Advanced Financial Instruments for Sustainable Business and Decentralized Markets

Prof. Abhinava Tripathi
Department of Management Sciences
Indian Institute of Technology – Kanpur

Lecture – 9
Week 3

In this lesson we will introduce the concept of portfolio management we will discuss the computation of expected returns and risk of a portfolio we will discuss the concept of portfolio construction with two security case and multi security case lastly we will examine the concept of risk diversification with portfolios (refer time: 00:31) portfolio construction with two securities expected returns till now we have understood the interpretation of expected returns for a single security case now we will discuss expected returns for portfolio securities let us start with the portfolio construction for two securities some of the important question that we are trying to answer is what is a portfolio and why to invest in it more specifically what happens

When you invest in a portfolio what happens to your expected return and what happens to the risk when you combine two securities or multiple securities in a portfolio what is this term called diversification what are the benefits of this diversification also what is investing in mutual funds and indexes and how they provide diversification lastly what is the difference of investing in a portfolio like nifty fifty versus a single stock like hdfc what are the risk involved how they are fundamentally different from each other (refer time: 01:40) let us consider a portfolio of two securities where the actual returns from the two securities are r one and r two also if the proportionate weights or amounts invested in these securities are w one and w two then w one plus w

Two equal to one because the entire amount is invested in the portfolio so the summation of all the weights have to be one we already know that expected return on security one expected or e r one equal to r one bar and expected return on security two e r two is r two bar now let us try to understand the return from the portfolio the actual returns from portfolio can be very easily computed as shown here in equation one where r p equal to w one into r one plus w two into r two that is a very simple mathematical notation for the actual returns from a portfolio with two security cases where w and w two are the proportionate amounts and r one r two are the actual returns observed on these two securities(refer time: 02:45) now let us try to understand

The expected returns for this portfolio the expression for expected return on portfolio r p can be written as e r p equal to expectation or w one r one plus w two r two or we can further expand this result as expected value of w one r one plus expected value of w two r two please note here w one and w two are constants which are fixed from outside from the investing side so we can take them out and therefore the resulting expression becomes w one into expectation of r one plus w two into expectation of r two as shown here in the equation four please note here that r one and r two are probabilistic random variables with finite distributions generally such distributions are approximated by a normal distribution (refer time: 03:41) and therefore

These random probabilistic variables which are returns for them expectations can be computed for example the expected return on security one r one is r one bar similarly expected return on security two is r two bar and if the weights investment weights are known that is w one and w two the expected return on portfolio can be computed as r p bar equal to w one times r one bar plus w two times r two bar so therefore it appears that expected returns from the portfolio are simply the weighted average of expected returns of individual securities in the portfolio we are able to achieve this result simply because r one and r two being random probabilistic variable their expectations can be easily computed as r one bar and r two bar (refer time: 03:45)

Now this result can be easily generalized for a three security case or even multi security case let us consider a simple three security case where the entire amount or entire wealth is nested in three securities r one r two r three the proportionate amounts are w one w two and w three and therefore the summation of w one w two w three should be one because this is the total amount of wealth and w one amount is invested in security one w two amount is invested in security two and w three is invested security three therefore when we generalize our previous result r p bar equal to w one times r one bar plus w two times r two bar plus w three times r three bar this can be further generalized to n security case where proportionate amounts that is w one w two and

So on up till w n are invested in securities one two three and so on up to security n respectively also the summation of all the w i should be equal to one which is the total amount of wealth invested and generalizing the result total expected returns on portfolio r p bar equal to summation i equal to one to capital n w i times r i bar where w i are the weights and i bar are the expected returns on security i to summarize in this video we discussed the expected returns on a portfolio of two securities which comprises simply the weighted average of returns on those two securities weights being the proportionate amount invested we arrived at the formula for these two securities and then we generalized this case for a multi security case where security is

One to n securities are there and the expected return on that portfolio is simply the weighted average amount of proportionate amounts invested in all these n securities which is multiplied by the expected returns on all these n securities which is w i times r i bar summation i equal to one to n (refer time: 06:41) in this video we will talk about expected returns from a portfolio with equal probability case and case with different probabilities let us start with the expected returns from portfolio case one with different probabilities on column one we have probabilities that are shown here these are the probabilities for different scenarios corresponding returns for security a are provided here and corresponding returns for security b are provided here

