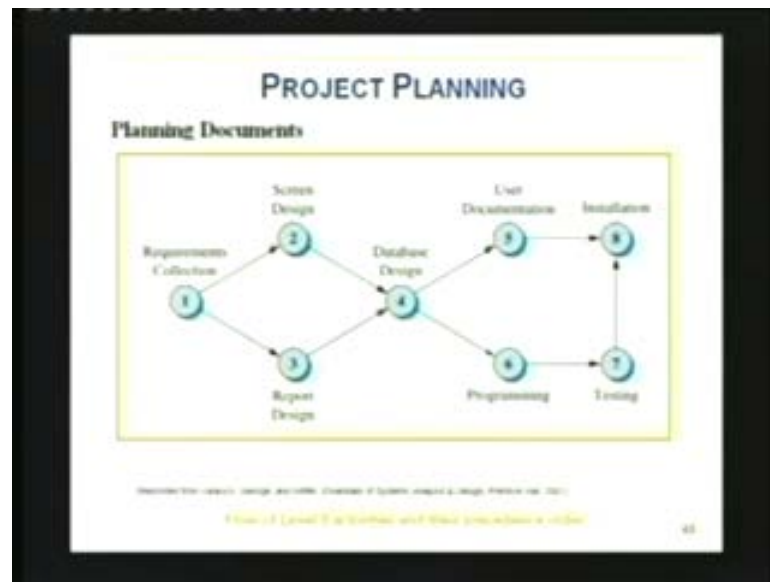


**Six Sigma**  
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**Module No. # 01**  
**Lecture No. # 16**  
**Critical Path Method**

Good morning **good morning** everyone, everyone is here pretty well and that is wonderful we did the introduction yesterday.

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And now, we get into some of the analytical stuff like project management and if I am not loud enough just you know **(( ))** like this and I will make sure I become audible to all of you. The part will be take off today is what you see in the slide in a couple of minutes you will see that; we have a sequence of tasks this is to achieve a particular mission in our simple language, which has a certain scope.

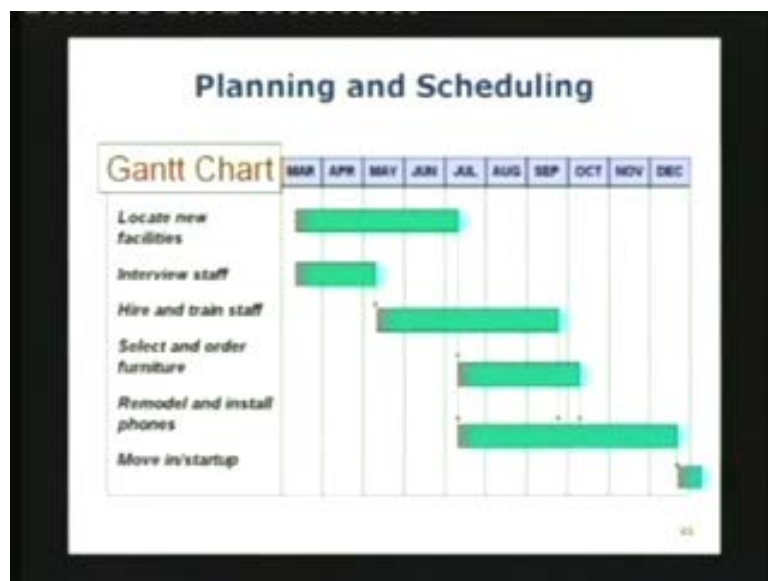
This project has a certain scope and that, when it is broken down into deliverables and tasks to be done, we end up with the list of tasks and this is called basically the Work Breakdown Structure, WBS that is then technologically sequenced, because you cannot start **the the** the rear end before you do the front end type of things. So, we got to make sure we sequence them **right** and for that we got to reach in to the expertise of domain experts, they tell us what comes first, what comes next.

Many times the project manager himself he does not have domain knowledge, but he is an expert **in systems** in the systems approach he is very much familiar with the framework and so on, in which things have to be cast.

Therefore, it is very important for him to recognize that, the domain expert has a big role here, he is the one who basically tells you in what sequence these tasks have to be done, you will see in a couple of minutes you will see on the slide we have this slide up here. There, we have a certain task to be done and that is about installation of a particular software completed, system design.

And these have been identified as nodes here you see the nodes here; those basically identify the task that have to be done and the sequence in which they have to be done. This is the beginning of your planning; your planning must begin with this. First of all, listing of the tasks which come from w b s and then, sequencing, where help is provided by the domain expert. What we do next is?

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We try to approach this; this is your Gantt chart. In fact, if you walk in to any project management office, you will probably see the Gantt charts on people's desk, on the walls and so on. In fact, the largest Gantt chart I saw was in Dallas, where they were actually showing how this space shuttle is to be launched; starting with sub contract work, fabrication, assembly, transport and then, putting out the test beds there, firing the engine, testing them and so on so forth and then, getting the astronauts ready and so on.

Making sure, they have all their checks and everything, they are (( )) well and all those things that have to be done.

Then of course, they get into the chambers, they are there for few hours then, the launch takes place. This whole thing was charted **this whole thing was charted** and would you believe it was a room that was about 100 feet by 100 feet and the wall was plastered with these charts.

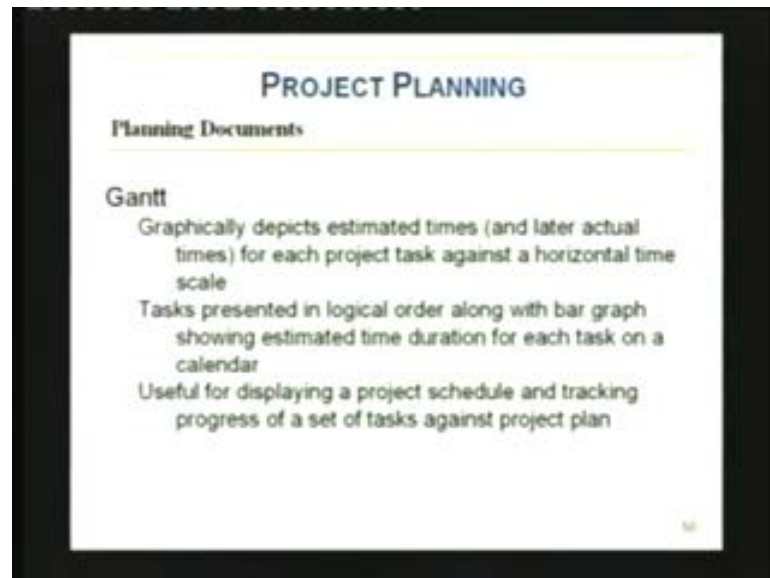
Now obviously, there was no one who was managing everything, these have been broken into **these have been broken into** smaller tasks. So, there were sub projects that eventually linked up and made the whole total project. So, here we have got a few tasks on this thing there, you probably have the p d f, locate new facilities **this is the** this is the project, where we are relocated, let us say, from the old campus to this new campus. So, department is to move here as good, we are creating some new facility including **you know** setting up the new offices, class rooms and so on. And also probably hiring some new staff and then of course, this **would be launched the** facility would be launched. So, locate new facilities.

So, here the decision that is to be made by people, who decide on the location itself then, alongside it would also be interviewing of the staff, because we know the mission of this project we know exactly what this facility is suppose to deliver, so there we have requirements. So, interview staff that of course, goes into **you know** it will have decomposition into reviewing the cv's, inviting cv's, getting **getting** your **your** feedback and so on, so forth. And then, eventually coming up with the list of people you want to interview and they interviewing the task, that has been shown here to start coincident with location search, those things because, the task has to be done. So, this would be also started alongside the other part that is going there.

Once this part is complete then you **go a head** go ahead and do your hiring and you start training the staff, that is what you would be doing right after that and the Gantt **Gantt** chart shows that. When you got select and order of furniture, which is to follow after you have got decisions made about the location the rooms that, you select and so on and so forth, then you order the appropriate furniture, that will take some time and that time is shown on the green bar there.

Then of course, you will remodel the facility and so on and so forth and the last part is, when you move in and do the start up that is like the thing, so this is the total project. Thus obviously, is a snapshot view, it does not have the kind of details that can let you find out with this you can only approximately find out how long the project is going to take, but to try to do that better you need to do C P M.

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And those are couple of techniques one is C P M and the other is pert and we will look at that in a couple of minutes we will be looking at that (Refer Slide Time: 05:36). So, the Gantt chart basically this was the Gantt chart; the Gantt chart is basically a graphic display of the total task involved. And basically the Gantt chart you show the main task, is the primary task that is what we show there. Later on, the subcontract may pick up the first part and he may blow it up into a large project and that is his project, where he will have all details of so on and so forth, that will be there, for you it is just one sub project or even one task to complete this total task that is there.

So, the **Gantt the** Gantt chart will graphically depict estimated times because, that is how you get the length of the bar and later on you might bring in some resource requirement also that will come later, but that is also something that you have to keep in mind. Once, I am going to be identify a particular task, I will require resources, I will probably have to budget them and I will have to do all kind of things later on with this, but at this time I just have a representation of the task **as a** as a green bar.

The task must be put in the logical order, the order in which they come and that will come from the domain expert, the domain expert is going to tell you what comes first, what comes next and so on. And this is the real easy way to display the overall schedule, it is a real easy way for example, you are a vice president and you are in a pharmaceutical company, you got four projects going, four molecules they are being researched on.

So, basically if you walk into the r and d area right upfront on the front wall, they have probably four Gantt charts, showing the status of these four different projects, that will be shown right there. So, that is like **it is a great way to communicate** it is a great way to communicate with the sponsor and those who are involved in the project and also let people know, what all projects are running in your company that is like a, it is a easy way to show that.

