Artificial Intelligence (AI) for Investments
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Lecture - 06

Introduction to risk and return. Till now, we have gone through different topics without discussing the concept of risk, but we need to take the term risk head-on now. We will no longer be satisfied with a slightly vague statement that the opportunity cost of capital depends on the risk of the projects. We need to understand how risk is defined and how to compute risk. We start the discussion by understanding the basics of the risk-return framework.

The basics of the risk-return framework are explained by comparing these very different instruments, that is, T-bills, government bonds, and common stock, then we discuss the empiricalestimation of risk. Some of the very important measures of risk that we discuss are variance and standard deviation. Then we discuss the concept of risk diversification in the context of portfolios.We discuss and understand the risk-return computation for a two-security case and extend the framework to a multi-security case.

We find that the risk of any security comprises two components, first stock specific diversifiable risk component and second, market risk or systematic risk component. This stock-specific idiosyncratic risk can be diversified by simply adding more securities to the portfolio. The marketrisk of a portfolio cannot be diversified away and therefore becomes the bedrock of risk for a well-diversified portfolio.

Next, we discuss the contribution of a security to a portfolio and how to estimate the same empirically. We discuss the concept of beta, which is the sensitivity of a security to market movements.Lastly, we discuss the concept of value additivity, which is the sum of present values of cash flows for different project assets is same whether considered individually or taken together as a unit in afirm.

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Basics of Risk-return framework

- Consider three instruments: T-Bills, Government Bonds, and common stock

- T-Bills are short maturity instrument with almost no risk of default

- Bond is a rather long-term instrument and fluctuates with interest rates

- Common stocks are infinite maturity instruments

We will discuss the basics of risk written framework by comparing the performance of three broadinstruments that is treasury bills, government bonds, and common stock. We will also discuss theconcept of the risk premium and the factors that affect this risk premium. Consider three broad instrument categories that are treasury bills, government bonds, and common stock. These investments offer different degrees of risk.

First, consider treasury bills. These are short-maturity instruments with no risk of default; that is, one can obtain a sure shot payoff for short periods, for example, three months, by investing in these instruments. Although there is still uncertainty due to inflation, the actual real value may also depend upon the prevailing inflation rates. Compare this to long-term government bonds. This is a long-term instrument and fluctuates a lot with interest rates.

Bond prices fall when interest rates rise, and bond prices rise when interest rates fall. Next, we have common stocks that are market portfolios like nifty 50 or SNP 500. Common stocks do not have a finite maturity; the investor in common stock shares the ups and downs of the issuing company.

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Basics of Risk-return framework
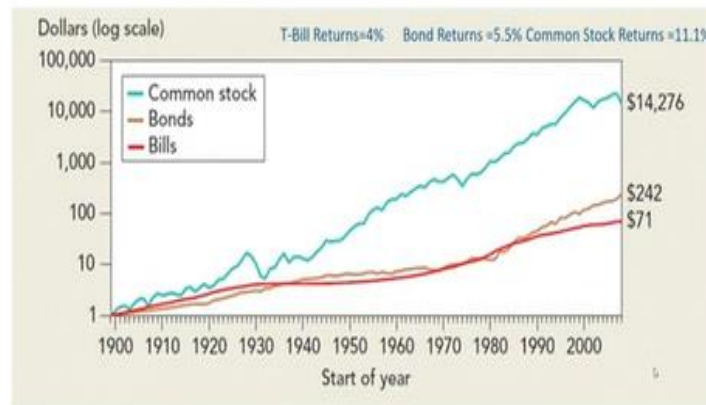
Consider the performance of these three instruments, that is, treasury bills, government bonds, and common stocks for the U.S. dollar over 100 years as shown in the figure here that is one dollar investment at the beginning of 1900 and its value by 2008. Assuming all the dividends and interest payments are reinvested in the same asset portfolios, the value shown here are nominal and do not account for inflation.

Observe the investment performance of different instruments in the figure shown here. It appears that the performance of these instruments matches their risk order. A dollar invested in treasury bills would have grown to 71 dollars by the end of 2008, which is a nominal return of 4 percent. A one-dollar investment in long-term bonds would have become 242 dollars, a nominal return of 5.5 percent.

Investors who place one dollar in common stocks would have received 14276 dollars, a nominal return of 11.1 percent. It seems that the common stocks provided the highest returns over this period. One important point to be noted here is that returns from common stock fluctuate a lot. Therefore, averages over a short period are not so meaningful.

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Basics of Risk-return framework

- Notice the difference between the returns on T-Bills and common stocks: 11.1-4=7.1%

- This additional return can also be said to be the risk-premium received by investors

- On a given year T-Bill rate was 0.2% and you are asked to estimate the expected return on common stocks. A reasonable estimate would be obtained by adding this 7.1% to obtain the total return of 7.30%

- However, this assumes that there is a stable risk premium on the common stock portfolio, that is, future risk premium can be measured by the average past risk premium

- But (a) Economic and financial conditions change overtime; (b) Risk perceptions change; (c) Investors' risk tolerance and return expectations also change over time

Notice the difference between the returns on treasury bills and common stocks. This additional return of 11.1% – 4 % = 7.1 %. This additional return can also be said to be the risk premium received by investors, that is additional returns for bearing the risk associated with their investmentin stocks. For example, on a given year treasury bill rate was 0.2 percent, and you are asked to estimate the expected return on common stocks.

A reasonable estimate would be obtained by adding the 7.1 percent to obtain a total return of 7.3 percent. However, this assumes that there is a stable risk premium on the common stock portfolio; that is, future risk premium can be measured by the average past risk premium. However, is it fairto say that the returns investors were demanding for bearing a given amount of risk 50 years ago are demanding the same returns for the amount of risk today as well?

There can be various reasons to argue against it. First economic and financial conditions change over time. Therefore, risk perceptions also change. And depending upon the current market conditions, investors' risk tolerance and risk expectations also change over time. For example, it isoften considered that dividend yields are a good proxy to measure this risk premium. When investors have a higher risk tolerance, their content with a lower return and a lower dividend yield is observed.

In contrast, when risk tolerance is low, investors' demands are written for the same levels of risk, and a higher dividend yield is observed.