The weights are fixed externally forty percent investment proportionate investment in security a and sixty percent proportionate investment in security b now the actual returns for each scenario are computed here which are simply w a r a plus w b bar r b and this will give us the actual return for each scenario with given probabilities as shown here now we can multiply each probability with the return scenario that is p t into r p for example zero point two into seven point two percent which is one point four four percent similarly each probability can be multiplied with corresponding return to get p t into r p and when we add this all up that is p one into r p one plus p two into r p two till p six into r p six we get the expected return on portfolio r p

This is expected to return portfolio r p which is eight point six five percent this was the case where all the associated scenarios have different probabilities (refer time: 08:14) let us consider another case where all the possibilities are equal that means all the scenarios have equal probabilities and if let us say we have n possible scenarios then each scenario will have one upon n probability in this case since we have six possible scenarios each scenario will have one by six probability and therefore to start with we have the corresponding returns weights are externally fixed so first we will compute the return from portfolio for each scenario which is w a into r a plus w b into r b so for first case it is seven point two percent which is

Nine percent into zero point four plus six percent into zero point six and w a r a which is three point six percent w b r b which is again three point six percent and the overall figure works out to seven point two percent in the similar manner we will compute the actual returns for all the possibilities all the six possibilities and the summation works out to eight point four three percent summation the average of the summation works out to eight point four three percent that means we sum up all these return numbers and then take the average which is eight point four three percent which is the average of all these six return figures and that becomes our expected return for equal probability case where all the probabilities are equal to summarize in this video

We discussed how to compute expected returns for a portfolio when case one given scenarios comprise different probabilities with each scenario and case two a scenario where all the possible all the possibilities or all the scenarios have equal probability (refer time: 10:00) portfolio construction with two security case risk in this video we will try to understand the risk of two security portfolio till now we have already understood how to compute the variance of a security for a case with given probabilities kindly have a look at this formula here if the probability of different return scenarios is p t that is p one p two and so on up till p t and different returns that is r i one r i two and so on as a generic term r i t then the variance of that

Security can be easily computed as p t into r i t minus r i bar raised to the power two summation over t equal to one two t this is a very simple and standard formula for computation of variance for a security with different probabilities now if the probability or the probability of observing each observation is equally likely then in that case p one equal to p two equal to p three and so on up till p t we also know that summation of p i is equal to one in that case this becomes one upon t so if there are t observations and all of them are equally likely then of all of them becomes one upon t in this case the variance of the security can be easily computed with this formula summation r i t minus r i bar raised to the power two upon t in some of

The textbooks they also note that since we are working with samples instead of using t they will be using t minus one here instead of using t they will be using t minus one (refer time: 11:46) now let us consider the variance for the risk of a two security portfolio in the case of a two security portfolio let us remember the very simple formula for a plus b raised to the power two which is a square plus b square plus two ab the risk of a two security portfolio can be very easily extended or understood in this with this help of this formula which is sigma p square is w sigma square which is corresponding to a square then w two square sigma two square corresponding to b square and for two ab we have a term very similar which is two times w one sigma one

Plus into w two sigma two into rho one two now here this extra term of rho one two rho one two here is the correlation between security one and two has some special meaning as we will see shortly here sigma p is the standard deviation or a proxy of risk for the portfolio sigma one and sigma two are the standard deviation of individual securities w one and w two are the proportionate amount of investment in each of these securities we already know if the portfolio carries only two securities then w one plus w two should be equal to one these are the proportionate amount invested and importantly rho one two is the correlation between these two securities this correlation varies from minus one to plus one now we will try to understand different scenarios what if

This rho two is equal to one or zero or minus one (refer time: 13:11) let us start with a very simple scenario where this rho one two is equal to one again the formula of the portfolio risk is already known to us which is sigma p square equal to w one square sigma one square plus w two square sigma two square plus two times w one w two rho one two sigma one sigma two now please note here this expression rho one two into sigma one sigma two is also called a covariance between the securities one and two and here rho one two is equal to rho two one and that is why you have a multiple of two in case this row one two or rho two one equal to one this then this formula considerably simplifies to sigma p square equal to w one sigma one plus w two sigma

