Advanced Algorithmic Trading and Portfolio Management
Prof. Abhinava Tripathi
Department of Management Sciences
Indian Institute of Technology-Kanpur
Week 3
Lecture - 9

In this lesson we will discuss CAPM. We will start with the assumptions of CAPM. Subsequently, we will introduce a simple proof approach to understand CAPM. We will understand the capital market line CML and security market line SML and their relationship with CAPM. However, there are real life phenomena that result in violation of CAPM. We will conclude the lesson with these fallings of CAPM.

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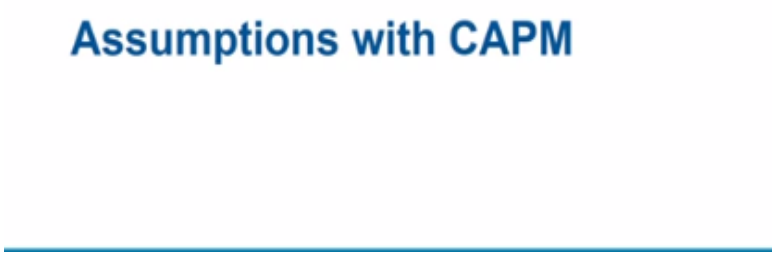

Assumptions with CAPM. Application of capital asset pricing model assumes certain assumptions that deviate from real life behavior. In this video, we will discuss some of those assumptions.

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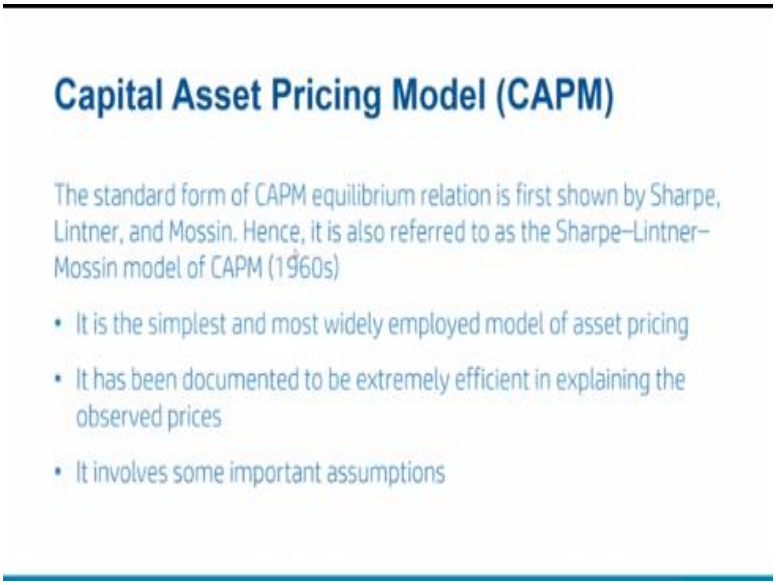

The standard form of CAPM or capital asset pricing model in equilibrium relationship is first shown by Sharpe, Lintner and Mossin. That is why it is also referred to as Sharpe-Litner-Mossin model of CAPM, and was developed in 1960s. CAPM or capital asset pricing model is arguably the simplest model of equilibrium asset pricing. The key point to note about any model like CAPM is the test of its efficacy.

That is, how better it can describe the real observed prices. CAPM while extremely simple has been found to be very efficient in describing the observed prices in capital markets. Models like CAPM are equilibrium models that allow us to determine the relevant measure of risk for any asset and the relationship between expected return and risk for any asset when markets are in equilibrium.

CAPM is the most simple model based on most stringent or less realistic set of assumptions. However, the test of the efficacy of any model is not how realistic the assumptions are, but how well the model describes the reality. Moreover, CAPM has been documented to be extremely efficient in explaining the observed prices. But it also has certain important assumptions as we will see shortly.

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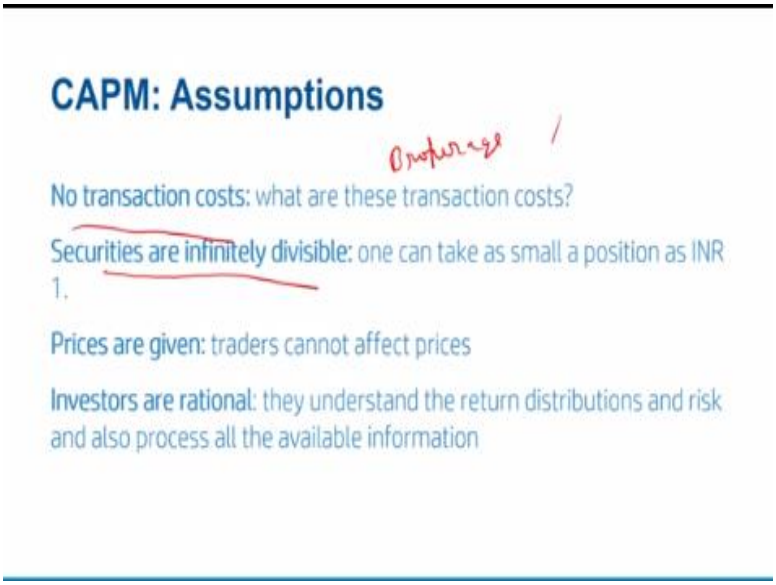

First CAPM assumes no transaction cost. There are no transaction costs associated with buying and selling of assets. Please note that this is rather harmless assumption up until certain years ago, but with algorithmic trading, there are millions of transaction costs each day and the transaction costs have become an important consideration. However, following the literature we will try to ignore these transaction costs.

Some of these transaction costs include like brokerage charges, security transaction taxes, bid-ask spread and so on. The next important assumption made by CAPM is that securities are infinitely divisible. That is one can buy stocks worth even rupees 1 or $1. And therefore investors can take any position is in assets howsoever small it may be.

The next important assumption is that prices are given or alternatively, investors or traders cannot affect prices. While this is also a sort of realistic assumption, but when certain investors like large institutional investors, they can affect prices with their own trades and therefore for them, prices are not exactly given and they can also estimate with some certainty how much prices will move based on their own rates.

Next another important assumption is that investors are rational that is, they understand the return distribution, the risk and also process all the available information. However, this assumption has certain deviations. Most often investors do not take into consideration all the available information and even sometimes they exhibit irrational behavior.

For example, they take excessive risk, as would have been predicted by the rational investor model. And sometimes they also play gambles in stock market.