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Basics of Risk-return framework

- Consider a stock with $12 dividend expected by the end of the year
- Investors are expecting a 10% return on this stock
- $PV = \frac{DIV_1}{r-g} = \frac{12}{0.10-0.07} = \$400$; Dividend yield = 12/400=3%.
- If dividend yield changes to 2%, and investors demand an expected return = 2%+7%=9%
- $PV = \frac{12}{0.09-0.07} = \$600$
- Expected returns on the stock reflect the dividend yields and the growth rate of dividends: $r = \frac{DIV_1}{P_0} + g$
- Risk-premium= $r - r_f$; this risk premium can change overtime
- Often dividend yield is a good indicator of risk-premium

Present value $PV = \frac{dividend1}{r-g} = \frac{12}{0.10-0.07} = \$400$ Here dividend yield is $\frac{12}{400} = 3\%$. Imagine that no investors revise their expectations and expect a dividend yield of 12 percent and demand an unexpected return of 9 percent.

The revised price estimate can be computed as shown here. Present value $PV = \frac{12}{0.09-o.07} = \$600$. Thus, a fall from 10 percent to 9 percent in the required returns leads to a 50 percent rise in the stock price. This is one case where the returns required by investors have come down. If we had relied on past returns, then we would have overestimated the premium desired by the investors today.

If markets are liquid and efficient and the stock is fairly priced, then the following relationship holds nicely that is $r = \frac{DIV_1}{P_0} + g$ that is, expected returns on the stock reflect the dividend yields and the growth rate of dividends. Here $r - r_f$ is the risk premium expected here r_f = risk period. This risk premium is expected and may be different from the actual realized risk premium. There can be many reasons this risk premium varies over time.

Investors are overly optimistic in some years and pessimistic in others about g that is expected dividend growth rates in the future. The second component is dividend yield which is $\frac{div_1}{P_0}$, prior literature suggests that this dividend yield is often less correlated with the dividend growth rates g. It is considered that this dividend yield is related to the risk premium. However, historical yields may not provide a good estimate of risk premiums in the future.

Therefore, one needs to have a more efficient and contemporaneous measure of risk. To summarize in this video, we compared the performance of three securities with different risks that are treasury bills, government bonds and common stock. We observed that these instruments offer different returns that are aligned with their risk. For now, we approximated the risk with the fluctuations observed in the instrument.

This excess return for bearing additional risk is often referred to as the risk premium of the instrument. This risk premium varies over time and depends upon a number of factors. These include prevailing economic and financial conditions, investor tastes, and risk preferences, among others. We also discussed that variation in dividend yield is one such measure of risk premium.
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Measures of risk

- A very prominent statistical measure of risk is variance (or standard deviation)

- $Variance\ (r_t) = Expected\ value\ of\ (r_t - \bar{r})^2$

- Where r_t is the actual return and $\bar{r}$ is the expected returns

- $Standard\ deviation\ (SD) = \sqrt{Variance\ (r_t)}$

- Standard deviation is often denoted by the symbol σ and variance by σ^2

We will discuss the empirical estimation of variance or standard deviation as a measure of risk. A very prominent statistical measure of risk is variance or standard deviation. Put simply, variance

of return is the expected square deviation from the expected returns, as shown here. That is the variance of $r_t = $ $expected\ value\ of\ (r_t - \bar{r})^2$ where r_t is the actual return and $\bar{r}$ is the expected returns.

Standard deviation is simply the square root of variance that is $standard\ deviation\ (SD) = \sqrt{variance(r_t)}$. Standard deviation is often denoted by the symbol σ and variance by σ^2.

(Refer Slide Time: 09:38)

Measures of risk

- Let us understand this concept with a small coin toss game

- The following probabilities are observed

  ➤ (a) H+H: Gain 40%; (b) H+T: Gain 10%;

  ➤ (c) T+H: Gain 10%; (d) T+T: lose 20%

- Thus, there is a 25% chance that your return will be 40%, 50% chance that your return will be 10%, and 25% chance that you will lose 20%

- Expected return : $\bar{r}$=0.25*40%+0.5*10%+0.25*(-20%)= 10%.

Let us understand this concept with a simple coin-tossing game. The following probabilities are observed first head plus head gain of 40 percent, head plus tail gain of 10 percent, tail plus head gain of 10 percent and tail plus tail loss of 20 percent. Thus, there is a 25 percent chance that your return will be 40 percent, 50 percent chance that your return will be 10 percent, and a 25 percent chance that you will lose 20 percent.

The overall expected return from this exercise can be easily computed as shown here, $\bar{r}$ that is $expected\ returns = 0.25 \times 40\ \% + 0.5 \times 10\ \% + 0.25 \times -20\ \% = 10\ \%$.

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Measures of risk

- Now let us compute the variance and standard deviation of these returns

Returns (%)	Mean Deviation $(r_i - r)$	Mean Square deviation $(r_i - r)^2$	Probability	Probability squared deviation
40	30	900	0.25	225
10	0	0	0.50	0
-20	-30	900	0.25	225
			Total	450

- Variance= 225+225=450 and Standard Deviation $(\sigma) = \sqrt{(450)} = 21\%$

- An event is considered to be risky if there are many possibilities of outcomes associated with it

- As these possibilities increase, i.e., the spread of possible outcomes increases, the event is said to have become riskier

- Standard deviation or variance is a summary measure of these possibilities, that is spread in the possible outcome

Now let us compute the variance and standard deviation of these returns as shown in the table here. $Variance = 225 + 225 = 450$ and standard deviation that is $\sigma = \sqrt{450} = 21\%$. Please note that standard deviation can be expressed in percentages. Thus, we have now computed the standard deviation of this game as 21 percent. How do we understand this standard deviation of variance financially? This variance is a measure of risk.

An event is considered to be risky if there are many possibilities or outcomes associated with it. As these outcomes or possibilities increase, that is, the spread of possible outcomes increases, the event is said to have become riskier. Standard deviation or variance is a summary measure of these possibilities that is spread in the possible outcome. For example, if there was one sureshot outcome of our coin tossing game that is one single possibility, whether positive or negative, we would have said that there is no risk with this event.