Two raised to power two which is again remember coming from the a plus b raised to the power two formula which was equal to a square plus b square plus two ab and in this case the sigma p works out to w one sigma one plus w two sigma two please note for all the values and we already know that the sigma this rho one two lies between minus one to plus one so the magnitude of this rho one two will always be less than one and therefore for all the values of rho one two they will be less than one they starting from minus one to plus one they will always be less than one and therefore for all the other values of rho one two the value of risk sigma p will be less than this quantity whatever it may be it will always be less than this quantity what are the implications of

This please understand that if the securities have perfect correlation that means they are perfectly moving in lockstep and therefore their correlation is one there is no diversification and portfolio risk is also very similar to the portfolio expected return similar to what we computed for expected return that was weighted average portfolio risk here is also the weighted average of risk of these two securities for all the other cases where this correlation is less than one and we note that this is a maximum correlation case which is rho one two equal to one for all the other cases this sigma p will be less than the weighted average of individual risk components multiplied by the proportionate weights and this reduction in the risk is precisely what

We call as diversification and risk why are we getting this diversification this diversification is appearing because the correlation between these two securities is less than one that means any correlation which is less than one results in diversification (refer time: 15:53) let us consider another extreme case where correlation is equal to minus one in this particular case again our good old formula of sigma p square equal to w one square sigma square plus w two square sigma two square and so on two times w one two rho one two sigma one sigma two considerably simplifies to this formula now in this formula if we further solve this expression we will achieve this this is a very special case for a correlation of minus one what are the implications of

This formula please notice although the correlation of minus one is usually not observed in financial markets or other commodity and derivative markets it leads to a special case where a particular w and w two can be achieved where this value can be made zero but please note this is only theoretical discussion because in no financial markets or between two securities negative correlation is observed over medium to long terms and therefore from theoretical and academy discussion purposes assuming that sigma rho one two correlation equal to minus one in that case a particular combination of weights can be achieved where sigma p becomes zero this is a more of a theoretical case but it suggests that a particular scenario can be obtained theoretically where

The overall risk can be made zero this is the case where maximum diversification is achieved where sigma p becomes equal to zero the risk of the portfolio becomes zero (refer time: 17:28) now let us understand about generic and easy to use thumb rule while constructing the risk for two security portfolio you need to create two cross two boxes like this where you have first w one sigma one and w two sigma two here and w one sigma one w two sigma two here now in the first box you will combine these two terms to get w one square sigma one square and again this cross term will give you w two square sigma two square in a very similar manner so these are called diagonal boxes if you look at odd off diagonal boxes or boxes that are opposite to the diagonal

You will combine w two sigma two with w one sigma one and you get w one sigma one w two sigma two but in addition to that you also get a correlation term which is rho one two similarly in this off diagonal box you combine w two w two sigma two with w one sigma one to get w one sigma one w two sigma two with again correlation term and if you add up all these four terms you get the generic expression that we saw here w one square sigma one square plus w two square sigma two square plus two times w one w two rho one two into sigma one sigma two which was our generic expression that we used earlier to compute the risk of two security portfolio to summarize in this video we understood how to compute the risk of a two security portfolio we also understood that

Securities that move exactly lockstep manner with a correlation of one there is no diversification and maximum diversification is achieved when the securities move exactly in opposite manner that is with a correlation of minus one for all the other cases correlation remains less than one and some amount of diversification is always achieved when two securities are added (refer time: 19:23) portfolio construction with multiple securities risk: in this video we will extend our understanding of portfolio risk for a multiple security case let us start with a portfolio of three securities we will extend our logic that we developed in the previous video for two security case we will try to implement that here for multi securities simply extending the formula for

A two security case to multi security case kindly have a look at this three cross three box and we will try to extend this formula that we developed there for this three cross three security case so we have w one sigma one for security one w two sigma two for security two w three sigma three for security three similarly we have rho one two for correlation between security one and two rho one three correlation between security one and three and then we have row two three for security two and three (refer time: 20:11) now the overall portfolio risk will be a combination of these diagonal terms that is w one square sigma square very similar to what we did earlier for two securities then w two square sigma two square w three square sigma three square and again