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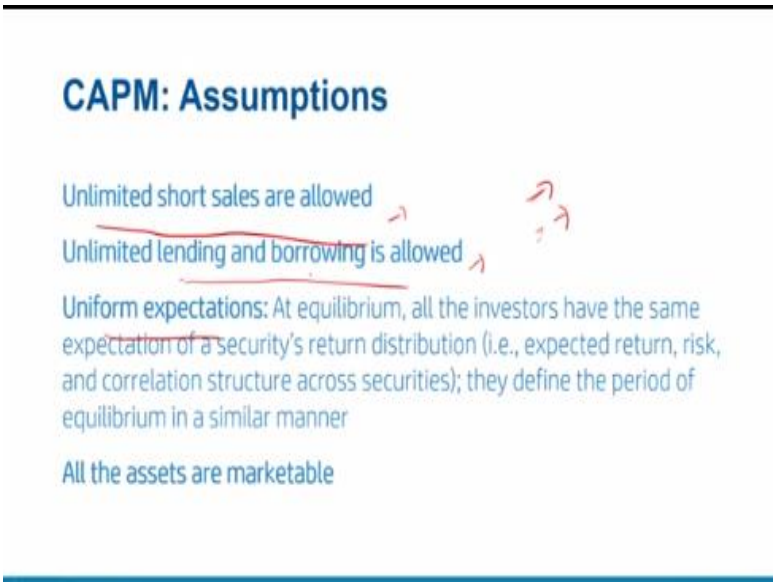

Some of the other assumptions include unlimited short sales are allowed. In most of the markets in the world today, there are some form of restrictions that are placed by regulatory authorities on short sales. So one cannot freely sell all the security short that are available, there are certain restrictions. For example, in India, you can in cash market you cannot carry forward the short position beyond one day.

Next is unlimited lending and borrowing is allowed at risk free rates. This is also not exactly a practical assumption. In any market, first and foremost, lending and borrowing are not at same rate. Moreover, the risk free, if you are borrowing the money, you cannot borrow unlimited amount, while unlimited lending maybe risk free, you can lend unlimited amount in a risk free manner.

But unlimited borrowing is not allowed in risk free manner as you keep on increasing the borrowing, the risk of the borrowing will increase and borrowing will become prone to default and therefore, it is not risk free anymore, you will be paying a hefty premium in the form of interest rates. Next another very important assumption is of uniform expectations.

It is assumed that in equilibrium, all investors have uniform expectations about a security in terms of its expected return and variance for a given period. Also they define

the period in a similar manner. For example, in the context of portfolio selection, all the investors have identical expectations about expected returns, variance of returns and correlation structure across all the pair of stocks.

However, this is not the case. Investors define period in a different manner. They have different expectations of return and risk from the similar securities. Last assumption is that all the assets are marketable, which is also not exactly true. The assumption suggests that all the assets can be sold and brought in the market, even the non-financial assets like human capital and real estate which we know it to be not true.

To summarize, in this video, we discussed the assumptions behind CAPM. These included no transaction costs and taxes. Securities are infinitely divisible. Presence of no personal taxes. Prices are given and did not change by individual investor's actions. Investors make investment decisions rationally based on their understanding of expected return and risk.

Unlimited short sales are allowed. Unlimited lending and borrowing at riskless or risk free rates. In equilibrium, all investors have uniform expectations and all assets are marketable. And this standard form of CAPM that we have discussed here is first shown by Sharpe, Lintner and Mossin. And that is why it is also called SLM or Sharpe-Lintner-Mossin version of CAPM.

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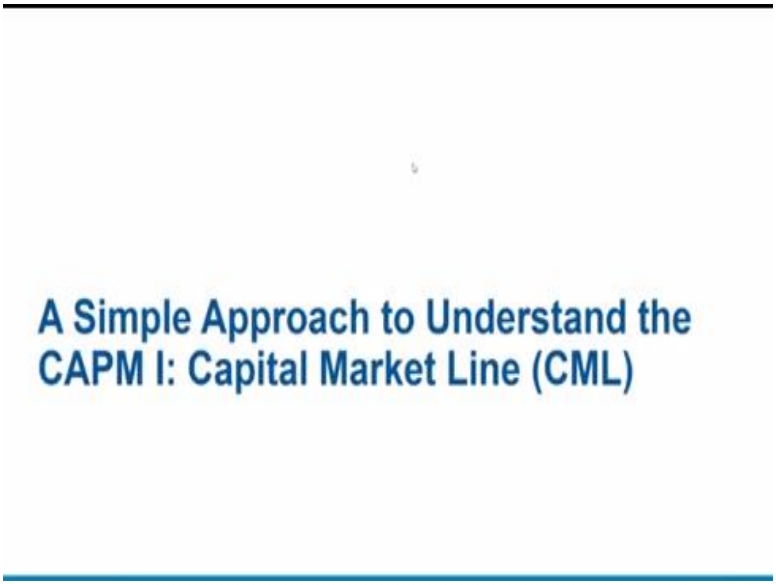

A simple approach to understand CAPM. In this video, we will examine a simple approach to understand the capital asset pricing model and capital market line CML.

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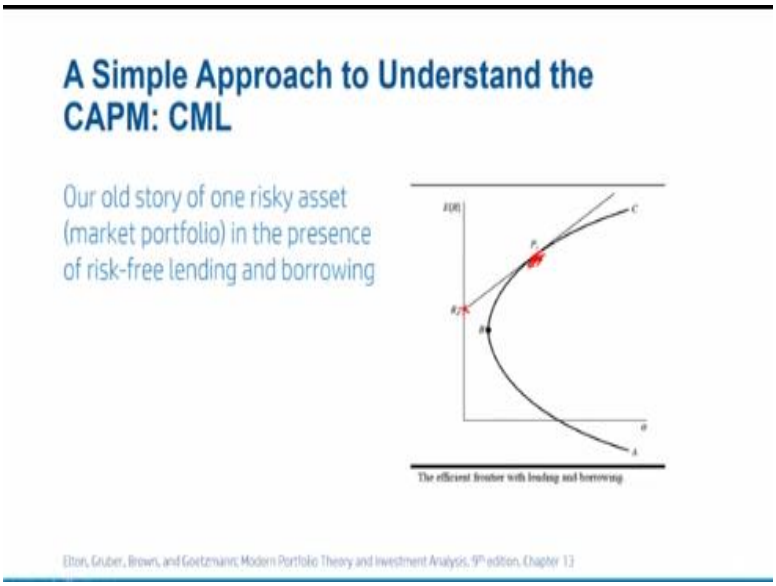

Let us go back to our discussion on risky asset portfolios in the presence of riskless lending and borrowing. Let us examine the figure shown here. We said that if all the investors have the same expectations and they face the same risk free lending and borrowing, then all of them will hold a portfolio of risky assets P_I, this portfolio P_I through which the tangent from point RF passes.

