Advanced Financial Instruments for Sustainable Business and Decentralized Markets

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Lecture – 5
Week 2

In this lesson we will introduce the economic theory of choice under certainty. Individuals behave according to their risk preference which determines the utility of wealth. In this backdrop, we will discuss the indifference curves for investors and how these concepts help us in arriving at the risk return framework in financial markets. We will also discuss the expected returns and actual returns, in financial markets. We will examine various measures of risk in financial markets and their computation.

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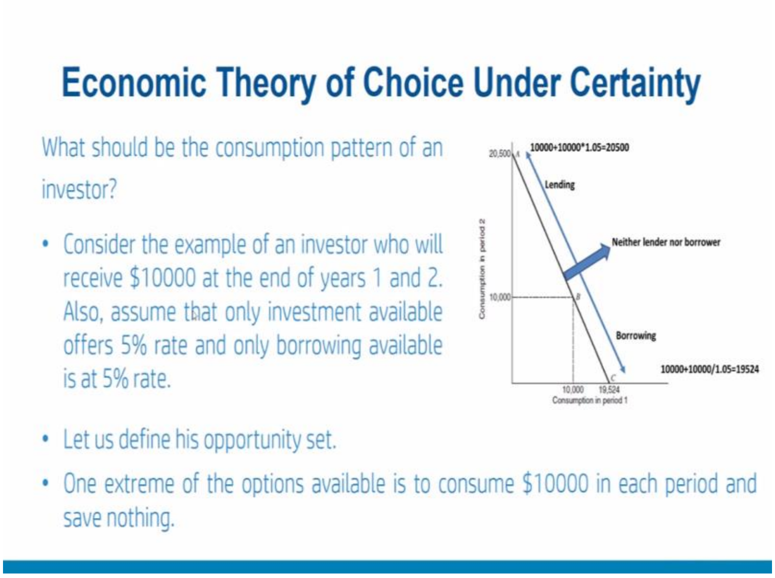

Economic theory of choice under certainty. In this video, we will discuss about economic theory of choice under certainty. We will examine how individuals can optimize their time pattern of consumption with the help of financial markets. Consider the example of an investor who will receive 10,000 rupees at the end of years 1 and 2. Also assume that the only financial instrument available can be borrowed or lent out at an interest rate of 5 %.

The question we are trying to explore here is as follows what should be the consumption and investment pattern of this investor. To answer this question we need to first define his opportunity set. At one extreme of the options available is to consume rupees 10,000 in each period and save nothing. This option is shown as point B in the figure here.

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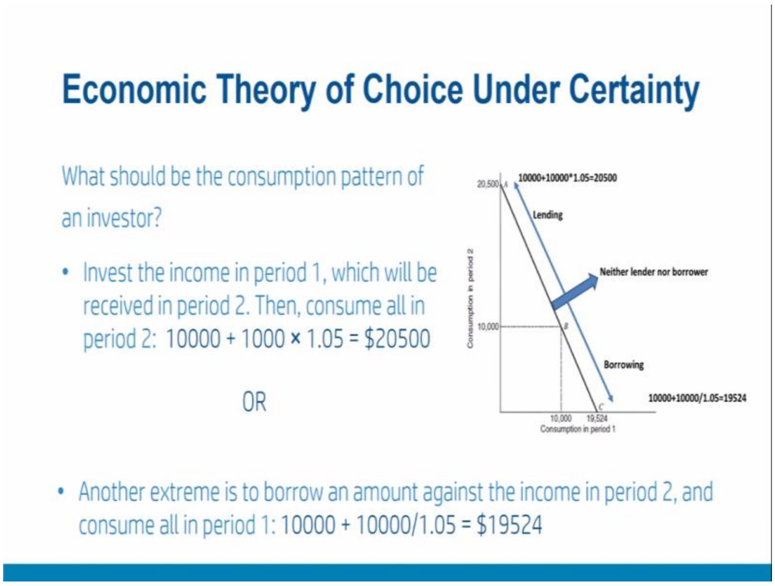

If he saves all the income that is invest at 5 % in period 1 and consumes everything in period 2 then the amount he invest in period 1 will become rupees 10,500 including interest in period 2. Adding this to the second period income the total amount he can consume becomes rupees 20,500 as shown here in point A. Another extreme is to borrow an amount against the income in period 2 this will be 10,000 divided by 1.05 = rupees 9,524. So, he can consume 19,524 in period 1 itself, this is shown by point C.

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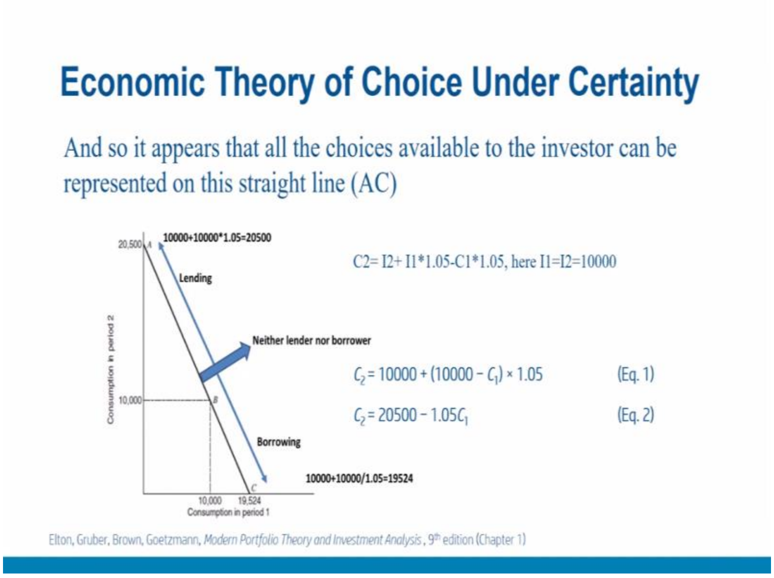

And so as we can see here in the diagram it appears that all the choices available to the investor or the individual can be represented on the straight line AC as shown here in the figure.

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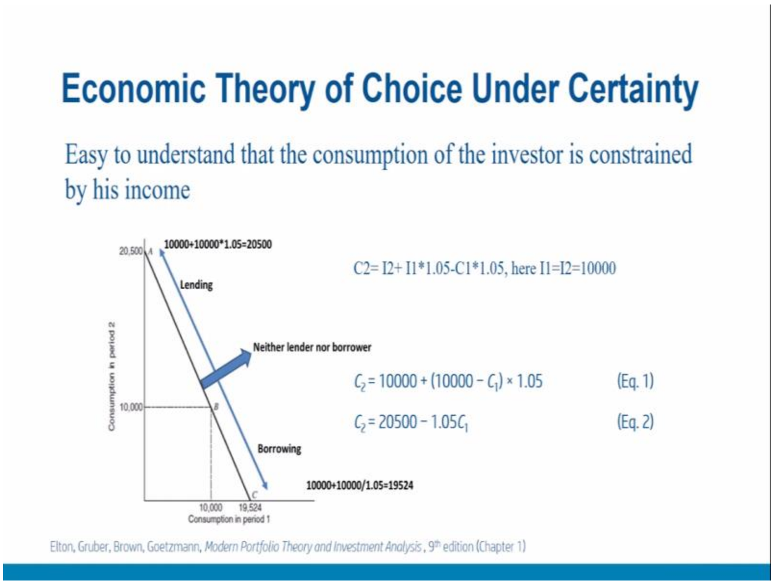

It is easy to understand that the consumption of the individual is constrained by his income. If the consumption in period 2 is C 2 and in period 1 is C 1 we can write a simple equation that defines the relationship between income I 1 and I 2 and consumptions C 1 and C 2 as shown here this will be as follows. $C_2 = I_2 + I_1 \times 1.05 - C_1 \times 1.05$ here I 1 = I 2 = 10,000.

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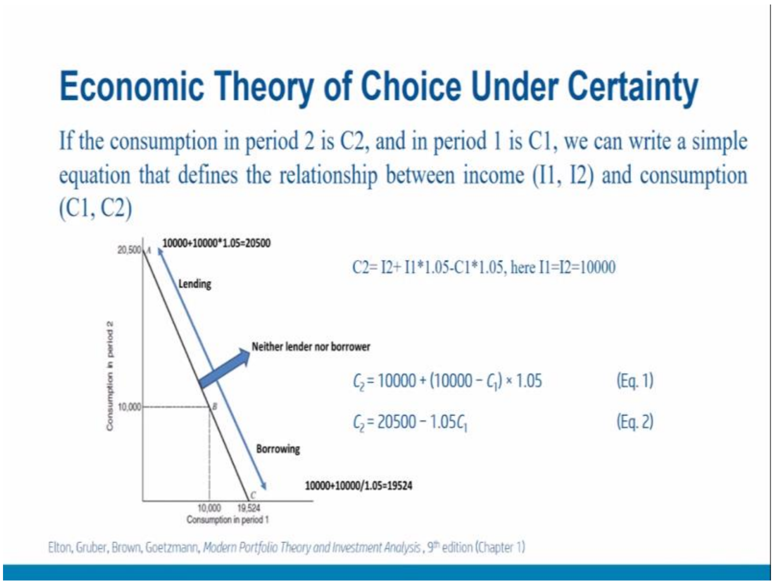

So, if the consumption period 2 is C 2 and in period 1 is C 1 we can write a very simple equation as shown here that defines the relationship between income I 1 and I 2 and consumption C 1 C 2. For example $(10{,}000 - C_1) \times 1.05$ is the amount available in period 2 which is equal to amount saved at 5 % interest rate after consuming C 1 from the income of 10,000.

