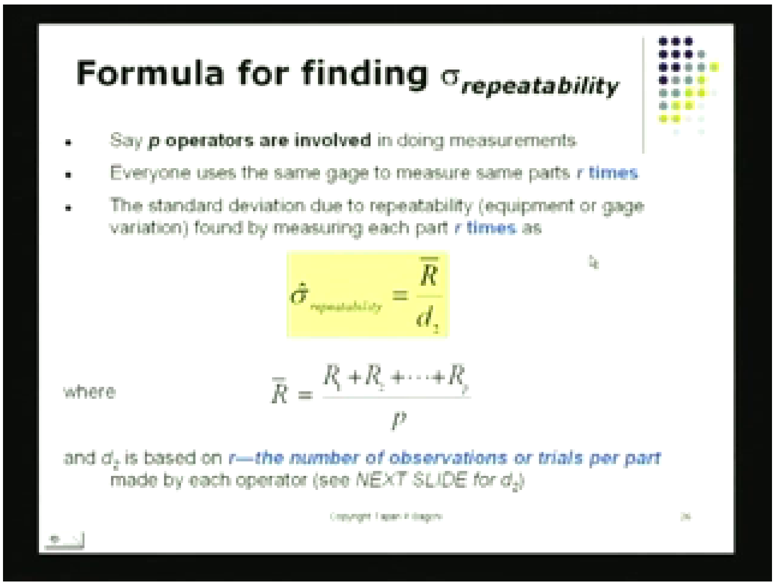

How do I actually find this? I will be showing you a couple of points here, I have remember my old picture was like this I have a total variation.

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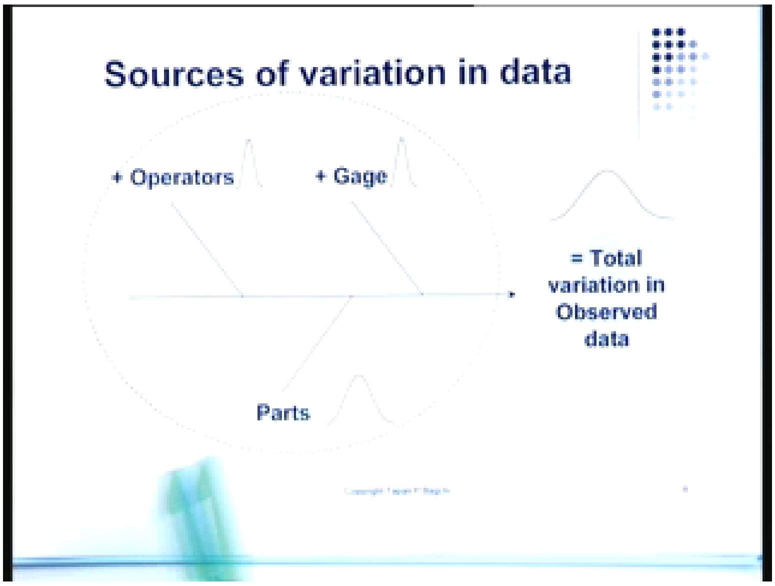

I have this total variation, which is due to partly due to the operators, partly due to the gage and partly of course, due to the true part. These three they contribute to this total variation what ideally I would like to see is, I would like to say this total variation to be comprising only a part variation, but what happens in real world. I am using a real gage which is got some repeatability problem, I also got an operator a bunch of operators, they are contributing reproducibility variance.

So the, what I observed there is partly due to this, partly due to this and of course, then I have got this true variation part to part variation also there. Right now, I am going to be telling you how to get an estimate of this, this is issue of repeatability.

Let us come back to the screen here again and I have here what we called repeatability. I notice the formula shows here that sigma, sigma is the standard deviation in gage to gage measurement, repeated measurements application of the same gage. What is this quantify call R bar to the right? I am going to show you some real data also. Therefore, you would be able to get an idea of the real return right away.

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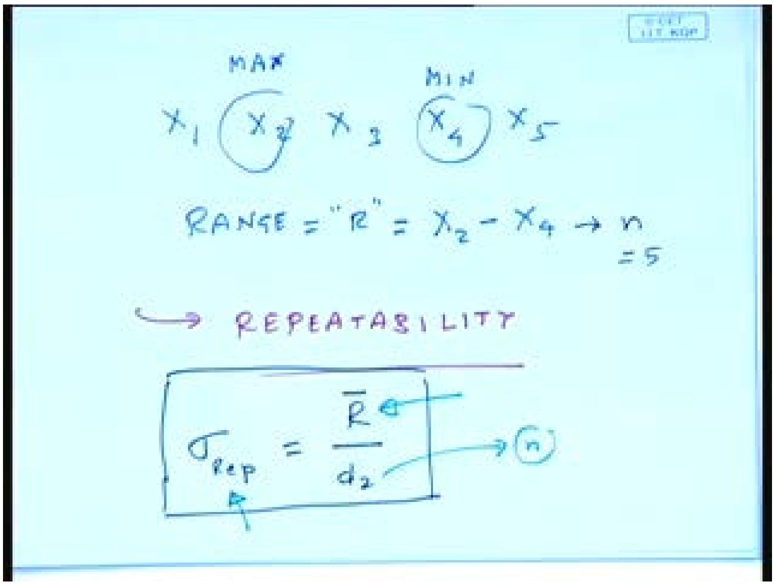

But let see I have made some measurements, with the same gage I have made some measurements and I am the signal operator. I have made some measurements using the same gage and my readings are x 1 x 2 x 3 x 4 x 5 that is I have made five measurements. Now, range, range which is also called R of the data that I measured here, is the difference between the largest one this is the max data value and the min data value.

So, what does this R comprise now x 2 minus x 4, that is the range of my measured data and if these data, if they are repeated application of the gage on the same object I have information, which is really going to capture my repeatability. So, this is going to give you repeatability, (No audio from 34:25 to 34:33) this is now application of the same gage, on the same part. I apply the same gage, on the same part repeatedly and I take readings and I write them down. Those readings turns out to be x 1 x 2 x 3 x 4 x 5 as I shown here, these readings they are individual readings, but they have slight variation and that variation is really due to repeatability.