Since everybody will hold this portfolio in equilibrium, then this portfolio is called market portfolio because everybody is holding this portfolio. Market portfolio is a portfolio of comprising all the risky asset in the market. And in market portfolio, each asset is held in a proportion that is the ratio of its market value in the actual market to the aggregate market value of all the risky assets.

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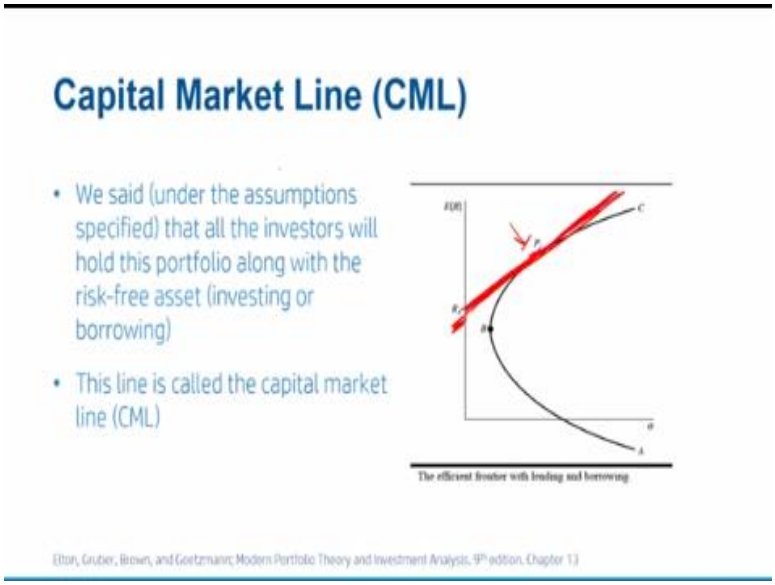

So all the portfolios or all the investor portfolios will be having this risky asset P_I. This will contain all the risky assets available in the market in addition to some amount investors in this R_F risk free asset or borrowed at this rate. So those who are investing some amount in R_F some in this market portfolio P_I they will be standing on this lending or investment segment of this line.

While those who are borrowing at R_F they will be standing here, and investing all the amount in market portfolio P_I will be standing here. All the efficient portfolios will lie on this line for investing or borrowing whatever it is, all the portfolios will lie on this line. And therefore, this line is called the capital market line as shown here, this line is our capital market line.

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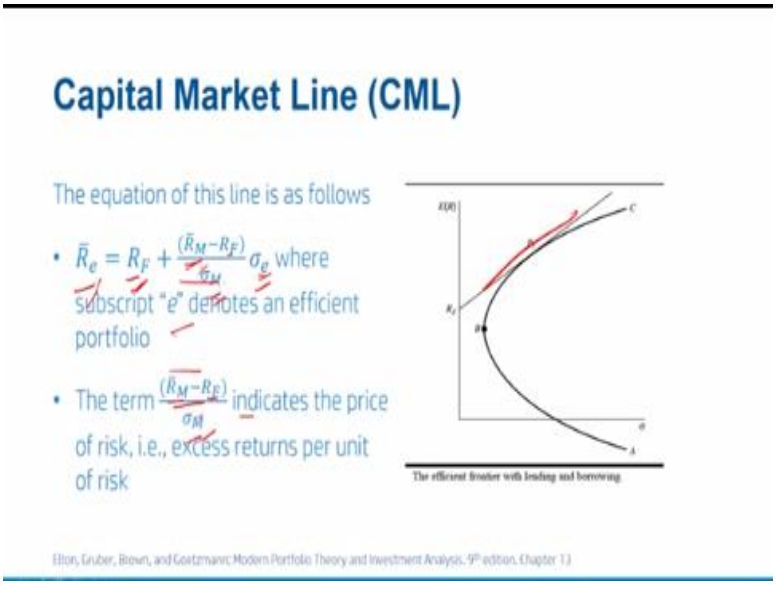

We also know that the equation of this capital market line CML, this line CML can be simply represented by this $\bar{R}_e = R_F + [(\bar{R}_M - R_F)/ \sigma_M]* \sigma_e$ where this subscript e is for efficient security or portfolio. Now $\bar{R}_e$ here is the expected return on that efficiency total. R_F is the risk free rate. $[(\bar{R}_M - R_F)/ \sigma_M]* \sigma_e$.

$\bar{R}_M$ is the expected return on market σ_M here is the standard deviation or risk of market. Σ is the standard deviation or risk of that efficient security. The term here, let us focus the term on the RHS which is $[(\bar{R}_M - R_F)/ \sigma_M]* \sigma_e$. Here we are saying that $[(\bar{R}_M - R_F)/ \sigma_M]$ indicates the price of risk that is excess returns $[(\bar{R}_M - R_F]$ per unit of risk, which is σ_M.

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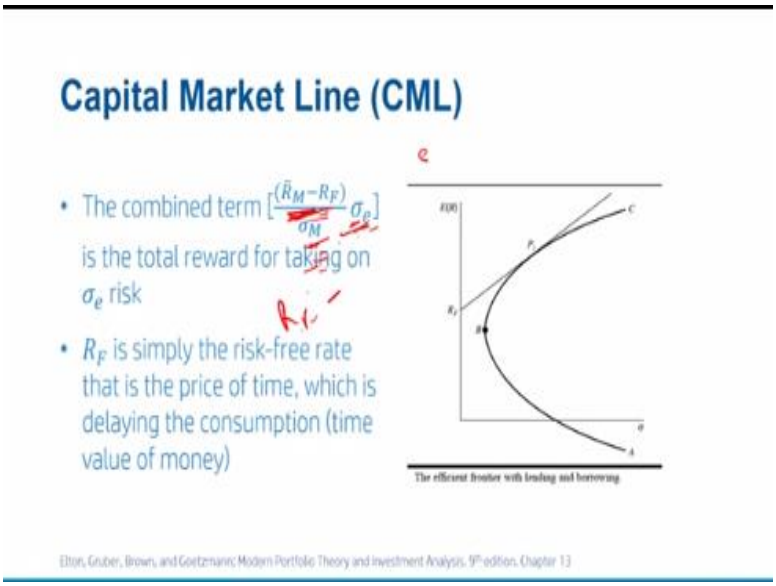

Let us focus on this term. $[(\bar{R}_M - R_F)/ \sigma_M]* \sigma_e$. This term has two components, the first part is $[(\bar{R}_M - R_F)/ \sigma_M]$ which indicates the price of risk that is for taking market risk. Please remember this free asset that is R_F has no risk. And how much extra return is available for taking that risk, which is $(\bar{R}_M - R_F)$ on efficient portfolio e.

