Advanced Algorithmic Trading and Portfolio Management
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Lecture – 6
Week 2

In this lesson, we will introduce the concept of portfolio management. We will discuss the competition of expected returns and risk of a portfolio. We will discuss the concept of portfolio construction with two security case and multi security case. Lastly, we will examine the concept of risk diversification with portfolios.

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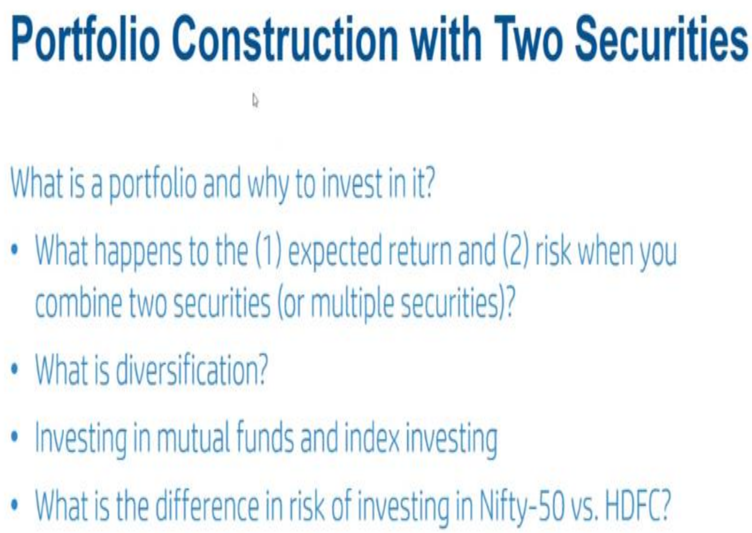

Portfolio construction with two securities expected returns. Till now, we have understood the interpretation of expected returns for a single security case. Now, we will discuss expected returns for portfolio securities. Let us start with the portfolio construction for two securities. Some of the important question that we are trying to answer is what is a portfolio and why to invest in it? More specifically, what happens when you invest in a portfolio?

What happens to your expected return and what happens to the risk when you combine two securities or multiple securities in a portfolio? What is this term called diversification? What are the benefits of this diversification? Also, what is investing in mutual funds and indexes and how they provide diversification? Lastly, what is the difference of investing in a portfolio like Nifty-50 versus a single stock like HDFC? What are the risk involved? How they are fundamentally different from each other?

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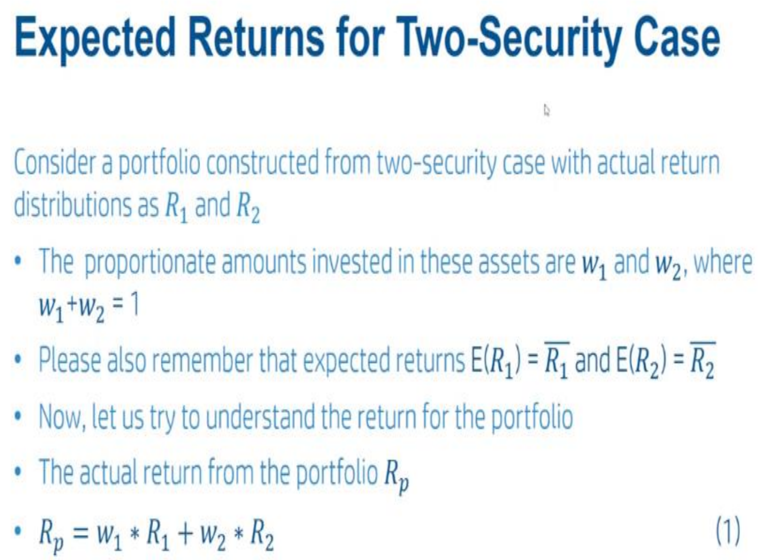

Let us consider a portfolio of two securities where the actual returns from the two securities are R 1 and R 2. Also, if the proportionate weights or amounts invested in these securities are w 1 and w 2, then w 1 + w 2 = 1 because the entire amount is invested in the portfolio. So, the summation of all the weights have to be 1. We already know that expected return on security 1 expected or E R 1 = R 1 bar and expected return on security 2 E R 2 is R 2 bar.

$w_1 + w_2 = 1$

$E(\bar{R}_1) = \bar{R}_1$ and $E(\bar{R}_2) = \bar{R}_2$

Now, let us try to understand the return from the portfolio. The actual returns from portfolio can be very easily computed as shown here in equation 1 where R p = w 1 into R 1 + w 2 into R 2 that is a very simple mathematical notation for the actual returns from a portfolio with two security cases, where w 1, w 2 are the proportionate amounts and R 1, R 2 are the actual returns observed on these two securities.

$R_p = w_1*R_1 + w_2*R_2$

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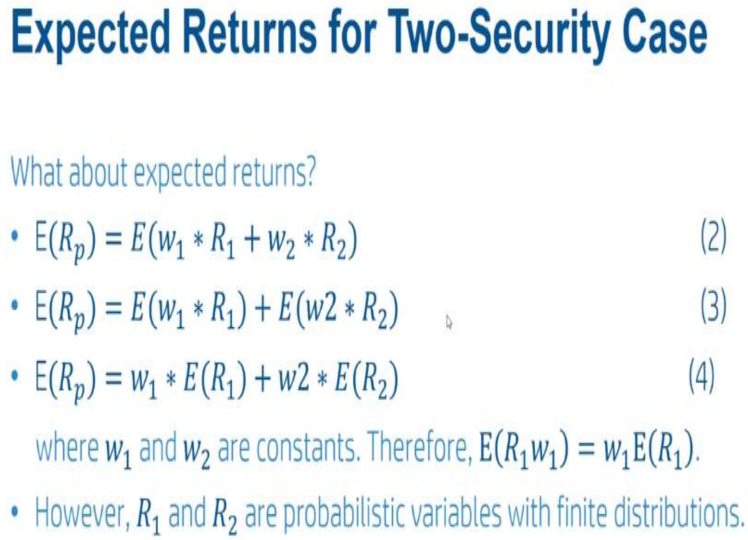

Now, let us try to understand the expected returns for this portfolio. The expression for expected return on portfolio R p can be written as E R p = expectation or w 1 R 1 + w 2 R 2 or we can further expand this result as expected value of w 1 R 1 + expected value of w 2 R 2. Please note here w 1 and w 2 are constants which are fixed from outside from the investing side, so we can take them out.

$E(R_p)= E (w_1*R_1 + w_2*R_2)$

$E(R_p)= E (w_1*R_1)+ E(w_2*R_2)$

$E(R_p)= w_1*E(R_1)+ w_2*E(R_2)$

$E(R_1 w_1) = w_1E(R_1)$

And therefore, the resulting expression becomes w 1 into expectation of R 1 + w 2 into expectation of R 2 as shown here in the equation 4. Please note here that R 1 and R 2 are probabilistic random variables with finite distributions. Generally, such distributions are approximated by a normal distribution.

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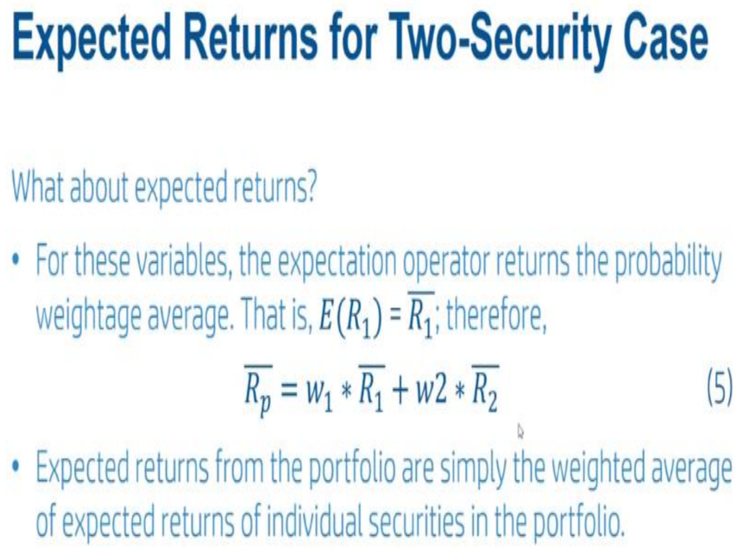

And therefore, these random probabilistic variables which are returns for them expectations can be computed. For example, the expected return on security 1 R 1 is R 1 bar. Similarly, expected return on security 2 is R 2 bar and if the investment weights are known that is w 1 and w 2, the expected return on portfolio can be computed as R p bar = w 1 times R 1 bar + w 2 times R 2 bar.