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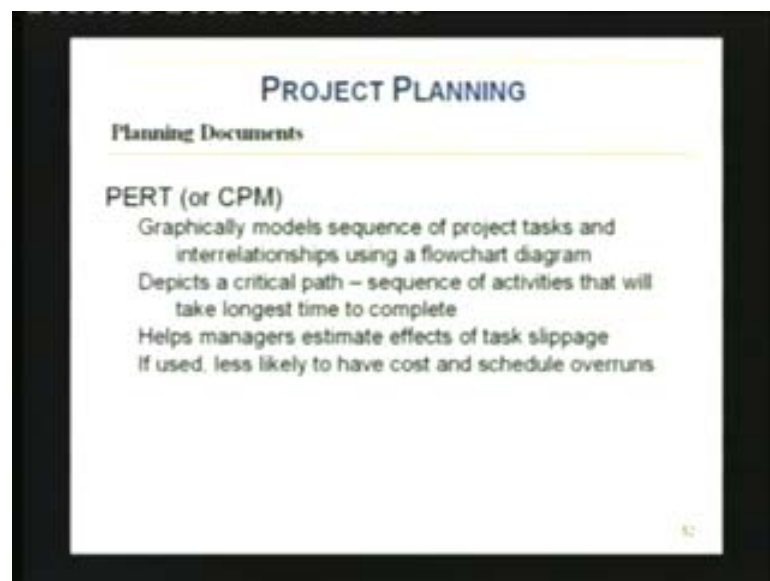


Of course, one should not stop with the Gantt chart, you need **much more in** a lot more details than just doing this, but the sad part of the story is not many people prepare even the Gantt chart **that is the** that is the very unfortunate part of, those are the reasons why many times projects fail, they fail because, good planning has not been done. And preparing the Gantt chart not at the conceptualisation stage, but defining and designing the project, the planning phase that is what I am preparing the Gantt chart.

So, you will see more details as you get deeper into it you will see more details, many times you utilize (( )) to try to capture the details to display the details and to have the relevant reports shown and so on, so forth. If you look at look at a project management tools such as primavera or you look at m s project; they do this tasks for you, they do this job for you. We will get some practice in using m s project by the way m s project it has been installed in our lab pc. So, they are about 40, 45 pc's there, they all have m s project involved and installed already there.

So, what you could do is, take a small project and just try it out try it out and those of you who have already worked in m s project, they are probably can hold the hands of others otherwise, come and see me and I will make sure I go out to the lab there and I show you how to actually get your your own project information put up on on a task you get your displayed like that so that that would be done later.

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Let us now get into the analytical part of project planning. The part will be focussing on right now is to come up with a proper schedule. So, looking at we are looking at only the time dimension here, there are three objectives that we got to worry about we got to have requirements, we got to have time how much we got the budget to take care of requirements of course, first and foremost you got to have requirements otherwise, you cannot really deliver the charter, you cannot deliver the scope of the project.

So, that is there once that is given to you then **you get into the you go and** walk up to the domain expert you ask him what details are involved in carrying out this project and they start then talking, they start giving you details, they give you the timings, they give you the costs and they also lend you what kind of expertise would be required to check and identify kind of resources required for carrying out certain tasks **you know** the same guys they cannot do everything. So, you got to make sure you match the requirements with people who are aware about.

Sometimes, if you are limited in resources that will stretch out the project and tools like m s project for example, they can actually show you the impact of having limited resources and the impact of that on the overall projects and then, we will also take a look at something called crashing; which is like putting additional resources on certain tasks so that we can compress the time.

Let us say, the dead line is only **(( ))** away and your project is going to take 8 months per your plan then, you take a look at tasks that might require additional resource, so that you can compress them, this task is called crashing. And it is got to be done in a manner, which optimizes things and I will try to do some of this on paper here. I am somewhat limited and handicapped, because the light pen is not working, but I will try to see if I could convey the things with **on** by writing on the sheet of paper here.

So, pert or C P M they are really they are used together, but they are not synonymous; pert has a particular goals, C P M has a particular goal, they both follow the chart, they both follow the activity chart and we will take a look at how it is done, what both pert and C P M charts do? They graphically model the sequence of project tasks that something the pert and C P M networks do.

Now, you end up with a network of activities that is what we end up with, it will also help you particulars of the C P M will help you find the critical path; the critical path is the sequence of tasks, if any task is delayed on this path, the whole project is going to be delay. So, this is the path that you got to watch, if there are any **any** kind of slippage of time.

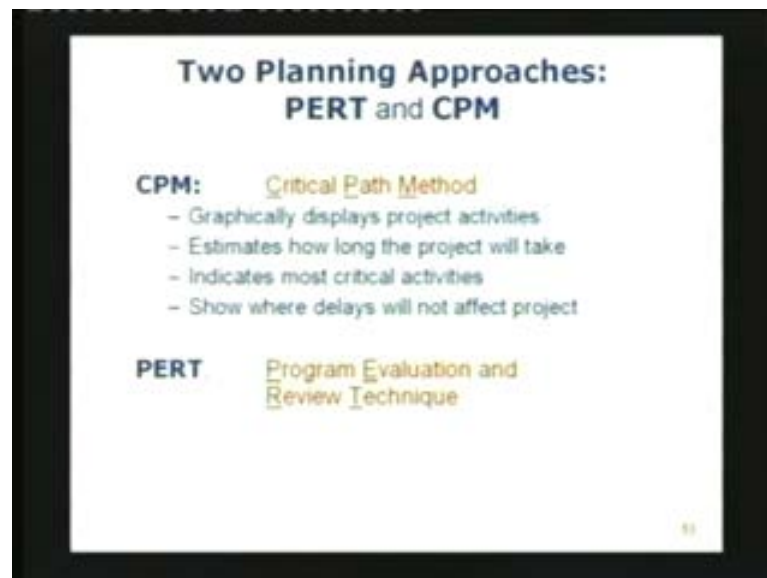
For example, on any of these tasks you should flag, it **it** is a sign that the project is going to be delayed and later on I will show you there are techniques like techniques called e v m for example, earned value management; where you basically try to find out, what is

the delay due to what is it due to, because of lack of resource or something else or say environment factor or whatever it is, if an activity on the critical path is affected you better make sure you have the resources and so on to be able to carry through the task.

This C P M and the pert network, they both let you let the managers estimate the total task, total total amount of work required to complete the length, total amount of time required and also any task that is also is tracked by doing this and it can actually, if you use C P M or pert you can reduce your cost, you can also finish your project on time, because, it keeps things visible.

In fact, if you look at a chart like this this display this is just showing the (( )) basic Gantt chart, but alongside you can also plot the progress of the project as you can track the variance of from actual you know, there is there is the actual then, there is a plan that variance also you can track by by visually looking at this thing which task is getting followed up which task is getting and which tasks are getting finished ahead of time we will take a look at all that once we get into this things.

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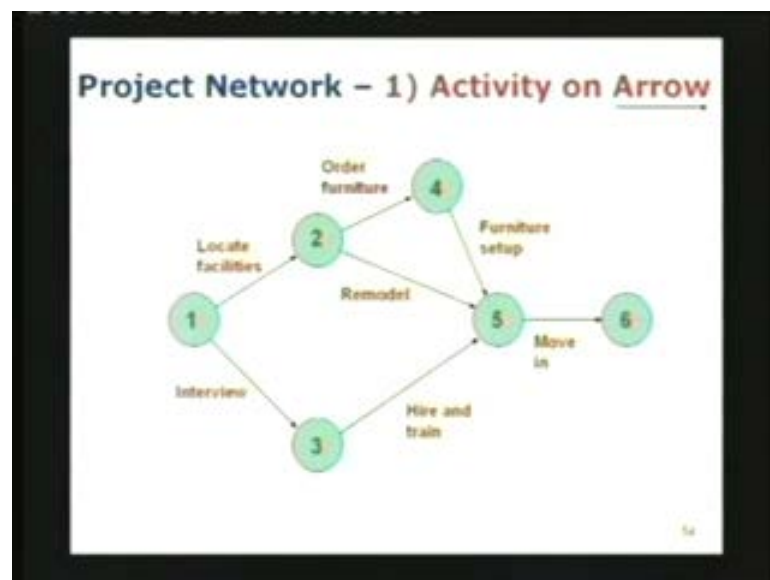
So, there are these two basic methods, one is called the critical path method, the other is called pert, they are quite different, they both use networks of activities, but their goals are different, C P M is totally deterministic; deterministic means I assume that, the times have no randomness the estimates the time estimates that are given to be they are fixed, they are known and they are fixed. And therefore, the activity time the total activity time



also turns out to be deterministic quantity with C P M, pert gets a little bit into time mismanagements or slippage of dead line for example.

So, it reaches the activity times as random variable and we use a simple model to capture that **that** randomness and I will **I will** show you that in a couple of minutes, how we capture the randomness that might be there in real life, how it captures that into a pert network, still the network is the same, but the timings used for pert analysis, these acknowledge the randomness and they exploit the randomness to try to make some predictions about, what is the chance of missing the dead line that is something that is done very easily using the pert network, we will see that in a couple of minutes.

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Now, here is the beginning of a little task we will be doing I have **a** the same project that you saw earlier, which is like moving from old campus to new campus and when I spoke with the I made a list of all the tasks then I spoke with the expert who is there an expert in moving **moving** equipment and moving people and relocating and so on, so forth. And he said, so if you want to do this right, this is the sequence which we will be doing it.

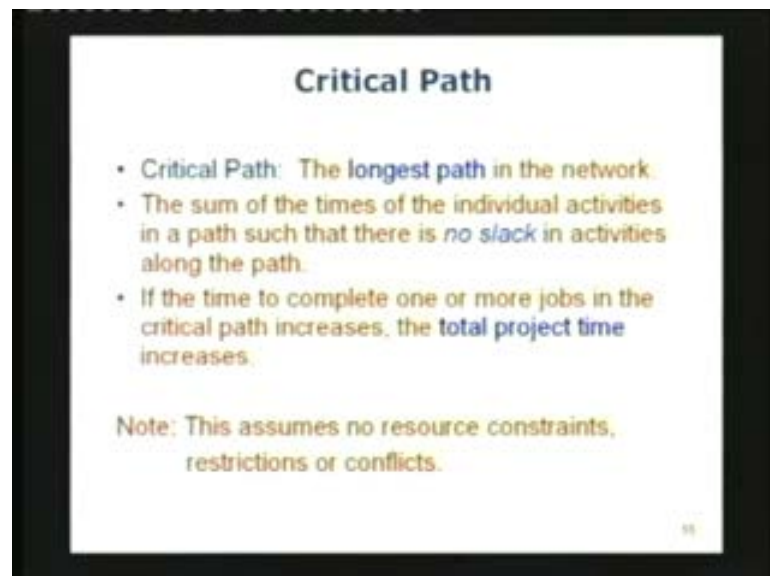
So, if you look at this thing what I would like to suggest to you is draw this on a sheet of paper draw **this draw** this on your network, because we will be working with this, draw this take a couple of minutes you have these nodes; nodes here are mile stones. So, the very first one node number 1 is starting the project, node number 2 is when you have accomplished location of facility that is a mile stone, node number 4 is when you have

ordered furniture ordering is complete that is a mile stone, node number 3 is when you an interview is complete that is a mile stone, again node number 5 is when you have got the furniture all setup, when you got remodelling of the facility is done, may be erecting some new walls, installing cameras and stuff like that you have done all that.