So, the resulting equations are $C_2 = 10{,}000 + (10{,}000 - C_1) \times 1.05$ which can be further simplified to write $C_2 = 20{,}500 - 1.05\ C_1$.

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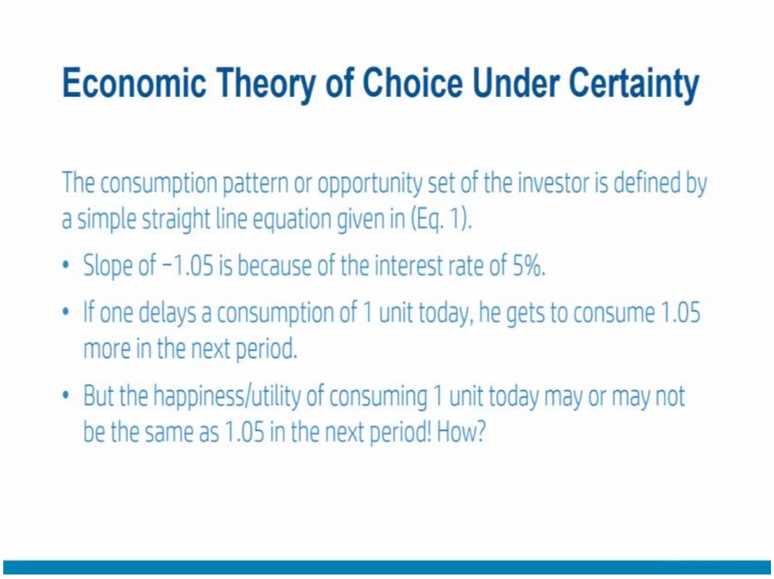

The consumption pattern or opportunity set of the investor or individual shown here is defined by a simple straight line as was given in equation 1 or equation 2. This is an equation of straight line AC as we saw in the figure that passes through point B. The slope of this line was – 1.05. The slope reflected that for each additional unit the investor consumes in period 1 1.05 unit is consumed that is less in period 2.

This means you could have saved this amount one unit to earn 0.05 unit interest and consume 1.05 units in period 2. This line AC that we saw it represents the set of choices and it is also referred to as the opportunity set. The question still remains here is; that is there any particular point on this opportunity set that is most optimum to an individual? We will have more to say about this in the next video.

To summarize, in this video we defined the opportunity set of individuals and individual who is more of consumer would like to borrow and consume as much as possible. Another individual who is a saver will consume less and would prefer to invest more. These two cases represent two extremes of the opportunity set and all the individuals would lie in between. The straight line joining these two extreme cases will represent the opportunity set of different individuals.

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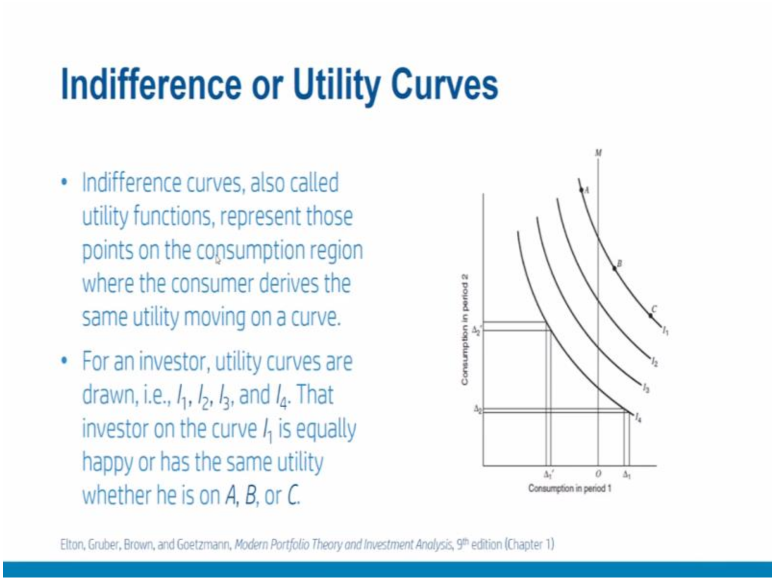

Indifference curves and interest rates. In this video we will discuss the indifferent curves and we will try to understand how individual maximize their utility on the opportunity side. Indifference curve or utility curves as they are called represent those points on the consumption region where the consumer derives the same utility by moving on the curve that is he is equally happy on any of the points on the curve.

For example in the figure shown here we plot utility curves along the consumption axis. For an investor four utility curves are drawn that is I 1, I 2, I 3 and I 4. The investor on the curve I 1 is equally happy or has the same utility whether he is consuming at points A, B or C.

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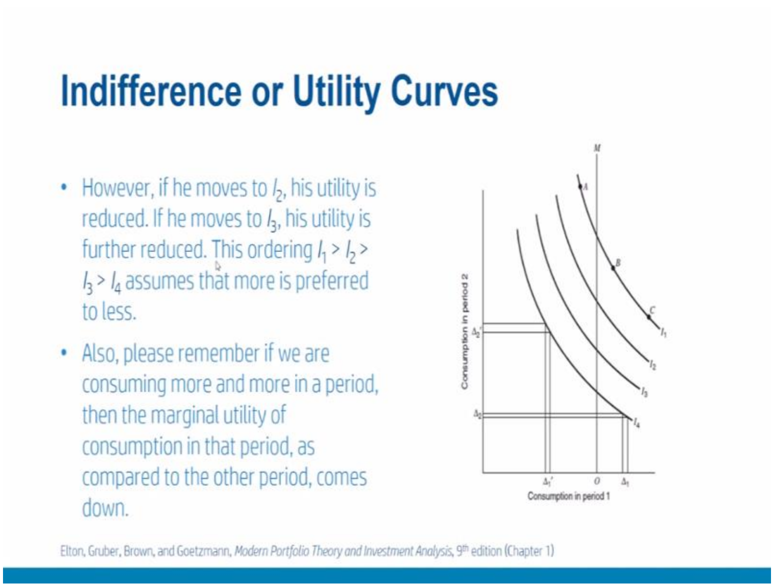

However, if we moves to I 2 his utility is reduced if he moves to I 3 his utility is further reduced. The ordering that is I 1 greater than I 2 greater than I 3 is greater than I 4 assumes that more is preferred to less. Also please remember if we are consuming more and more in a period than the marginal utility of the consumption in that period as compared to the other period comes down.

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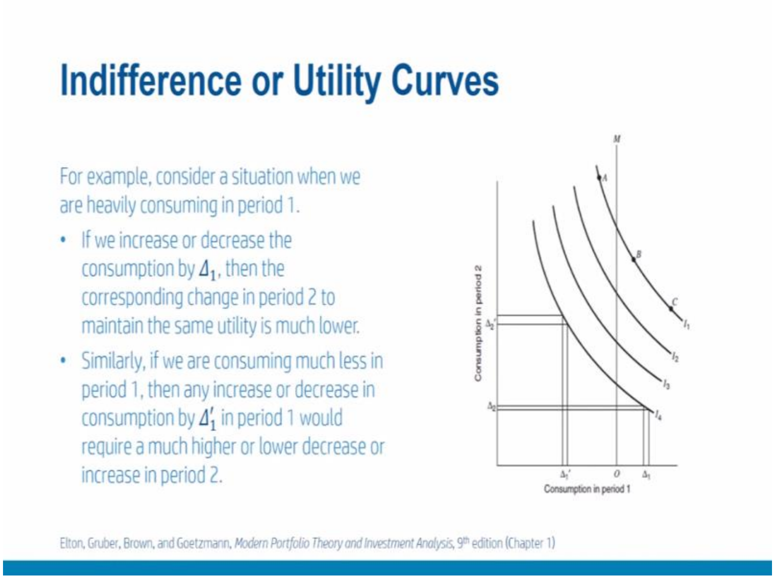

For example consider situation where we are heavily consuming in period 1. If we increase or decrease the consumption by delta 1 then the corresponding change in period 2 to maintain the same utilities much lower. Similarly, if we are consuming much less in period 1 then any increase or decrease in consumption by delta 1 dash in period 1 would require a much higher or lower decrease or increase in period 2.