$\bar{R}_P = w_1 * \bar{R}_1 + w_2 * \bar{R}_2$

So, therefore it appears that expected returns from the portfolio are simply the weighted average of expected returns of individual securities in the portfolio. We are able to achieve this result simply because R 1 and R 2 being random probabilistic variable, their expectations can be easily computed as R 1 bar and R 2 bar.

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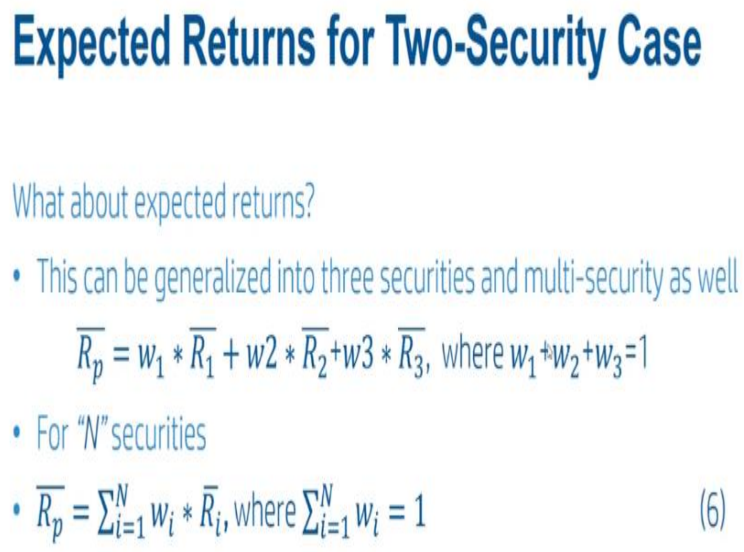

Now, this result can be easily generalized for a three-security case or even multi-security case. Let us consider a simple three-security case where the entire amount or entire wealth is nested in three securities R1, R2, R3. The proportionate amounts are w1, w2 and w3 and therefore the summation of w1, w2, w3 should be 1 because this is the total amount of wealth and w 1 one amount is invested in security 1, w 2 amount is invested in security 2 and w 3 is invested security 3.

Therefore, when we generalize our previous result R p bar = w 1 times R 1 bar + w 2 times R 2 bar + w 3 times R 3 bar. This can be further generalized to N security case where proportionate amounts that is w 1, w 2, and so on up till w N are invested in securities 1, 2, 3 and so on up to security N respectively. Also, the summation of all the w i should be equal to 1 which is the total amount of wealth invested.

$\bar{R}_P = w_1 * \bar{R}_1 + w_2 * \bar{R}_2 + w_3 * \bar{R}_3$ where $w_1 + w_2 + w_3 = 1$

And generalizing the result, $\bar{R}_P = \sum_{i=1}^{N} w_i * \bar{R}_i$, where $\sum_{i=1}^{N} w_i = 1$. To summarize, in this video we discussed the expected returns on a portfolio of two securities, which comprises simply the weighted average of returns on those two securities weights being the proportionate amount invested.

We arrived at the formula for these two securities and then we generalized this case for a multi-security case where security is 1 to N securities are there and the expected return on that portfolio is simply the weighted average amount of proportionate amounts invested in all these N securities which is multiplied by the expected returns on all these N securities, which is w i times R i bar summation i = 1 to N

$\bar{R}_P = \sum_{i=1}^{N} w_i * \bar{R}_i$

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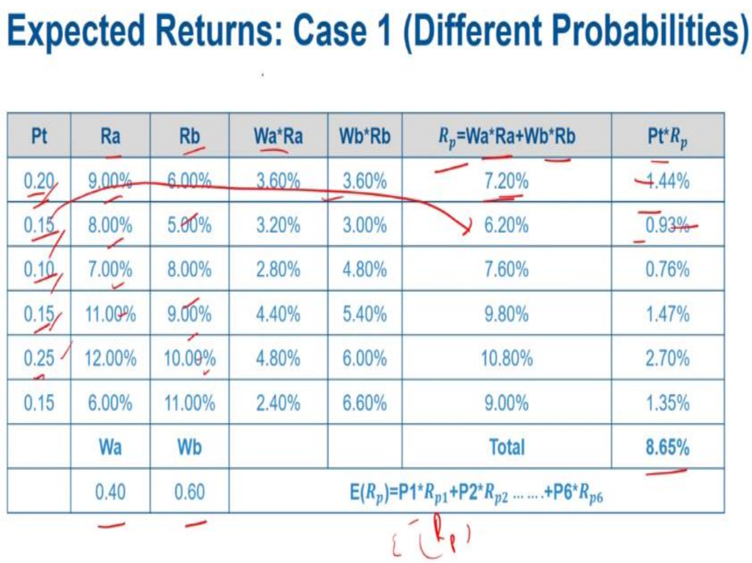
Expected Returns: Case 1 (Different Probabilities)

Pt	Ra	Rb	Wa*Ra	Wb*Rb	R_p=Wa*Ra+Wb*Rb	Pt*R_p
0.20	9.00%	6.00%	3.60%	3.60%	7.20%	1.44%
0.15	8.00%	5.00%	3.20%	3.00%	6.20%	0.93%
0.10	7.00%	8.00%	2.80%	4.80%	7.60%	0.76%
0.15	11.00%	9.00%	4.40%	5.40%	9.80%	1.47%
0.25	12.00%	10.00%	4.80%	6.00%	10.80%	2.70%
0.15	6.00%	11.00%	2.40%	6.60%	9.00%	1.35%
	Wa	Wb			Total	8.65%
	0.40	0.60	E(R_p)=P1*R_{p1}+P2*R_{p2} ….. .+P6*R_{p6}			

In this video, we will talk about expected returns from a portfolio with equal probability case and case with different probabilities. Let us start with the expected returns from portfolio case 1 with different probabilities. On column 1, we have probabilities that are shown here. These are the probabilities for different scenarios. Corresponding returns for security a are provided here and corresponding returns for security b are provided here.

The weights are fixed externally, 40 percent investment proportionate investment in security a and 60 percent proportionate investment in security b. Now, the actual returns for each scenario are computed here which are simply $w_a*R_a + w_b*R_b$ and this will give us the actual return for each scenario with given probabilities as shown here. Now, we can multiply each probability with the return scenario that is Pt into Rp, for example 0.2 into 7.2 percent which is 1.44 percent.

Similarly, each probability can be multiplied with corresponding return to get Pt into Rp. $E(R_p) = P_1*R_{P1} + P_2*R_{P2} + \ldots + P_6*R_{P6}$. This is expected to return portfolio Rp which is 8.65 percent. This was the case where all the associated scenarios have different probabilities.