And also you got the human side taking care of, you got the hiring done, you got the training done, when all these three things are done, only then, you can move in (( )) the node here which is 5 that is a milestone, when I have got this task furniture setup done, remodelling done and hiring and training done I have accomplished three milestones I have got I have reached this node that is the designated 5.

Then of course, I can give do the last part of the project, which is moving in and when I reach the node 6 when I reach node number 6 I have accomplished the total project and the mile stone the project is complete. Now, this is just basically the description of it. Now, that is not good enough for management I understand how the project will be done, but how long it is going to take. This is how we will need to bring in the time dimension of it. So, my basic network is setup here, now I have to imprint I have to bring in now additional information, which would be time information.

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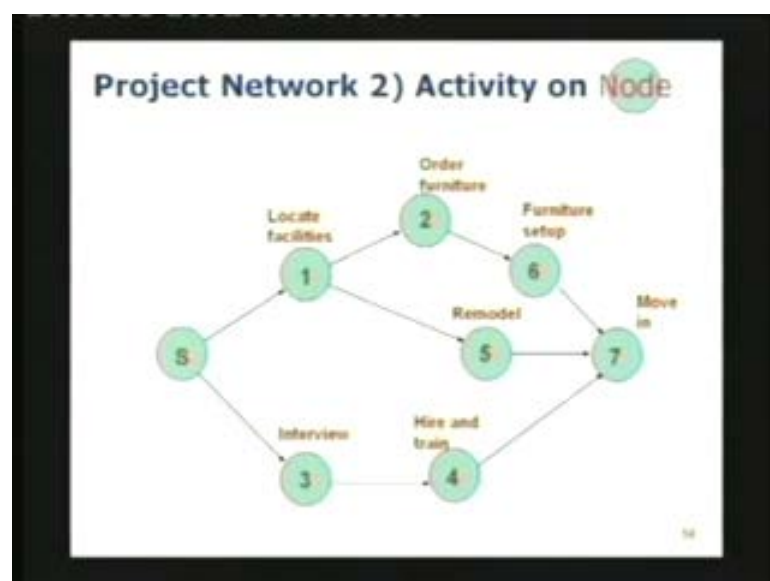
In fact, the thing that I would like to discover is the longest time taken to complete all the tasks, because that is going to be the duration of the project.

Now, some of you who have been working in project management, it is kind of obvious that you need to find this out. So, that I can then **then** you can actually commit a particular deadline you can actually tell them all be done by let us say, march 15 all will be done with the project to reach that point you have to identify the critical path, which means you have to draw a network like the kind of network we have there, but if we augment this **with times** with activity times you are also have to identify for example, are there any slacks in a in some particular tasks are there some slacks there; slacks are very handy in terms of reallocating resources.

If a particular task has a lot of slack probably you can spare those people and they can go out and help on a task that is on the critical path; this way you end up keeping everything up **up** to speed pretty well and you make the good use of make **make** the best use of resources **at your at your** at your disposal.

The total project time of course, is the goal that is where we want to reach. Of course, in the C P M scenario; in the C P M scenario and also in the pert scenario you do not assume any kind of constraint of resources due to resources; you assume lots and lots of resources are available. The only dimension you are looking at here is the time to finish given lots and lots of resources, no limit on budgets, no limits on basically resource availability. We just want to see most optimistically, when I can finish the project that is the goal let us see how we do that.

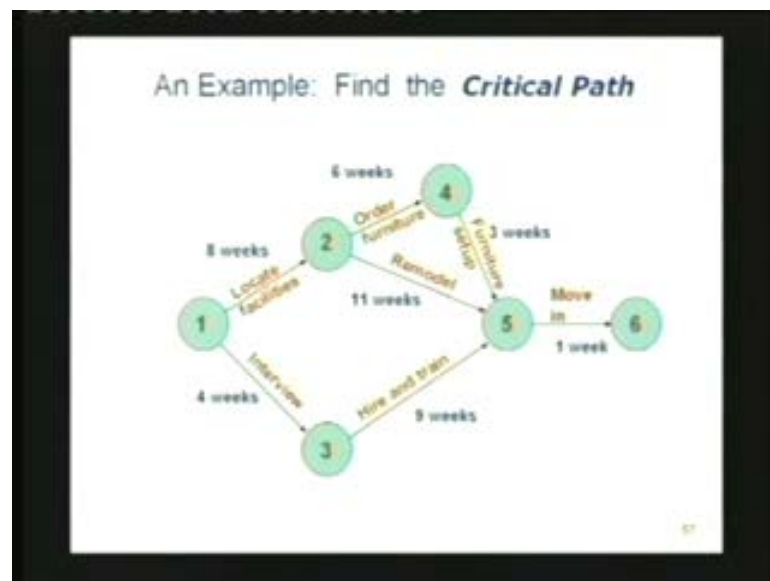
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I have to bring in a particular topology. Now, there are two topologies used the one that I showed you earlier, notice here the activities are on arrows; the activities here basically if you see them, when I switched to the paper here you can actually see this is the activity locate facilities it is on an arrow, order furniture is also on an arrow, this sort of diagram is called activity on arrow diagram **activity on arrow diagram** this is that one way to represent this C P M network.

There is another approach, which is called activity on node and that is this one, the same information what **what** I have done here is, I have made the nodes the activities themselves. So, here some people prefer this mode, some people prefer this topology. But, I personally prefer the activity on **arrow** arrow type. So, that is the one we will be using for our analysis. But you should really know how to switch back and forth, because there are certain things that are possible with the activity of node topology, which is not possible with the activity on arrows topology.

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Given that what we do is, we try then to talk to people, who have done similar work before and we try to come up with time estimates. So, if I look at time estimates I bring the next sheet which should be right here and that is your next slide. And now, if you look at the sheet you will notice their times. So, there is 8 weeks there for this task, 6 weeks for this task, 3 weeks for this task, 11 weeks for this task, 4 weeks here, 9 weeks here, 1 week there.

Now, given this and someone who is just on high school; that means, **he's** he can add and so on, so forth, he can compare and add, if you ask him please tell me the total finish time for this project, what he will do is? He would trace all the path from beginning to the end, he will trace all the path. So, I have got 8 plus 6 plus 3 plus 1 that would be some number, 8 plus 11 plus 1 that will be another number, 4 plus 9 plus 1 that will be the third number. The longest path whichever one it turns out to be is going to be the duration of the project, the other task are they are going to have the slack. So, can you quickly tell me what the longest path is?

**(( ))**

1 2 5 6 that is the longest path, which actually means even if I am doing two activities here on this front, it looks like 6 plus 3 it is going to be smaller than 11 and this is the guy who is going to be that is going to be delaying this one and this one. I got to make sure **I now trace the** I can trace the critical path here, but just doing it this way does not tell me one small thing, it does not tell me the amount of slack in here and the amount of slack in here and also it does not tell me the slacks available at these activities it does not do that.

So, for that we have got to do little bit of calculation and let us see how easily that can be done, what we do is we will start with the blank sheet and we go back this our writing sheet here. I say, if this is the activity and this is my starting node, what is the earliest time I could start this activity? Time is 0 **right**, so earlier starts is going to be 0. Earliest finish is going to be earlier start plus 8, so earliest finish for this task is going to be 8, what is the earlier start time for this guy, earlier start?

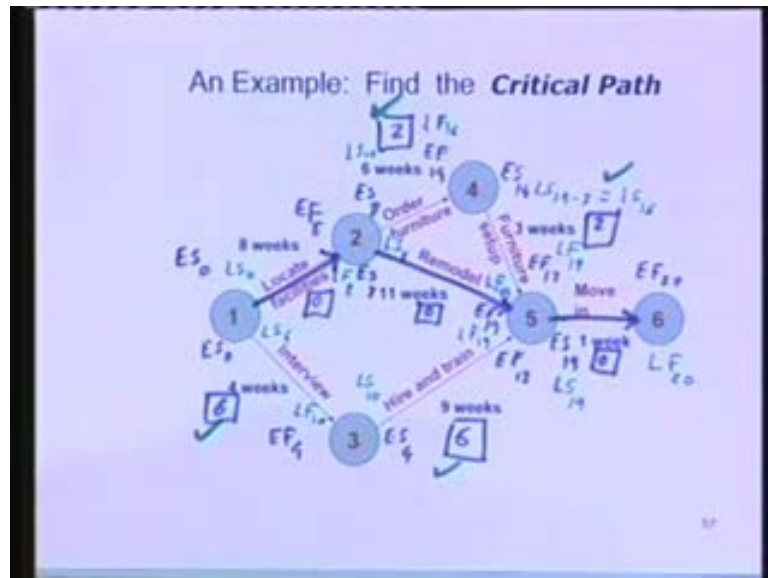
8

It is going to be 8. So, earlier start here is 8, then I look at this one earliest finish for this I only have to complete this task 8 plus 6 **6** 14, earliest finish is 14. I come to this side and I say my god that task, it could start only when this task is done therefore, the earlier start here for this task is going to be 8, earliest finish here is going to be 19 that is 8 plus 11 19.