The off diagonal terms that are here they will be put together w one sigma two sigma one w two sigma two and along with that there is a correlation term rho similarly if you look at this particular box here you are combining w one sigma one with w three sigma three so this term will result and then in addition you are putting correlation term and this goes on for all the nondiagonal or off diagonal boxes like this and this each of these nondiagonal boxes they will include a correlation term because here we are combining the variance of two different securities unlike diagonal terms where we have same security and the variance numbers computed (refer time: 21:15) let us extend this understanding to a more generic and security case here again you have w one sigma one w

Two sigma two and so on up till w n sigma n and w one sigma one here on the column and row aside as you would have already guessed these diagonal boxes sorry these diagonal boxes will include all the various terms that are combination of w one square sigma one square w two square sigma square so this will be something like w one square sigma one square and so on so up to w n square sigma n square so these are all the diagonal boxes on the off diagonal terms you will have combination of two different securities for example here you have w two sigma two w one sigma one along with the correlation terms similarly here you will have w one sigma one w n sigma n along with the correlation term rho one sigma rho one n and so on and so forth up till here where

For example you may have something like w n minus one sigma n minus one into w n sigma n and the correlation term between security n n minus one so in this fashion we will fill all the off diagonal terms which will include the covariances between two different securities these off diagonal boxes while the dark diagonal boxes like this they will include only the variance terms (refer time: 22:46) now let us understand the variance of this n security portfolio and let us focus on the diagonal terms so how many diagonal terms you observe there will be total there will be n into n that is n square terms while out of these n square terms there will be n diagonal terms and these diagonal terms will carry only variance terms like sigma i square w i square

So if i sum up all these diagonal terms something like sigma i equal to one to n summation w i square sigma i square will result this is because all those diagonal terms if you notice this diagram here all these diagonal terms there are n such diagonal terms and all these diagonal terms will include an expression like w one square sigma one square or w n square sigma n square so if i add up all these diagonal terms an expression like this will emerge now to simplify the situation let us assume that all these securities have equal investment that is one by n proportion that means since there are n securities and if i invest equal amount in each security the proportionate amount invested in each security will be one by n with that assumption my w i which is

Equal to one upon n and therefore w i square will become one upon n square and therefore a term like this will be simplified to this now let us take this one upon n outside the expression so we are left with this please note here n is not the varying term only i subscript is varying so we can take the n outside and here w i is one upon n so we get something like this expression inside which we can define as average variance so if let us say define average variance something like this so we will have something one upon n into average variance down which is defined simply like this so our overall variance for the portfolio becomes one upon n times sigma square average where this is average variance term defined as this (refer time: 24:43) now let us focus on

The off diagonal boxes as we noticed that there are total n square boxes and n are the diagonal boxes so we are left with n square minus n off diagonal boxes which include covariance terms or the cross products of securities including the weights invested in these securities and the resulting term we have already seen this that will appear something like w i w j sigma i sigma j into rho ij so these are all off diagonal terms now the overall multiplication will appear something like this if i sum up all the covariance terms off diagonal terms all those n square minus n terms i will end up with something like this w i w j sigma i sigma j rho ij summation i equal to one to n summation j equal to one to n where i is not equal to j why this special subscript because

For all the cases that i equal to j they will become the diagonal terms with variance terms now we already made a simplifying assumption that w i equal to w j equal to one upon n that means we are investing equal amount in all the securities and therefore those proportionate amounts w i equal to w j are same as one upon n in that simplifying case our overall expression becomes this one upon n square into sigma i sigma j rho ij we can take n out of this expression because n is constant so we are left with this expression now let us define an average quantity of covariance or average covariance we already know that there are n square minus n boxes or n into n minus one boxes so an average covariance can be easily defined something like number of or summation of all

These covariance term divided by total number of these covariance boxes which are n into n minus one or n square minus n so let us define our average covariance as we did for various terms also we can define our average covariance as total number of boxes or total number of terms which is n into n minus one multiplied by summation of all these covariances so this is the average covariance or a simplifying assumption (refer time: 26:46) now let us rearrange these covariance terms a little bit we already defined that summation of all these covariance term is this which further simplified to this and we defined something called average covariance like this which is one upon n into n minus one times the summation of all these covariance terms