This is called market price of risk. This is divided by σ_M. So this is the per unit of market risk. You are getting this much extra return which is $\bar{R}_M - R_F$ on the efficient portfolio σ_e. So it is multiplied by σ_e which represents the total risk of the efficient portfolio. In simple terms, the first term here the R_F, which we saw R_F, the first term is simply the price per unit of risk.

And the second term is the risk. The combined term is the total return for taking the risk σ_e, the σ_e. So R_F is the simply the risk free rate that is the price of time that is the return for delaying the consumption or simply you can say reflecting the time value of money.

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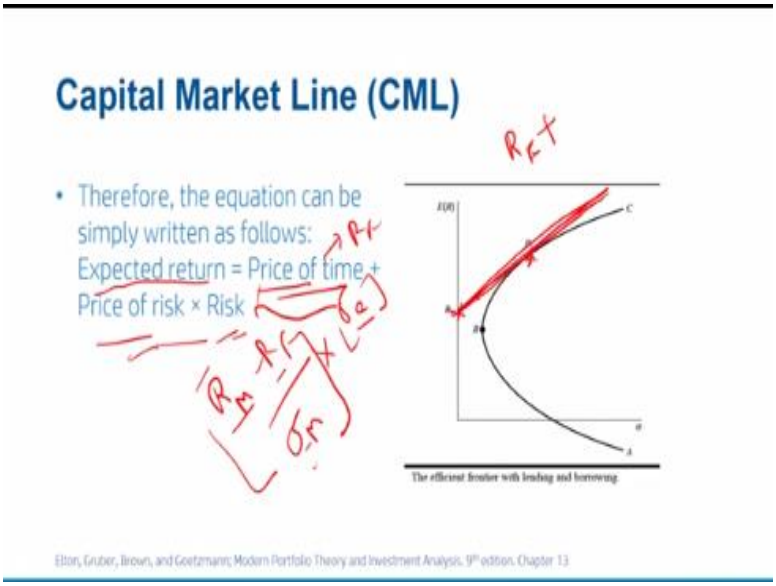

So the resulting equation is expected return on any efficiently priced security is price of time plus price of risk into risk. Price of time is nothing but risk free rate. Price of risk we have already discussed which is $\bar{R}_M$ - R_F, which is the additional return for taking the risk standardized by market risk σ m into risk which is σ_e. Σ e is the risk of efficient portfolio and this is the price of risk.

This component is the price of risk. This is the total risk. To summarize, we discussed that all the investors in market if they behave rationally, all of them will hold this market portfolio P_I which is the tangency portfolio in combination with this risk free asset. So they can either borrow or invest in this risky asset if they are borrowing and investing the entire amount in this market portfolio here, tangency portfolio they will be standing here.

If they are investing some amount in R_F and some on the market portfolio, they will be standing on this. But in all cases, they will be standing on this line. This line will contain all the efficient portfolios in the presence of R_F. For any security lying on this line, the expected return can be broken into two components. First is the price of time which is nothing but delaying consumption or time value of money which is represented by R_F.

The second term is the price of this which is taking on the risk of this risky asset, which is price of risk [($\bar{R}_M$ - R_F)/ σ_M] multiplied by total risk of that efficient security. Depending upon where on this line segment you stand, you will have some value of σ_e, which is the risk of that efficient security. And this line, this tangency line eventually becomes your capital market line.

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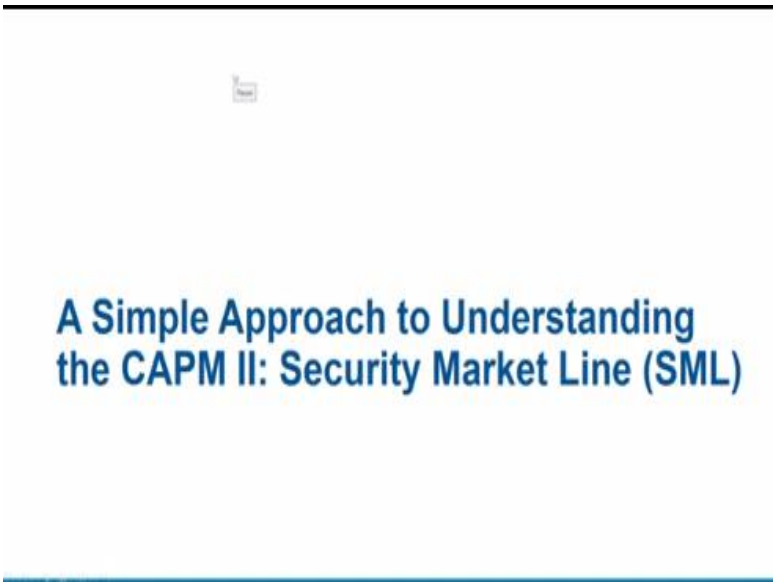

A simple approach to understanding CAPM part II. In this video, we will examine the part II of simple approach to understanding capital asset pricing model. We will also discuss security market line SML.

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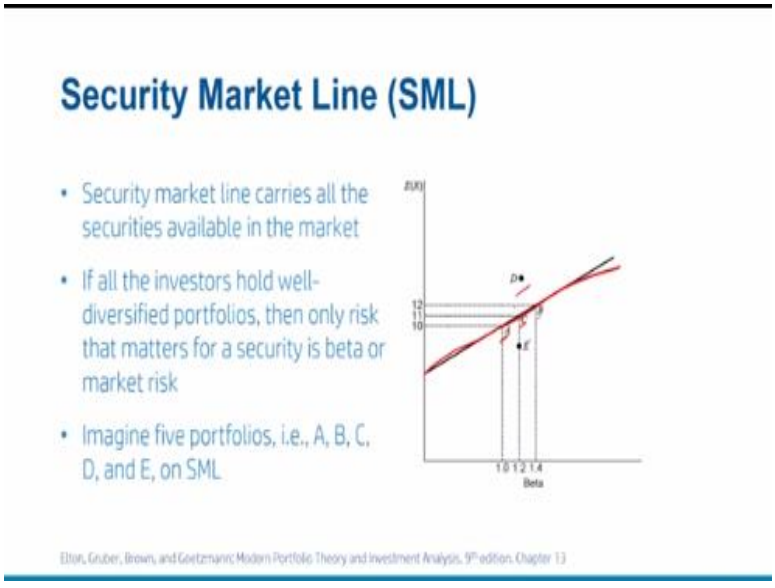

For well diversified portfolios, nonsystematic risk tends to become zero. The only relevant risk is the market risk. For each security in the portfolio, β represents the

contribution to this risk. Therefore, to a well-diversified investor that stands on the efficient frontier, the only consideration in selecting a security is its β.