Now, look at this task, the earliest it could begin is 14, earlier start is 14 because, that is when this guy gets finished and I start from that milestone and I add 3 to 14 I end up

with 17, earliest finish here is 17. I come to this side, what is the earlier start for this guy? 0 earlier start is 0, earliest finish for this guy is going to be 4 plus 0 that is 4. Earliest start on this side is going to be 4; earliest finish from this branch is going to be earliest finish is going to be 9 plus 4 that is 13.

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Now, look at this task, this task cannot be started till this is done, this is done, this is done and the earliest finish turns out to be this guy. So, the earlier start for this task is going to be 19, earliest start is going to be 19, there earliest finish for the project is going to be now 19 plus 1, earliest finish is 20, 20 is the length of the critical path; 20 turns out to be the length of the critical path, this is one way to do it, one way to get the what we call the forward pass through the network.

Of course, what we have to do is, we got to identify the slacks we have not done that yet I mean it could be probably done somehow, but let us see, how we do it precisely, how we do it using some analytical work this **this** number is fixed, the n number is fixed. Therefore, quite easily I could actually start here, the **latest finish** latest finish for this task is going to be 20, if I finish it very later, and the project will be delayed.

So, the latest finish required now, if I commit to 20; 20 weeks as the duration of the total project, it is going to be latest finish is going to be 20, the latest start for this task, latest start it will be finish plus this task minus this task so it is going to be latest start for this guy is going to be 20 minus 1 that is 19.

Try to check now the latest finish for this one this task, this needs to begin the earlier start is 19, even if I finish at 17 I have got some extra time there. So, the latest finish is going to be, when this guy starts that is all I need not really **you know**, when there is no reason for me to start it any early. So, **latest finish** latest finish is going to be 19, for this start.

**(( ))**

**Yes** surely see I am trying to find for this particular task I know I am going to be finishing at 20 weeks; at the end of 20 weeks I will be done and this task takes, so what is the latest finish for this task; however, much time it might be taking, it must be finished by 20 weeks. The latest start of this is going to be that 20 minus the duration of this task. So, 20 minus 1 that is latest start for this task is 19.

Now, look at this guy this one must be completed, this one must be completed and also this one must be completed in order for this to begin, for the latest start of this is 19, if any of these gets delayed even beyond that therefore, this task will get delay; that means, the whole path is going to be delayed, whole **whole** project is going to be delayed.

So, the latest finish for this guy is like saying the latest arrival time of a student here is 8:30, if anybody comes after that, the class is going to be delayed. So, imagine this to be the start of the class, start of the project management class, now these are different people arriving I said the latest for you to arrive latest arrival time is going to be basically when the class starts just before that and **and** instant before that.

So, the latest finish is going to be 19, what about this one, what do you think will be the latest finish for this? 19 **19** again, latest finish is 19; latest finish for this tasks also is going to be 19 **latest finish is going to be 19**. Please do not feel rush I am going to give you a very simple example after this and I will let you work on this, but right now just copy this down.

So, you can at least see what the mechanical process is, the latest start of this guy **latest start of this guy** the task is going to take 19 and the latest I can finish is latest finish is 19 and the task is going to take you 9 weeks, so what is the latest start 19 minus 9 that is going to be 10. Let us think through this again I must finish by 19 at the end of the 19

week I must finish and I have to make sure I give it enough time for this task to be completed.

So, how late can I start this task? 19 minus the duration of this task that is 10 is this clear or should I slow down more, look at this one look at this tasks **look at this task** it must finish by 19, because this guy has to start by 19 that is the latest start for this otherwise, the project would be delayed, it takes 1 week for to go from here to here.

So, I got to make sure I really have enough time for it to have the latest start as late as possible, the latest finish for this is going to be then something that should not delay this task so that is 19, this task would take 11 weeks, what should be the latest start of this? Latest start, 19 minus 11 is this clear, not clear, somebody should say not clear I feel happy you are ok with then, 19 minus 11 that is going to be 8 there.

I will look at the side now this task takes 3 weeks latest finish is 19, what would be the latest start of this, how late do you think I could 16 because, the task takes 3 weeks I must finish in by 19 weeks. So, it will be 19 minus 3 **19 minus 3**, which is equal to latest start at 16, latest finish for this guy if it is anything beyond this I will again have trouble. So, latest finish here it is going to be 16 and this guy takes 6 weeks.

So, the latest **latest** start here, latest start is going to be 16 minus 6 **right** that is going to be 10. I have got the latest start here, latest start, latest finish there and so on, so forth. Look at this guy now, this is the tricky one, latest start is 8, I cannot delay it anymore. When do you think this task should finish? 8, so latest finish for this is going to be 8.

Let us debate this a little bit suppose, I have finished in 10 weeks; I took 10 weeks to get to this point, what would happen down the row everything will be delayed. So, I have got the backward calculation done to try to make sure, when is the class going to start and I start backward planning from that. So, the class is going to start at this point and I find that, if it goes beyond any of them I cannot finish at later start that would just mean that, this task would be ready, but this would be delayed. So, I finish this task by 8, what will be the latest start time for this? 8 and 8, 8 minus 8 are going to be 0 I cannot delay it any more.

Let us look at this side, latest finish here is going to be how much, latest finish?



(())

Only 10 and this guy takes 4 weeks, what will be the latest start of this, 6. So, therefore, the project can start when it can start at time 0. Now, notice something this task has latest start at 0 also the earliest start is 0, there is no slack, look at this task latest start is 8 and earlier start is 8 again I have got no slack, because because it is sort of like this, how late can you arrive you got to be there at 8:30 and it turns out the start of the task the, it turns out the earlier start is going to be 8, if they coincide then; that means, you have got no slack there I have no slack here in fact, I can now say, I can take my pen and I can draw the critical path, this this path has no slack, this path has no slack, this path also has no slack. If, you count up now, 8 plus 11 plus 19 plus 1 that is 20, this is the path that is going to determine the duration of the project.

So, what about the other tasks; let us take a look at what is happening there, the latest start on this task is 10 and the earliest start is 4. So, I could start at the end of week 4, but I could wait as late as 10, which means I have got some slack I have got some slack there, what is the amount of slack here, the difference between those two.

So, slack I am just going to put it in a box, slack here is 6, the slack here is going to be 0 slack here is going to be 0 slack; obviously, on this is going to be 0 on the critical path you will have 0 slack, there is no room at all to play around with time, what about this guy, how much slack is there, latest start is 6 and I could start immediately I could wait till 6 without delaying any problem.

So, therefore, there is some slack do you understand the concept of slack, slack means I have got some room to play, this is like the old ambassador steering wheel you all have played right, so before you start turning you better watch out you get into the point when it almost begins to a sort of move and then, it turns forward, if you just hold it in a middle, it will never turn, it will probably go all over the place. So, here I have some room there again the slack here is going to be 6 please tell me the slack on this, there is no slack there, what about the slack here?

2

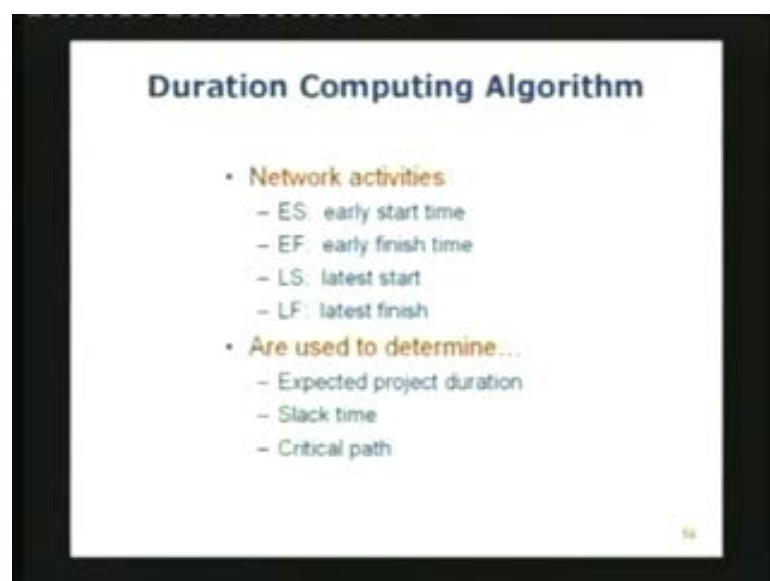
2 is the slack there and on this side 2 again, 2 is the slack there. So, there is something very interesting this guy something very interesting going on here, it is got a black cap

and blue colour that is ok we are all human **right**. This diagram here it basically tells you, how the project is going to progress.

Now, you find that, there are certain task this one, this one, this one and this one, they have some slack, which is non zero, non zero slack you do have **do have some** slack, they may have spare resources and you could swing that resource probably to this task, you could swing that resource to this task, you can swing that resource to this task and thereby you could speed up **up speed up** some of these tasks and make sure it probably, would **would** get down may be in 18 weeks or 19 weeks something like that without spending any extra money all you have done is good project management we have reallocated people.

So, you have got some interview you could have probably some capable people there perhaps, they can give you an opinion on site location; first thing, they could give us some assistance here in remodelling **if there**, if there is any physical work to be done that sort of things should be exploited. Whenever you got a diagram like this, you should try to locate those opportunities, when you got some slack that is more than 0. See, if you could spare some resources from there, this is not going to hurt the project in any way; it is actually going to help you speed up the project. So, this is point number one, then we go a little beyond this of course, have my calculation and exactly what we did the same thing is defined here.

(Refer Slide Time: 34:31)



Again to give you the definition I have got earlier start time **yes**, I have got earliest finish time E F, I have got latest start time L S, I have got latest finished time, these have to do with activities and these are required to find the expected project duration that is like something we got to do, slack time and also critical path. So, the product did the results for my doing this, this little diagram here and so on, so forth. The project the **the the the** **the the** benefits of doing this is these things we **we** result we ended up with these things.