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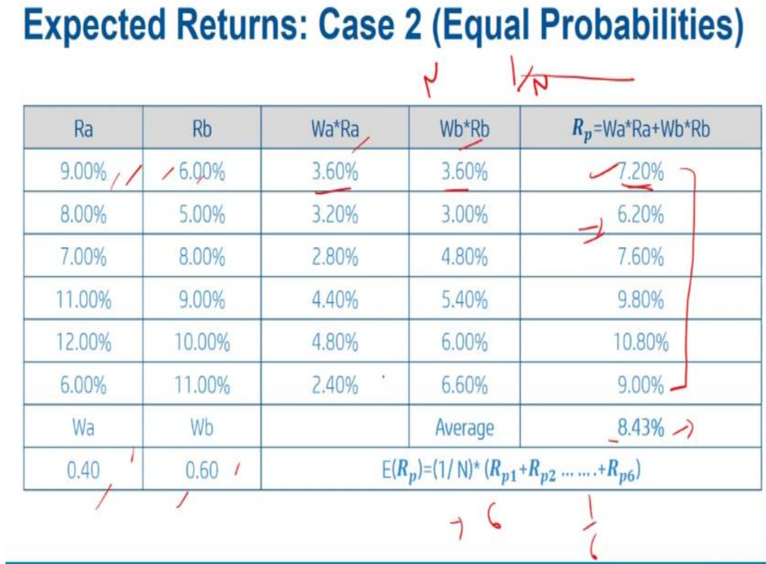

Expected Returns: Case 2 (Equal Probabilities)

Ra	Rb	Wa*Ra	Wb*Rb	R_p=Wa*Ra+Wb*Rb
9.00%	6.00%	3.60%	3.60%	7.20%
8.00%	5.00%	3.20%	3.00%	6.20%
7.00%	8.00%	2.80%	4.80%	7.60%
11.00%	9.00%	4.40%	5.40%	9.80%
12.00%	10.00%	4.80%	6.00%	10.80%
6.00%	11.00%	2.40%	6.60%	9.00%
Wa	Wb		Average	8.43%
0.40	0.60	$E(R_p)=(1/N)*(R_{p1}+R_{p2} \ldots \ldots .+R_{p6})$		

Let us consider another case where all the possibilities are equal that means all the scenarios have equal probabilities. And if let us say we have N possible scenarios, then each scenario will have 1 upon N probability. In this case, since we have 6 possible scenarios, each scenario will have 1 by 6 probability and therefore to start with we have the corresponding returns, weights are externally fixed.

So first, we will compute the return from portfolio for each scenario which is $w_a*R_a + w_b*R_b$. So for first case it is 7.2 percent, which is 9% X 0.4 + 6% X 0.6, and w_aR_a which is 3.6 percent, w_bR_b which is again 3.6 percent. And the overall figure works out to 7.2 percent. In the similar manner, we will compute the actual returns for all the 6 possibilities and the summation works out to 8.43 percent. The average of the summation works out to 8.43 percent.

That means we sum up all these return numbers and then take the average which is 8.43 percent, which is the average of all these 6 return figures and that becomes our expected return for equal probability case where all the probabilities are equal. To summarize, in this video we discussed how to compute expected returns for a portfolio when case 1 given scenarios comprise different probabilities with each scenario and case 2 a scenario where all the possibilities or all the scenarios have equal probability.

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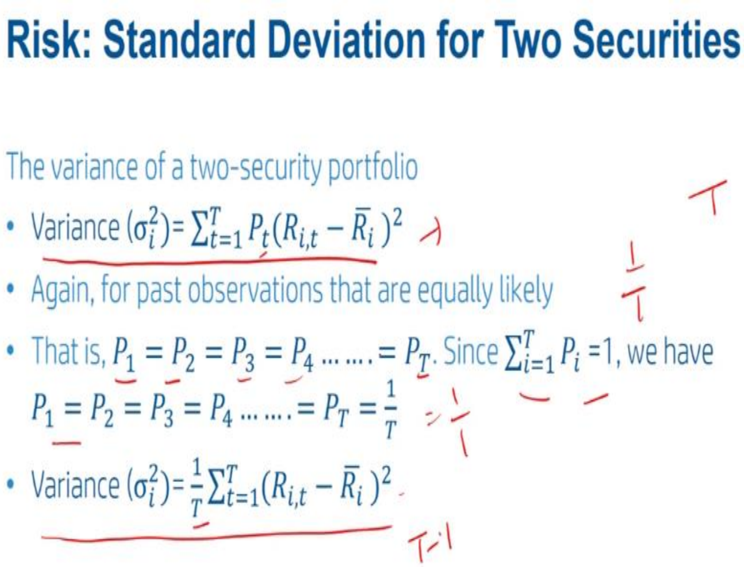

Portfolio construction with two security case, risk. In this video, we will try to understand the risk of two security portfolio. Till now, we have already understood how to compute the variance of a security for a case with given probabilities. Kindly have a look at this formula, here if the probability of different return scenarios is Pt that is P_1, P_2 and so on up till P_T and different returns that is R_{i1}, R_{i2} and so on as a generic term $R_{it.}$

Variance (σ_i^2) = $\sum_{t=1}^{T} P_t(R_{i,t} - \bar{R}_i)^2$ this is a very simple and standard formula for computation of variance for a security with different probabilities. Now, if the probability or the probability of observing each observation is equally likely, then in that case P1 = P2 = P3 = P4 = …=P T. We also know that $\sum_{i=1}^{T} P_i = 1$

So, if there are T observations and all of them are equally likely, then of all of them becomes 1 upon T. In this case, the variance of the security can be easily computed with this formula summation R i t – R i bar raised to the power 2 upon T. In some of the textbooks, they also note that since we are working with samples, here instead of using T they will be using T – 1.
Variance (σ_i^2) =1/T * $\sum_{t=1}^{T}(R_{i,t} - \bar{R}_i)^2$

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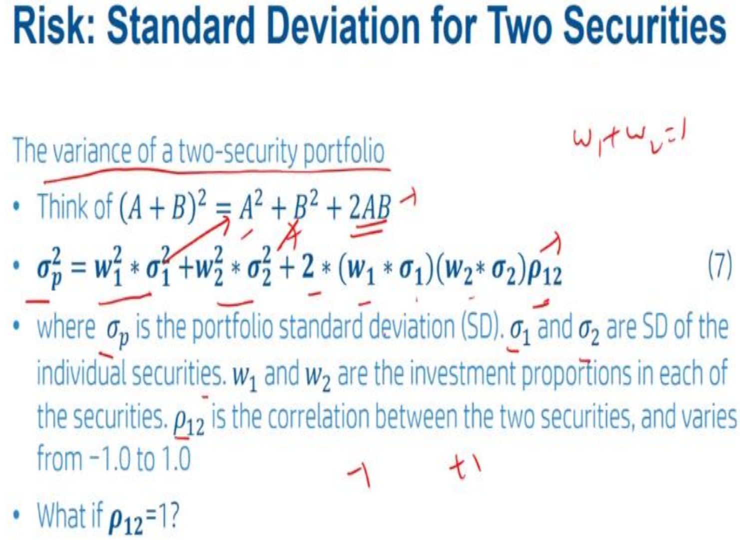

Now, let us consider the variance for the risk of a two-security portfolio. In the case of a two-security portfolio, let us remember the very simple formula for A + B raised to the power 2 which is A square + B square + 2 AB. The risk of a two-security portfolio can be very easily extended or understood with this help of this formula which is σ p square is w1 σ square which is corresponding to A square, then w 2 square σ 2 square corresponding to B square.

$(A + B)^2 = A^2 + B^2 + 2AB$

$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * (w_1 * \sigma_1)(w_2 * \sigma_2)\rho_{12}$

And for 2 AB we have a term very similar which is 2 times $w_1\sigma_1$ into $w_2\sigma_2$ into ρ_{12}. Now here this extra term of ρ_{12}here is the correlation between security 1 and 2 has some special meaning as we will see shortly. Here, σ_p is the standard deviation or a proxy of risk for the portfolio. σ_1 and σ_2 are the standard deviation of individual securities, w_1 and w_2 are the proportionate amount of investment in each of these securities.