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Measures of risk

- The risk of an asset can be completely expressed, by writing all the possible outcomes and the possible payoffs associated with each of the outcome

- If the outcome was certain, i.e., no risk, then the standard deviation would have been zero

- One of the challenges in performing such computations is the estimation of probability associated with each outcome

- One way to go about this is to observe past variability

- For example, consider the historical volatilities of three different kinds of securities

Portfolio	Standard Deviation (σ)	Variance (σ^2)
Treasury Bills	2.8	7.7
Government Bonds	8.3	69.3
Common Stocks	20.2	406.4

- It appears that T-Bills are the least variable and common stocks are the most variable

Thus, one way to define risk is that there are various possible outcomes in an event, and in a similar manner, the risk of an asset can be completely expressed by writing all the possible outcomes and the possible payoffs associated with each of the outcomes. These outcomes are summarized using variance or standard deviation measures. If the outcome was certain that is no risk, then the standard deviation would have become zero.

The positive value associated with the standard measure precisely indicates this uncertainty. One of the challenges in performing such computations is the estimation of probability associated with each outcome. One way to go about this is to observe past variability. It is assumed that those securities or portfolios that have shown high variability will also be more volatile in the future. For example, consider the historical volatilities of three different kinds of securities as observed over the period 1900 to 2008, as shown here in the table.

Treasury bills have a 2.8 percent standard deviation, government bonds have an 8.3 percent standard deviation, and common stocks have a 20.2 percent standard deviation. It appears that T bills are the least variable, and common stocks are the most variable. There is no reason to believe that this variability will remain the same in the future. Still, though it is an important indicator, for example, there will be periods of excessively high volatility, such as the 2008 financial crisis.

At the peak of the crisis in 2008, October, and November, the standard deviation was 70 percent per annum. The risk premium increases during these times of volatile periods and comes down during normalcy. To summarize in this video, we discussed the computation of standard deviation or variance as a measure of risk. Any event with more than one output is considered a risky event. We examined this with the help of a simple coin-tossing game.

However, one needs to assign the probabilities in order to estimate the standard deviation. Historically empirically observed values, for example, pass returns, are often employed for this purpose.

(Refer Slide Time: 13:53)

Diversification of risk

- One can compute the measure of variability for individual securities as well as the portfolio of securities

- The standard deviation of selected U.S. Common stocks (2004-08) such as Amazon (50.9%), Ford (47.2%), Newmont (36.1%), Dell (30.9%), and Starbucks (30.3%) was much less than the standard deviation of market portfolio, i.e., 13% during this period

- It is well known that individual stocks are more volatile than the market indices

- The variability of market doesn't reflect or is same as the variability of individual stock components

- The simple answer to this question is that diversification reduces variability

We will discuss the concept of diversification with the help of portfolios. We will also discuss the concept of specific risk and market risk. One can compute the measure of variability for individual securities as well as the portfolio of securities. The standard deviation of selected U.S common stocks for the period 2004 to 2008, such as Amazon at 50.9 percent Ford, at 47.2 percent Newmont, at 36.1 percent Dell, at 30.9 percent, and Starbucks at 30.3 percent, was much less than the standard deviation of a market portfolio that is 13 percent during this period.

A market portfolio is a collection of many stocks in the form of indices such as the S&P 500 or nifty50. It is well known that individual stocks are more volatile than market indices. This raises an important question the market portfolio is made up of individual stocks. So, the variability of

the market does not reflect or is the same as the variability of individual stock components. The simple answer to this question is that diversification reduces variability.

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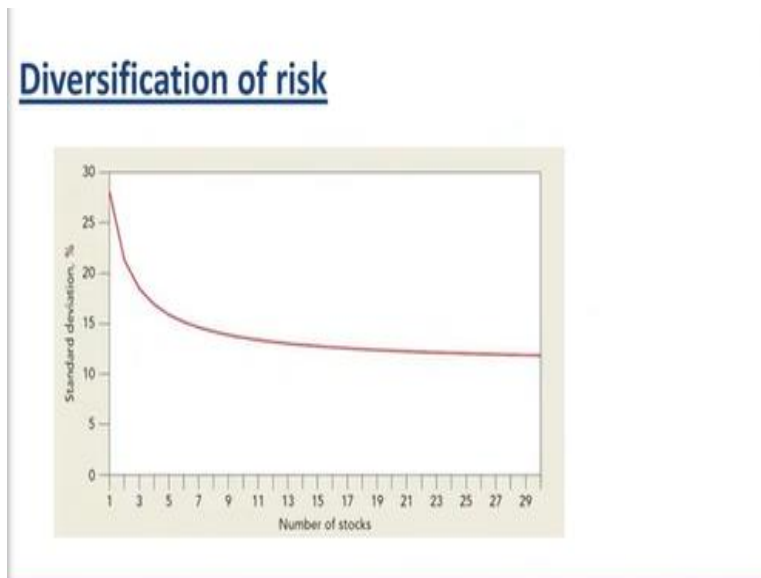
Consider the figure shown here it shows the change in standard deviation of the portfolio as the number of stocks are increased. Notice how portfolio standard deviation or risk decreases as 1, 2, 3 and more stocks are added. The average risk, that is, the standard deviation of the portfolio, decreases as the number of stocks in the portfolio is increased. This is called diversification. Please note that diversification reduces risk rapidly at first, and then the process is more gradual.

We can see that even with an addition of a small number of stocks, one can cut the portfolio variability by almost half. This concept of diversification arises because stocks do not move exactly together. This is so because stock prices are not perfectly correlated.

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Diversification of risk
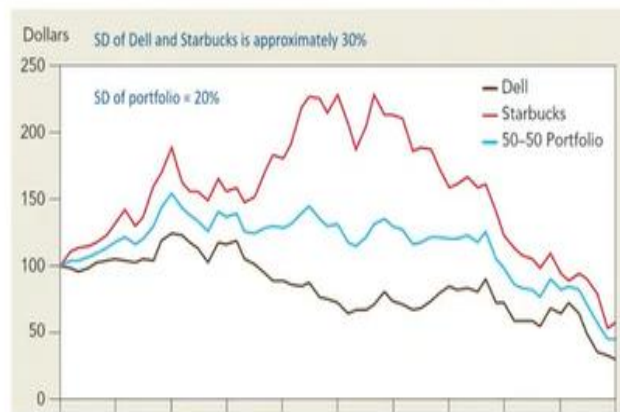
Consider the example where two stocks Dell and Starbucks and the 50 50 portfolio of these stocks is plotted over time. Assume an initial investment of 100 dollars, it is very easy to see here that diversification provides a substantial reduction in variability. The standard deviation of both of these stocks was about 30 percent. However, these stocks did not move in a lockstep manner. The decline of one stock was often compensated by the rise of the other.