(Refer Slide Time: 35:07)

**Calculating ES, EF, LS, LF,  
and Completion Time**

**Forward Pass Method**

- $ES(a) = 0$  for the starting activity
- $EF(a) = ES(a) + t(a)^*$
- $ES(a) = \max [EF(\text{all predecessors of } a)]$
- Project completion time** =  $\max [EF(\text{all ending activities})]$

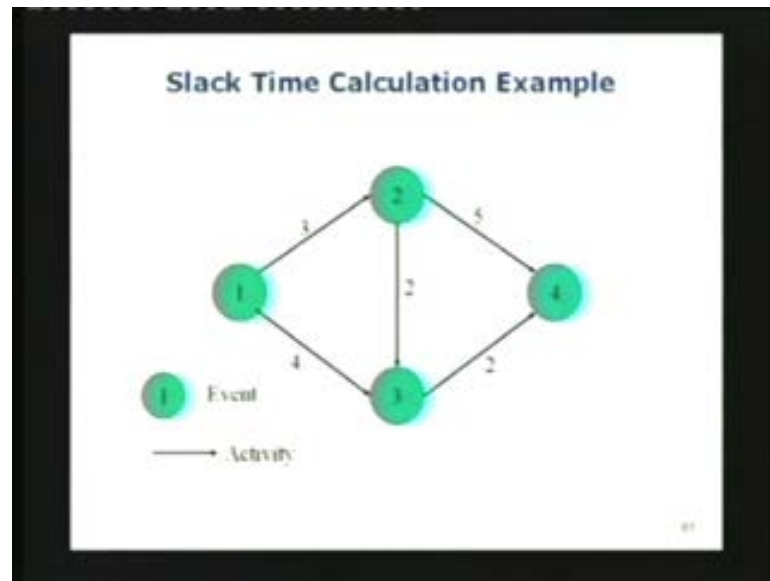
**Backward Pass Method**

- $LF(a) = \min [LS(\text{all successors of } a)]$

\*  $t(a)$  denotes the duration of activity  $a$

And of course, whatever we did its clarified here, the formulas are given then nothing like e is equal to m c square this is just a b c we add them up and we get the numbers.

(Refer Slide Time: 35:18)



Now, I will give you this example just copy this on the thing. See, if you could get me the slack times and also if you could give me the project finish time, (()). What kind of diagram is this activity on node or activity on arc?

Activity on arrow

Activity on arrow very good

How many milestones are there that you can see here?

4

4 milestones milestones of course, are they are event, so they have no time, but the activities end up having time and I have got the times here whatever unit it is 3 2 5 4 and 2 those are the ones that between the milestones that is what I am giving here you try to see now if you could; obviously, you could easily calculate the longest path, but that is not the total answer. Let us also try to see, if you could find the earlier start and so on, so forth, take a couple of minutes (No audio from 36:22 to 36:34) just to get you rolling here; the task from 1 to 2, it can start any time and the earlier start is going to be, earlier start time for the activity 1 2?

0

0 very good similarly, the earlier start time for activity 1 to 3 is going to be 0 you got to start there, then you work out the earliest finish and so on, so forth (No audio from 37:02 to 37:34), how many paths are possible to reach from 1 to 4, how many paths?

3

3 different paths very good and roughly, the length of these paths?

(( ))

8, 7 and 6 very good, so I know the duration of the project now. (No audio from 37:52 to 38:44) somebody has the answer; couple of you have the answer good very **good**. First of all, have you identified the critical path, can you just call the sequence of milestones?

(( ))

1 2 4 that is the critical path and that is the path, where the activities have no slack time is this clear, then that is the other guys must have some slack and you have to just work out their slacks, what is the slack on activity 2?

(( ))

1 is there good. Activity 1 connecting to 3?

2

2 that is a good slack there. And connecting 3 to 4?

1

So, we know now and; obviously, 1 to 2 there is no slack, 2 to 4 there is no slack.

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### Slack Times

**Total Slack Time**  
=  $LS(a) - ES(a) = LF(a) - EF(a)$   
= time of activity can be increased *without delaying the project*

**Free Slack Time**  
=  $\min [ES(\text{all immediate successors of } a)] - EF(a)$   
= time activity can be increased *without delaying the next activity*

41

So, we have identified pretty quickly by doing the same thing that we did earlier.

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### Slack Time Calculation

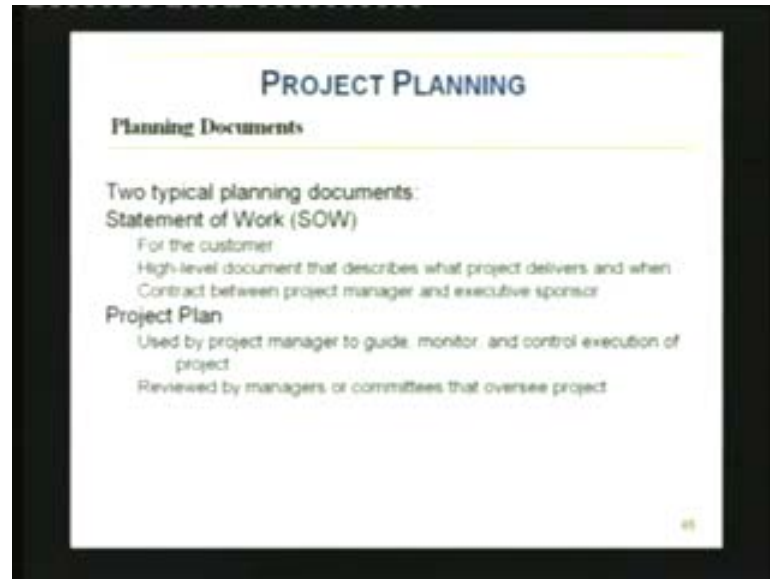
Activity	time	Immediate predecessor	ES	EF	LS	LF	slack
1-2	3	-	0	3	0	3	0
1-3	4	-	0	4	2	6	2
2-3	2	1-2	3	5	4	6	1
2-4	3	1-2	3	6	3	6	0
3-4	2	1-3, 2-3	5	7	6	8	1

Paths: 1-2-3-4 Completion time = 7  
**1-2-2-4** Completion time = 6 → **critical path** - longest path - no slack  
1-3-3-4 Completion time = 8

41

And if you walk through this thing you can verify **you can verify** your answer, if you want to see, what the answer is, the answer is here.

(Refer Slide Time: 39:53)



And let us see, you have done that, then you go back and you try to see, what the total task is going to be, you will identify the critical path. So, let us try to make sure we show the critical path, it is from 1 to 2 to 4 that is the critical path, the other guys have slack what is the utility of understanding, where the slacks are, what is what is the usefulness I can swing resources?

The allocation of the resources.

Very good I can swing resources and I also know now which all activities must resource completely in order for not to delay the project alright that was one picture that was our C P M calculation. Let us now see, what other things are required; obviously, the total planning is not complete here I have a statement of work which is basically what is given to contractors they are given the (( )), they are supposed to be responsible for and they are suppose to come up with these time estimates.

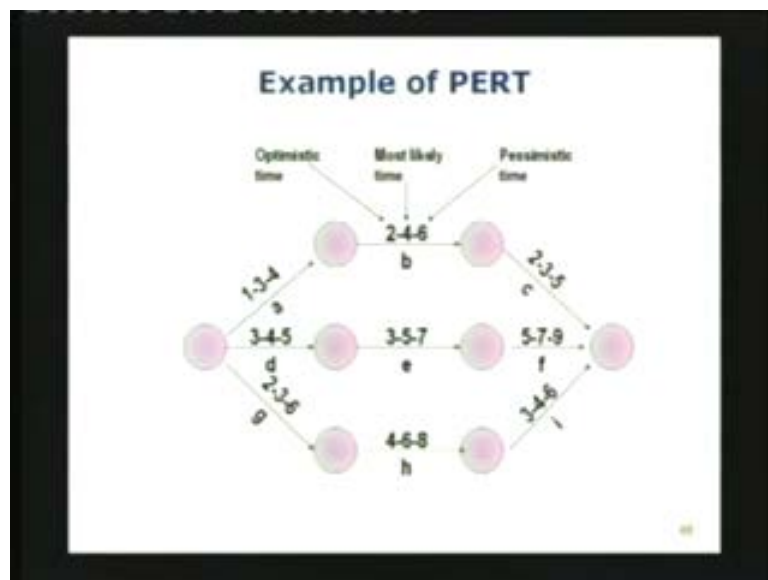
Of course, many times people work this out ahead of time, they try to get an idea if there is like an over path to be built or some building has to be constructed or some engineering product has to be basically designed, they have an idea or some software has to be created people from historical sort of sources, they can work out roughly, what the task is going to be, the total task is going to take, but they also do the C P M path and the C P M path; obviously, is more exact you identify the slacks and all those things, but

what have we assumed when you did the C P M work what did we assume, we assumed two things; first was hint is a letter starting with a word starting with d?

Deterministic

Deterministic very good number one, the other has to start with r resource **resource**, there is no resource constraint in the C P M that is like something we assume only then we can go ahead and do our C P M because, if you bring in constraints of resource that becomes a pretty complicated problem or we will take a look at that later on. We will try to see, how we utilize information on resource availability let me do that.

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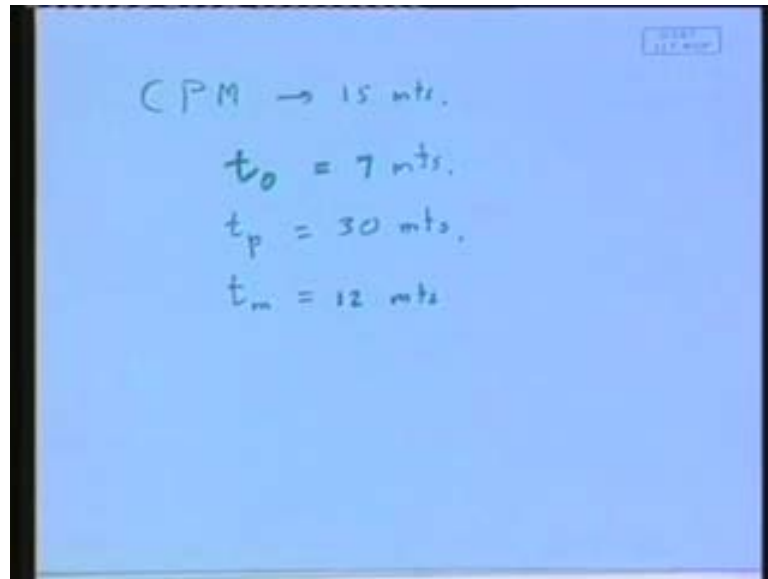
Let us, now take a look at this technique called pert, how much time tell me, how much time does it take you to come from let us say a hostel to the site here?