$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * (w_1 * \sigma_1)(w_2 * \sigma_2)\rho_{12}$

We already know if the portfolio carries only two securities, then $w_1 + w_2$ two should be equal to 1. These are the proportionate amount invested. And importantly, ρ_{12} is the correlation between these two securities. This correlation varies from –1 to +1. Now, we will try to understand different scenarios what if this ρ_{12}= 1 or 0 or –1.

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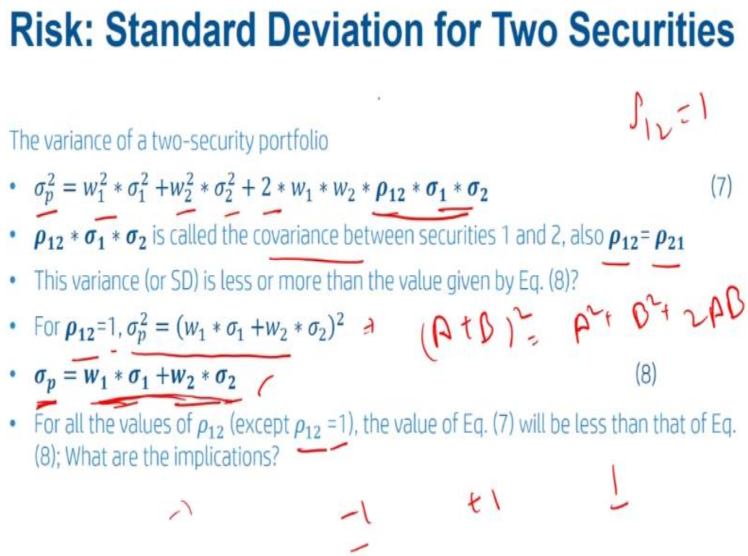

Let us start with a very simple scenario where this $\rho_{12} = 1$. Again the formula of the portfolio risk is already known to us which is $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * \sigma_1 * w_2 * \sigma_2 * \rho_{12}$

Now, please note here this expression ρ_{12} into $\sigma_1 \sigma_2$ is also called a covariance between the securities 1 and 2 and here $\rho_{12} = \rho_{21}$ and that is why you have a multiple of 2.

In case this ρ_{12} or $\rho_{21} = 1$, then this formula considerably simplifies to $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2$ which is again remember coming from the A + B raised to the power 2 formula which was equal to $A^2 + B^2 + 2AB$. And in this case, the σ_p works out to $w_1\sigma_1 + w_2 \sigma_2$. Please note for all the values and we already know that this ρ_{12} lies between –1 to +1.

So the magnitude of this ρ_{12} will always be less than 1 and therefore for all the values of ρ_{12} they will be less than 1, starting from –1 to +1 they will always be less than 1 and therefore for all the other values of ρ_{12} the value of risk σ_p will be less than this quantity, whatever it may be it will always be less than this quantity. What are the implications of this? Please understand that if the securities have perfect correlation that means they are perfectly moving in lockstep.

And therefore their correlation is 1, there is no diversification and portfolio risk is also very similar to the portfolio expected return, similar to what we computed for expected return that was weighted average, portfolio risk here is also the weighted average of risk of these two securities. For all the other cases where this correlation is less than 1 and we noted this is a maximum correlation case, which is ρ_{12}= 1.

For all the other cases this σ_p will be less than the weighted average of individual risk components multiplied by the proportionate weights. And this reduction in the risk is precisely what we call as diversification and risk. Why are we getting this diversification? This diversification is appearing because the correlation between these two securities is less than 1, that means any correlation which is less than 1, results in diversification.

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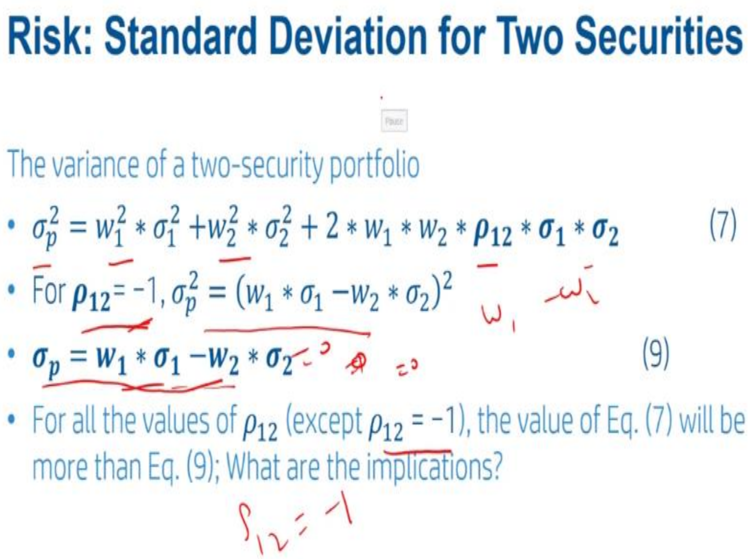

Let us consider another extreme case where correlation is equal to –1. In this particular case, again our good old formula of $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2* w_1* \sigma_1*w_2* \sigma_2 * \rho_{12}$, considerably simplifies to this formula. Now, in this formula if we further solve this expression, we will achieve this. This is a very special case for a correlation of –1. What are the implications of this formula?

Please notice although the correlation of –1 is usually not observed in financial markets or other commodity and derivative markets, it leads to a special case where a particular w_1, w_2 can be achieved where this value can be made 0, but please note this is only theoretical discussion because in no financial markets or between two securities negative correlation is observed over medium to long terms.

And therefore from theoretical and academy discussion purposes, assuming that ρ_{12} correlation equal to –1, in that case a particular combination of weights can be achieved where σ_p becomes 0. This is a more of a theoretical case, but it suggests that a particular scenario can be obtained theoretically where the overall risk can be made 0. This is the case where maximum diversification is achieved where σ_p becomes equal to 0 the risk of the portfolio becomes 0.

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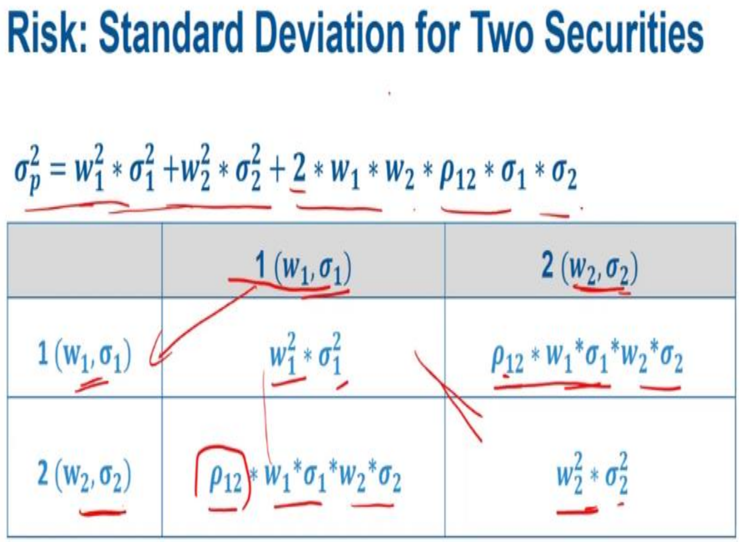

Now, let us understand about generic and easy to use thumb rule while constructing the risk for two-security portfolio. You need to create 2 cross 2 boxes like this where you have first w_1, σ_1 and w_2 σ_2 here and w_1 σ_1, w_2 σ_2 here. Now, in the first box you will combine these two terms to get $w_1^2 * \sigma_1^2$ and again this cross term will give you $w_2^2 * \sigma_2^2$ in a very similar manner.

So, these are called diagonal boxes. If you look at off-diagonal boxes or boxes that are opposite to the diagonal, you will combine w_2 σ_2 with w_1 σ_1 and you get w_1 σ_1 w_2 σ_2. But in addition to that, you also get a correlation term which is ρ_{12}. Similarly, in this off diagonal box, you combine w_2 σ_2 with w_1 σ_1 to get w_1 σ_1 w_2 σ_2 with again correlation term.