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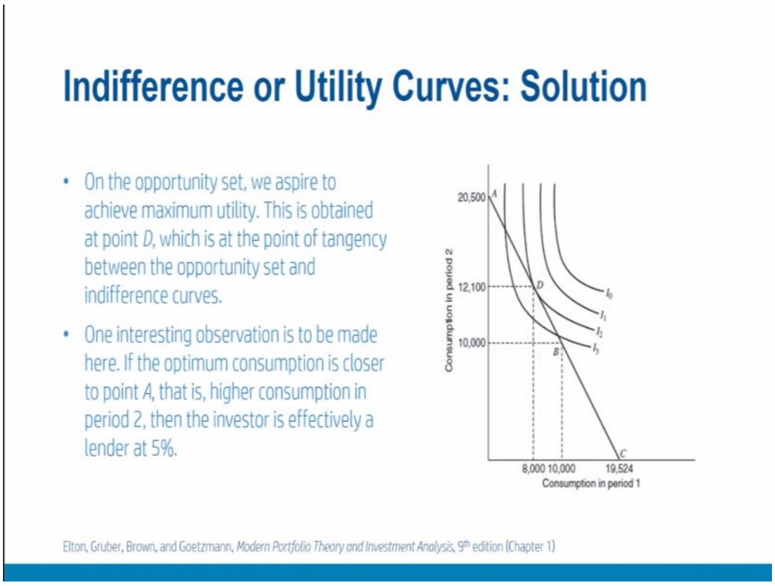

A common ground or solution to this problem is determined by the opportunity set and utility curves. This is shown in the figure here. On the opportunity set we aspire to achieve maximum utility this is obtained at point D which is at the point of tangency between the opportunity set and indifference curves. Any other indifference curve will be above the opportunity set.

One interesting observation is to be made here if the optimum consumption is closer to point A that is higher consumption in period 2 then the investor is effectively a lender at 5 %.

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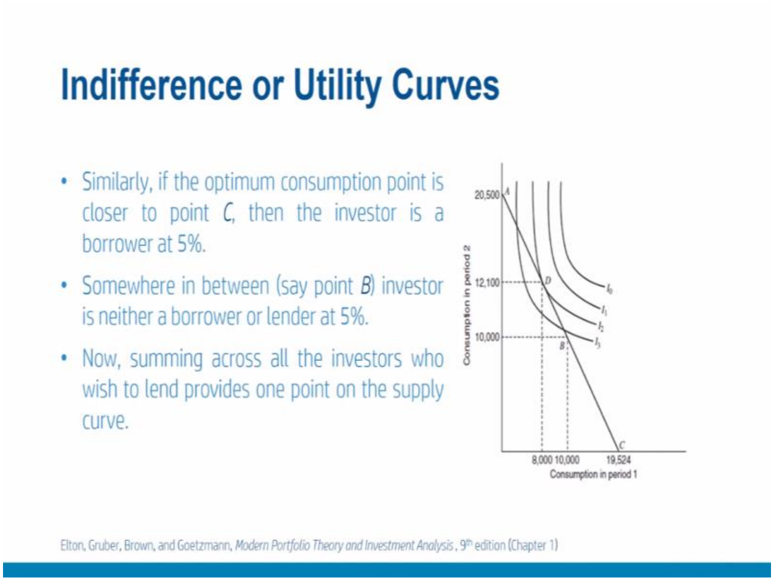

Similarly, if the optimum consumption point is closer to point C then the investor is effectively a borrower at 5 % somewhere in between say point B the investor is neither a borrower nor a lender at 5 %. Now summing across all the investors who wish to lend provides one point on the supply curve.

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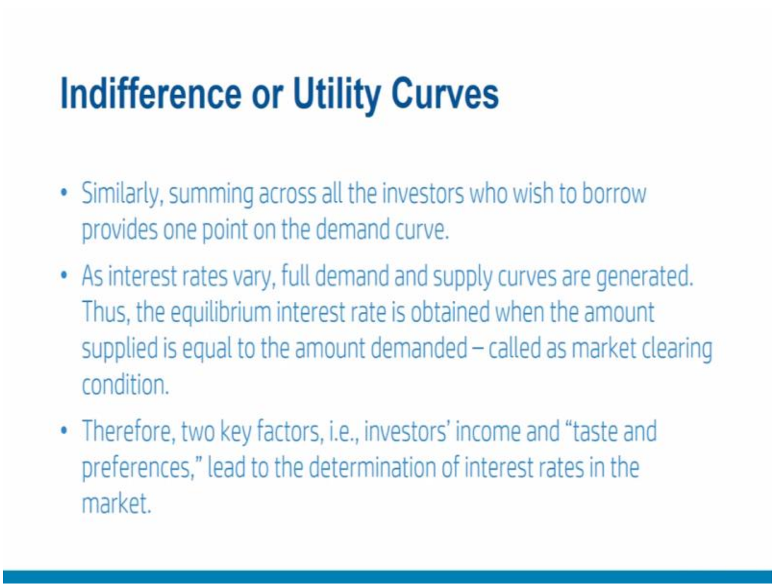

Similarly, summing across all the investors who wish to borrow provides one point on the demand curve as interest rates vary full demand and supply curves are generated. Thus, the equilibrium at interest rate is obtained when the amount supplied is equal to the amount demand called as the market clearing condition. Therefore, two key factors that is investors income and taste and preference lead to the determination of interest rates in the market.

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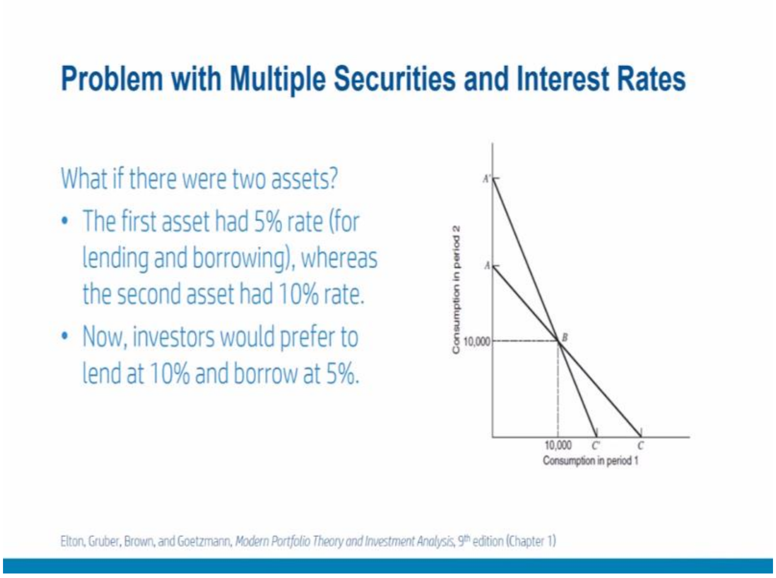

The above case sustains to a single financial instrument what if there were two financial instruments? The first instrument has 5 % interest rate for lending and borrowing and the second instrument has 10 % interest rate. Now, investor would prefer to lend at 10 % and borrow at 5 % and therefore the optimum region here would be A dash B C dash as shown in the figure here. However, this seems to be not feasible.

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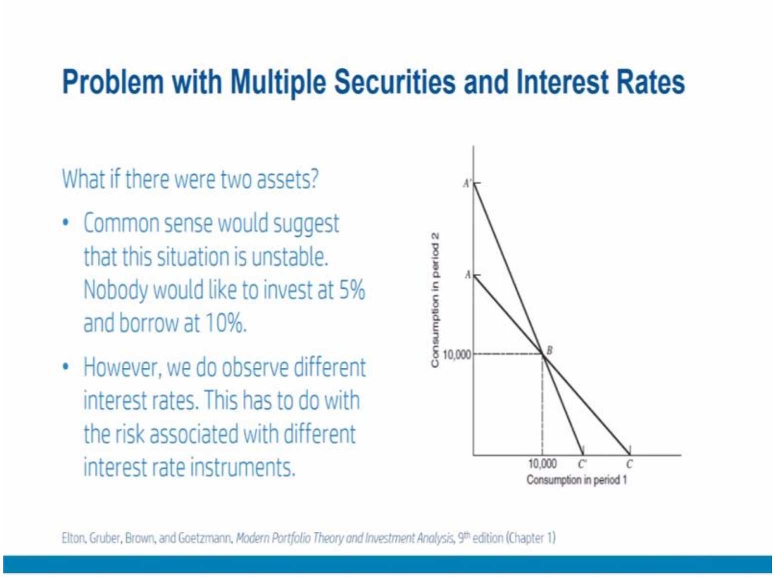

Common sense would suggest that the situation is unstable and nobody would like to invest at 5 % and borrow at 10 %. However, we do observe financial instruments with different interest rates. Now this has to do something with the risk associated with the different interest rate instrument. To summarize, in this video we examine the indifference curves for individuals.

And their role in utility maximization for individuals with respect to their opportunity set. These maximum utility points determine whether individual is a borrower or investor. When aggregated across all the individuals these maximum utility points provide demand, supply curve for a market and this demand and supply in funds help us in generating market clearing interest rates.