So, this is an issue here, I what did I find here I found the value of R. How do I get sigma out of R? I am just going to call this sigma repeatability, this will be equal to you take many such readings and you take many such samples, you will end up with r 1 r 2 r 3 r 4 and so on. So, you have like an R bar, R bar is the average range that you found in many such measurements, that are done parts are different parts, but again each part is measured five times or n number of times. And from that you find your R, how many times you measure is very important because that will you an idea of n, n is the number of times you have measure the same parts.

So, here n in our case is equal to 5, to try to find sigma what I do I divide that R bar, which is the average of repeated, such repeated estimates of the range I divide that guy by quantity called d 2, d 2 is quantity this is the formula. This is the formula which allows you to start from range, it has allows you start from quantity called range. Which is this quantity I need to allows you to determine this quantity of course, the unknown here is d 2 what does d 2 dependent on? d 2 is a constant, d 2 dependence on this is dependent on n, n get number of times you made measurements.

Let me show you how this is done I have the formula on the screen here. And notice here I says sigma repeatability is equal to R bar divide by d 2 this is the R bar. Now, they are the average of every part has been measured a certain number of times and I have taken an average of that I have got this thing. Now, suppose I have got p such R bar is found then of course, I will have R double bar therefore, I have gotten I have gotten estimate here, which is the average R bar. Once I have that all I need is the value of d 2 and d 2 dependence on the number of times made by measurements on each part.

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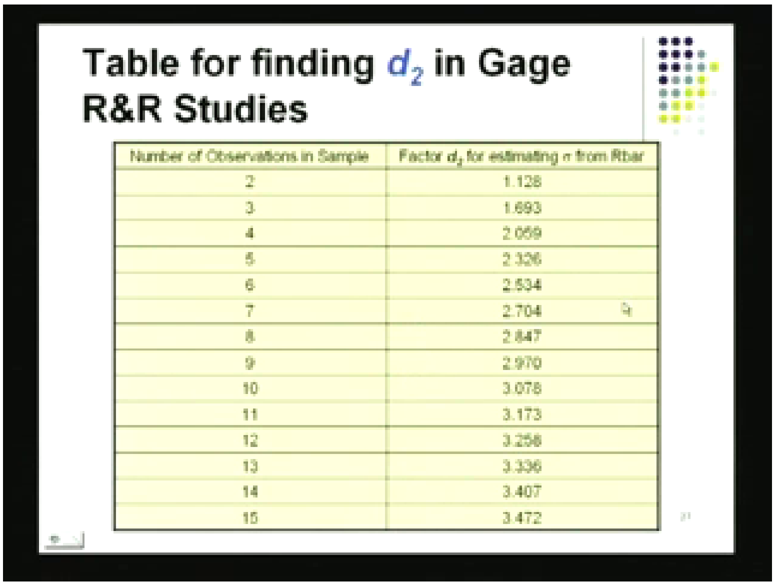

Table for finding d_2 in Gage R&R Studies

Number of Observations in Sample	Factor d_2 for estimating σ from Rbar
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970
10	3.078
11	3.173
12	3.258
13	3.336
14	3.407
15	3.472

So, when I do that, I actually have a table, I have a table and this table I am showing you here, the number of observations in a sample, what is the sample? A sample again like I mentioned to you in the chart, that I was showing you the diagram that I was showing you x 1 x 2 x 3 x 4 x 5, there are five measurements there. Therefore, the number of observations are made here is 5, n is equal to 5 there and that put down here. This n is used on this chart here and I used that right here and I read off the value of d 2. So, d 2 in our case is going to be 2.326. So, the value of d 2 here is going to be actually 2.326 that.

And of course, I have got my R bar on top, this is going to give me an idea of sigma. Sigma is what am opted, what is this sigma due to? This sigma is due to the gage, this is sigma is due to what you call repeatability, that is what this is for. So, now, you have an idea how I can go from repeated measurements to the calculation of R bar. Then I used the information about d 2 to get my sigma. This sigma is a very important one it gives me an idea of what is the amount of variation, that is actually contributed by what we call the gage apply repeatedly on the same object.

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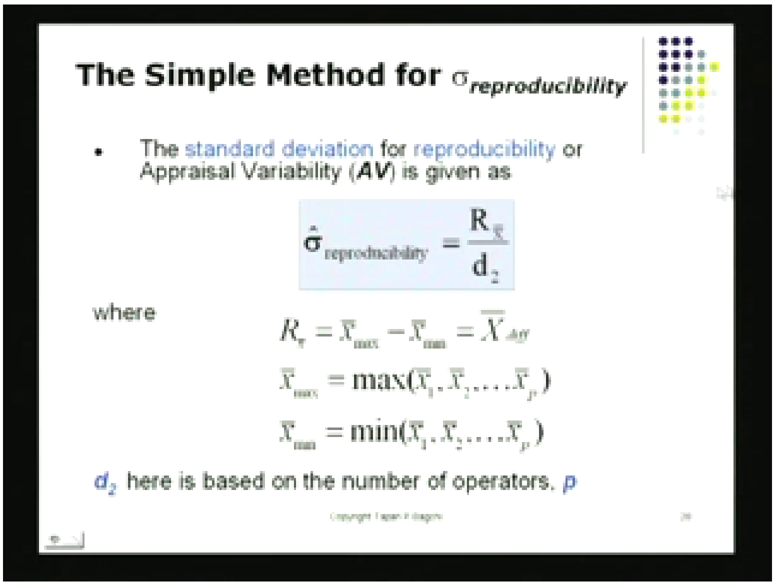

I have now the other dimension and in this case what I have done is now I am talking about reproducibility. And try to recall reproducibility is cause by the operator repeatability is cover by the gage, it is cause by the gage and reproducibility sigma cause by the operator. Now, one person using the same gage will probably not introduce much of an error that is captured under what we call repeatability. But, if the multiple individuals and they using the same gage and producing different readings then, an issue of reproducibility, people are not able to reproduce what somebody else did I must have an idea of that also.