If you believe that individuals hold only these efficient portfolios, then for other securities including individual stocks, the only risk that should matter to an investor is market risk, which is measured by β. Thus, one can plot all the risky securities on the expected return β space. The assumption is that market risk is the only factor that affects the expected returns.

Let us examine five portfolios A, B, C, D, E here as shown in the figure. These investments and their βs are also plotted on the figure. And the security market line SML is plotted like this. We will discuss this SML in more detail. This SML plots all the securities available in the market and these A, B, C, D, E are these are securities.

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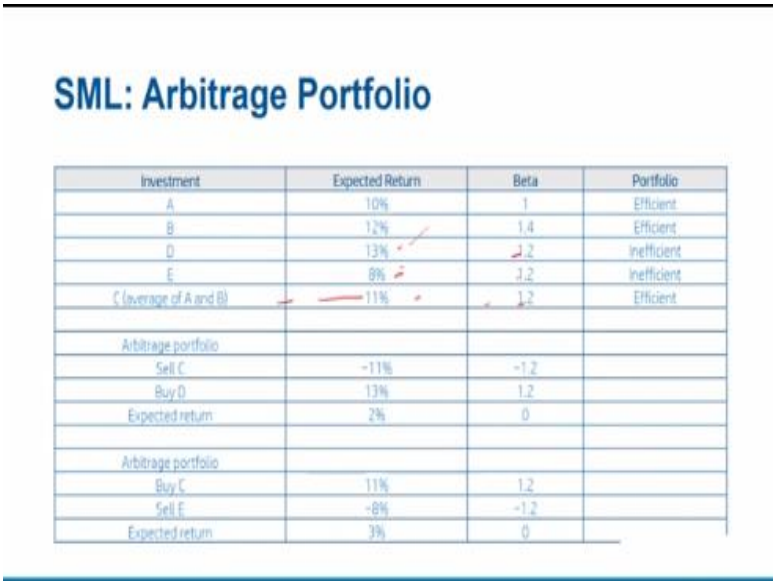

Now let us look at their expected return profile and risk profile, risk measured by β. C is a combination of A and B. So C is the average of A and B 50-50%. So it is expected return and β is also 50-50 combination of A and B. It appears that portfolio D and E are slightly anomalous. This should also be clear by this diagram here, that B and D are slightly anomalous because they do not fall on the SML.

We will have more to say about this now. Notice the risk of D and E. Both of them have β 1.2 same as C, which is the average of A and B but their expected return is not aligned to their β as per the SML. For example, for a given level of β same as C, D offers a high

return. While the price of D will rise because it cannot go like this for long, investors will buy D either from own funds or shorting C.

Thus the price of D will rise and expected returns will fall. The converse is true for investment E. This is called arbitrage and its implications are as follows. Two assets with the same risk profile and cash flows should sell at the same price. Thus, in equilibrium, all the securities will fall on the same straight line SML in the return β space, as we will see shortly.

If any security is above or below this line, there will be an opportunity for riskless arbitrage, which will drive these securities or portfolios back to this SML line. Let us visualize this.

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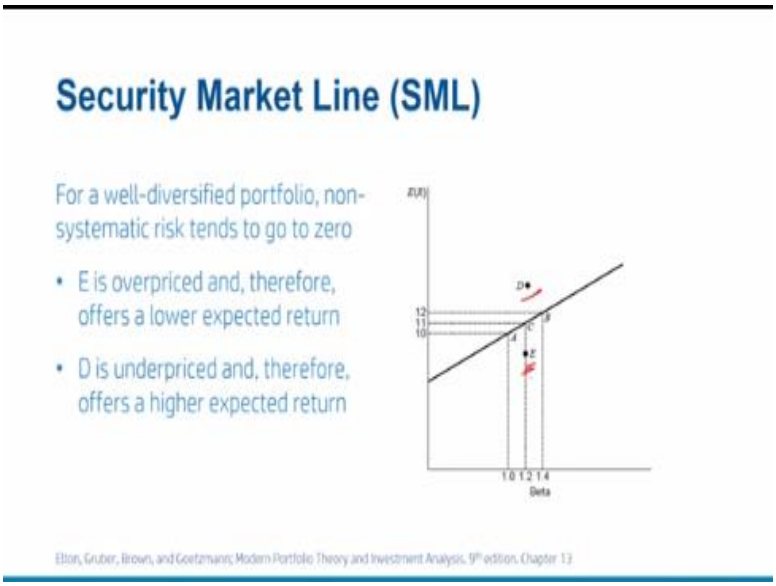

Here for a well-diversified portfolio nonsystematic risk tends to go zero and market risk is the only relevant risk which is measured by β. So we plot all the securities on expected return and β space. Securities A, B C, D E are plotted. As we discussed earlier D and E are anomalous, D is above SML and E is below SML. So they are not aligned to this SML line. E clearly shown here is below the SML.

So it is overpriced and therefore, offers a lower expected return. So it is overpriced and offers a lower expected while D is underpriced and offers a higher expected return.

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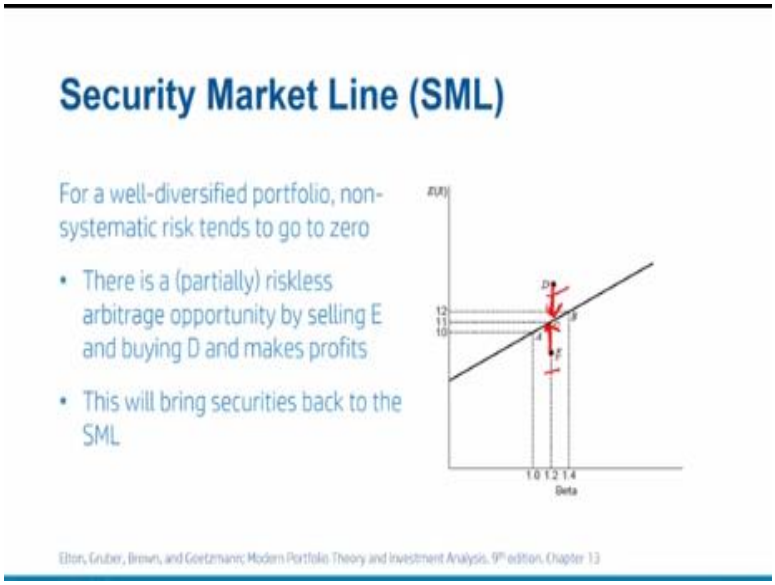

Now it appears that there is a riskless arbitrage opportunity by selling E and buying D to make riskless profits. If this happens, then E will be driven towards this SML because more and more people will be selling E. So its price will fall and expected return will rise. So it will move up while D as more and more people will buy D its price will rise and expected return will fall so it will be driven towards the SML as well.