15 minutes.

15 minutes, now is that your average time, is that your optimistic time, is that your pessimistic time you gave me a good number and that is a very good input on C P M.



(Refer Slide Time: 42:24)



So, if I did C P M analysis your 15 minutes would be very handy, that would be very good **very good** sort of input in doing that. But, now let me ask you things that can actually can fall up between your **your** walk from your hostel to this place, what is your optimistic time, how quickly do you think you could get here?

**(( ))**

Optimistic means?

**(( ))**

The best time the least time, how much is that do you think?

7 minutes.

7 minutes. So, t optimistic is going to be 7 minutes, what about pessimistic, what **what** kind of things can follow up in your walk; traffic, bicycles, weather, little accidents here and there and so on, the road being blocked because, **there is a** there is a truck that is trying to back up the same way or the gate is closed or whatever it is, the security people do not show up, these things can actually probably they can force you to retrace your path, if things do get fouled up, how long do you think it might take you t pessimistic?

**(( ))**

30 minutes, may be 30 minutes I would not really worry too much about it and there is something called most likely and the most likely may be slightly different from the average, but there is a time called t most likely, what is the most likely time I was given a time of 15 minutes is that most likely?

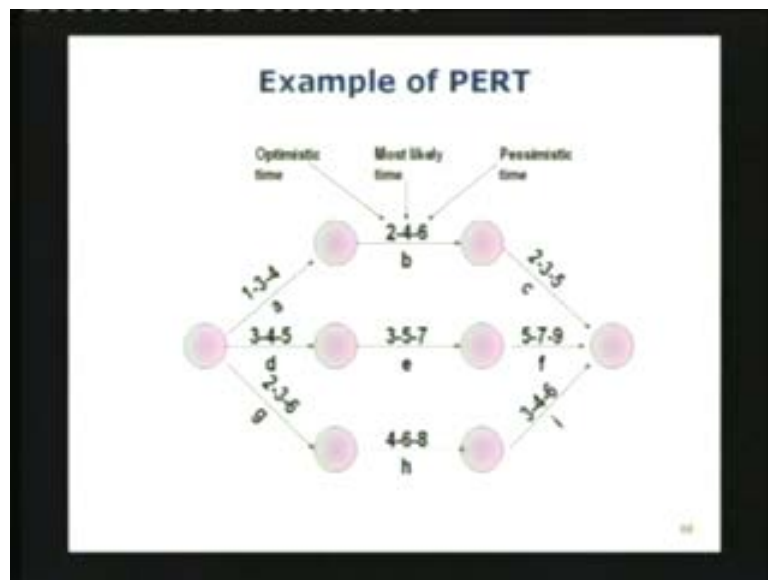
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No problem **no problem** I could take 15 minutes or if you want to change it, I could also change it.

(( ))

12 minutes **let us put it just for** let us put it as 12 minutes. Now, notice here just by asking a few questions I established couple of things on my C P M assumption **was not** was not full proof was not 100 percent, there is this variation of real life, if you look at the real project like the one I have got on my screen here this is the project that has got three phases three parts that had to be completed.

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And just take a look at this part here I have got task here, which are like this is activity on arc. I have got activity a then, I have got activity b then, I have got activity c on the top path, then the middle path has middle chain has d, e and f and then, I have got g, h and i.

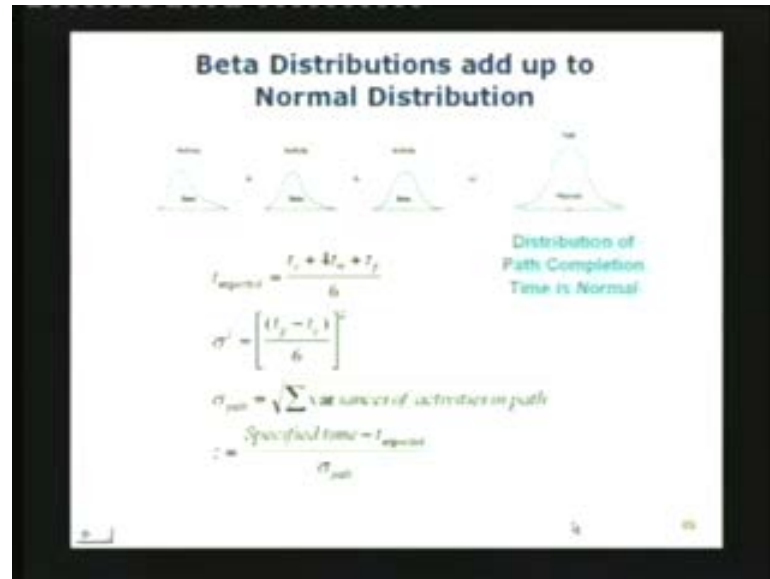
Let us, just recall for a minute, how did I construct the diagram, how did I construct this **this** little picture here, what was my starting point; starting point started with the charter then, tell me identified the scope good from scope I got to w b s; w b s, work breakdown structure and the work breakdown structure eventually brought me down to work packages, the work packages are now my tasks. Those are the tasks I will be assigned to different contractors, sub contractors.

Those are the tasks that I see here a, b and c; these are basically the different task that have to be completed by people. And then of course, I talked to the domain expert **I talked to the domain expert** and the domain expert told me that, suppose you are going to be doing this; **the if** if you are going to be doing this, you first have to do a then, you have to b then, you have to do c. Then, if you look at b, e and f that way you will be getting the second task; second part of the project done. Then, you do the last one; which is the g, h and i that is the lowest part thank you good job, sometimes managers have to act **right**. So, **So**, we did that.

So, I have here, if you look at this total project, how many tasks are involved in this, nine tasks; nine tasks are involved in this. I have got the sequence done there, what do you notice about the timings, what have they done, they have given you the optimistic estimate, they given you the most slightly estimate, they have given you the pessimistic estimate, this is the information about uncertainty **this is the information about uncertainty**.

What I can do? Now, I have to kind of give a timeless projection for this project; that means, I have to from this incident uncertainty I have to figure out, how long this project might take let us see how we do that. The first thing that I have to do is; I have to convert these times which have been given as three point estimates a triangular estimate I have to convert them to random variables.

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And it tells out, if you look at any one of those tasks it will have a distribution, the time will have a will have a distribution. So, I have that for activity a then, I have that same thing for activity b and I got the same thing for activity c, that path a, b, c would end up finding a distribution of its own, the sum will have its own distribution that is actually composed from the individual random compound.

Now, the it turns out you cannot just add them, because there is a random number there, there is a random number here, there is a random number here, you cannot just take random numbers add them up you cannot do that easily. What you have to do is, you have to go through a process called Monte carol. So, if you have been doing a bit of simulation, what I do in that is? I express the randomness in the first task; task a and I capture that by specifying two things I assume that, the distribution is approximately normal and I specify its expected time, which is the mean and the formula is given here;  $t_o$  plus 4 times  $t_m$  the most slightly time plus  $t_p$  divided by 6 plus the expected time to finish that task a.

How could you find this because, this is given to you for triangular distribution, if there is a little bit of theory in this then, I will explain that to you later on. I can actually come back and I can explain that to you this is one way to take your three estimates to your  $t_p$  and  $t_m$  and from that figure out the expected time and for standard deviation what they what they say is the total spread is suppose to be something like 6 sigma.

So, they take the difference between the **the the** backend; which is  $t_p$  minus  $t_o$  divided by 6 that is one sigma in a squared **(( ))** it is an approximation. See, many times you cannot really exactly replicate real life in your models. So, you do an approximation which is good enough and then, we check it out against reality, if say good not fit you say fine I will walk with this no, with this what I have done is, I have got now uncertainty captured on fact on task a, then task b, then task c. I have done that, not to compose them and the part that I am looking at is; obviously, if you have the three means you will end up with the total estimated mean of that past a plus b plus c.

For variance what you have to do is, you have to take the individual variances and basically then, you have to square, you have to add the variances and you take the square root of the sum that will give me sigma for the total path. So, the sigma for the total path is going to be in fact, the variance of the total path is going to be the sum of the variances of the individual **individual** activities effecting this **(( ))** square root of this I end up with my sigma, that the beauty of this is once I know expected we expected quantity **and the** and the quantity which is basically my variance and then, from that I get my sigma I can construct a little variable called the zee variable.

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$$Z = \frac{t - E_{x_{path}}}{\sigma_{path}}$$

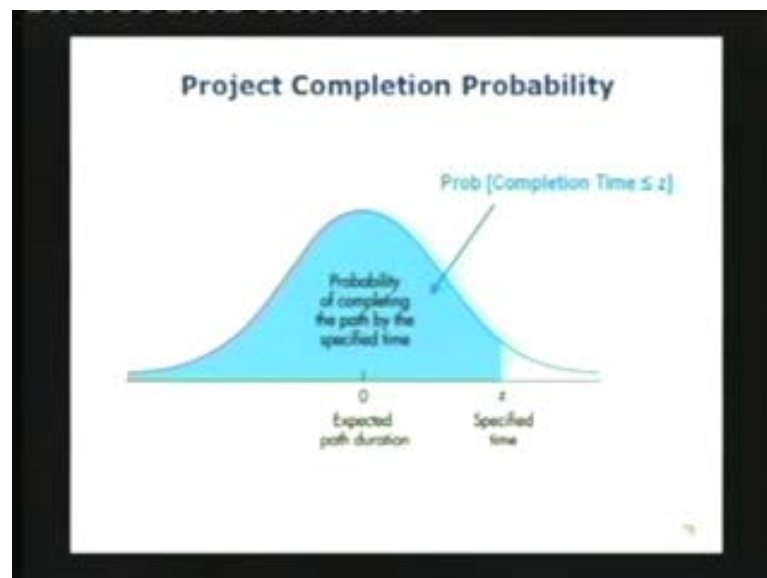
↓  
 $\mu = 0$   
 $\sigma = 1$

Zee is going to be, zee is a standard normal variant, which basically **takes you** take **any time** any time  $t$  minus  $t$  expected divide by sigma. And this is of course for the path, this is also for the path, this little formula converts the total finish time for the project **into a**

into a zee variable and what is the distribution for this zee variable, it is the standard normal  $(0,1)$ , what is it mean; mean is 0 excellent mean is 0 and sigma 1.