And if you add up all these four terms, you get the generic expression that we saw here, $w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * \sigma_1 * w_2 * \sigma_2 * \rho_{12}$ which was our generic expression that we used earlier to compute the risk of two-security portfolio. To summarize, in this video we understood how to compute the risk of a two-security portfolio.

We also understood that securities that move exactly lockstep manner with a correlation of 1, there is no diversification and maximum diversification is achieved when the securities move exactly in opposite manner that is with a correlation of –1. For all the other cases, correlation remains less than 1 and some amount of diversification is always achieved when two securities are added.

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Risk: Standard Deviation for Multiple Securities

$$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$$

	1 (w_1, σ_1)	2 (w_2, σ_2)	3 (w_3, σ_3)
1 (w_1, σ_1)	$w_1^2 * \sigma_1^2$	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$
2 (w_2, σ_2)	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$w_2^2 * \sigma_2^2$	$\rho_{12} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$
3 (w_3, σ_3)	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$	$\rho_{12} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$	$w_3^2 * \sigma_3^2$

Portfolio construction with multiple securities, risk: In this video, we will extend our understanding of portfolio risk for a multiple security case. Let us start with a portfolio of three securities. We will extend our logic that we developed in the previous video for two-security case, we will try to implement that here for multi-securities. Simply extending the formula for a two-security case to multi-security case, kindly have a look at this 3 cross 3 box.

And we will try to extend this formula that we developed there for this 3 cross 3 security case. So we have w_1, σ_1 for security 1 w_2, σ_2 for security 2 w_3, σ_3 for security 3. Similarly, we have ρ_{12} for correlation between security 1 and 2, ρ_{13} correlation between security 1 and 3 and then we have row 23 for security 2 and 3.

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Risk: Standard Deviation for Multiple Securities

$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$

	1 (w_1, σ_1)	2 (w_2, σ_2)	3 (w_3, σ_3)
1 (w_1, σ_1)	$w_1^2 * \sigma_1^2$	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$
2 (w_2, σ_2)	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$w_2^2 * \sigma_2^2$	$\rho_{23} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$
3 (w_3, σ_3)	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$	$\rho_{23} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$	$w_3^2 * \sigma_3^2$

Now the overall portfolio risk will be a combination of these diagonal terms that is $w_1^2 * \sigma_1^2$ very similar to what we did earlier for two securities, then $w_2^2 * \sigma_2^2$, $w_3^2 * \sigma_3^2$. And again the off diagonal terms that are here they will be put together w_1 σ_2 σ_1, w_2 σ_2 and along with that there is a correlation term ρ. Similarly, if you look at this particular box, here you are combining w_1 σ_1 with w_3 σ_3.

So, this term will result and then in addition you are putting correlation term and this goes on for all the nondiagonal or off diagonal boxes like this and this, each of these nondiagonal boxes they will include a correlation term because here we are combining the variance of two different securities, unlike diagonal terms where we have same security and the variance numbers computed.

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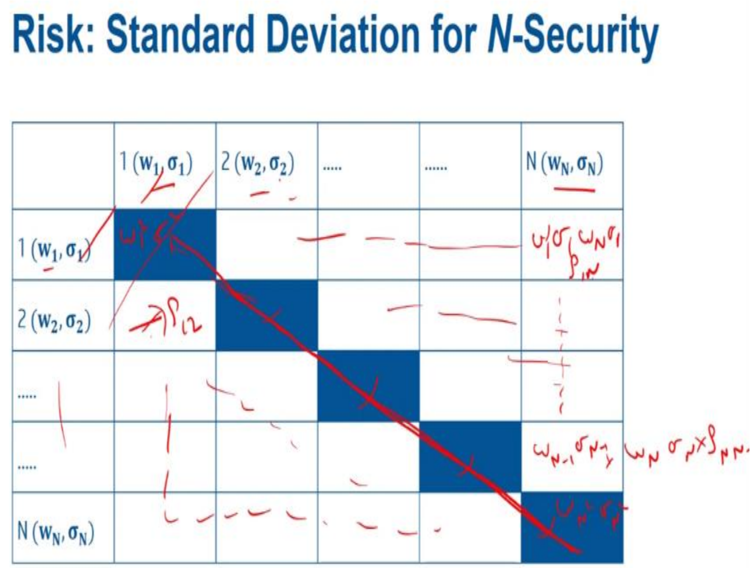

Let us extend this understanding to a more generic and security case. Here again you have w_1 σ_1 w_2 σ_2 and so on up till w_N σ_N and w_1 σ_1 here on the column and row aside. As you would have; already guessed these diagonal boxes will include all the various terms that are combination of w_1^2 $* \sigma_1^2$, $w_2^2 * \sigma_2^2$. So, this will be something like $w_1^2 * \sigma_1^2$and so on so up to $w_N^2 * \sigma_N^2$.

So, these are all the diagonal boxes. On the off-diagonal terms, you will have combination of two different securities, for example here you have w_2 σ_2 w_1 σ_1 along with the correlation terms. Similarly, here you will have w_1 σ_1 w_N σ_N along with the correlation term ρ_{1N} and so on and so forth up till here where for example you may have something like w_{N-1} σ_{N-1} into w_N σ_N and the correlation term between security N N–1.

So, in this fashion, we will fill all the off diagonal terms which will include the covariances between two different securities these off diagonal boxes while the diagonal boxes like this they will include only the variance terms.

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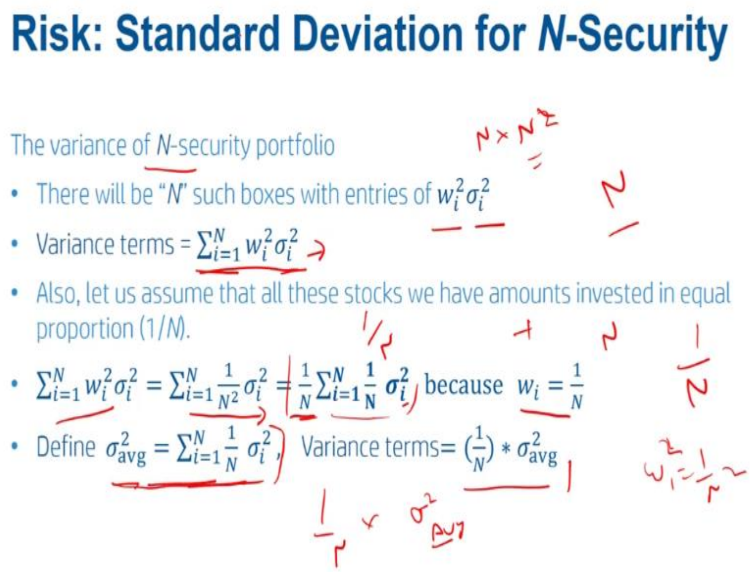

Now, let us understand the variance of this N-security portfolio and let us focus on the diagonal terms. So, how many diagonal terms you observe? Total there will be N into N that is N square terms, while out of these N square terms there will be N diagonal terms and these diagonal terms will carry only variance terms like σ i square w i square. So, if I sum up all these diagonal terms, something like Variance terms = $\sum_{i=1}^{N} w_i^2\, \sigma_i^2$

This is because if you notice this diagram here all these diagonal terms, there are N such diagonal terms and all these diagonal terms will include an expression like w 1 square σ 1 square or w N square σ N square. So, if I add up all these diagonal terms, an expression like this will emerge. Now, to simplify the situation, let us assume that all these securities have equal investment that is 1 by N proportion.$\sum_{i=1}^{N} w_i^2\, \sigma_i^2 = \sum_{i=1}^{N}\frac{1}{N^2}\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\mathrm{N}}\sigma_i^2$