Therefore, the standard deviation of the 50-50 portfolio of these stocks was about 20 percent per year. The rest that can be eliminated by diversification is called specific risk. The specific risk arises due to issues that are predominantly specific to the company or its close players. There is another category of risk called a market risk that cannot be diversified. Market risk arises due to the factors that are affecting the entire economy and the market in a similar manner.

This risk causes securities in a market to move together all and sundry. Thus, market risk affects security movements in a similar manner, up or down, and therefore cannot be diversified away through the addition of more securities in the portfolio.

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Diversification of risk

This division between market risk and specific risk is shown here in the figure. It can be seen that if you have a single stock, the specific risk is very important. But if you have a portfolio of 20 or more stocks, the market risk becomes more important. That is, for a reasonably well-diversified portfolio, only market risk matters. That is the predominant source of risk for such a diversified portfolio is the movement of the market, whether it will rise or fall, taking the portfolio with it.

To summarize, in this video, we discussed the concept of diversification. Individual securities do not move in a lockstep manner; therefore, adding more securities to a portfolio leads to diversification. This diversification reduces the stock-specific component of the risk. However, the market risk cannot be diversified away. This is so because market risk affects most of the securities in a market in a similar manner.

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Computing portfolio risk

- We now know that diversification reduces the risk of a portfolio

- Consider a portfolio comprising stocks A (60%) and B (40%)

- A has expected returns of 3.1% and B has expected returns of 9.5%

- *Expected portfolio return = 0.6 • 3.1 + 0.40 • 9.5 = 5.7%*

- Standard deviation of A is observed as 15.8% for A and 23.7% for B

- Standard deviation of this portfolio: 0.6*15.8%+0.4*23.7%=19.0%??

- This would be incorrect

Computing portfolio risk. In this video, we discussed the computation of risk and return for a portfolio of multiple stocks. We built the concept with a two-stock portfolio and extended the argument for a portfolio of multiple securities. We mathematically show how, for a well-diversified portfolio comprising a large number of stocks, the stock-specific risk is eliminated, and only market risk remains relevant.

We now know that diversification reduces the risk of a portfolio. To develop this understanding, we need to know how the risk of a portfolio depends on the risk of individual shares. Consider a portfolio of stocks comprising stock A with 60 % weight and stock B with 40 % weight. Also, assume that A has expected returns of 3.1 percent and B has expected returns of 9.5 percent. The expected returns on this portfolio can be easily computed, as shown here.

$Expected\ portfolio\ return\ =\ 0.60\ \times\ 3.1\ +\ 0.40\ \times\ 9.5\ =\ 5.7\ \%.$ Calculating the expected returns is very intuitive and easily done. However, computing the risk of the portfolio is not as easy. In the past, the standard deviation of A is observed as 15.8 percent for stock A and 23.7 percent for stock B. If you think that the standard deviation of this portfolio is simply the beta average of the two stocks, that is $0.6\ \times\ 15.8\ \%\ +\ 0.4\ \times\ 23.7\ \%\ =\ 19\%$, this would be incorrect.

As a special case, only if the prices of two stocks moved in a perfect log step, then only this competition would be true. For any other case, the diversification, as we see shortly, will reduce the risk below this figure.

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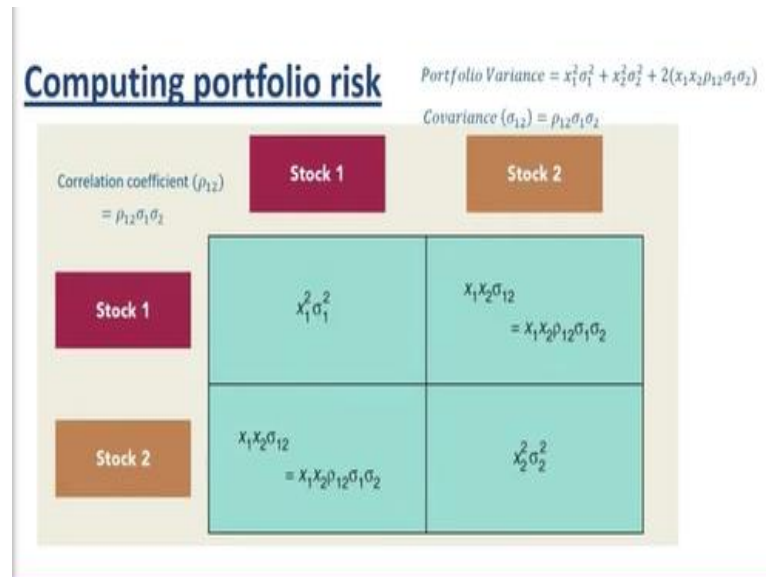
For a simple two-stock case, the standard deviation of the portfolio is simply computed, as shown in the figure here, by filling all the boxes and adding them up. The simple principle followed here is this. The risk or variance of each stock is weighted by the square of the proportion invested in those stocks, that is $W_1^2 \times \sigma_1^2 + W_2^2 \times \sigma_2^2$ and that portion of the rest that is on account of the covariance between these stocks.

That is their common movement due to the effect of the market. This covariance is expressed as the product of the correlation coefficient, that is ρ_{12} and the two standard deviations that are the covariance $\sigma_{12} = \rho_{12} \times \sigma_1 \times \sigma_2$. If the two stocks move together, then you would expect the correlation coefficient that is ρ_{12} to be positive, and therefore the covariance to be positive as well.

Similarly, if the two stocks move in the opposite direction, then you would expect the correlation to be negative. If the stocks are unrelated, then we can expect the correlation coefficient and covariance to be closer to zero. Thus, the resulting expression for r2 stock portfolio can be computed as shown here. $Portfolio\ variance = x_1^2\, \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \rho_{12} \sigma_1 \sigma_2$. The portfolio standard deviation would be the square root of the portfolio variance.