However, this analysis suggest a single clearing interest rate while in real markets we do observe financial instruments with multiple interest rates indicating the presence of uncertainty or risk in market conditions.

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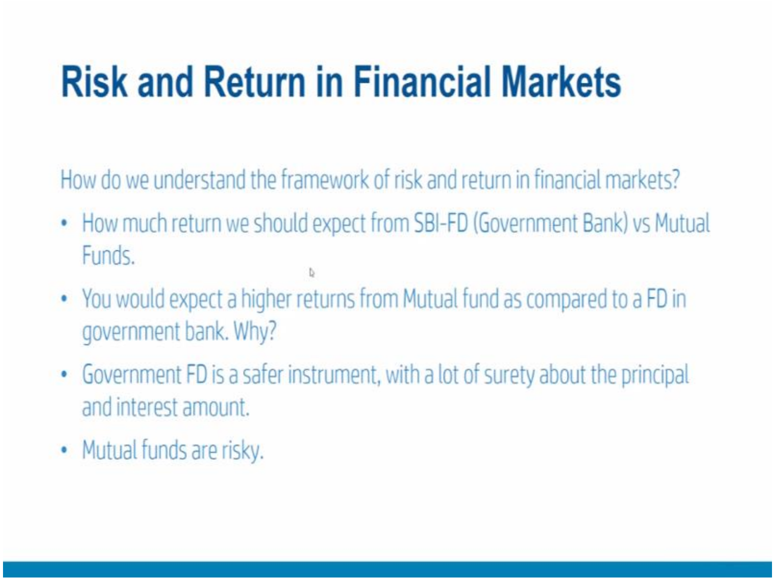

In this video, we will introduce the concept of expected returns and see how it is different from actual returns. Consider an SBI fixed deposit and investment into mutual funds how much return we should expect from these investments? From SBI fixed deposit probably a fixed return of say 7 % what about investment into mutual funds may be 10 % or even higher.

Why should we expect different returns from these instruments because these instruments offer different degrees of risk to the owners of the instruments. For example SBI fixed deposit is safe and for a very high degree of surety you know that your principal and interest amount will be paid back at the time instrument matures that is the holding period. However, in case of mutual funds there is no such surety and you may even get an amount that is lesser than what you invested. This uncertainty of outcome is called risk.

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Let us come to the computation of returns. Generally speaking return from an instrument has two major components first is capital appreciation or capital gains which is ending price minus opening price and then interest income or cash inflow from holding that instrument that is dividends or interest from fixed deposit. Consider simple example where price P 0 that is the price at which you bought the instrument is rupees 10.

$$Return = \frac{Capital\ appreciation + Interest\ Income}{Inititial\ Investment}$$

And the instrument is held for one year. The price at the end of the year is P 1 which is rupees 15 also during the year the stock gave you a dividend of rupees 5 then please note the following computations. Capital gains in the form of change in the price of security which is 15 – 10 / 10 which is 50 % and cash inflow in the form of dividend or interest income which is 5 / 10 is 50 %.

Assuming there are no taxes the overall return that you have is 50 % plus 50 % which is 100 %. This can be easily computed as follows. Total return on the instrument or holding period return is equal to cash inflow plus capital appreciation divided by initial investment. Here the initial investment is the value of the investment at the beginning of the period in this case:

$$Return = \frac{5 + (15 - 10)}{10} = 1\ or\ 100\%$$

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Consider the following statement. Stock A offers 10 % return and stock B offers 20 % return which one is the better investment? Is there some piece of information missing? Of course, we need a holding period, for example, if stock A offers 10 % in a week and B offers 20 % in two years surely we are not comparable still we are not saying that A is better than B we will come to that later once we introduce the risk.

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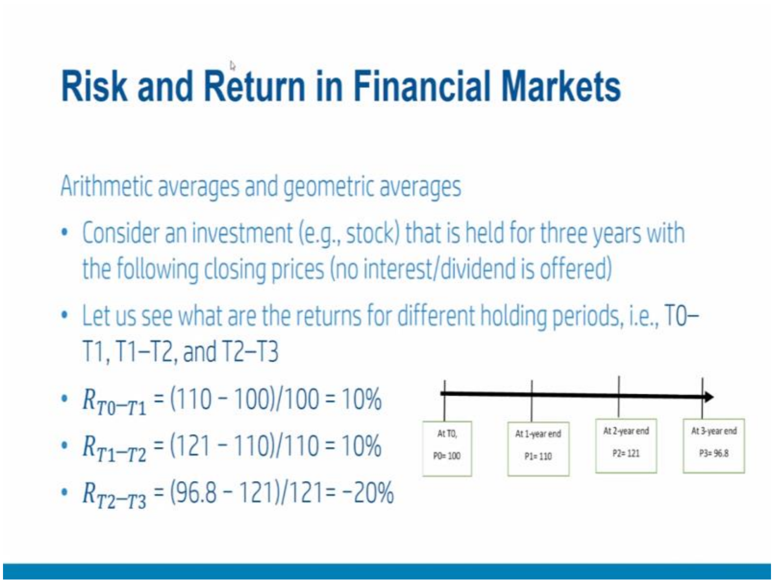

Now, we will come to the computation of actual returns and comparison between arithmetic averages and compounded returns. Consider investment in stock that is held for three years with the following closing prices assume that no interest or divided is offered so we can see the timelines at T = T0 prices P0 which is 100 at one year end price P1 = 110 at 2 year end price P2 = 121, at 3 year end P3 = 96.8.

Let us see what the returns are for different holding periods that are T0 to T1, T1 to T2, T2 to T3.

$R_{T0\text{-}T1} = (110 – 100) / 100 = 10\%$

$R_{T1\text{-}T2} = (121 – 110) / 110 = 10\%$

$R_{T2–T3} = (96.8 – 121) / 121 = – 20\%$.

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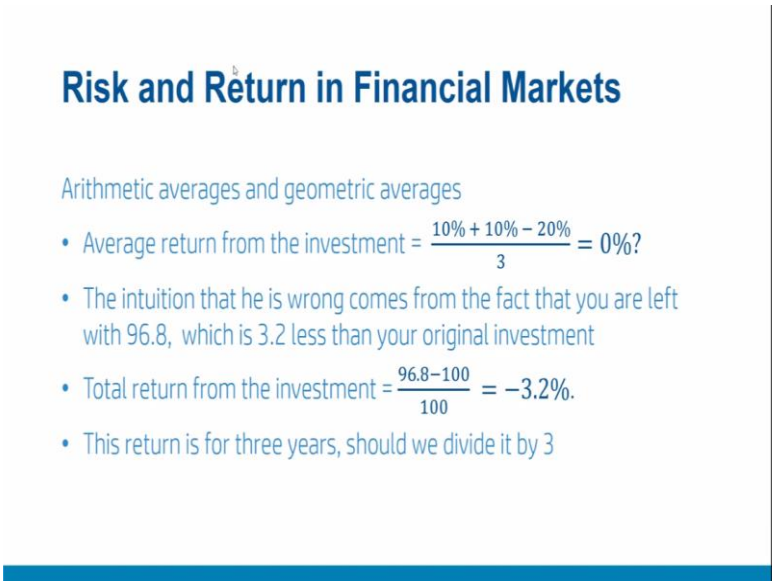

Now you are expected to compute average return from the investment. Your accountant tells you that average return from the investment = (10% + 10% – 20%) / 3 = 0% is he correct? Please note although we will discuss it later this is called expected return, but we will explore this concept in more detail shortly. The intuition that he is wrong comes from the fact that you are left with 96.8 at the end of the holding period which is rupees 3.2 less than your original investment.

Surely your investment yielded you a negative return, but how much negative that we will see. These suggests that in this context arithmetic averages are not exactly correct and so we will focus on the compounded returns or geometric averages. From our definition of holding period return total return or holding period return in this case amounts to with no dividend or interest income at (96.8 – 100) / 100 = – 3.2%.

This return however is for 3 years what about annual returns should we divide it by 3 similar to the simple arithmetic averaging scheme to get the annual returns, but have not we seen that arithmetic averages are not correct in this context.