So both of these securities will be driven towards this SML or security market line.

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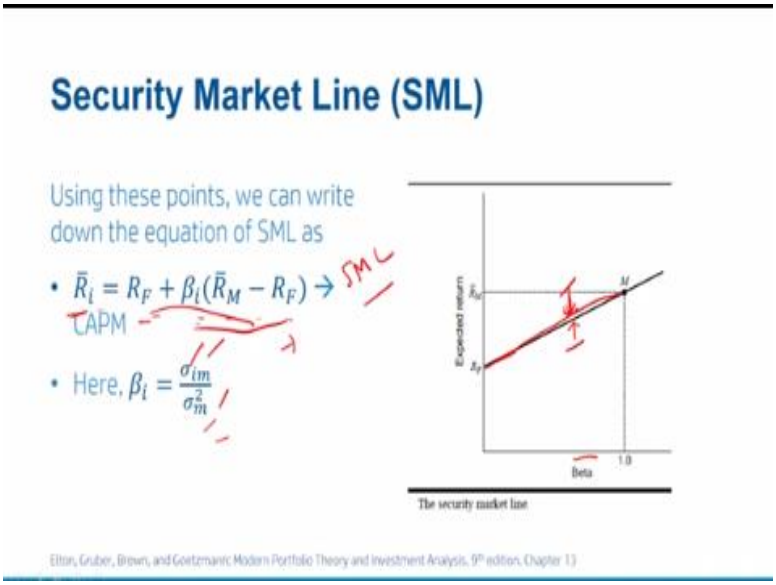

Now a very important question is how to identify this SML. SML here can be easily identified using two points through which it passes. One is the risk free investment. The first point is corresponding to the risk free investment which is this R_F. It has been equal

to zero. It is the risk free asset or risk less asset. The second point is corresponding to the market portfolio which is this endpoint.

It has a bit of 1 and expected returns of $\bar{R}_M$. So we can easily identify the equation of this line as follows. $\bar{R}_i$ for any security the expected return if they are aligned to this SML $\bar{R}_i = R_F + \beta_i (\bar{R}_M - R_F)$, which is precisely the CAPM. So the equation of this SML or security market line is precisely the CAPM where $\bar{R}_i = R_F + \beta_i (\bar{R}_M - R_F)$

Here β is σ_{im}/σ_m^2. This equation defines the expected return for any security asset portfolio in the market. Σ m is the covariance between individual security i and market. Σ square m is the variance of market; covariance, variance. The higher the value of β for a security, higher are the expected returns from it.

This relationship also suggest that the differences in the expected returns between two securities can be explained only using this β. This equation confirms our previous discussions that it is not the variance of returns but the portion of the variance of returns that cannot be diversified determines the expected return of a security. Please note here that $\bar{R}_M$ and R_F are not the functions of asset itself.

Thus as per CAPM, the differences between the expected returns of two securities can be ascribed to their βs only. The equation also validates that systematic risk is the only component that is relevant and not the non-diversifiable part of overall risk. However, this statement does not mean that all times high β stocks will offer higher returns than low β stocks.

What it means is that the stocks with high β are risky. So at times they will also produce lower returns. However, on average, they will offer higher returns for long periods. This discussion though not very mathematical, but still provides intuition for CAPM model as shown here in this equation.

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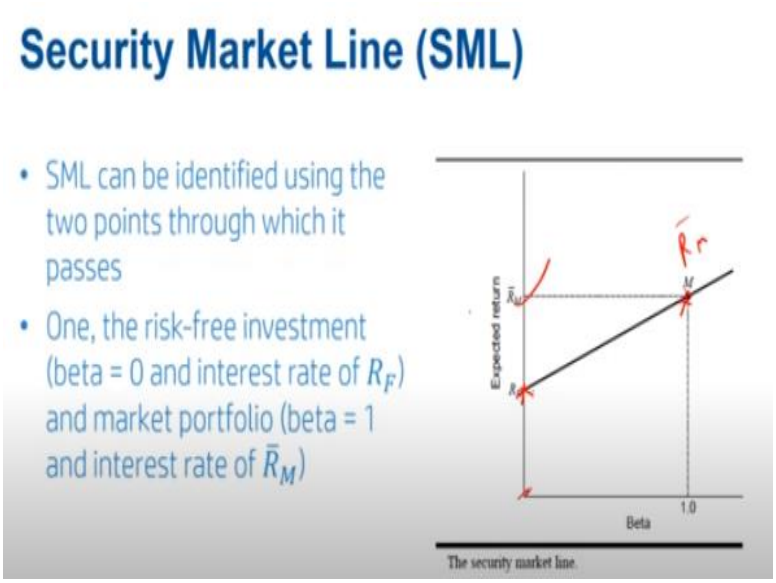

To summarize, in this video, we discussed that equation of security market line which is this line is precisely the capital asset pricing model. All securities whether efficient or not in capital market should lie here. That is their expected return on these securities are determined by the systematic risk which is β.

If there are securities, which are not in equilibrium with the security market line, that is they fall either above or below this line, then process of arbitrage or forces of arbitrage will drive them towards the security market line. And gradually as these securities will fall on this SML, they will reach their equilibrium.

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Fallings of CAPM. In this video we will talk about certain assumptions of CAPM that are violated in real life scenarios.

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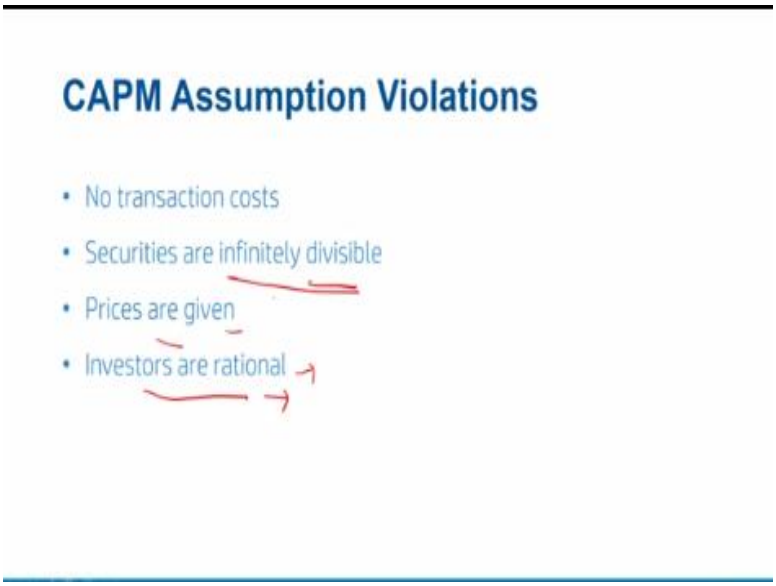

First CAPM assumes no transaction costs. That is absence of friction such as bid-ask spread, security transaction taxes and brokerage charges, commissions. However, we know that in real life, such transaction costs do exist and many times these transaction costs comprise a large fraction of trading costs. Second, securities are infinitely divisible. CAPM assumes that one can trade small quantities as small as one rupee or $1.