Computing portfolio risk

- Let us fill the above box with some numbers; Assume a correlation coefficient of 1

	Stock A	Stock B
Stock A	$x_1^2\sigma_1^2 = 0.6^2 \cdot 15.8^2$	$x_1 x_2 \sigma_1 \sigma_2 = 0.6 \cdot 0.4 \cdot 1 \cdot 15.8 \cdot 23.7$
Stock B	$x_1 x_2 \sigma_1 \sigma_2 = 0.6 \cdot 0.4 \cdot 1 \cdot 15.8 \cdot 23.7$	$x_2^2\sigma_2^2 = 0.4^2 \cdot 23.7^2$

- Portfolio variance = $0.6^2 \cdot 15.8^2 + 2 \cdot 0.6 \cdot 0.4 \cdot 1 \cdot 15.8 \cdot 23.7 + 0.4^2 \cdot 23.7^2 = 359.5$

- The standard deviation is $\sqrt{359.5} = 19\%$

- Let us now assume a correlation coefficient of $\rho_{12} = 0.18$

- Portfolio variance = $0.6^2 \cdot 15.8^2 + 2 \cdot 0.6 \cdot 0.4 \cdot 0.18 \cdot 15.8 \cdot 23.7 + 0.4^2 \cdot 23.7^2 = 212.1$

- The standard deviation is $\sqrt{212.1} = 14.6\%$

Let us fill the above boxes with some numbers. Assume a correlation coefficient of 1. The portfolio risk box will look something like this as shown here in the table. The resulting portfolio variance will be the sum of all these boxes that is $portfolio\ variance = 0.6^2 \times 15.8^2 + 2 \times 0.6 \times 0.4 \times 1 \times 15.8 \times 23.7 + 0.4^2 \times 23.7^2 = 359.5$.

The standard deviation is the square root of 359.5, that is 19 percent. This is exactly the same as the weighted average of these two variances. This can be ascribed to the fact that we have considered a correlation of 1. Let us now assume a correlation coefficient of $\rho_{12} = 0.8$, the resulting portfolio variance can be simply computed as shown here, that is, portfolio values $0.6^2 \times 15.8^2 + 2 \times 0.6 \times 0.4 \times 0.18 \times 15.8 \times 23.7 + 0.4^2 \times 23.7^2 = 212.1$.

The standard deviation is $\sqrt{212.1} = 14.6\%$. The risk is reduced from that computer earlier. In fact, this is not less than both the stocks A and B considered individually. This is a very interesting result; it indicates that as the correlation or covariance between the two stocks comes down, the risk also comes down, which means the highest levels of diversification occur if the correlation in the stocks is close to zero or negative.

A consistently negative correlation over long periods is not observed. At best, fund managers try to select stocks that are very low correlations over time.
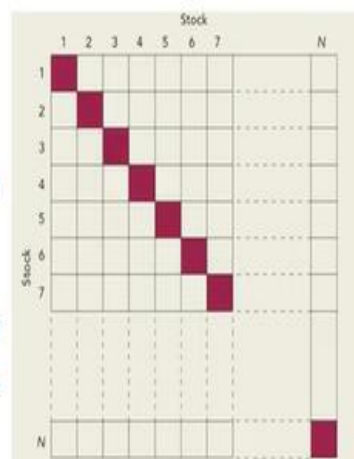
Let us consider a very hypothetical case of extreme negative correlation that is $\rho_1 = -1$. In this case, the portfolio variance is computed as shown here that is $portfolio\ variance = 0.6^2 \times 15.8^2 + 2 \times 0.6 \times 0.4 \times (-1) \times 15.8 \times 23.7 + 0.4^2 \times 23.7^2 = 0.$ In a theoretical world where a perfect negative correlation is available, there is a set of security weights that result in zero variance risk.

However, perfect negative correlations do not exist in real markets. The concepts developed here also hold for a portfolio of multiple stocks.

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Let us develop the formula to compute the portfolio risks for a portfolio of multiple stocks. We will approach this problem in the same manner as filling boxes with respective risk components. As earlier, the diagonal boxes here represent the risk weights associated with variances that is $x^2 \times \sigma^2$. The off-diagonal boxes represent risk weights associated with covariance terms that is $x_i \, x_j \, \sigma_{ij}$.

Let us consider a case of N securities. On a more practical note, when we are adding securities to a portfolio, chances are that investments will not be extreme on either site up or down; that is, one would invest amounts closer to average that is $\frac{1}{N}$ of the proportion. Taking this assumption that is a more realistic assumption, portfolio variance can be computed in the form of two components that is variance component and the covariance component.

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Computing portfolio risk

- There will be N variance terms; then portfolio variance can be simply written as $N \cdot \frac{1}{N^2} \cdot (Average\ variance)$

- Remember $w_1 \cdot w_2 \cdot \sigma^2$. Here $w_1 = w_2 = \frac{1}{N}$; and $\sigma = average\ variance = \sigma_{Avg}$

- Also, $N^2 - N$ covariance terms where average covariance term $= \sigma_{Cov-Avg}$

- The sum of covariance terms is $\left(N^2 - N\right) \cdot \frac{1}{N^2} \cdot \sigma_{Cov-Avg}$

- $Portfolio\ Variance = N \cdot \left(\frac{1}{N^2}\right) \cdot (Average\ Varinace) + \left(N^2 - N\right) \cdot \left(\frac{1}{N^2}\right) \cdot (Average\ Covariance)$

- As the number of securities, N, in the portfolio increase, the specific-risk term, $N \cdot \left(\frac{1}{N^2}\right) \cdot (Average\ Varinace)$, approaches to a value of zero

Thus, there will be N variance terms. Let us represent the variance of these terms as an average variance measure. The portfolio variance can be simply written $N \times \frac{1}{N^2} \times \sigma_{avg}$. Why so? Remember $W_1 \times W_2 \times \sigma^2$ where $W_1 = W_2 = \frac{1}{N}$ and σ = average variance that is σ_{avg} . The average variance is defined in a manner that if summed up across all the inboxes.

It will result in the variance or specific risk component of the portfolio that $N \times \frac{1}{N^2} \times$ $avreage\ variance = N \times \frac{1}{N^2} \times \sigma_{avg}$. Similarly, we can define the average covariance term that

is $\sigma_{cov-avg}$ here this average covariance reflects the market risk component, that is, the risk of the portfolio driven by the covariance across securities.

If summed up across $N^2 - N$ columns that is $N^2 - N \times \frac{1}{N^2} \times \sigma_{cov-avg}$. The overall combined portfolio variance term becomes $portfolio\ variance\ = N \times \frac{1}{N^2} \times average\ variance\ + N^2 - N \times \frac{1}{N^2} \times average\ covariance$. Please note that as the number of securities that is N in the portfolio increase, the specific risk term that is $N \times \frac{1}{N^2} \times average\ variance$ approaches to a value of zero.