Now let us rearrange them a little bit so here i can take the n square from this expression to this side so my summation of all these terms is equal to covariance terms into n square now here i already know that this expression is equal to because of this this expression this expression is also same as sigma average covariance times n into n minus one so this is equal to this because of this expression and we already defined that summation of these covariance terms is equal to this one where n square we can take here so a very simple expression summation of all these covariances become this covariance term summation into n square which is equal to again sigma average covariance term n into n minus one and therefore what we are interested in is

The summation of all the off diagonal terms or the risk of off off diagonal terms which is covariances which simply becomes n square into n minus minus n which is this into one upon n square into sigma square average covariance because we can take this n square here and therefore the equality becomes very simply n square minus n into one upon n square times sigma average covariance or more simply n minus one upon n into average covariance (refer time: 28:21) now that we have a simplified expression for covariance as well as variance terms let us put put this expression together to find the overall risk of the security so first we have set of variance term for which we found a simplifying as simplifying expression in the form of average variances

So our simplified variance terms is equal to one upon n times sigma square average and our covariance term summation is equal to n minus one upon n into average covariance so we will put them together to find the overall risk of the portfolio or sigma p square which is one upon n into sigma square average plus n minus one upon n into average covariance now now if you have large number of securities fairly large number of securities where n is tending to infinity for practical purposes you do not need to go up to infinity even with twenty thirty securities a reasonable amount of diversification is achieved so theoretically if n is fairly large please notice that this term will approach to zero because your denominator is n while if n tends to infinity

This term will approach to one and therefore we can easily say that the when n value is sufficiently large or tending to very large values the variance term or the diagonal terms will come zero so the overall portfolio risk or portfolio variance will be simply equal to the average covariance and all the variance terms are canceled out because of this property that n is fairly large and therefore this term will approach to zero and this term will approach to sigma average covariance or average covariance of the portfolio what are the implications so here we find that just by adding sufficient number of securities we can cancel out or neutralize the idiosyncratic or stock specific or variance part the diagonal terms that were there they are canceled out while

The average covariance term did that is not nullified and therefore the overall portfolio risk tends closer to the average covariance term this is precisely what we achieved with the help of diversification simply by adding more and more securities we achieve or neutralize the idiosyncratic or variance terms that are stock specific equal to zero to summarize in this video we discussed how addition of securities neutralizes the stock specific or variance terms or diagonal terms part of this these are the variance terms and as we keep on adding security the off diagonal terms which include covariances they are not neutralized they still remain and therefore the portfolio risk tends towards this average covariance for a large number of security portfolio

We also discussed how to extend our logic of two security portfolio to generate expression for multi security portfolio and how this multi security portfolio with reasonably large number of securities achieve diversification (refer time: 31:26) risk diversification with portfolios: in this video we will conclude our discussion on this diversification with portfolios with the help of a few numerical examples now we have understood that in a portfolio with large number of securities the summation of variance term is close to zero for example if we start with a portfolio with one security like in the diagram shown here a large part of the portfolio risk comprises security specific or idiosyncratic risk which is driven by the variance terms

The diagonal terms that we saw which are variance terms however as we keep on adding more and more securities this particular component risk variance or idiosyncratic or stock specific component of risk tends to become nullified or canceled in contrast there is another component of risk which is on account of covariances that is correlations across securities this component of risk is even though it is small in a single stock portfolio as we keep on adding more and more securities it is not diversified or nullified in the portfolios so as we keep on adding more and more securities this component remains still and for a very large number of securities only this part of portfolio risk remains while the specific or idiosyncratic risk tends to go down to zero and

The remaining component which is this average covariance risk this is also called bedrock of risk as it cannot be eliminated by adding more and more securities the diversification is achieved only because of cancellation or neutralization of this stock specific risk we also noted that this average covariance risk or risk that is on account of those off diagonal terms or covariances that is driven by the correlations across security now if we want to further decrease the risk which is given by these covariances we would like to select securities that have low correlations with each other as you keep on adding more and more securities in a portfolio like we discussed this specific risk is almost close to zero and if you add all the securities available in