So, for that part I do I set up a table and I get different operates I am going to show you that tablet in a minute. I am going to ask different operators to make measurements and from that I try to again calculate averages. Average for each person, if these averages different from each other, again I have got a range of data and again I can utilize the same formula or the same d 2 type formula. And I am going to get an idea of what I call this time sigma reproducibility.

And that is given by the formula which is on the gray box here, the gray box basically says, this standard deviation for reproducibility and this also called appraisal variability. Because, appraisal is the inspector, appraisal variability is A V, which is equal to this, you look at the standard deviation of that is equal to R x double bar. And I am going to tell you exactly what that extra divided by d 2, this d 2 is different from the d 2 that I used before.

So, this d 2 should not be confused with the d 2 that I used before. The first d 2 actually comprise, it came from the numbers of times are repeated my measurements using the same gage I measured the same part n number of times. And here I have got different operators I have got p different operators and notice that I have got p here, p different operators have used the same gage. They produce readings what are the readings like? I have, I you know when I look at the readings, I look at the X bar value. And within X bar value I will end up with, if I show you the real data look at really good idea how I go about doing this. So, I am going to come back to this formula again I am going to come back to this formula in a couple of minutes.

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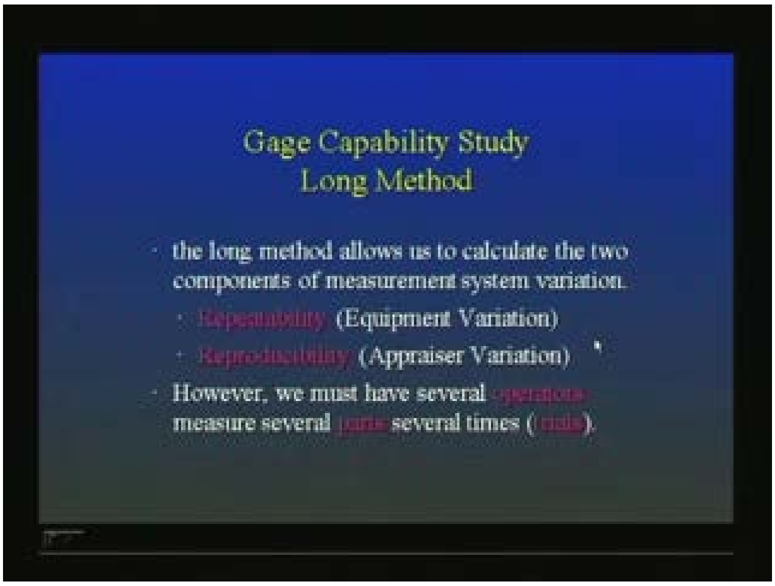

Let us go and see what really data looks like, say I have got calliper and I produce my data there.

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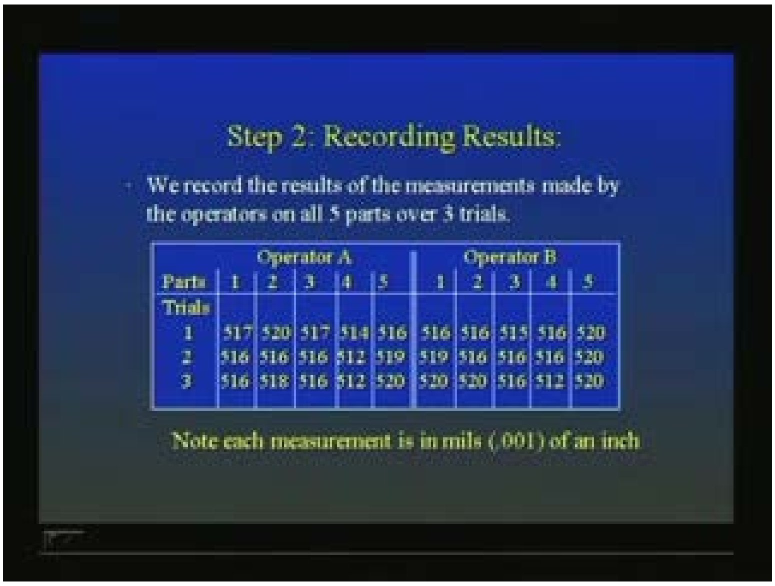

I use this and I apply my data and there is some data. Now, I am going to have a cleaner display of the same data there, just take a look at what kind of things we got in this, we got two sets of part. Actually is the same part I got five parts, part number 1, part number 2, part number 3, part number 4, part number 5. And i ask operator A to repeat the measurements on part 1 three times, repeat the same kind of reading on part 2 three times, repeat the readings of part 3 three times and so on and so forth.

Then I take the same parts and I know exactly which those, which actually those parts are, what is the identification? I know that. So, part 1 here is again given to another operator part 1 is given to operator B and he makes three measurements there and again. He makes three measurements of part 2, three measurement of part 3 and so on and so forth. Now, notice something here, notice a couple of things, there are variations in what we call reading to reading, there is a variation measured by the same operator on the same part. What would this be attributed to? This is the repeated application of the gage So, this will give me an idea of repeatability.

And now, if I workout the overall average of all this I will get an idea, if I compared that for operator A to operator B I will get an idea of what we call reproducibility. And will show that to you the movement I convert this in to a sheet, that I can start doing this thing.

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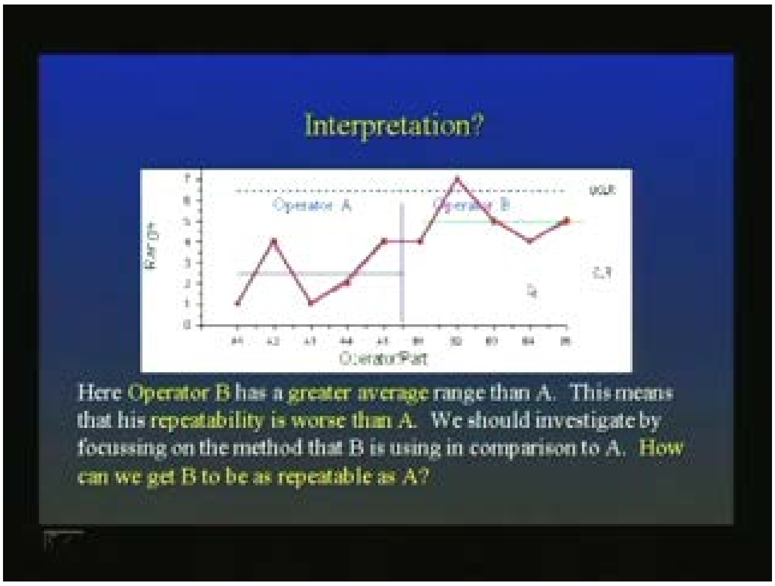

Now, this is just a pictorial view of what we showed you earlier, I have the data there in a table. Now, this is just a pictorial view of it, what is this now? What is this point? This is actually the range of measurements done on part 1, this is the range of readings taken by operator A on part 2, this is the range of and we are going to, where we find this? Let us look at this thing. For example, let us look at the first one, the first column is suppose, to readings taken by operator A of part 1.