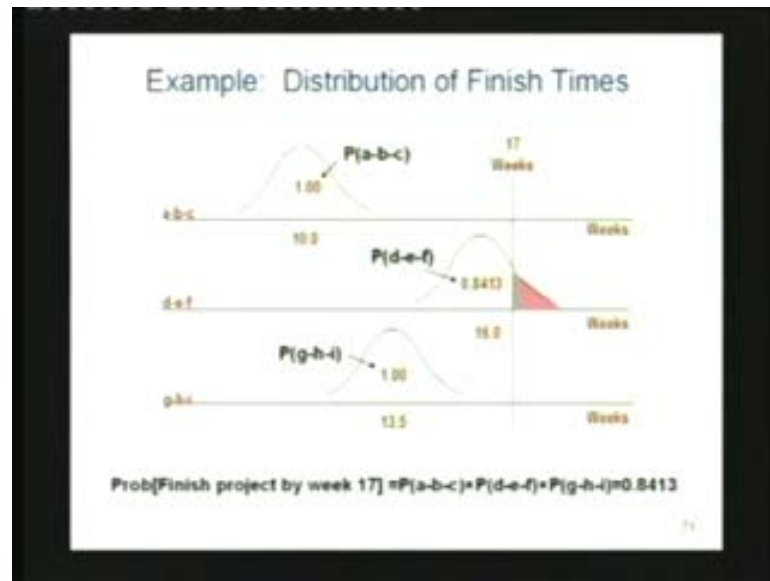
And I know most of the books of statistics or even project management books; they provide you with tables if you go to the back of the table you would be able to find a table that gives you the distribution of normal variables. Once you have that in front of you, it will tell you a particular value of zee; if you look at a particular value of zee, what is the chance for my finding a true value of zee, a real value of zee beyond that value, this gives me the dead line. This way I can locate the dead line there and this is the **the** dead line is the total project finish time. That means, I flip the slide here and you will see how that thing works out.

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This is basically what you will end up with and this is now the distribution of the transformed total **total** completion time converted on to zee scale by doing the little formula that we used to convert your standard normal to convert the normal distribution into a standard normal distribution. Now, of course, I can say the white area here in this diagram that, white area is the chance of my true project going beyond the dead line.

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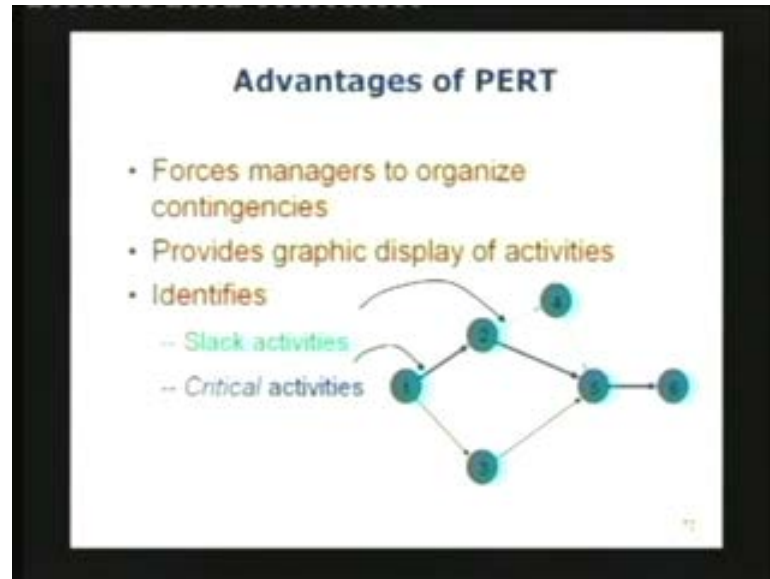


Let me show you how that is done, how many parts did we have in our diagram? Three parts I had a, b, c. When I did my Monte Carlo, when I did my calculation I ended to up with a distribution of a b c you said that, P a b c that is to the left you see the dead line what do you think is the chance of part a b c getting delayed beyond 17 weeks almost 0, what is the chance of g h i part getting beyond 17 weeks?

(( ))

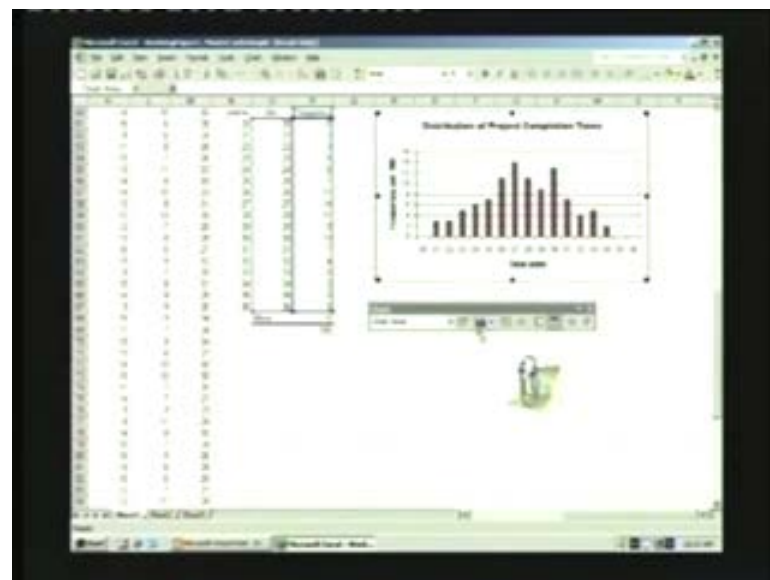
This one look at the look at the look at the bottom one, g h i almost no chance, but more chances than a b c a b c (( )), but look at d e f this carries the trouble some trouble someone this guy clearly tells me that, there is a substantial about about 15 percent chance of my exceeding the dead line. Here again, I can come back and say this is the guy that is going to be affecting my project very likely. So, what I do is, I swing resources I try to see, if I could bring them back here.

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So, this is in fact **what is done**, what I have done is, I have converted this into a excel worksheet and I am going to mailing you that worksheet you will actually be seeing this Monte Carlo.

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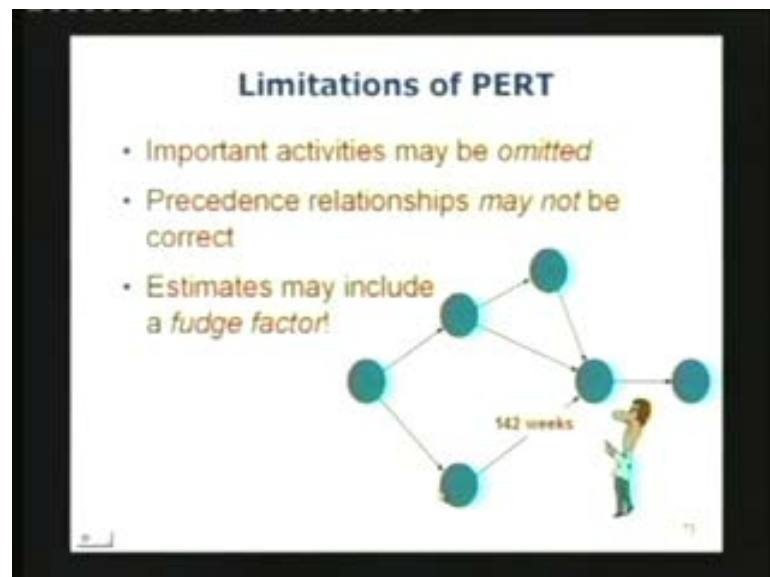
I will let me just see, if I could go there for a second, the Monte Carlo will eventually end up giving you a distribution like this and here again I began with my a b c **I began with my a b c** and this I am going to give you sheets like this, a b c are basically the three



parts then, I have got the total finish time for this and these are of course, random variables, if we click on that, you will be able to see numbers there.

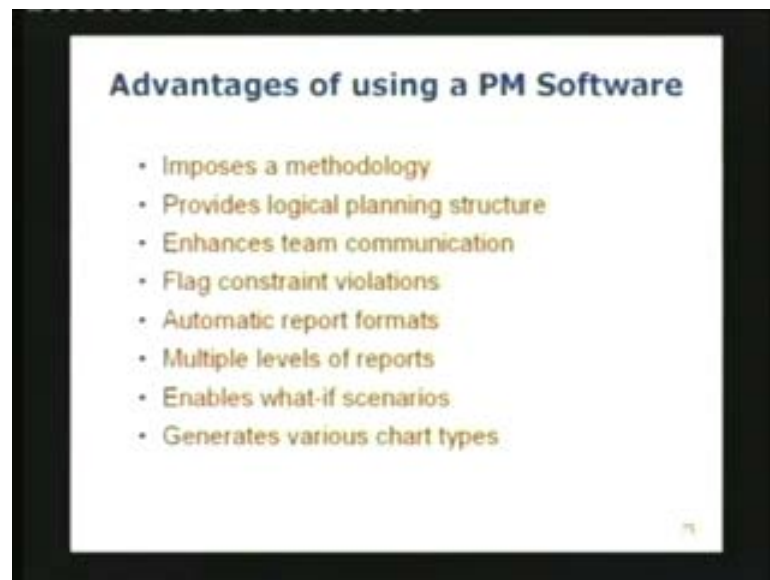
And they have been powered by utilizing a particular formula **and that formula is built into excel** that formula is built into excel and that is over here, the formula is given and I will show you again at some other time I will show you how exactly do it. You see the formula their you probably cannot see it, but never mind I will give you this worksheet with that you would be able to very easily workout any kind of projects that you have their how to go about doing this lets resume here just a little bit.