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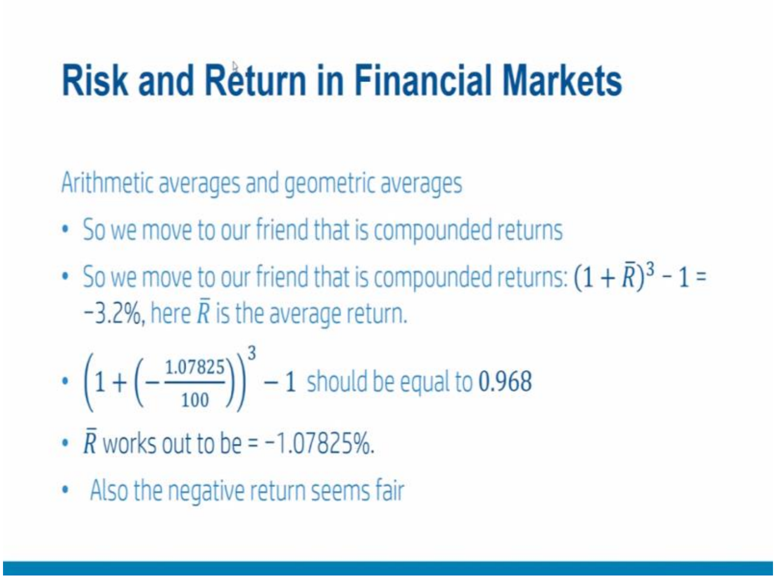

So, we move to our friend that is compounded returns. As per the compounding schemes consider an average annual return of R bar in this investment then the total return or holding period return for 3 year should be $(1+\bar{R})^3 - 1 = -3.2\%$. Solving this equation we can easily find that R bar works out to – 1.07825%. We can back calculate this number to check if we are correct that is $\left(1+\left(-\frac{1.07825}{100}\right)\right)^3 - 1$ should be equal to 0.968.

Also the negative return of – 1.078 % seems fair unlike the 0 % return that we obtain from the simple averaging scheme.

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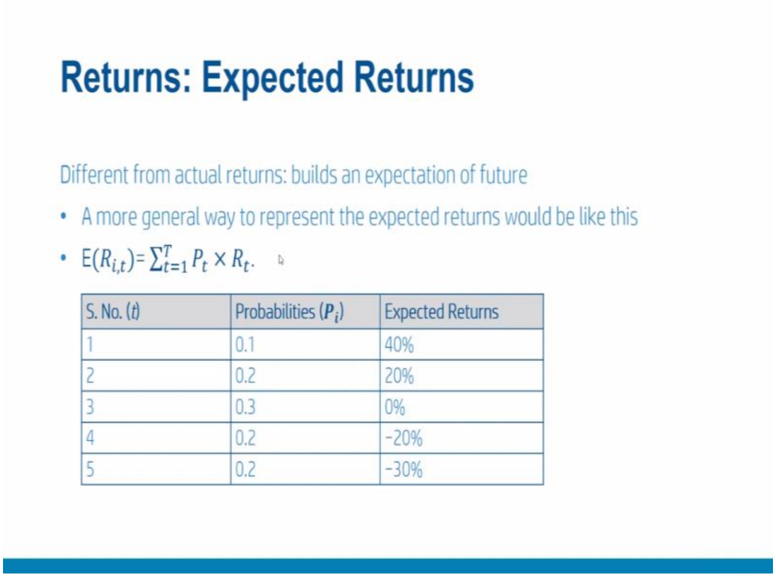

Please note that the above discussion holds in the context of or the calculation of actual returns. The actual returns are different from expected returns and we will come to the expected returns in a short while for that we need to understand the expectation operator. Consider the table shown here in the slide and a simple game like this. If you invest rupees 100 there is a 10 % probability that you will end up with 140 which is 40 % return.

20 % probability of ending up with rupees. 120 that is 20 % return, 30 % probability of 100 that is 0 % return, 20 % probability of 80 that is – 20 % return and 20 % probability of 70 that is – 30 % return as we can see here from the table. Now a very simple way to compute expected returns with the formula shown here is

$$E(Rit) = \sum_{t=1}^{T} Pt \ X \ Rt$$

We will explore this formula in more detail now.

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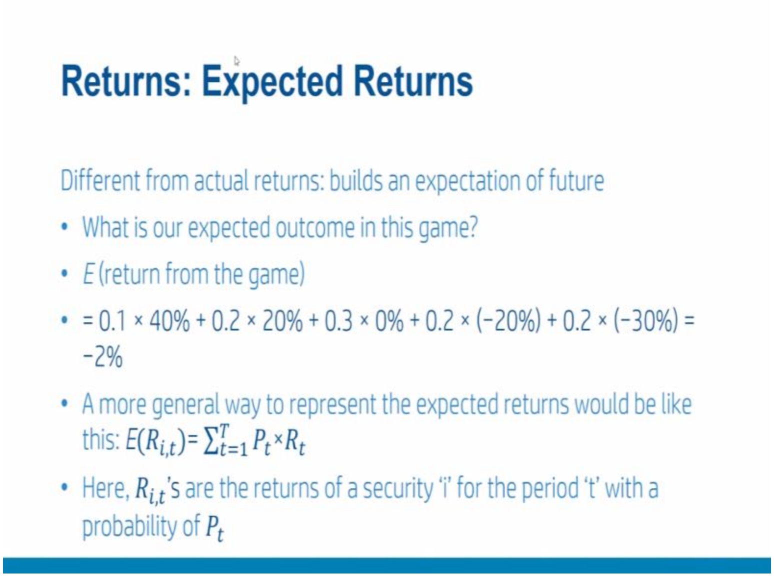

What is our expected outcome from this game? From our understanding of simple probability computations we know that the expected value of this game can be shown as follows. Expected return from the game =0.1 x 40 % + 0.2 x 20 % + 0.3 x 0 % + 0.2 x (– 20 %) + 0.2 x (– 30 %) = – 2 %. We are simply multiplying returns with probabilities.

Please note that we will get the same result even if you work with absolute value of outcomes that is rupees such as 98 or 100 however in financial markets it is prefer to use returns instead of prices. A more general way to represent the expected returns would be something similar to expected return, expectation of

$E(Rit) = \sum_{t=1}^{T} Pt\ X\ Rt$. Here Rit are the returns on security Ri for period t with the probability of Pt. In this case we have t discrete outcome of returns for which we are given the probabilities.

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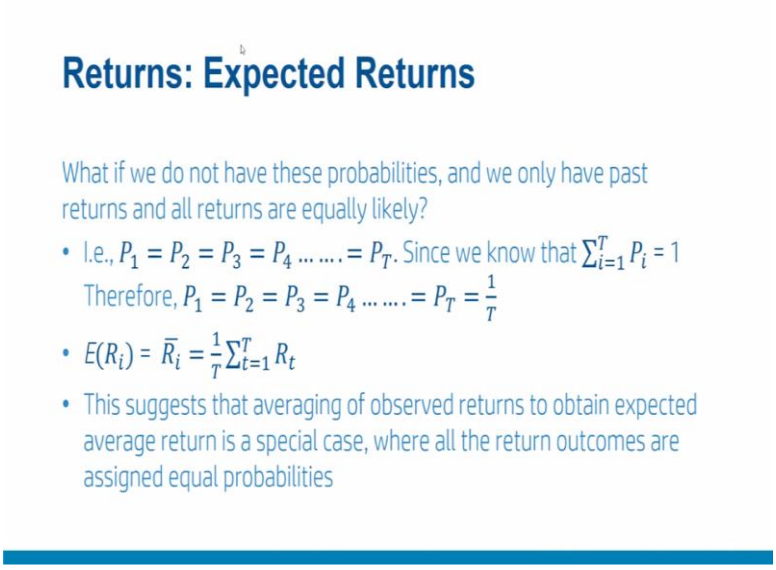

What if we do not have these probabilities and we only have past returns and all the returns are equally likely. Consider a special case if all the scenarios have the same probabilities that is $P_1 = P_2$ and so on up till P_T and

$$\sum_{i=1}^{T} Pi = 1$$

and therefore $P_1 = P_2 = P_3$ and so on equal to $P_T=1/T$.

Therefore, our formula of expected return becomes $E(Rit) = \bar{R}i = \sum_{t=1}^{T} Rt$ this we saw earlier as well. This suggests that averaging of observed returns to obtain expected average return is a special case where all the return outcomes are assigned equal probabilities. To summarize, in this video we introduced the risk return framework in financial markets.

We examine the computation of actual returns and expected returns and discuss the contrasting differences in these. Lastly, we also discuss that averaging of returns is a special case of expected returns where we can assign equal probabilities to all the possible historical return outcomes.