And the range here is indicated as one, means go back to the data and look at the data again the max value of the readings say 17, 517 min value is 516 the difference of that is one. And that is exactly what I got here, I got one as the gage there and the reading there. Look at part 2 range is given here as 4. Let us go back and see what the data says I see the max value as 520 and the min value is 516 the difference between these two is 4.

So, the range here was 1, range here is 4, range here is 1 again, range here is 2, range here is going to be the difference between 520 and 516 that is 4. It is 4 here, 4 here and 1 here and 4 here and 0. The 5th data, the 5th set of data that is, these are measurements taken on part 5 by operator B it has no range. Just take a look at that chart here again, it has got no range, once I have this data this is the average and staying right there. Once I have this data I will take a look at some of the average is also. First of all I try to find out the range in each of these cases, I got the range calculations done and because the data is given here the range calculations are not going to be very different.

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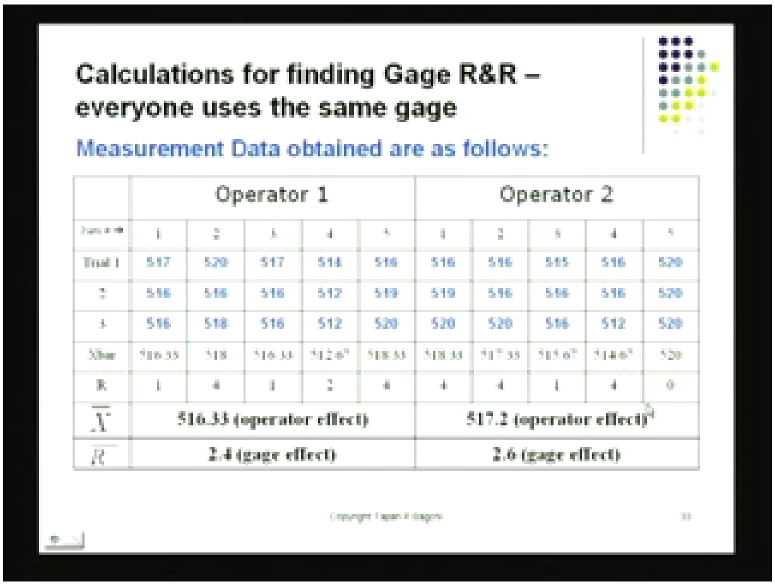

Now, if I look at this quantity here that is the average of the readings taken on part 1. This is the average of the readings taken out part 2, part 3, part 4 and so on and so forth. There is an the overall effect, overall value for this average. This average there is an overall average there and that turns out to be 516.33. Now, that average is the average which has now got a couple of things cancel out. One it has got the gage application, the application in a reading to reading variation that is applied by the gage.

That is that is produce by the gage. So, that has been averaged out by this and also have average out the different across parts. So, I have got this data here has been adjusted from both. It is got the part to part variation cancelled out and also gage to variation has been cancelled out. It is very very important that we do this. It is absolutely important, it is (No Audio Time: 45:47 to 45:54) it has got part to part variation that is there. Part to part variation has been neutralized by a crossing by cross and of course, gage to gage to variation has been also averaged out by doing this.

Now, this is the over average that is produce by the operator. It turns out to operator b produces operator 2 produces the different reading there. Now, the difference between these two readings is really attributed, attributable only to the difference between the two operates. Therefore, this would give me an idea now the variation that is attributable to reproducibility and we will take a look at that. The gage effect that is actually the average of the ranges there.

So, the average of the ranges is right there. I got the gage there, these reading there. I have got the gage reading here also. Actually if I average out these two, that 2.5 number is going to be the gage effect contribution and the difference between this number and this number that is going to be effect of the operator.

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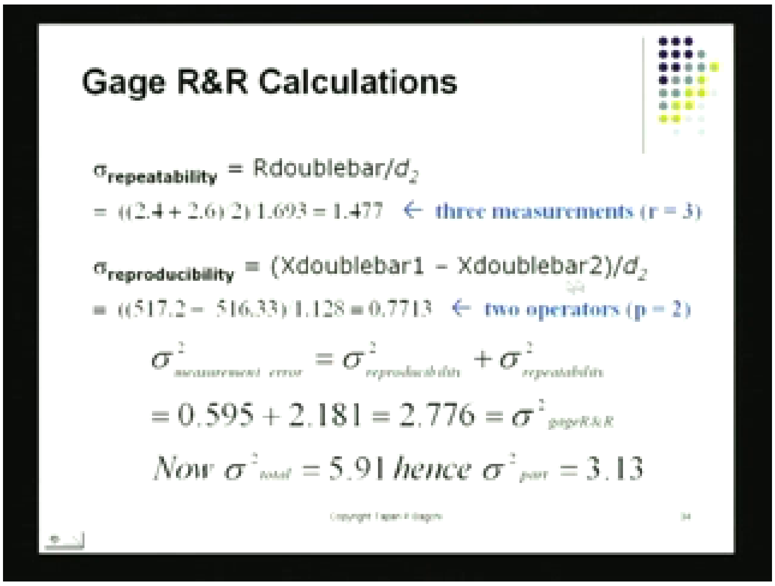

How do I do that? Let us take a look at the calculations done. Repeatability that has that is the contribution do to the gage and do the gage system itself. And I have two readings here and I will show you exactly where these readings came from. These readings came from the R value, R bar, R bar value the range value that I produce by the gage and remember now this is the average of all these different ranges.