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Computing portfolio risk

- Thus, the overall portfolio variance approaches the average covariance term
- This is also often referred to as portfolio diversification
- Thus, if these securities have very low correlation, then one can obtain a portfolio with very low risk
- That is, just by increasing the number of securities in a portfolio, one can eliminate the idiosyncratic (specific or diversifiable risk)
- The remaining risk is often called market risk or non-diversifiable risk
- That is why, this market risk (or average covariance or non-diversifiable risk) is what constitutes the bedrock of risk, that is risk that is there after eliminating all the specific risk

Thus, the overall portfolio variance approaches the average covariance term. This is also often referred to as portfolio diversification. Thus, if these securities have a very low correlation, then one can obtain a portfolio with very low risk; that is, just by increasing the number of securities in a portfolio, one can eliminate the idiosyncratic or specific, or diversifiable risk. The remaining risk is often called market risk or non-diversifiable risk.

That is why this market risk or average covariance or non-diversifiable risk is what constitutes the bedrock of risk, that is, the risk that is there after eliminating all the specific risks. To summarize in this video, we discussed the computation of risk and return for a portfolio of stocks. We started

the discussion by showing how to compute the return and risk that is the standard deviation for a two-stock portfolio.

We noted that for a two-stop portfolio, the risk of the portfolio is lower than the weighted average of the risk of individual portfolio constituents. We further observed that this reduction in risk is determined by the correlation between these two portfolios. For example, if the correlation is negative and high in magnitude, then the risk of the portfolio is considerably lower. Conversely, if the correlation is positive and high in magnitude, then the risk of the portfolio is considerably higher.

We extend our discussion of two stock portfolio to a multi-stock portfolio and observed that as the number of stocks in a portfolio are increased the stock specific idiosyncratic risk dies away. In this multi-stock portfolio, the only relevant risk is market risk.

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Impact of individual securities on portfolio risk

- Investors usually add many securities in their portfolio to diversify the stock-specific idiosyncratic risk
- It is not the risk of a security held individually but in a portfolio that is important
- To measure the impact of a security to the risk of portfolio, one needs to measure the market risk component of the security
- The market risk of a security is measured through its beta
- Stocks with beta of more than 1.0 tend to amplify the movements of market
- Stocks with beta between 0 to 1.0 tend to move in the same direction as market, but are considered less sensitive
- The market portfolio has a beta of 1.0 and reflects the average movement of all the stocks in the market

Impact of individual securities on portfolio risk. In this video, we discussed the contribution of a security to the risk of the overall portfolio. We also discussed the computation of beta, a security sensitivity to market movements. Lastly, we also summarize the concept of value additivity. Individual securities are volatile and risky. Investors usually add many securities to their portfolios to diversify the stock-specific idiosyncratic risk.

Thus, it is not the risk of security held individually but in a portfolio that is important. If the portfolio is well diversified, then only the market risk component of the stock that is non-diversifiable or systematic risk matters to the investors. Thus, to measure the impact of security on the risk of a portfolio, one needs to measure the market risk component of the security, that is, how sensitive a security is to market movements.

The market risk of a security is measured through its beta, which is the sensitivity of the stock to market movements. Stocks with a beta of more than one tend to amplify the movements of market stocks with a beta between 0 to 1 tend to move in the same direction as the market. But are considered less sensitive. The market portfolio has a beta of one and reflects the average moment of all the stocks in the market. This is so because a market portfolio is often defined as the portfolio of all the stocks in a market.

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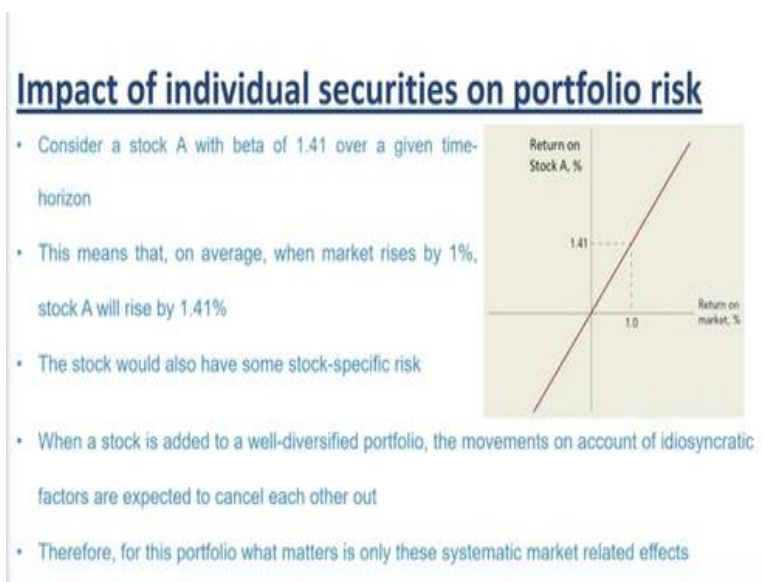
Consider a stock with a beta of 1.41 over the given time horizon. This means that on average, in that period when the market rises by 1 percent, the stock will rise by 1.41 percent. If the market falls by 2 percent, then stock a is expected to fall by 2.82 percent on average. If this stock does not have any stock-specific idiosyncratic risk, then its written profile can be easily plotted as shown here. However, the stock would also have some stock-specific risks.

That means the actual returns are scattered around this line and may not exactly fall in a straight line. Nonetheless, when a stock is added to a well-diversified portfolio, the movements on account of idiosyncratic factors are expected to cancel each other out. Therefore, for this portfolio, what matters is only the systematic market-related effects.