That means since there are N securities and if I invest equal amount in each security, the proportionate amount invested in each security will be 1 by N. With that assumption my w i which is equal to 1 upon N and therefore w i square will become 1 upon N square and therefore a term like this will be simplified to this. Now, let us take this 1 upon N outside the expression, so we are left with this. Please note here N is not the varying term, only i subscript is varying

$$\sum_{i=1}^{N} w_i^2\, \sigma_i^2 = \sum_{i=1}^{N}\frac{1}{N^2}\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\mathrm{N}}\sigma_i^2$$

So, we can take the N outside and here w i is 1 upon N. So we get something like this expression inside which we can define as average variance. So, if let us say define average variance something like this, so we will have something 1 upon N into average variance down which is defined simply like this. So, our overall variance for the portfolio becomes 1 upon N times σ square average where this is average variance term defined as this.

$$\sigma_{avg}^2 = \sum_{i=1}^{N} \frac{1}{\mathrm{N}} \sigma_i^2$$

$$Variance\ terms = \frac{1}{N} * \sigma_{avg}^2$$

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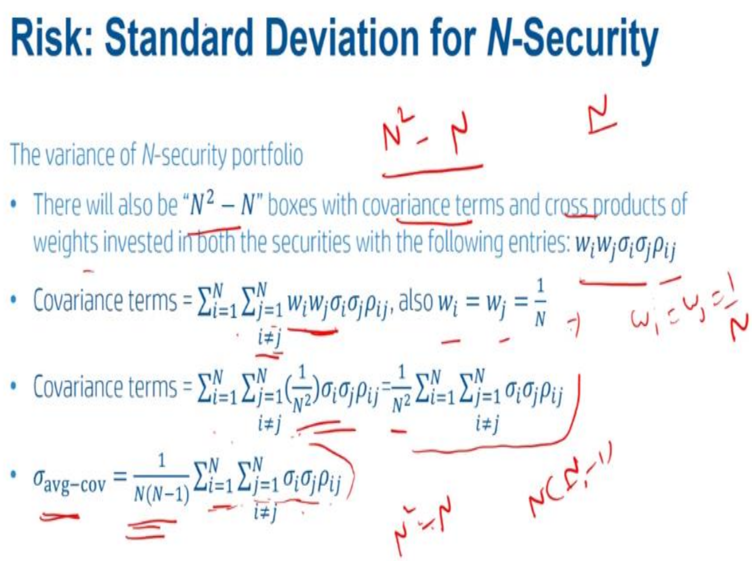

Now, let us focus on the off-diagonal boxes. As we noticed that there are total N square boxes and N are the diagonal boxes. So, we are left with N square – N off-diagonal boxes which include covariance terms or the cross products of securities including the weights invested in these securities and the resulting term we have already seen this that will appear something like w_i w_j σ_i σ_j into ρ_{ij}. So, these are all off-diagonal terms.

Now, the overall multiplication will appear something like this if I sum up all the covariance terms, off-diagonal terms, all those N square – N terms, I will end up with something like $\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j\, \sigma_i \sigma_j \rho_{ij}$, where i is not equal to j. Why this special subscript because for all the cases that i = j, they will become the diagonal terms with variance terms.

Covariance terms = $\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \, \sigma_i \sigma_j \rho_{ij}{}_{i\neq j}$, also $w_i = w_j = \frac{1}{N}$

Now, we already made a simplifying assumption that $w_i = w_j = 1$ upon N that means we are investing equal amount in all the securities and therefore those proportionate amounts $w_i = w_j$ are same as 1 upon N. In that simplifying case, our overall expression becomes this 1 upon N square into σ i σ j ρ ij. We can take N out of this expression because N is constant, so we are left with this expression.

Covariance terms = $\sum_{i=1}^{N}\sum_{j=1}^{N}(\frac{1}{N^2}) \, \sigma_i \sigma_j \rho_{ij}{}_{i\neq j} = (\frac{1}{N^2})\sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_i \sigma_j \rho_{ij}{}_{i\neq j}$

Now, let us define an average quantity of covariance or average covariance. We already know that there are N square – N boxes or N into N – 1 boxes. So, an average covariance can be easily defined something like number of or summation of all these covariance term divided by total number of these covariance boxes which are N into N – 1 or N square – N. So, let us define our average covariance as we did for various terms.

Also we can define our average covariance as total number of boxes or total number of terms which is N into N – 1 multiplied by summation of all these covariances. So, this is the average covariance or a simplifying assumption.

$$\sigma_{avg-cov} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_i \sigma_j \rho_{ij}{}_{i\neq j}$$

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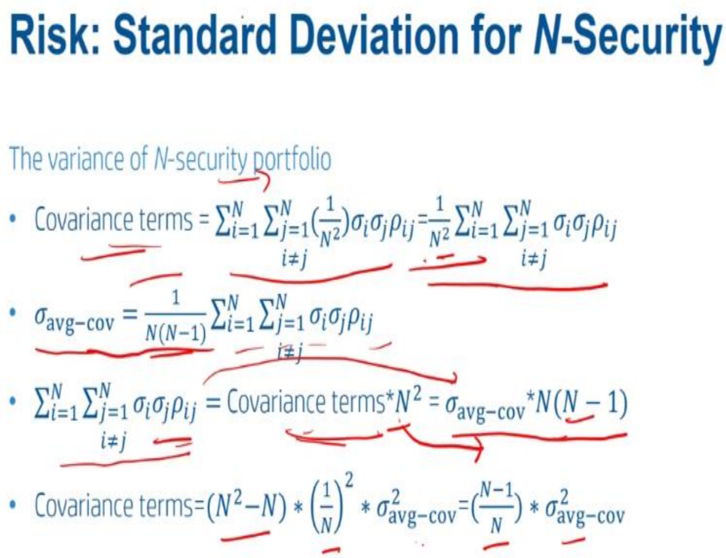

Now, let us rearrange these covariance terms a little bit. We already defined that summation of all these covariance terms is this, which further simplified to this and we defined something called average covariance like this which is 1 upon N into N – 1 times the summation of all these covariance terms. Now, let us rearrange them a little bit. So, here I can take the N square from this expression to this side.

Covariance terms = $\sum_{i=1}^{N}\sum_{j=1}^{N}(\frac{1}{N^2})\,\sigma_i\sigma_j\rho_{ij}{}_{i\neq j} = (\frac{1}{N^2})\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\rho_{ij}{}_{i\neq j}$

So, my summation of all these terms is equal to covariance terms into N square. Now, here I already know that this expression is equal to because of this expression, this expression is also same as σ average covariance times N into N – 1. So, this is equal to this because of this expression and we already defined that summation of these covariance terms is equal to this one where N square we can take here.

$$\sigma_{avg-cov} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\rho_{ij}{}_{i\neq j}$$

So, a very simple expression, summation of all these covariances become this covariance term summation into N square which is equal to again σ average covariance term N into N – 1. And therefore, what we are interested in is the summation of all the off-diagonal terms or the risk of off-diagonal terms which is covariances, which simply becomes N square – N which is this into 1 upon N square into σ square average covariance.

$\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\rho_{ij_{i\neq j}} = Covariance\ Terms * N^2 = \ \sigma_{avg-cov}$* N(N-1)

Because we can take this N square here and therefore the equality becomes very simply N square – N into 1 upon N square times σ average covariance or more simply N – 1 upon N into average covariance.