And these ranges were there because in any each of these repeated readings I end up, ended up with a variation, ended up with a variation there, ended up with variation there and a variation there and a variation there. And I when I average these out I get the idea of the contribution due to the gage there. I do the same thing for the other part; I end up with 2.6 as the effect of the gage there.

If I look at the calculation here now, R double bar which is now the overall average on the range calculation there. The range calculation 1, range calculation 2. This is going to capture the effect of the gage and that is what I got captured there. When I that when I divide that by d 2. This d 2 would have come this time from r. Why r? Because I applied the gage three times. I applied the gage three times when I calculated one R, my R calculation came actually from the variation of these three gage gages there and that is what I got there.

If I go back to the calculation again these three measurements they lead to my estimate of my sigma repeatability. That is the standard deviation of repeatability. That is cause by the gage. Reproducibility would come by when I look at the X double bar on one side and X double bar on the other side. I divide that this is now a range. This range is attributable right now to the two operators. And therefore, this d 2 would have to be dependent on p equal to 2 and I go back to the table again and I can actually do that.

Let us take a look at d 2here, d 2 are the values that are used here is 1.128 and that should correspond to my value of n to be equal to which in this case is p to be equal to 2. Let us go back to the table there of d 2 and here is the d 2 table there. Notice there for sample size is equal to 2, I have got 1.128 and for sample size is equal to 3 which is the attributable to the gage it turns out to be 1.693 and this is what I have got gage, I have got shown there. I have for r equal to 3 or three sample, three sample of size 3 taken that turns out to be 1 .693 and that is what I got there.

Now, this is the sigma now that is attributable now to the gage and this is repeatability. And reproducibility would come the estimate for that is going to be this. So, in this case for I want to be at an idea of the total variance contributed by my gage which is the repeatability contribution and my operators which is the reproducibility contribution. I would have sigma square measurement error.

The total measurement error contributed part by the gage and part by the operator. And the those if you square these quantities there, square this quantity and square this quantity end up with 0.596 plus 2.181. That is 2.776. That is that contribution by the gage R and R which is the gage reproducibility and repeatability not being perfect. I realize this should be 0, but it was not so.

Therefore, the total variation now, the total variation that the total variation was found by going back to the gage, going back to the total overall data and I found out the overall variation and I called that the total variation. That is contributed part by the in part by the part variation part 1 2 3 4 5 part due to the operator and part due to the gage. With these three different contributors of variance when they are ended up, we end up with the sigma total that turns out to be this.

Now, if gage variance is so much, if I subtract this quantity from this quantity we will end up with the part to part variation. This what I am really after? I am really after part to part variation which I have captured like this.

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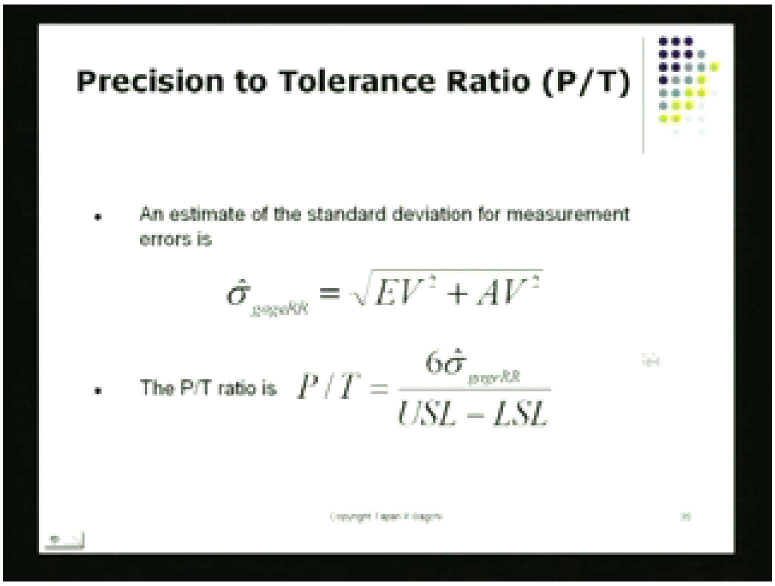

Now, there are some other ratios also which are used by industry. For example, this is exactly the same formula that we had there before. The only thing we have done here is we got the estimate of the variation that is contributed by equipment and also by the operators. So, those we have got there and this precision to tolerance ratio this is also shown here. This also very important and this case instead of that five point number I used six point number. So, slightly wider this is also used by many industries. The upper spec limit or lower spec limit these of course; come this specification of the part itself or the product itself.

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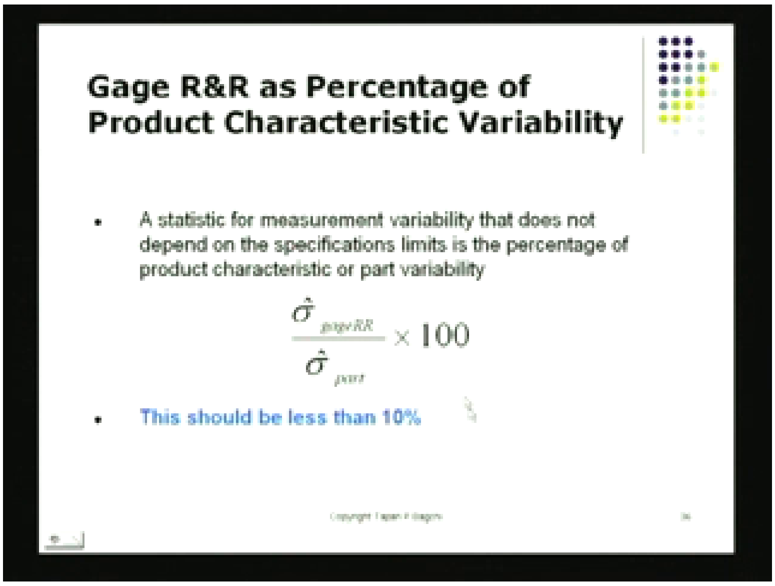

Again if you look at this ratio there that is the sigma contributed by gage R and R divided by the sigma. That the sigma of the true part variation. This ratio should not be should be less than 10 percent. It should not be much it should be less than 10 percent.