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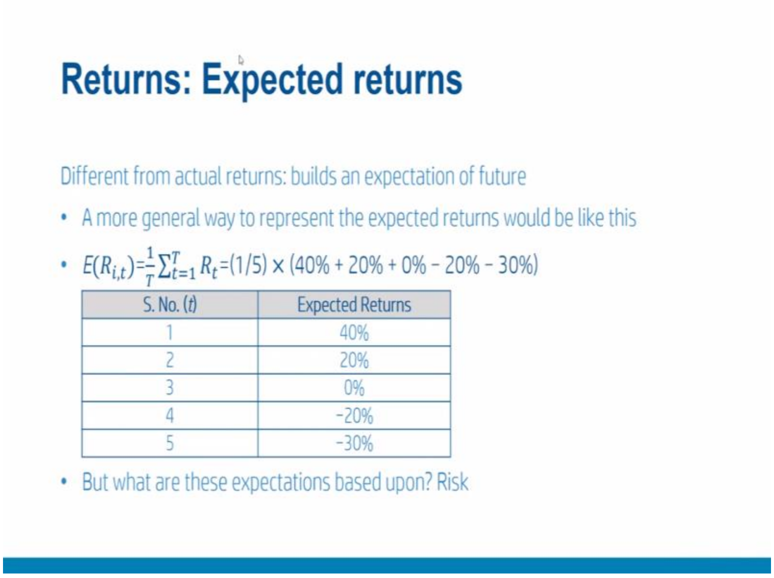

Let us have a look at one simple example and try to compute this. The numbers are exactly similar here and only difference is that now the probabilities are not given and we are assuming that all the probabilities are equal. So, therefore we will add up all the 5 observations of return and divided them by 5. The key question that remains and that we would discuss in next video is what are these expectations are based upon which is risk.

$$E(Rit) = \bar{R}i = \sum_{t=1}^{T} Rt = \left(\frac{1}{5}\right) X\ (40\% + 20\% + 0\% - 20\% - 30\%)$$

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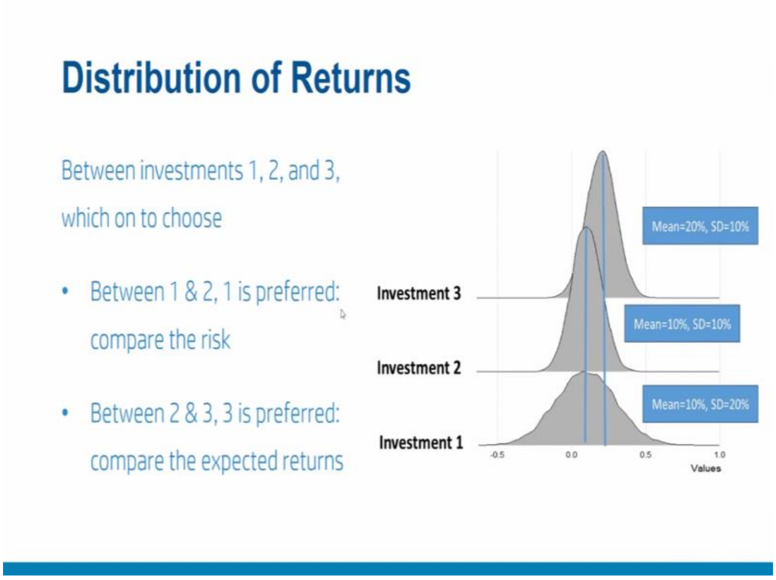

Please examine the return distributions for three investment opportunities investment 1, investment 2 and investment 3. If these three opportunities are given to you which one would you choose between investment 1, 2 and 3? Let us examine and compare investment 1 and 2. If you notice the mean or expected return for both the investments that is investment 1 and 2 are same.

However, the prospects of investment 1 are more scattered and more deviated from its mean as shown or as is visible in the more scattered nature of returns around the mean. This is also reflected in the fact that the standard deviation of investment 1 is much higher at 20 % as compared to investment 2 at 10%. We will wait to see the computation of this standard deviation for now.

The standard deviation is a measure of risk, the variance and standard deviation of returns are two important measures of risk or spread around the mean or mean deviation. They represent the risk associated with the stock, we will see the formula shortly. However, given that their mean or expected return is same, but the standard deviation of investment 2 is less you would prefer 2 over 1.

Now, let us compare investment 2 and 3. For both of these instruments or securities standard deviation is same at 10 % however the mean of investment 2 is lower while the mean of investment 3 is higher at 20 % and therefore you would prefer as a rational investor you would prefer investment 3 over investment 2. So, while comparing 1, 2 and 3 we notice that an investor would prefer higher expected return or mean return for a given amount of risk or standard deviation and for a given amount of expected return they prefer lower risk or lower standard deviation.

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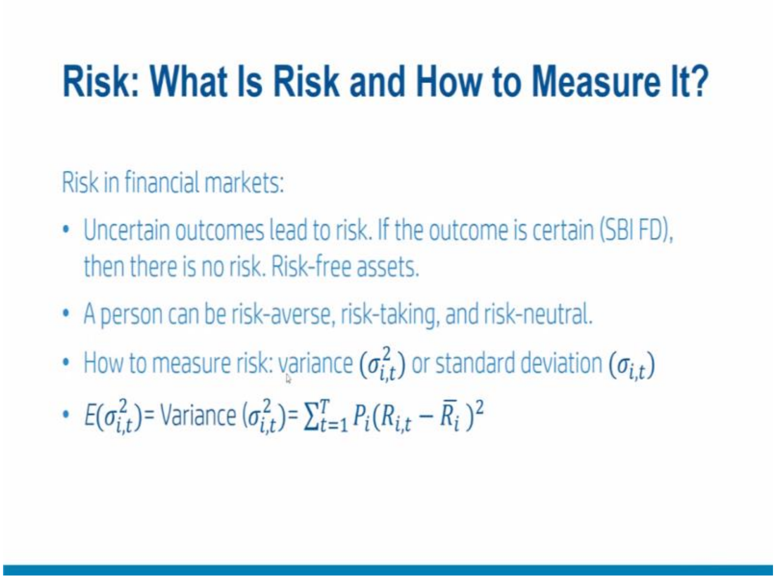

Now, let us move to the computation of variance and standard deviation. It is quite similar to computation of expected returns. Please remember that uncertain outcomes in financial market lead to risk. For example if the outcome is certain as we discussed about SBI fixed deposits then there is no risk and that kind of instruments are called risk free assets. The second important thing to remember here that a person can be risk averse, risk taking or risk neutral.

Generally in financial markets are more practical case is that investors are risk averse and these investors are therefore called operational investors. For these investors risk is a very important property and this risk is measured through variance or standard deviation which is very similarly given similar to the computation of return this is given with the formula here where

$$E(\sigma^2 i, t) = Variance\ (\sigma^2 i, t) = \sum_{t=1}^{T} Pi\ (Ri, t - \bar{R}i)^2$$

and this is your variance square root of this number will give you standard deviation.

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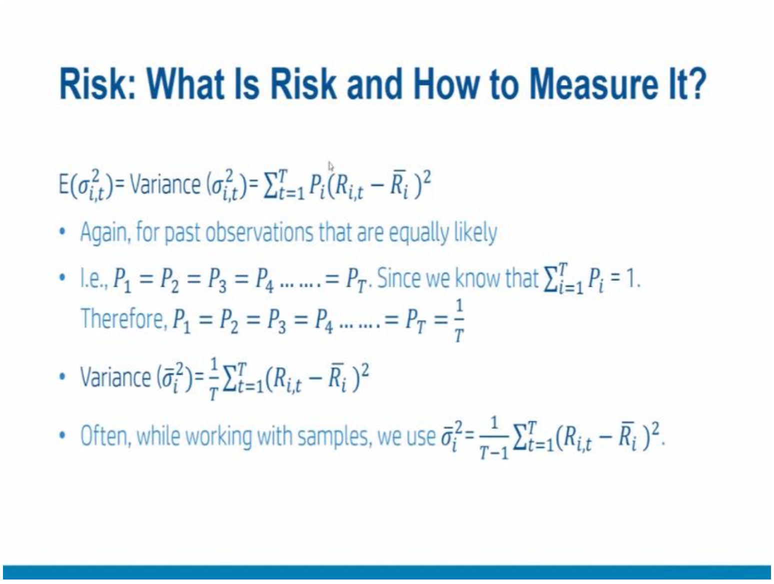

Now, in this formula of variance where we are multiplying probabilities with squares of returns deviated from mean we will again make similar assumptions as we made in case of returns that is if all the observations are equally likely if all the past observations are equally likely that is $P_1 = P_2$ and so on and we also know $\sum_{i=1}^{T} Pi = 1$

And therefore $P_1 = P_2$ and so on up till $P_T = 1 / T$ we can simplify the variance formula as shown here which is

$$Variance\ (\sigma^2 i) = \frac{1}{T}\sum_{t=1}^{T}(Ri, t - \bar{R}i)^2$$

often while working with the samples instead of using 1 / T we tend to use 1 / T – 1 however detailed position of this formula is not needed here. The idea here with this formula is that when the variance is computed for sample of observed returns a more efficient and unbiased measure is provided when 1 / (T – 1) is used.