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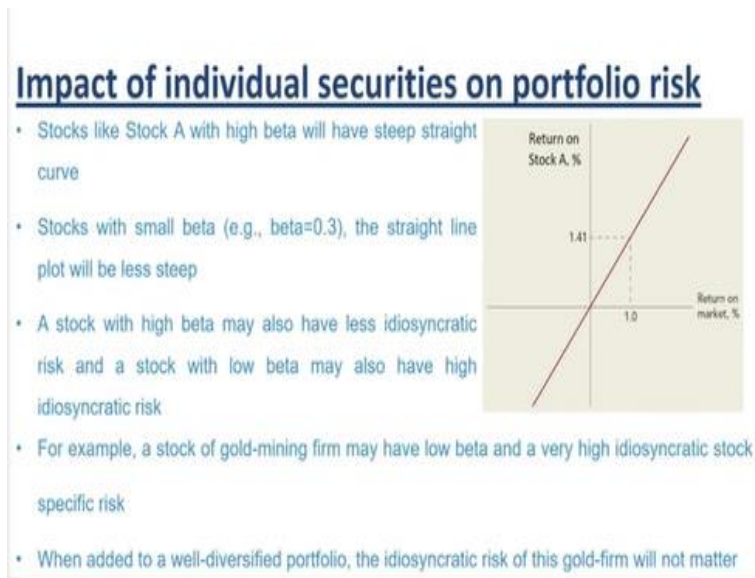
Stocks like stock A with high beta will have a steep straight curve. For stocks with a small beta, for example, a beta equal to 0.3, the straight line plot will be less steep. Many times, a stock with a high beta also has a high standard deviation, but this is not always the case. A stock with a high beta may also have less idiosyncratic risk, and a stock with a low beta may also have a high idiosyncratic risk. For example, a stock with a gold mining firm may have a low beta and a very high idiosyncratic stock-specific risk.

However, when added to a well-diversified portfolio, the idiosyncratic risk of this gold firm may not matter. What will matter is the market risk which is expected to be very low since the beta of this firm is expected to be very less.

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Impact of individual securities on portfolio risk

- So, let us now answer this question how security betas affect the portfolio risk

- Market risk accounts for most of the risk of a well-diversified portfolio

- Beta of an individual security measures its sensitivity to market movements

- Examine the figure shown here: the standard deviation (total risk) of the portfolio depends on the number of securities in the portfolio

- As the number of securities increase in the portfolio, more diversification is achieved
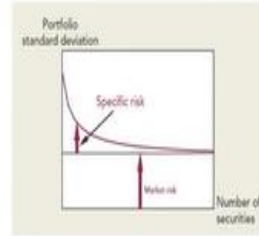
So, let us now answer this question of how security betas affect portfolio risk. First, market risk accounts for most of the risk of a well-diversified portfolio. A beta of individual security measures its sensitivity to market movements. Thus, it would be appropriate to summarize now that in the context of a portfolio, the beta would be a more appropriate measure of risk. Examine the figure shown here.

The standard deviation or the total risk of the portfolio depends on the number of securities in the portfolio. As the number of securities increases in the portfolio, more diversification is achieved.
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Impact of individual securities on portfolio risk

- With addition of more and more securities, the specific risk declines until all the stock specific risk is eliminated and only market risk remains

- Market risk depends on the average beta of the securities, i.e., the portfolio beta

- If one selects a fairly large number of securities from a market, you diversify all the idiosyncratic risk

- Thus, you get the market portfolio with beta= 1.0

- If the market portfolio has a standard deviation of 20%, then this portfolio is expected to have a standard deviation of close to 20%
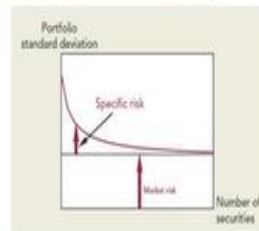
The specific risk here declines until all the stock-specific risk is eliminated and only market risk remains. That is why market risk is often referred to as the bedrock of risk, as it remains steady. The market risk depends on the average beta of the securities, that is, the portfolio beta. Therefore, if one selects a fairly large number of securities from a market, one diversifies all the idiosyncratic risk and are left with the market risk only.

Thus, you get a portfolio that is very close to the market portfolio. The beta of this portfolio is expected to be close to one. For example, if the market portfolio has a standard deviation of 20 percent then this portfolio is expected to have a standard deviation of close to 20 percent.

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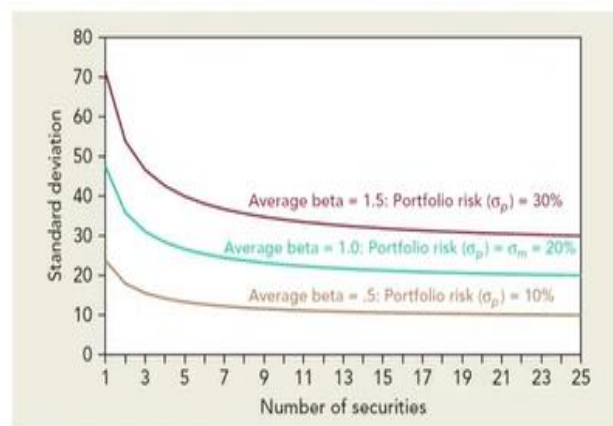
Consider the figure here. Here the green line shows a well-diversified portfolio with a beta of 1.0. If the market portfolio has a standard deviation of 20 percent, then this portfolio is also expected to have a standard deviation of 20 percent, which is the green curve. For a well-diversified portfolio with a beta of 1.5, one can expect the standard deviation to be closer to 30 percent. Similarly, if the portfolio beta is 0.5, then one can expect the portfolio standard deviation to be closer to 10 percent, which is the brown curve.

In summary, the risk of a well-diversified portfolio is equal to the average beta of securities in the portfolio.

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Impact of individual securities on portfolio risk

- Beta of a stock 'i' can be computed using the following formula. $\beta_i = \sigma_{im}/\sigma_m^2$. Here σ_{im} is the covariance between the stock returns and market returns. σ_m^2 is the variance of the returns on the market.

1 Month	2 Market return (%)	3 Deviation in Market Returns	4 Squared Market Deviation	5 Stock A	6 Deviation in Stock A returns	7 Deviation Product (3*6)
1	-8	-10	100	-11	-13	130
2	4	2	4	8	6	12
3	12	10	100	19	17	170
4	-6	-8	64	-13	-15	120
5	2	0	0	3	1	0
6	8	6	36	6	4	24
	Avg=2		Sum=304	Avg=2		Sum=456

Variance $\sigma_m^2 = \frac{304}{6} = 50.67$

Co-variance $\sigma_{im} = \frac{456}{6} = 76$

Beta $\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{76}{50.67} = 1.5$

Beta for stock i can be computed using the following formula, that is $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$. Here σ_m is the covariance between the stock returns and market returns and σ_m^2 is the variance of the returns on the market. The ratio of this covariance and variance is the contribution of the stock to the portfolio risk that is beta. Consider the example shown here in the table. The returns from the market and stock A are provided here.