That means they are infinitely divisible. Even a very small fraction of them at smallest price available can be traded, bought and sold, which is not true. Various stock exchanges have limits or called lot sizes, which number of, minimum number of shares that you need to purchase or sell. Therefore in real life securities are not infinitely divisible. And this can also act as a innovation or as a hindrance in buying and selling the stocks.

Prices are given. Investors do understand that their trading activity impacts prices, particularly large institutional investors, their large volume of trades, move prices and they also estimate this movement in prices because of their own trading activity. However, CAPM assumes that investors take prices as given and their trading activity does not affect prices. Investors are rational.

CAPM assumes that investors are rational. They factor in all the available information, process it and based on their expected risk and return framework, they take their trading

decisions. They do not like risk and if a security is risky, they expect or demand certain high return premium for that risk as a compensation.

However, real life scenario and behavioral finance have indicated that investors sometimes do like to gamble with their money and they take risk that are unwarranted and often not well compensated in the form of return premium. And many times they do not factor in all the available information. They have a limited set of information and even within that information set their processing capabilities or the ability to process that information is also limited.

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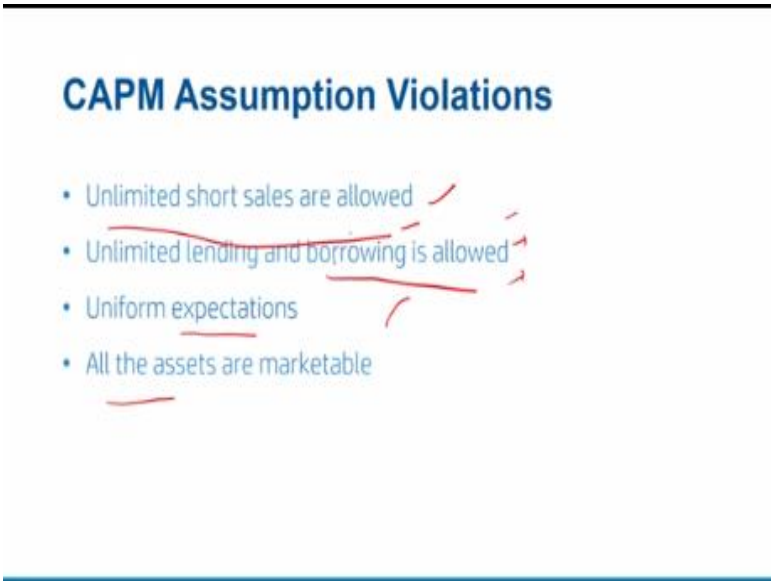

Next CAPM assume that unlimited short sales are allowed that one can short sell unlimited amount of security. However, different markets restrict short selling through different regulations. And most of the markets do not allow short selling in a very free and fully free manner. So this assumption is also well not exactly a depiction of real life scenario. Unlimited lending and borrowing at risk free rate is allowed.

All of us know that we cannot lend and borrow unlimited amounts at risk free rate. Particularly while you can lend a large amount of money in a risk free manner you cannot borrow it. As you keep on borrowing your leverage increases and the interest rate charged on you also increases which reflects the premium that you are paying or the risk that you are bearing.

Uniformly expectations. CAPM assumes uniform expectations. For example, CAPM assumes all the investors in an economy or in a market are forming their expectations based on the risk of the security and their expected returns are similar. They are also defining their periods in a similar manner. The mean variance frontier for all the investors are same.

That is the risk return tradeoff for all the investors are same, which is of course not true. Another very important assumption is that all the assets are marketable, which suggests not only the assets that are traded in financial markets, but human capital and various other real physical assets are also marketable completely, which is not true. There are a lot of challenges.

For example, human capital is of course, not marketable. Second, even the assets like land and real estate, while they are tradable, but there are huge transaction costs involved and also you cannot sell or buy them immediately, it takes a lot of time. So unlike financial market assets, these are not exactly marketable in a swift manner.

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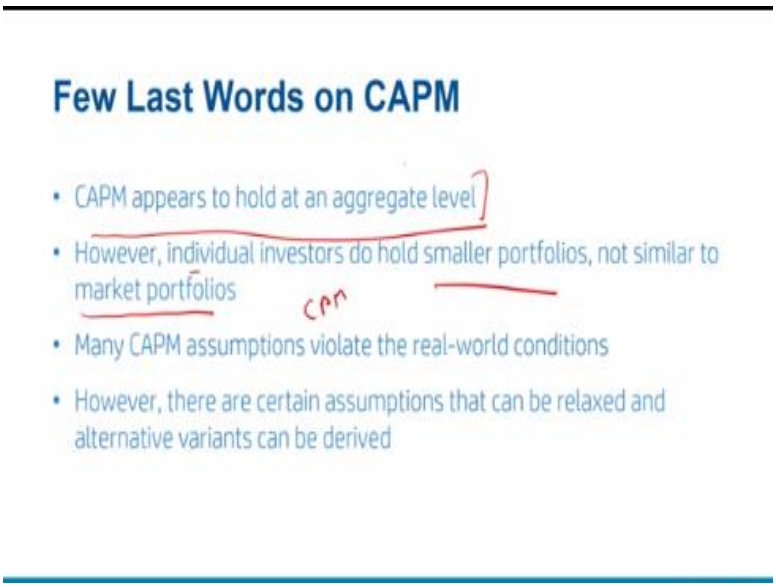

However these violations do not necessarily imply that CAPM does not hold. The efficacy of CAPM does not lie in its assumptions and their ability to match real life scenario, but its ability to describe security price behavior in financial markets. And it appears that CAPM do hold at an aggregate level. That is, if you look at a market as aggregate it appears it follows the postulations of CAPM.

However, it is also found that individual investors do hold smaller portfolios that are not similar to market portfolio. CAPM assumes that all investors should hold is a market portfolio. However, investors are often found to hold smaller portfolios that are not similar to market portfolios. And therefore, these assumptions are violative of real life conditions.

But it has been found that some of these assumptions can be relaxed and alternative form of variants of CAPM can be obtained. While these variants are slightly more complicated than simple CAPM they attempt to restrict some of these assumptions and incorporate the resulting changes in the risk return behavior or risk return relationship.