Covariance terms = $(N^2 - N) * (1/N)^2 * \ \sigma_{avg-cov}^2$ = (N-1)/N * $\sigma_{avg-cov}^2$

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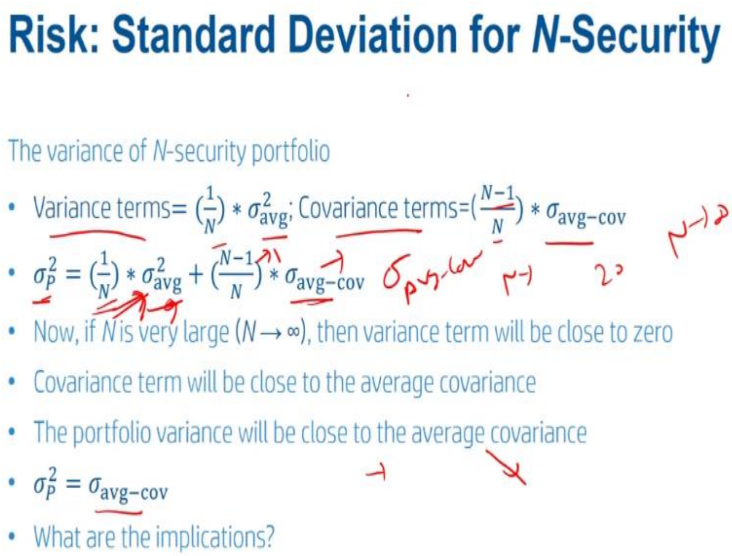

Now, that we have a simplified expression for covariance as well as variance terms, let us put this expression together to find the overall risk of the security. So, first we have set of variance term for which we found a simplifying expression in the form of average variances. So, our simplified variance terms is equal to 1 upon N times σ square average and our covariance term summation is equal to N – 1 upon N into average covariance.

Variance terms = $(\frac{1}{N}) * \ \sigma_{avg}^2$; Covariance terms = $(\frac{N-1}{N}) * \ \sigma_{avg-cov}$

So, we will put them together to find the overall risk of the portfolio or σ_P square which is 1 upon N into σ square average + N – 1 upon N into average covariance. Now, if you have fairly large number of securities where N is tending to infinity, for practical purposes, you do not need to go up to infinity even with 20-30 securities a reasonable amount of diversification is achieved.

$$\sigma_P^2 = \left(\frac{1}{N}\right) * \sigma_{avg}^2 + \left(\frac{N-1}{N}\right) * \sigma_{avg-cov}$$

So, theoretically if N is fairly large, please notice that this term will approach to 0 because your denominator is N, while if N tends to infinity this term will approach to 1 and therefore we can easily say that when N value is sufficiently large or tending to very large values, the variance term or the diagonal terms will come 0. So, the overall portfolio risk or portfolio variance will be simply equal to the average covariance.

And all the variance terms are canceled out because of this property that N is fairly large and therefore, this term will approach to 0 and this term will approach to σ average covariance or average covariance of the portfolio. What are the implications? So, here we find that just by adding sufficient number of securities, we can cancel out or neutralize the idiosyncratic or stock specific or variance part, the diagonal terms that were there they are canceled out.

$$\sigma_P^2 = \sigma_{avg-cov}$$

While the; average covariance term that is not nullified and therefore the overall portfolio risk tends closer to the average covariance term. This is precisely what we achieved with the help of diversification simply by adding more and more securities, we achieve or neutralize the idiosyncratic or variance terms that are stock specific equal to 0. To summarize, in this video, we discussed how addition of securities neutralizes the stock specific or variance terms or diagonal terms part of this.

These are the variance terms and as we keep on adding security, the off-diagonal terms which include covariances they are not neutralized, they still remain, and therefore the portfolio risk tends towards this average covariance for a large number of security portfolio. We also discussed how to extend our logic of two-security portfolio to generate expression for multi-security portfolio and how these multi-security portfolio with reasonably large number of securities achieve diversification.

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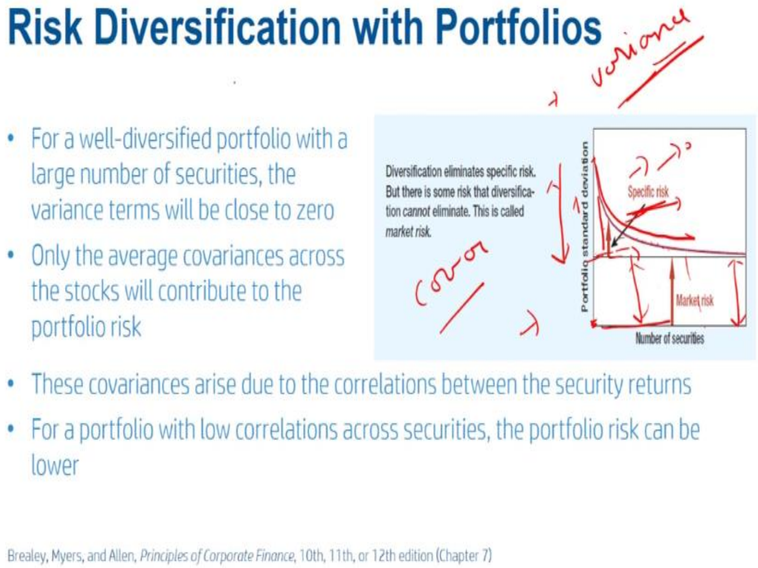

Risk diversification with portfolios: In this video, we will conclude our discussion on this diversification with portfolios with the help of a few numerical examples. Now, we have understood that in a portfolio with large number of securities, the summation of variance term is close to 0. For example, if we start with a portfolio with one security like in the diagram shown here, a large part of the portfolio risk comprises security specific or idiosyncratic risk.

Which is driven by the variance terms, the diagonal terms that we saw which are variance terms. However, as we keep on adding more and more securities, this particular component risk variance or idiosyncratic or stock specific component of risk tends to become nullified or canceled. In contrast, there is another component of risk which is on account of covariances that is correlations across securities.

This component of risk is even though it is small in a single stock portfolio, as we keep on adding more and more securities it is not diversified or nullified in the portfolios. So, as we keep on adding more and more securities, this component remains still and for a very large number of securities only this part of portfolio risk remains while the specific or idiosyncratic risk tends to go down to 0.

And the remaining component which is this average covariance risk this is also called bedrock of risk as it cannot be eliminated by adding more and more securities. The diversification is achieved only because of cancellation or neutralization of this stock specific risk. We also noted that this average covariance risk or risk that is on account of those off-diagonal terms or covariances that is driven by the correlations across security.

Now, if we want to further decrease the risk which is given by these covariances, we would like to select securities that have low correlations with each other. As you keep on adding more and more securities in a portfolio, like we discussed this specific risk is almost close to 0, and if you add all the securities available in the market, the overall risk that is remaining will only constitute this risk that is driven by these covariances and often it is called market risk or bedrock of risk.

Because that portfolio with all the securities available in the market the risk which may be called as market risk is driven by these covariances and often referred to as market risk or bedrock of risk. This is also called systematic risk or non-diversifiable risk because it cannot be diversified by adding more securities. In contrast, the specific risk that is called diversifiable risk or nonsystematic risk because it can be diversified simply by adding more securities to the portfolio.

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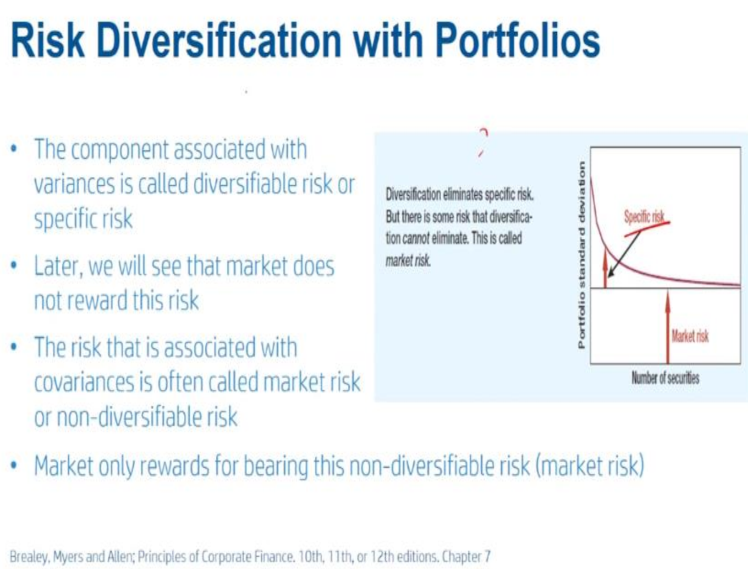

In financial markets, fund managers and investors are only rewarded for bearing this market risk. Often you would have heard the statement on mutual fund ads that mutual funds are subject to market risk. The statement precisely reflects this fact that this risk cannot be eliminated. Even if you had all the stocks and securities available in the market, you would still be exposed to this systematic risk or market risk.