Returns from market and stock A are pointed for six month period. The average returns for both these investments are 2 percent, the variance of market return is 50.67, and the covariance between the stock and market is 76; the beta for the stock works out to 1.5. This suggests that stock A is particularly sensitive to market movements. Column 3 computes the deviation in market return, and column 4 computes the square of these deviations.

Column 6 provides the deviation of stock A returns from their mean; column 7 provides the deviation product that is column three into column 6, that is, the deviation in stock A and market from their respective means multiplied with each other.

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Impact of individual securities on portfolio risk

- Can we say that a diversified firm is more attractive to investors than an undiversified firm

- If diversification is a good objective for a firm to pursue then each new project's contribution to firm diversification should also add value to the firm

- This seems to be not consistent with what we have studied about present values

- Investors can diversify for themselves more easily than firms

- If investors can diversify on their own, they would not be paying anything extra to firm for this diversification

- The present value of any number of assets is equal to the present value of their parts. That is, PV (ABC)=PV(A)+PV(B)+PV(C): Value Additivity

By now, we have established that diversification reduces risk and makes sense for investors. However, can we say that a diversified firm is more attractive to investors than an undiversified firm? If diversification is a good objective for a firm to pursue, then each new project's contribution to firm diversification should also add value to the firm. The diversification package of firm projects should be greater than the sum of the parts.

That also means that the present value of all these projects, if added together, would result in a higher value. However, this seems to be not consistent with what we have studied about present values. This is true for the simple reason that investors can diversify for themselves. In fact, they can do so more easily than firms. For example, they can diversify in different industry sectors by directly buying these securities.

However, for a firm to diversify in different industry sectors is extremely difficult. For example, to diversify the steel industry, the investor only needs to pay security transaction charges and broker fees. However, for a firm to enter into a new domain such as steel would require a huge amount of resources and time; this leads us to the following conclusion. If investors can diversify on their own, they would not be paying anything extra to the firm for this diversification objective.

However, there is an assumption of deep, liquid, and efficient financial markets that allow investors to choose securities from different industry sectors and riskiness as they wish. That is

why countries where large capital markets are available that facilitate firms to diversify investors do not pay any additional price to firms that are diversified in their projects and businesses; that is, the total value of the firm is equal to its parts.

This is the concept of value additivity, that is, the present value of any number of assets is equal to the present value of their parts that represent a value of A, B, and C took together equal to the present value of a plus present value of B plus the present value of C separately. This means that if a firm has three assets A, B, and C, then the present values of A, B, and C cash flows taken together are the same as the sum of their cash flow present values individually.

To summarize, in this video, we discussed the concept of beta, which is the sensitivity of a stock to market movements. We discussed that for a well-diversified portfolio, the stock-specific idiosyncratic risk is eliminated. Therefore, only the market risk component of the security matters , if the security is part of a well-diversified portfolio. This market component of security is measured using its beta.

For example, if the beta of a security is 1.5, then if the market moves by 1 percent the movement in security on account of this market movement is 1.5 percent. Of course, the security will also have some idiosyncratic risks that will not be related to this market movement. However, when the security is added to the portfolio comprising a large number of securities, then these idiosyncratic movements will be eliminated through the process called diversification.

Lastly, we also discussed the concept of value additivity that is some of the different assets or project cash flows is the same whether taken together or separately, which means investors do not give any additional value to a firm if it obtains diversification with the help of adding more projects. This is so because investors can diversify themselves by adding more securities from different industry sectors in their portfolios.

Assuming that deep and liquid financial markets are available to them to choose securities from different industry sectors and asset class groups. If this is the case, then the sum of different components that is projected assets cash flow present values, is the same whether considered

separately or taken together as a unit that is present value of A, B, C equal to present value of A plus present value of B plus present value of C separately.

To summarize in this lesson, we understand that returns to investor vary depending upon the risk bond by them. At one extreme very safe instruments such as treasury securities provide the lowest returns as compared to this, equity securities are considered to be more riskier asset class and offer higher expected returns. This also indicates that while evaluating a safe project, the discount rate to be applied to cash flows should be closer to the risk-free instrument such as treasury bill rates.

In contrast if you are valuing a risky project cash flows then the appropriate discounting rate should be closer to the more risk instruments like equity securities. This also gives us two benchmarks for the opportunity cost of capital for evaluating project cash flows. Risk of a security means that there are many possible return outcomes for that security. The more possibilities there are the higher the risk that security carries.

Thus, risk is a measure of spread of possible outcomes of a security price that is returns and is often measured as the standard deviation of a stock. The risk or standard deviation of a stock has two components. First stock specific risk and second market risk. Risk is often evaluated more appropriately in the portfolio context. Investors eliminate a sizable portion of their risk simply by adding more securities to their portfolio.

This diversification takes place due to the elimination of stock-specific risk or idiosyncratic risk. The second component that is market risk cannot be eliminated through the diversification process. A well-diversified portfolio is only exposed to market risk that is unanticipated in market movements. A security's contribution to a well-diversified portfolio is measured as the sensitivity of the security to market movements.

The sensitivity to market movements is often measured as the beta of the security. Beta measures that percentage change in the stock price for a 1percent change in the market value. The market is often measured or proxied through broad market indices such as SNP 500 or Nifty 50. The beta of

market is one which represents a portfolio carrying a large number of stocks in market. Thus, beta becomes the average of all the betas of the security in that market and hence is approximately one.

Any security with a beta of more than one is more sensitive to market movements. A stock with a beta of less than one is less sensitive to broad market-wise movements. For a well-diversified portfolio, the standard deviation is proportional to its beta, which means that for a well-diversified portfolio with a beta equal to 2 will have a standard deviation that is twice of the standard deviation of a portfolio with a beta equal to one.

While this diversification is a noble objective for investor this does not mean that form should also diversify by adding more projects simply to obtain diversification. This is so because investors can obtain diversification on their own account if deep and liquid financial markets are available. That means they will not pay any extra price for a firm just because it is diversifying by adding more projects.

This also means that product value of a project assets present value of cash flows are same whether considered separately or taken together as a unit for a firm. This is called the concept of value additivity.