The only change here is that instead of T we are dividing by T - 1 this accounts for the loss of one degrees of freedom because we are working with samples. We also note here that the use of subscript i in representing R i t and standard deviation sigma i of returns for security i. When we will work with portfolios we will use the subscript as P. For example for a market portfolio the subscript will be m.

$$\sigma^2 i = \frac{1}{T-1}\sum_{t=1}^{T}(Ri, t - \bar{R}i)^2$$

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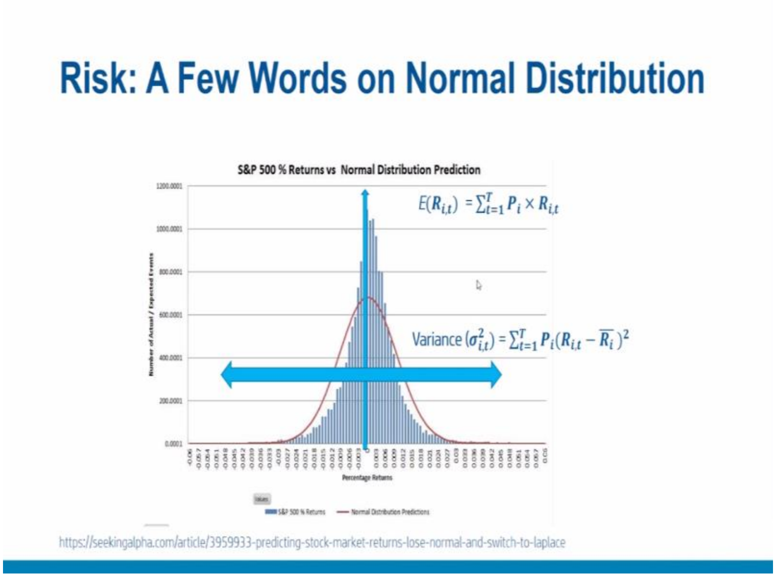

At this stage it is important to have few words about normal distribution. Security returns are often found to be very closely aligned to the properties of normal distribution. For example here we have plotted S and P 500 returns and superimpose normal distribution along with the S and P 500 returns. It appears that the frequency distribution and probability distribution plot of security returns looks very similar to normal distribution.

And that allows us to employ normal distribution in various applications related to forecasting and prediction and risk management of financial instruments. The reason is that normal distribution can be very easily defined by two parameters which is mean and variance. Mean or expected return for any security can be easily computed using the formula shown here which is P i into R i t or the average of historical returns as we have seen earlier.

$$E(Rit) = \sum_{t=1}^{T} Pt\ X\ Ri,t$$

$$Variance\ (\sigma^2 i,t) = \sum_{t=1}^{T} Pi\ (Ri,t - \bar{R}\mathrm{i})^2$$

And similarly variance can also be computed as P i into R i t – R i bar raise to the power 2 and the square root of this number is the standard deviation. Now, once we have this mean and variance we can easily define a normal distribution convert into standard normal distribution as well and then once we have the normal distribution we can make all the predictions about security returns.

For example we can easily predict with how much confidence let us say 95 % we are expecting returns to lie in a certain interval. So, all the concepts that probably you would have heard about interval estimation, hypothesis testing, statistical inference we can perform by assuming that security returns follow normal distribution which is a reasonable approximation.

And given the fact that normal distribution can be easily defined with its mean and variance it becomes much more convenient to employ a normal distribution in modeling security price returns. We will see some of these applications as a part of these courses. This includes wealth management and portfolio investing course and algorithm portfolio management course as well.

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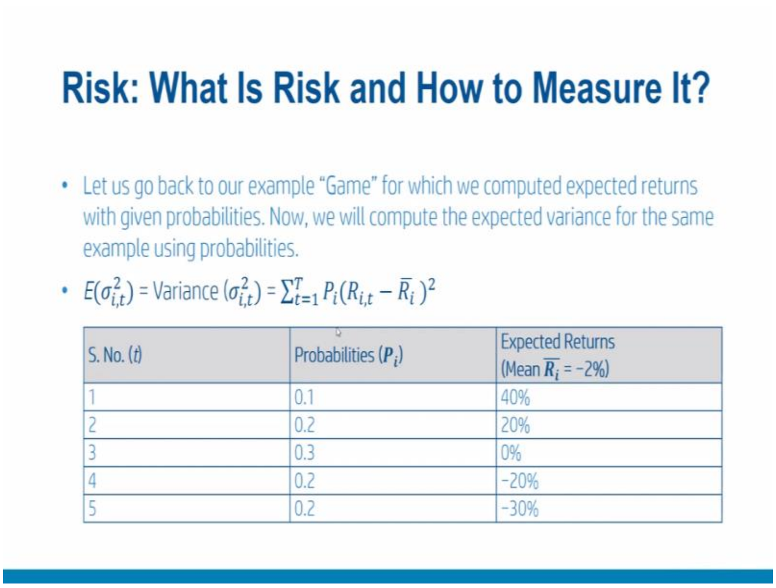

Now, let us look at the computation of the variance. Let us go back to our example game where we had certain probabilities associated with certain return observations. Now, using this formula of variance that is P i into R i t – R i bar raise to the power 2 let us try and compute the variance and standard deviation of this event. We have already computed the mean R i bar as – 2 % for this.

$$E(\sigma^2 i,t) = Variance\ (\sigma^2 i,t) = \sum_{t=1}^{T} Pi\ (Ri,t - \bar{R}i)^2$$

So, we have the mean R i bar as – 2 %, we have the probabilities and we have the expected returns.

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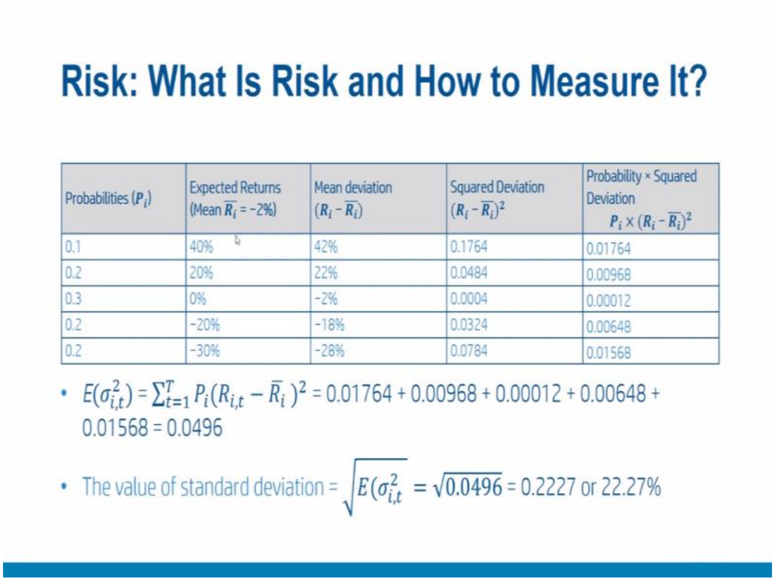

Risk: What Is Risk and How to Measure It?

Probabilities (P_i)	Expected Returns (Mean $\bar{R}_i$ = −2%)	Mean deviation ($R_i - \bar{R}_i$)	Squared Deviation $(R_i - \bar{R}_i)^2$	Probability × Squared Deviation $P_i \times (R_i - \bar{R}_i)^2$
0.1	40%	42%	0.1764	0.01764
0.2	20%	22%	0.0484	0.00968
0.3	0%	-2%	0.0004	0.00012
0.2	-20%	-18%	0.0324	0.00648
0.2	-30%	-28%	0.0784	0.01568

- $E(\sigma^2_{i,t}) = \sum_{t=1}^{T} P_i(R_{i,t} - \bar{R}_i)^2$ = 0.01764 + 0.00968 + 0.00012 + 0.00648 + 0.01568 = 0.0496
- The value of standard deviation = $\sqrt{E(\sigma^2_{i,t}}$ = $\sqrt{0.0496}$ = 0.2227 or 22.27%

So, let us look at the computation. First, we will compute mean deviations that is R i – R i bar which is, for example, 40 % minus, minus 2 % which is 42 % and so on up till – 30 % minus, minus 2 % which is 28 %. So, we have mean deviations now we will need to square them each of these mean deviations when we square that is Ri – Ri bar raise to the power 2 will be computed.

Then we will multiply them with corresponding probabilities. For example, 0.1764 into 0.1 which gives us 0.01764 which is nothing, but P i into R i – R i bar raise to the power 2 so on so forth we will move up till 0.0784 into 0.2 which is 0.01568. Now, we have probability into square deviations that is Pi into Ri – Ri bar raise to the power 2 if we sum up all of them as we can see in the slide we get the variance as 0.0496.

$$E(\sigma^2 i, t) = \sum_{t=1}^{T} Pi\ (Ri, t - \bar{R}i)^2 = 0.01764 + 0.00968 + 0.00012 + 0.00648 + 0.01568 = 0.0496$$

$$Value\ of\ Standard\ deviation = \sqrt{E\ (\sigma^2 i, t)} = \sqrt{0.0496} = 0.2227 \text{ or } 22.27\%$$

And the square root of this variance is standard deviation which is 22.7 %. With this, we conclude the computation of variance and standard deviation as a risk measure. To summarize, in this video we discussed how to compute risk in financial markets. Risk in financial market represent the scattered nature of returns around the mean or expected returns, more spreaded or scattered observations indicate a higher variance and therefore the high amount of risk associated with that security.