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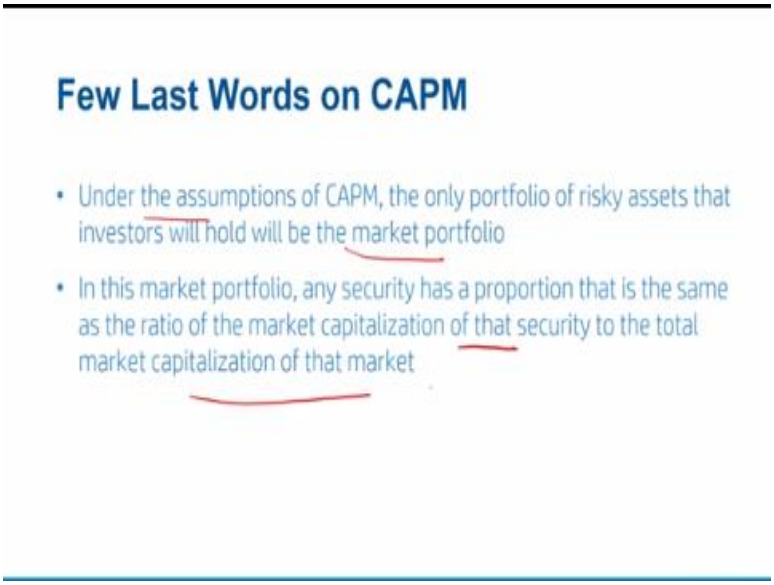

As we discussed, under the assumption of CAPM, the only portfolio of risky asset that investors should be holding is the market portfolio. And in this market portfolio, any security has the same proportion as the ratio of the market capitalization of that security to the total market capitalization of that market. This is theoretically the construction of market portfolio and the proportion of constituent securities in that market portfolio.

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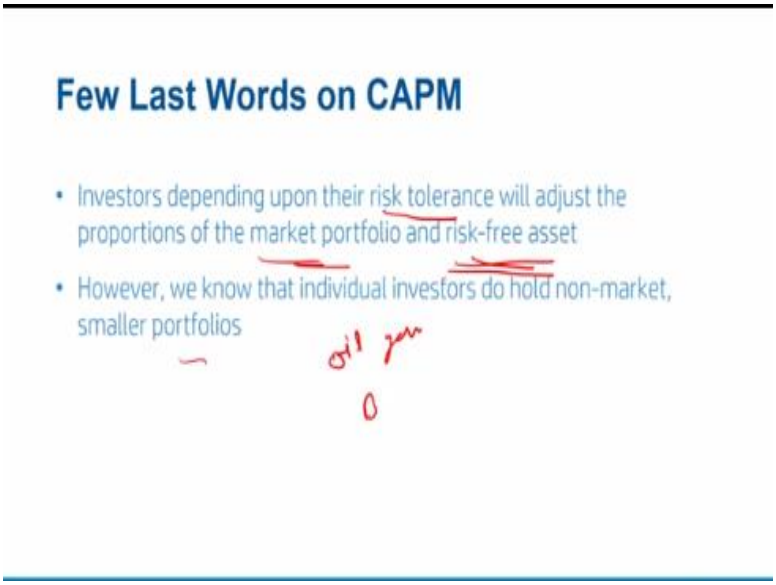

CAPM also postulates that all the investors need not have same risk tolerance. They will be having different risk tolerance and it will be reflected, this difference in this tolerance will be reflected in their combination of market portfolio and risk free asset. Remember, we said that investors will not only hold this market portfolio, but may also invest certain proportion at risk-free asset.

Or some of those that are more risk tolerance may heavily borrow at this risk-free rate and then invest their own wealth not only their own wealth, but the borrowed amount in the market portfolio. But as we have found and in real life scenarios depict investors do hold nonmarket smaller portfolios for example, set of oil and gas stocks or set of bank stocks. For example, in India you have an index called bank nifty.

So there are instances where individuals hold non-market portfolios, which seems to violate the postulations of CAPM.

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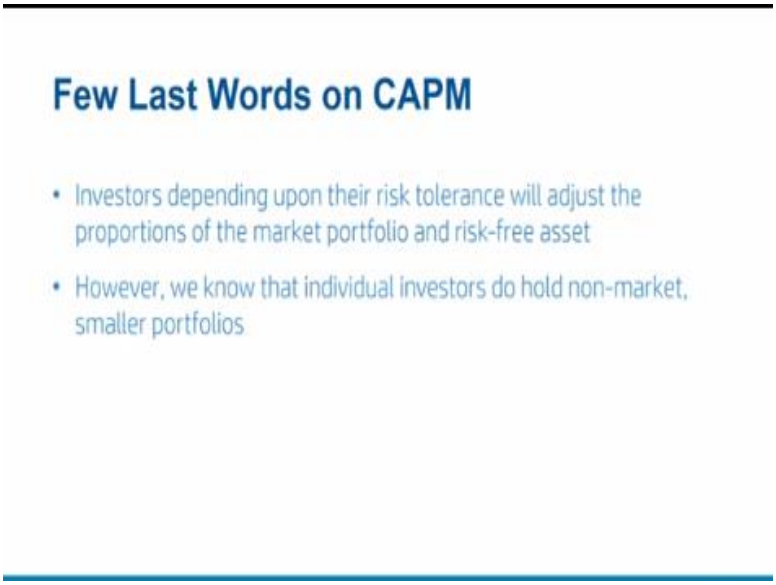

To summarize, in this video, we discussed certain assumptions of CAPM that are violative of real life conditions. For example, CAPM assumes that there are no transaction costs. Securities are infinitely divisible. There are no taxes. Prices are given to investors. Investors make investment decisions rationally and factor all the information available. Unlimited short sales are allowed.

Unlimited lending and borrowing at risk-free rate. All the investors have uniform expectations. Assets are marketable. All the assets including real estate, human capital and so on are marketable. So as we can see, most of these assumptions are violative of real life conditions. However, some of these assumptions can be relaxed, and slightly more complex form of CAPM can be obtained.

CAPM is a very simple yet powerful model of equilibrium asset pricing. CAPM is based on certain assumptions that violate real life situations. However, its efficacy lies in its ability to describe the real world observed prices. All the efficient portfolios lie on the capital market line that is the CML. The CML describes equilibrium prices in terms of price of time and price of risk.

Security market line that is SML describes the behavior of all the securities available in the market at equilibrium. Essentially this SML is the equation of CAPM. It passes through the market portfolio and risk free security. Various assumptions of CAPM violate the real world scenarios. CAPM postulates that all individuals should hold the market portfolio.

However, individual investors hold various small portfolios that are different from the market portfolio. The violation of a few assumptions individually may not necessarily invalidate CAPM.