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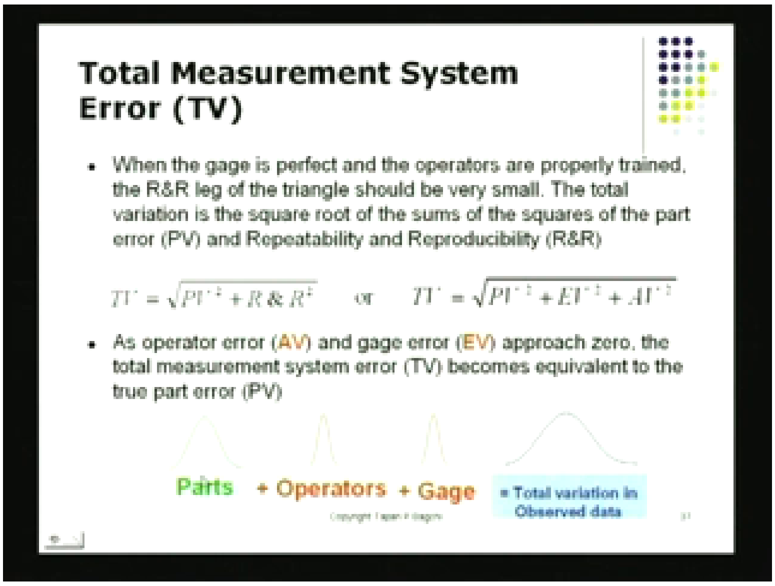

So, therefore, the gain the total picture is going to be like this. Part variation this is what I am really after is distorted by the operator cause variation in the data and also the gage cause variation in the data. So, what I observed in the readings of the gage is this and I have got to estimate these two quantities in order for me to get an idea by difference. Difference between this and the some of these quantities to get an idea of the true part to part variation. This is very important.

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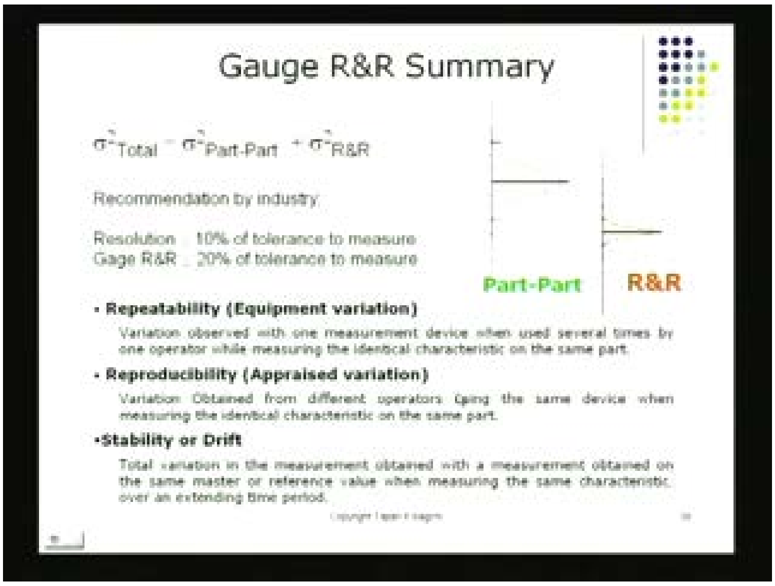

And of course, the industry given a couple of slides. Industry has it is own way to specify these things. When it comes to real a use of the real measurement system.

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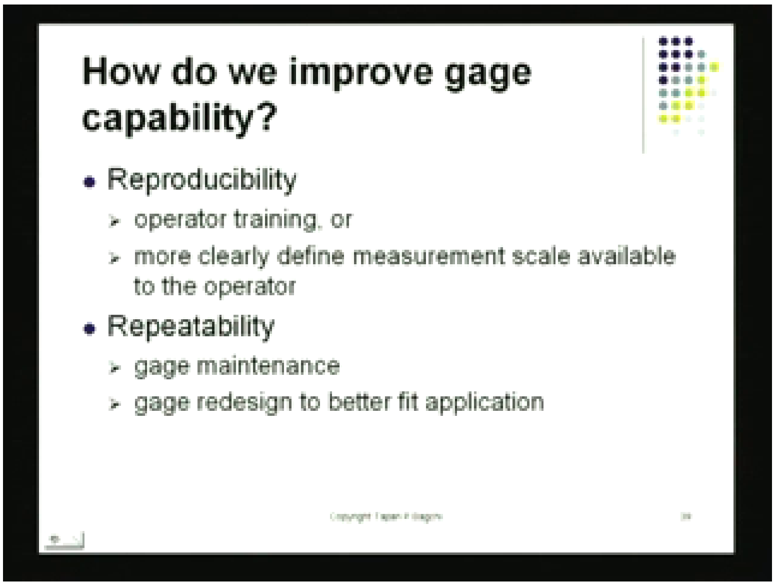

So, to try to tell you again reproducibility is due to operator. How can you improve it? It could be through training, it could be through you know really going to through the procedures over and over again and again to make sure that they understand what they have to do. And they should contribute as little as they can they should really act like each other. Repeatability that would come through gage maintenance and also notice here I have not used calibration. Calibration may impacts accuracy, calibration does not impact precision.