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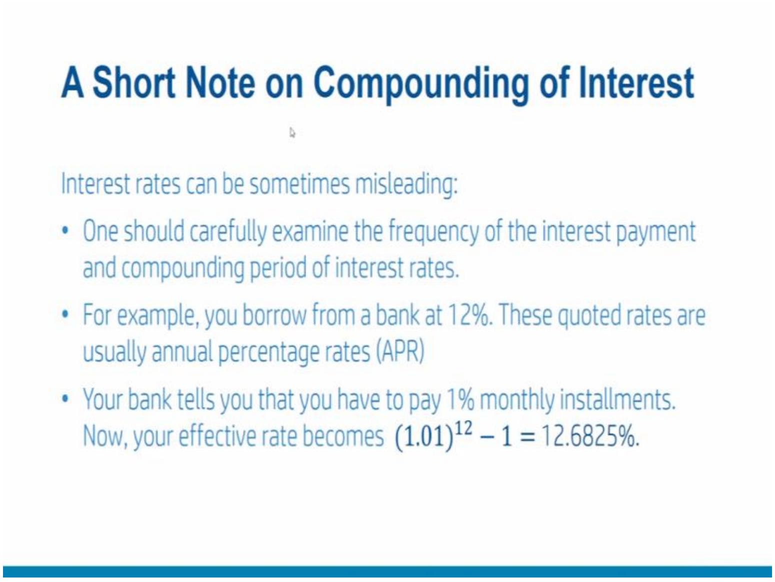

In this video, we will discuss the effect of compounding on interest rates. Interest rates can be sometimes misleading. One should carefully examine the frequency of the interest payment and compounding period of interest rates. For example you borrow from a bank at 12 % these quoted rates are usually annual %age rates or APR rates. However, in a short while you will observe that these are not effective rates.

Your bank tells you that you have to pay 1 % monthly installments now your effective rate becomes (1.01)^12 – 1 = 12.6825 %. While this near increase of 0.6825 % may seem like pocket money when you associated with thousands of crores and millions it will be a huge sum.

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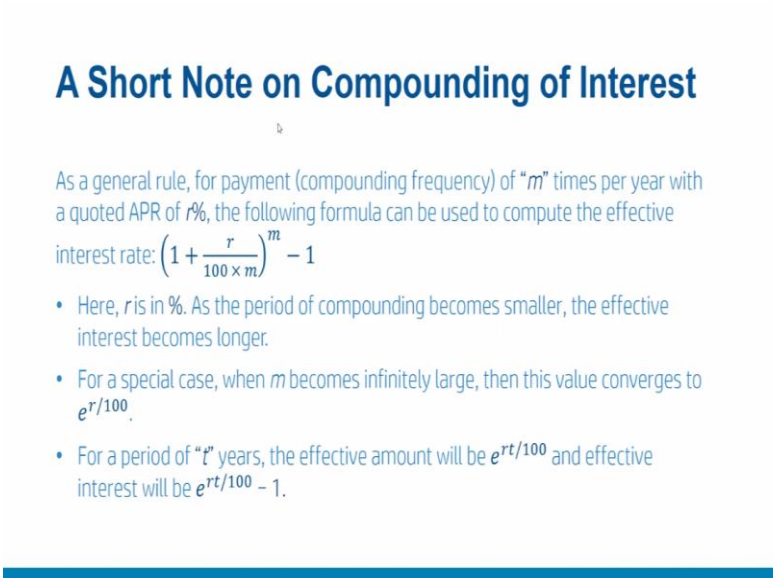

As a general rule for payment or compounding frequency of m times per year with a quoted annual %age rate of r % the following formula can be used to compute the effective interest rates that is $\left(1+\frac{r}{100\,X\,m}\right)^{m}-1$. Here r is in %age as the period of compounding frequency becomes smaller the effective interest becomes larger.

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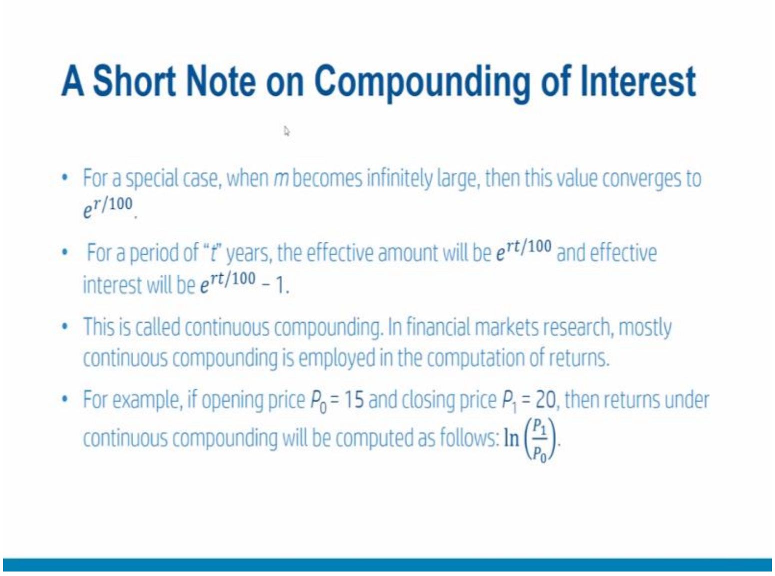

For a special case when m becomes infinitely large then this value converges to e^ (r/100) for a period of t years the effective amount will be e^(r t/100) and effective interest rate will be e^(rt/100)–1. This is called continuous compounding. In financial market research predominantly continuous compounding is employed in the computation of returns.

For example if the opening price P_0 = Rs 15 and closing price P_1 = Rs 20 then returns as per the continuous compounding will be computed as natural log (P_1/P_0).

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Example on Compounding of Interest

Compounding Frequency
1 year = 1.12
6 months (2 times)
4 months (3 times)
3 months (4 times)
1 month (12 times)
1 day (365 times)
Very small period (less than a day – continuous compounding)

Let us consider this example of compounding interest. If your compounding frequency is 1 year and 12% annual %age rate APR then your effective interest rate is 1.12 which is 1 into 1.12. Now, if you increase the compounding frequency and compound it two times a year that means interest is paid every 6 months then it is semiannual compounding and two times compounding will happen.

If interest is paid every 4 months then over the period of 1 year the compounding will take place 3 times. So, compounding frequency increases to 3 times. Similarly, as we keep on increasing let us say we take interest payment intervals as 1 month then compounding frequency is 12 times and as we keep on increasing a very small period that is less than 1 day is often referred to as continuous compounding because money is being continuously compounded or interest is paid as extremely small intervals.

And therefore we can see that as the period of interest increases compounding decreases as the period of interest decreases compounding frequency increases.

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Example on Compounding of Interest

Compounding Frequency	Effective Interest Rate	Excess Over 12%
1 year = 1.12	1.12 – 1 = 12%	0%
6 months (2 times)	$1.06^2 - 1 = 12.36\%$	0.36%
4 months (3 times)	$1.04^3 - 1 = 12.4864\%$	0.4864%
3 months (4 times)	$1.03^3 - 1 = 12.551\%$	0.551%
1 month (12 times)	$1.01^{12} - 1 = 12.6825\%$	0.6825%
1 day (365 times)	$(1 + 0.12/365)^{365} - 1 = 12.7475\%$	0.7475%
Very small period (less than a day – continuous compounding)	$e^{.12} = 12.75\%$	0.75%

Let us conclude this with a simply example. First if the compounding frequency is 1 times that means period is 1 year then effective interest rate is (1.12 – 1) = 12%. So, there is no excess over 12%. However, if the interest paid at interval of 6 months and compounding frequency is 2 times then your effective interest rate is $1.06^2 - 1 = 12.36\%$.

So, the excess over 12% is 0.36%. As we keep on doing this let us say we compound it each day 365 times a day then your effective interest rate is (1 + (0.12 / 365)) ^ 365 – 1 = 12.7475 % and an excess interest rate over 12% of 0.7475%. For extremely small period with continuous compounding you have e^0.12 = 12.75 % and an excess rate of 0.75%.

So, this is the maximum frequency that means interest is paid almost at as small interval as (()) (33:36). So, as the compounding frequency increases the excess interest rate over 12% also increases. To summarize, the effective interest rates are function of compounding frequency as the compounding frequency increases interest is paid more frequency and therefore the effective interest rate also increase.

The maximum frequency available or that can be generated is called continuous compounding. To summarize this lesson financial market provide a very important conduit for individuals to optimize the time pattern of their consumption. The demand and supply of funds determine the efficient clearing prices and therefore the interest rate environment that prevails in the economy, expectations of interest rate from security are determined based on the risk of the security.

Risk of a security represents the increase in certainty in outcomes the wider the possible outcomes the more the risk for a given level of risk investor prefer instruments with higher expected returns and for a given level of expected returns investor prefer instruments with lower risk. We discuss the computation of standard deviation and variance the two most important measures of risk in finance.

We also discuss the properties of normal distribution which is most often employed in modeling the financial market returns. Lastly, we would also discuss the concept of interest rate compounding.