The market the overall risk that is remaining will only constitute this risk that is driven by these covariances and often it is called market risk or bedrock of risk because that portfolio with all the securities available in the market the risk which may be called as market risk is driven by these covariances and often referred to as market risk or bedrock of risk this is also called systematic risk or non diversifiable risk because it cannot be diversified by adding more securities in contrast the specific risk that is called diversifiable risk or nonsystematic risk because it can be diversified simply by adding more securities to the portfolio (refer time: 34:31) in financial markets fund managers and investors are only rewarded for bearing

This market risk often you would have heard the statement on mutual fund ads that mutual funds are subject to market risk the statement precisely reflects this fact that this risk cannot be eliminated even if you had all the stocks and securities available in the market you would still be exposed to this systematic risk or market risk and this cannot be diversified and therefore markets only reward for bearing this market risk or systematic risk therefore as we will see shortly number of asset pricing models they price this market risk only they do not price this specific stock specific idiosyncratic risk they only price this market risk and therefore fund managers and investors are rewarded only for bearing this market risk component (refer time: 35:24)

Let us perform a simple example of expected portfolio return let us say if you have two securities you invest sixty percent of the money or wealth available with you in security one and forty percent of the wealth in security two now you also know that expected returns from security one and two are eight percent and eighteen point eight percent respectively therefore the computation of expected return as we have already seen the formula is very easy and is simply w one times r one bar plus w two times r two bar where w one is sixty zero point six and w two is forty percent which is zero point four multiplied by the respective returns expected returns which is eight percent and eighteen point eight percent and therefore we get twelve point three percent as expected

Return on the portfolio a portfolio that comprises forty percent of security two and sixty percent of security 1 wealth invested will give you twelve point three percent as expected returns (refer time: 36:23) let us consider in the previous example some additional information that is the risk of security one is thirteen point two percent is and risk or sigma two is thirty one percent is and the weights invested remain the same at sixty percent in security one and forty percent in security two now for different cases five cases in fact with correlations of and 1 minus one zero minus zero point five zero zero point and one let us let us try to compute the risk of the portfolios we already know that the risk of a two stock portfolio can be easily computed

With the help of this formula which is w one square sigma one square w two square sigma two square plus two times w one w two sigma one sigma two all that the quantities are fixed and we are only varying this correlation from minus one to plus one let us see how these numbers worked out (refer time: 37:10) so first look at the first case where rho one two equal to one and the formula is simply the same formula that we are using w one sigma square one and so on and in this case the overall risk works out to be twenty point three two percent which is also same as the weighted average of individual risks this is because now in this case as we have seen the rho one two no diversification occurs and the formula is simply standard deviation or variance

Formula is simply this because no diversification is there and securities are moving in perfect lockstep manner now as we keep on decreasing the correlation using the same formula please notice the portfolio risk or standard deviation comes down gradually seventeen point seven four percent for rho one two here the correlation number is zero point five all the other things remain same for correlation equal to zero the standard deviation is fourteen point seven one percent for correlation is equal to zero minus zero point five the portfolio risk is ten point eight eight percent please note that the portfolio risk is minimum when correlation is minus one so the shows that as correlation decreases diverse more and more risk is decreased and diversification is achieved as

The correlation is lower to summarize in this video we discussed the two components of risk which is one which is stock specific and idiosyncratic component or diversifiable risk or nonsystematic risk this component of risk can be easily nullified or neutralized by adding more and more stocks to the portfolio next we also discussed the systematic or market component of risk which cannot be diversified and it acts as a bes bedrock of portfolio so even if you include infinite number of securities this market risk or systematic component of risk cannot be diversified and it will depend on the correlation across securities in the portfolio we observed that as we keep on decreasing the correlation from minus one plus one to minus one we notice how standard

Deviation of the portfolio decreases so in order to decrease the risk of portfolio a well diversified portfolio with large number of securities if you pick and choose those securities where correlation is less than one you can achieve more and more diversification to summarize this lesson adding more securities that are less correlated or have lower covariance in the portfolio leads to diversification diversification here means the reduction of stock specific risk the part of the risk that is non diversifiable is on account of the covariances across securities often this risk is called market risk or systematic risk markets do not reward for bearing stocks specific diversifiable risk since these risks can be easily mitigated and diversified when we say that we expect certain return for bearing risk that risk is systematic non diversifiable risk or often referred to as market risk