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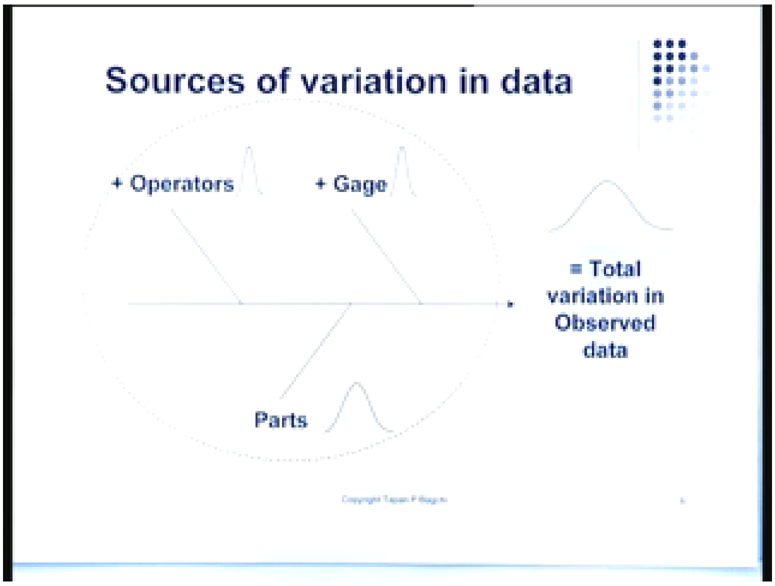

Gage reproducibility and gage repeatability these have to do with variation and that has to be really taking care of when you look at the variation. Variation what we are looking at? Say for you look at my total picture again. I am really; when it comes to precision and look in the total variation which is contributed part by part, in part by the parts that I am trying to measure of the process and part due to operator and part due to the gage.

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Of course, to try to improve this you could apply the DMAIC procedure that is like one way I have given you some details there.

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That you could take a look at.

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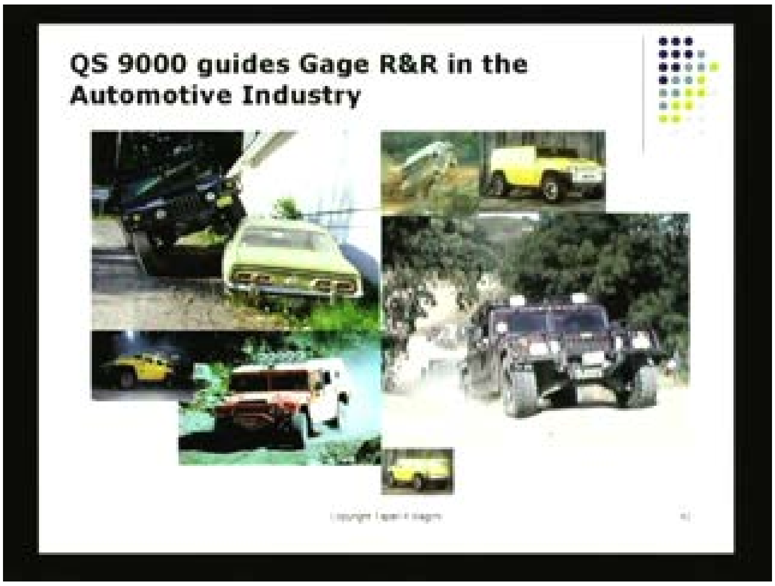

Industries come up with some specific guidelines in automotive industry people. Particular guys who make cars they come up with your own system and they use a different slightly different calculation procedure.

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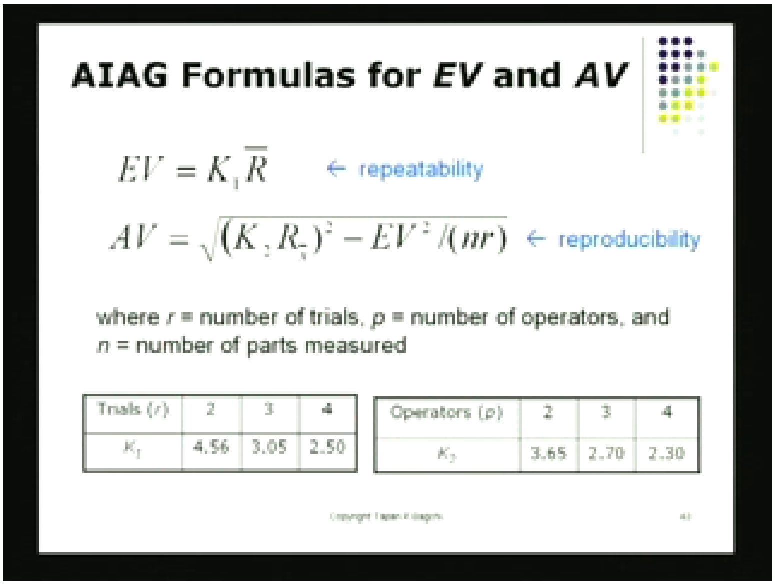

And I have given in the slides there. In place of using d 2 they use two other factors called k 1 and k 2, but exactly the plus procedure is exactly the same. And they also use the Pythagoras theorem and they also try to get an idea of starting with the gage. Then doing some measurements with the operator they try to get an idea of how much is the measurement system contributing to the total variance in the data?

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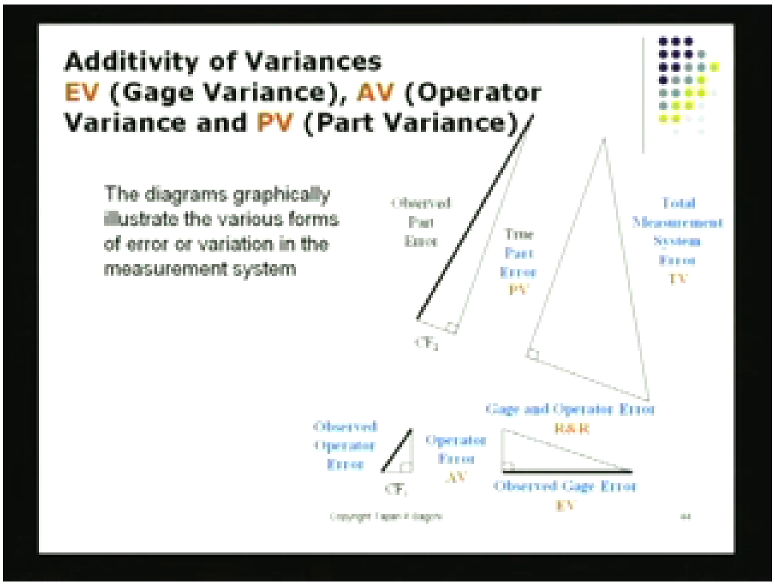

So, you look at the total variance in the data which is this one how much of that is contributed by the operator? How much is contributed by the gage? So, that they get an idea of this. Ideally they would like to have this contribution as small as possible. So, that what they see is what it really is. This is what they would like to be able to see.