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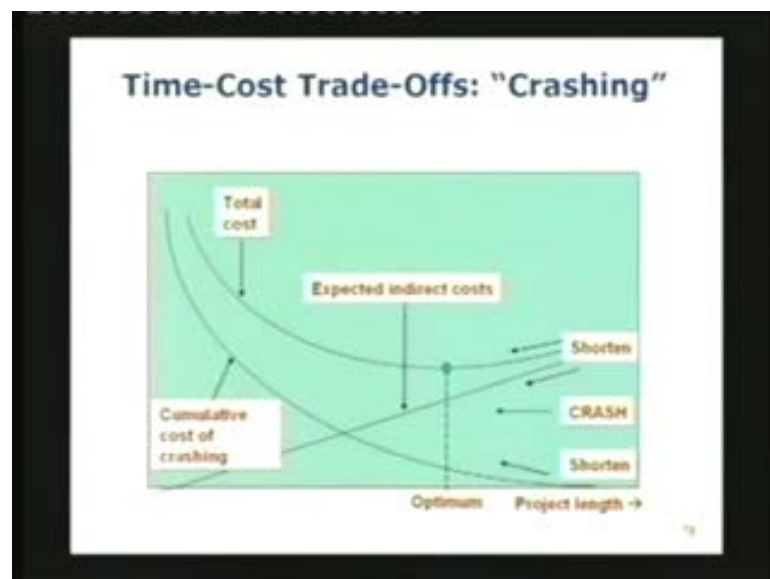
There are some limits of course, our problem is **you know** many times details are ignored sometime important activities may be ignored from this thing, people shy away from saying everything and they might be including fudge factors also. So, those are the real problems.

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When you are wherever you are looking at projects like this.

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Then, I want to get to one topic; which has do with crashing, crashing **is the** is the opportunity that you utilize, suppose your projects are getting delayed and you want to really know is there an activity that I could resource more I could bring in more resources I could compressive little bit. So, for that what you do is; obviously, it is going to cost you some money to compress it, **what do you gain when you finish the project**

early what do you think you gain when you finish a project early, what is the reason for you doing the project?

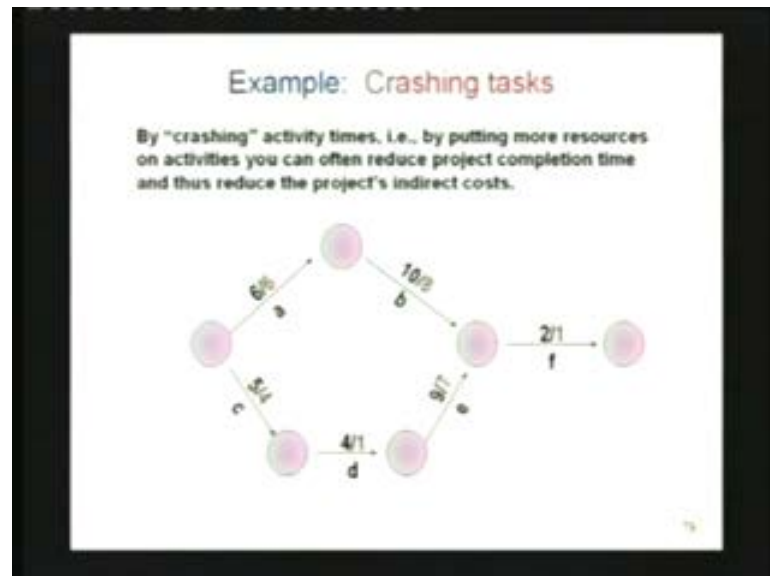
(( ))

So, what is the business effect of finishing a project early?

(( ))

Yes cash flow in future your revenue starts earlier. So, that is the trade off I can spend money on the project I can compress things and I can start my revenues coming in early so in fact, you see this is what is done whenever you are trying to optimize crash.

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Let us, take a look at this little diagram there, how many minutes I have 5 minutes, 3 minutes, 2 minutes, couple of minutes good. Let us let us just work on this. Here is a very simple diagram very simple project very simple project, I have task a and the black number here is the real time you probably cannot see it from there, but the black number is the real time, the estimated base time and the same thing would you applying for task b, task f, task e and so on, so forth.

Now, there is another number given here, which is in red colour they say, if you spend a 1000 rupees you could reduce the black number to the red number. See, if I on on task a for example, if I spend 1000 rupees I have no effect, I cannot reduce the task, it is like

growing a tree or letting the flower bloom it is going to take its time, it does not matter how much water I provide. But, task b something that can cut back that can actually reduce from being 10 weeks to 8 weeks.

Same thing is true for c, same thing is true for d; d has a straight jump, it can come from 4 to 1. Now, what you actually do is, you work out the deltas **the**, they returns from all of these and if I give you 1000 rupees, what is the first thing you would do to try to reduce the duration of this project, what is the first thing you would do?

**(( ))**

You will actually work out the critical part with those are the there is critical activities are the ones that really deserve the time. So, if I do that I will end up with the identified task and the next thing is going to be, I work out the trade off. If, I spend some money what is the reduction, I am going to be giving you a little exercise on this later on and this **this** procedure is called crashing.

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Now, what **I what** projects you are looking at you have to worry about resources, staffing and these things.

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**PROJECT PLANNING**

**Staffing**

**Counterproductive Characteristics**

1. Poor planning, direction, and control
2. Improper organization
3. Excessive staffing
4. Inadequate attention of management to productivity and the elimination of counterproductive elements
5. Internal communication problems
6. Insensitivity to people
7. Improper use of employees
8. An inadequate personal performance evaluation system
9. Ineffective interface with customers
10. Too many internal political machinations

Based on principles slide by Hughes Aircraft Company in 1960.

Counterproductive Characteristics of Project Team Effectiveness

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If you do not do them right, lot of things go wrong, those will be taking a look at.

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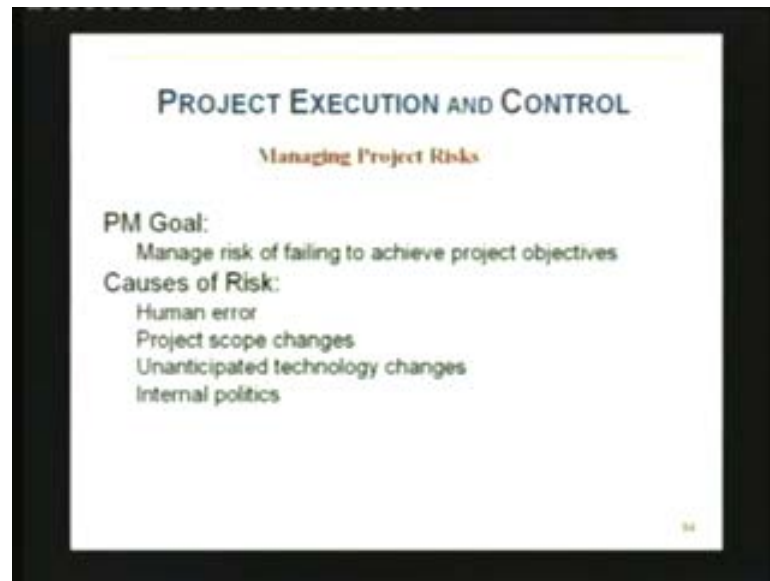
**PROJECT EXECUTION AND CONTROL**

- Project plan needs to be refined and reassessed throughout life of project
- Software project management tools commonly used to help initiate and monitor project tasks
- Communication among project team members critical for task coordination and integration
- Communication throughout project to all stakeholders is key to project success

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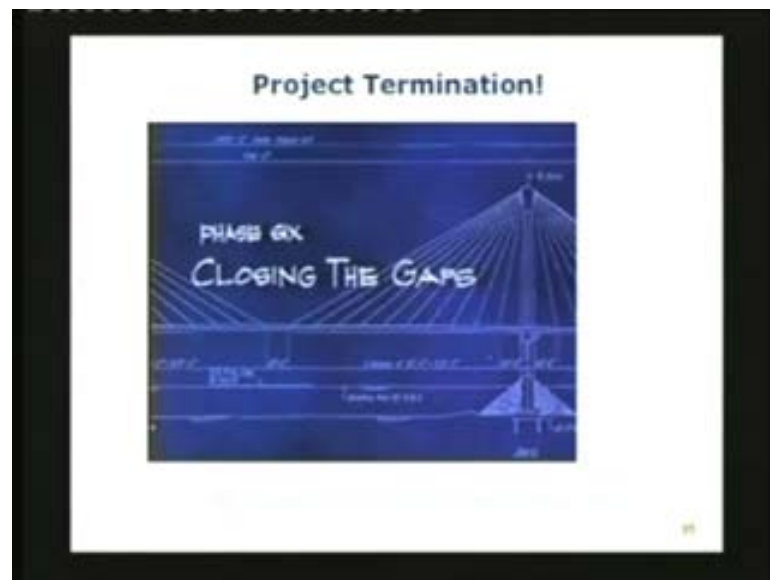
This is to be done during execution, this also be done in controls.

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The other important things, if you do it right, the project gives turn well ahead of time and so on, and so forth the customer is also happy.

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And you got to make sure you come to finish point, when the stake holders have been taking care of you have not exceeded the dead line and also **you have** you have under spent probably, if possible. That actually does not happen with too many projects, because of all the things that can fall up for that we will be studying risk management later on, thank you very much, any questions?

We have assumed that at part I have been when I combine all the activities in a path it becomes normal, but for calculating slack I need to have information about each particular activity to find.

Yes

So, for me the the distribution is not normal when I go to the activity level?

Very good

So, to get if I have to attach a probability value towards then, I need to model distribution for that not normal activity. So, how do we actually go about it?

Two things actually clarified; one, the concept of slack belongs to C P M and the concept of randomness belongs to pert; pert looks up the risk of over run, pert really does not really worry too much more criticality, C P M worries about criticality. So, pert is more complex and pert with resource limitation is very complicated and that is when you got to have good analysis done and so on, so forth. To really locate the right task that is what we did, when we looked at that task the task that had the distribution there.

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We wanted to make sure we understood the difference is and we only focussed on the path that really had the chance of forward run, we did not really get into C P M type of

thing, we did not really calculate slack, but we identified the candidate for resourcing by doing this, this is the issue, is that ok for you anybody, any other question?

Thanks a lot, we will talk to you tomorrow talk to you next week.