And this cannot be diversified and therefore markets only reward for bearing this market risk or systematic risk. Therefore, as we will see shortly number of asset pricing models, they price this market risk only, they do not price this stock specific idiosyncratic risk. They only price this market risk and therefore fund managers and investors are rewarded only for bearing this market risk component.

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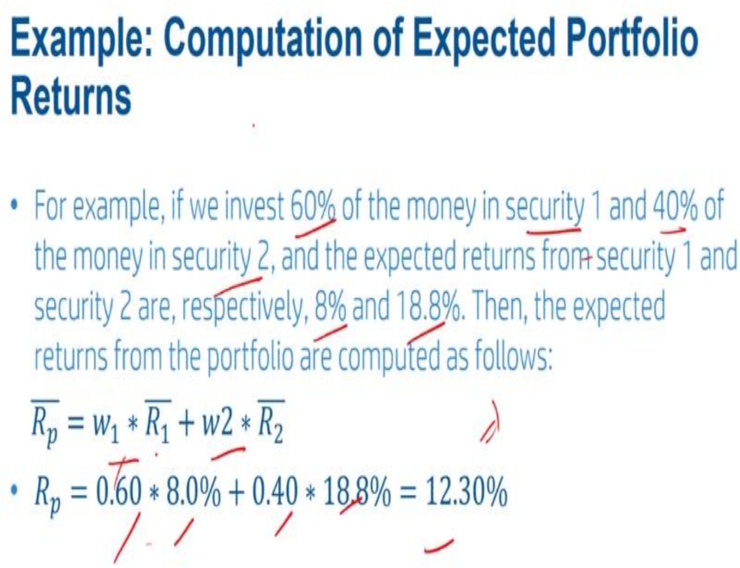

Let us perform a simple example of expected portfolio return. Let us say if you have two securities, you invest 60 percent of the money or wealth available with you in security 1 and 40 percent of the wealth in security 2. Now, you also know that expected returns from security 1 and 2 are 8 percent and 18.8 percent respectively. Therefore, the computation of expected return as we have already seen the formula is very easy and is simply $\bar{R}_P = w_1 * \bar{R}_1 + w_2 * \bar{R}_2$

.

Where w_1 is 60, 0.6 and w_2 is 40 percent which is 0.4 multiplied by the respective expected returns which is 8 percent and 18.8 percent and therefore we get 12.3 percent as expected return on the portfolio. A portfolio that comprises 40 percent of security 2 and 60 percent of security 1 wealth invested will give you 12.3 percent as expected returns.

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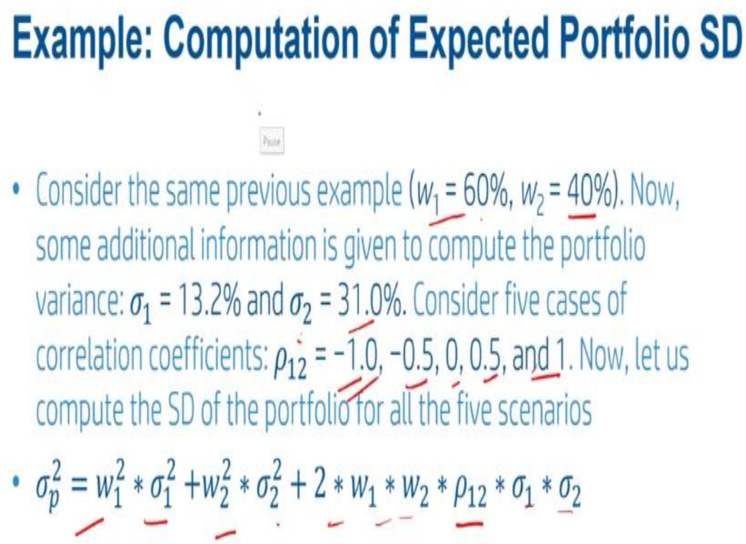

Let us consider in the previous example some additional information that is the risk of security 1 is 13.2 percent and risk or σ2 is 31 percent and the weights invested remain the same at 60 percent in security 1 and 40 percent in security 2. Now, for different cases, 5 cases in fact with correlations of –1.0, –0.5, 0, 0.5, let us try to compute the risk of the portfolios. We already know that the risk of a two-stock portfolio can be easily computed with the help of this formula. Which $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2*$ ($w_1 * \sigma_1$) ($w_2 * \sigma_2$)ρ_{12}, all that the quantities are fixed and we are only varying this correlation from –1 to +1. Let us see how these numbers worked out.

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Example: Computation of Expected Portfolio SD

Case	Variance (σ_P^2)	Standard Deviation (σ_P)
ρ_{12}=1	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 1 * 0.132 * 0.31 = 0.0413$	20.32%, which is same as = 0.6*13.2%+0.4*31.0%
ρ_{12}=0.5	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 0.50 * 0.132 * 0.31 = 0.0315$	17.74%
ρ_{12}=0.0	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 0.00 * 0.132 * 0.31 = 0.0217$	14.71%
ρ_{12}=-0.5	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * -0.5 * 0.132 * 0.31 = 0.0118$	10.88%
ρ_{12}=-1.0	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * -0.5 * 0.132 * 0.31 = 0.0020$	4.48%

So, first look at the first case where $\rho_{12} = 1$ and the formula is simply the same formula that we are using w 1 σ square 1 and so on. And in this case the overall risk works out to be 20.32 percent which is also same as the weighted average of individual risks. This is because now in this case as we have seen the ρ_{12}, no diversification occurs and the formula is simply standard deviation or variance formula is simply this because no diversification is there and securities are moving in perfect lockstep manner.

Now, as we keep on decreasing the correlation using the same formula, please notice the portfolio risk or standard deviation comes down gradually 17.74 percent for ρ_{12}. Here the correlation number is 0.5, all the other things remain same. For correlation equal to 0, the standard deviation is 14.71 percent. For correlation is equal to –0.5, the portfolio risk is 10.88 percent. Please note that the portfolio risk is minimum when correlation is –1.

So, the shows that as correlation decreases more and more risk is decreased and diversification is achieved as the correlation is lower. To summarize, in this video we discussed the two components of risk which is one which is stock specific and idiosyncratic component or diversifiable risk or nonsystematic risk. This component of risk can be easily nullified or neutralized by adding more and more stocks to the portfolio.

Next, we also discussed the systematic or market component of risk which cannot be diversified and it acts as a bedrock of portfolio, so even if you include infinite number of securities, this market risk or systematic component of risk cannot be diversified and it will depend on the correlation across securities in the portfolio. We observed that as we keep on decreasing the correlation from +1 to –1, we notice how standard deviation of the portfolio decreases.

So, in order to decrease the risk of portfolio, a well-diversified portfolio with large number of securities, if you pick and choose those securities where correlation is less than 1, you can achieve more and more diversification. To summarize this lesson, adding more securities that are less correlated or have lower covariance in the portfolio leads to diversification. Diversification here means the reduction of stock specific risk.

The part of the risk that is non-diversifiable is on account of the covariances across securities. Often, this risk is called market risk or systematic risk. Markets do not reward for bearing stocks specific diversifiable risk since these risks can be easily mitigated and diversified. When we say that we expect certain return for bearing risk that risk is systematic non-diversifiable risk or often referred to as market risk.