Then there are some other matrix also which are used and I have shown them there. And again of course, I have given you picture again of the data itself and mind you the data. The data is a data collection scheme. Here I got multiple operators involved. I have got two operators involved in this case. I have multiple measurements made. I use the same gage by the way I use the same gage to try to get an idea of how much is the contribution of this repeatability issue by the gage itself?

I get that then of course, I try to estimate the various things using the appropriate either ranges or averages to try to get an idea of what it is? So, again I show the calculation these are the calculations which are done by the automatic people. They are shown here.

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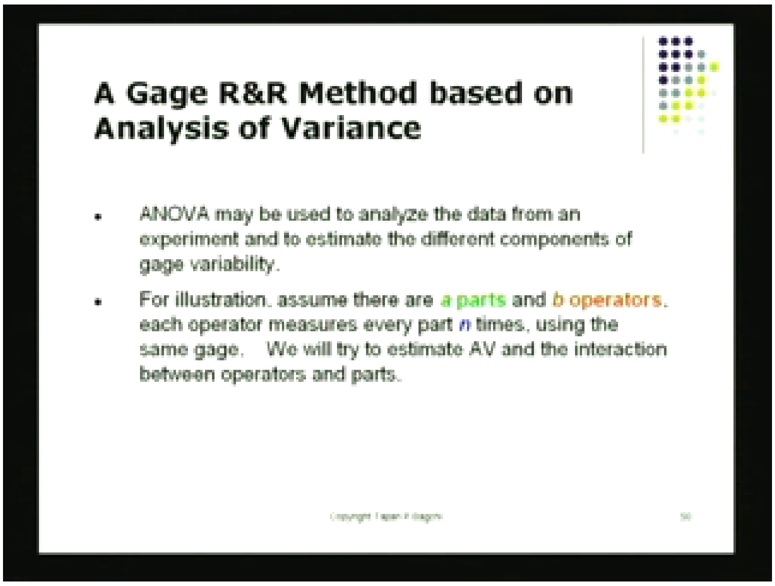

Certain cases the measurement system itself can be totally unacceptable and which is like when the gage readings they turn out to be such. Then they contributing almost 20 percent of the error that is in the final variation we have contributing to measurement system. That is not such a good picture and you got to make sure your parts which you got a really after those are measured correctly. Correctly means there is no inaccuracy.

That means, the gages are calibrated you got well trained operators. So, your reproducibility system is issue is as small as possible and also you measure end times.

When you take the average and you try to make sure the gage readings when you go from reading to reading to reading that repeatability issue is also not quite there.

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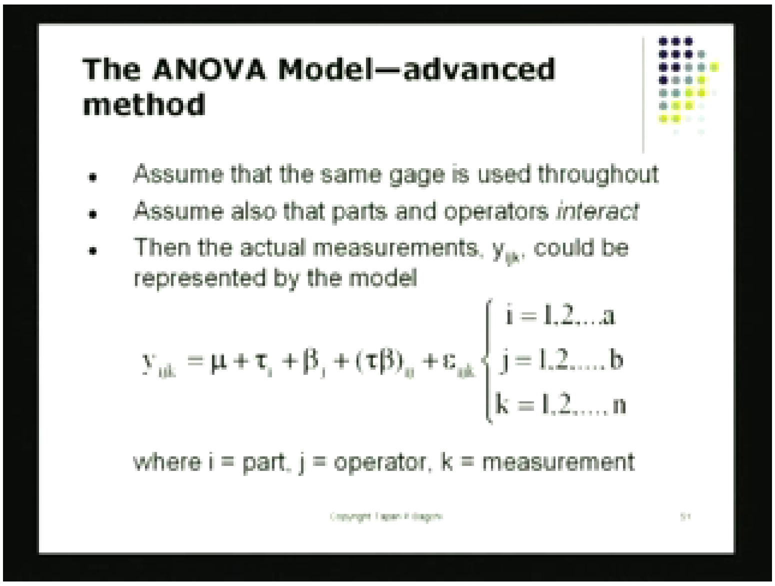

And of course, there are fancy methods called ANOVA. Those are also can also be applied and they can also be used to do measurement system analysis.

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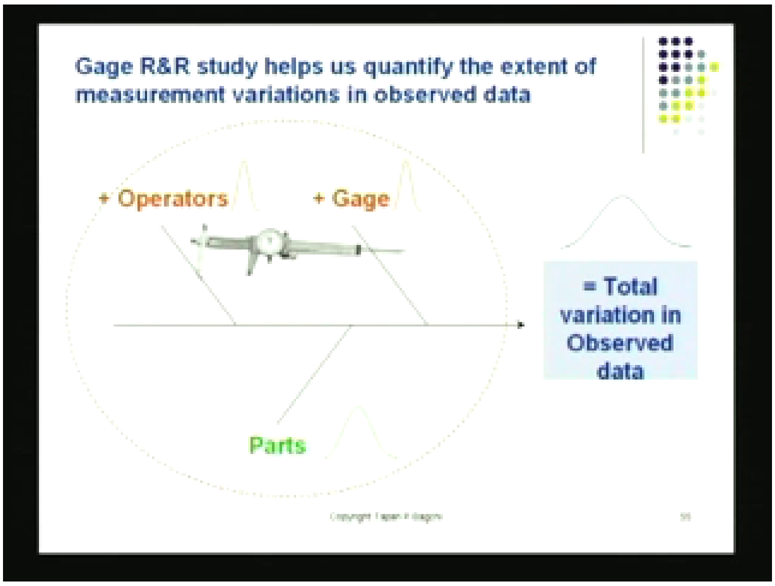

The ultimate idea of course, is you got to make sure that the measurements system whatever gages you are using plus the procedure, if they are using. They do not contribute much, and what you see? What you observe is exactly what that true variation is. So, therefore, you can rely then on the data that you procedure, because your decisions have to be data driven. I will continue with the six sigma series in the next lectures. Thank you very much.