Artificial Intelligence (AI) for Investments
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Lecture - 04

Investment in real assets requires a lot of money. Cash generated from internal accruals, and equity contribution from shareholders may not often be sufficient. To meet these financing requirements, firms use bank loans and issue bonds. These are called fixed-income securities or simply called loans. There is always a risk that the issuer of the loan may not be able to generate sufficient cash to repay the interest and principal obligations.

In the context of bonds, these interest obligations are often called coupons. If this happens, the issuer of the instrument, that is borrower, may default on their obligations. Generally, if the issuer is a government body or municipality, the investors are confident about the great worthiness of the security. However, if it is a private entity with not-so-strong fundamentals and a sound financial position, then there is always a possibility of default.

We start our discussion of fixed-income securities by examining the valuation of bonds. The interest or coupons on these bonds are not the same as the cost of capital for the issuing corporation. Corporations cannot borrow at the same risk-free rate as those availed by governments and quasi-government bodies. They have to pay some additional interest that is spread, which reflects that firm's business risk and financial condition.

Moreover, if the risk-free rates, that is, rates at which governments borrow, go up or down, this would also affect the interest rates for corporates. Thus, corporate bonds are considered riskier due to the possibility of default; also, they are less liquid than government bonds. In this lesson, we will discuss the valuation of fixed-income securities with the help of discounted cash flow valuation tools.

We will analyze the term structure of interest rates and its implications for the valuation of fixed-income securities. We will try to understand various theories that explain the dynamics of a rising or falling term structure of interest rates. Using the tools acquired in the discounted cash flow techniques, we will understand and compute a very important measure of the bond market instrument called yield to maturity.

We will examine the interest rate risk associated with fixed-income securities with the help of the duration measure. We will understand the theory behind this duration measure and also the computation of this measure.

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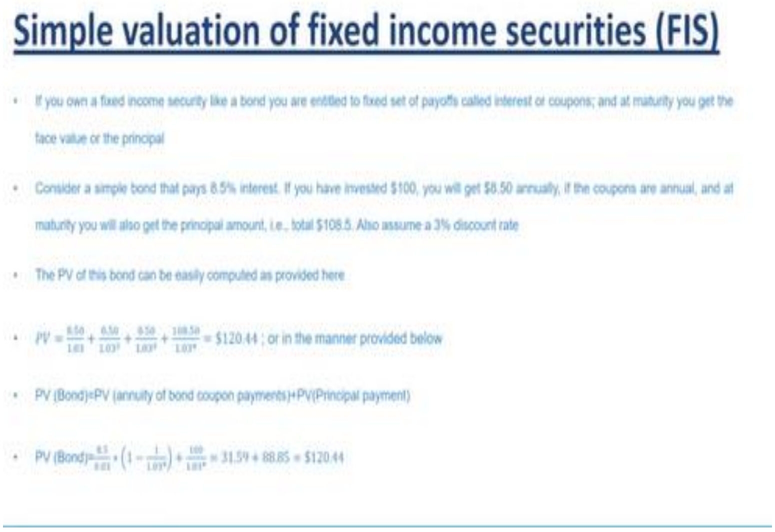

Simple valuation of fixed income securities FIS. In this video, we will be introduced to fixed-income securities FIS. We will understand the cash flow profile of fixed-income securities such as bonds. We will also discuss the concept of yield-to-maturity YTM for fixed-income security. If you own a fixed-income security like a bond, you are entitled to a fixed set of payoffs called interest or coupons.

These payments will continue until the security matures. Till then, you collect regular interest payments, and at maturity, you get the face value or the principal of that fixed-income security. Consider a simple bond that pays 8.5 percent interest. If you have invested hundred dollars, you will get 8.5 dollars annually if the coupons are annual. And at maturity, you will also get the principal amount of 108.5 dollars. Assume a 3% discount rate.

So, for whatever we have discussed in the present value concept and cash flow discounting, it is not too difficult to compute this bond's present value. It can be easily calculated as follows,

$$PV = \frac{8.5}{1.03} + \frac{8.5}{(1.03)^2} + \frac{8.5}{(1.03)^3} + \frac{108.5}{(1.03)^4} = \$120.44$$

Also, given our knowledge of valuing annuities, we can separate the coupon payments and principal payments and compute the value of this bond in the manner provided here.

$$PV(Bond) = PV(annuity\ of\ bond\ coupon\ payments) + PV\ (principal\ payment)$$

$$PV(Bond) = \frac{8.5}{0.03} * \left(1 - \frac{1}{1.03^4}\right) + \frac{100}{1.03^4} = 31.59 + 88.85 = \$120.44$$

Put simply, the bond can be valued as a combination of annuity, that is, coupon payments and a single final payment of principal.

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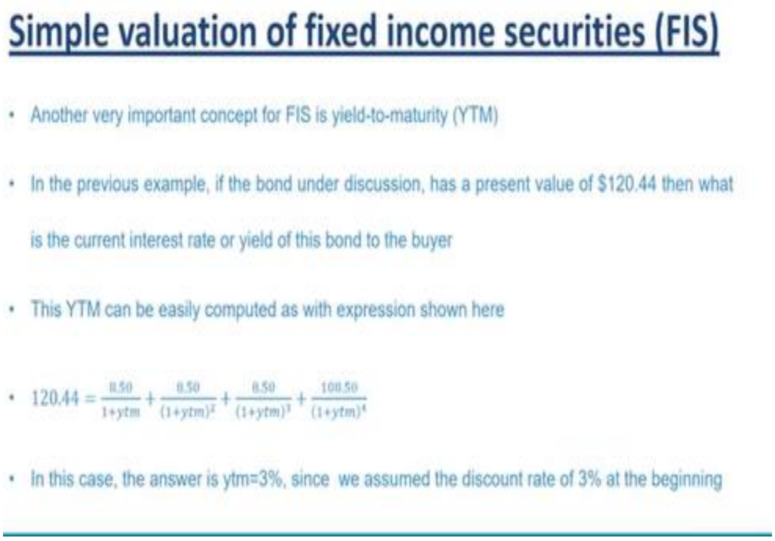

Next, there is another very important concept related to fixed-income securities, such as bonds that is yield to maturity. For example, if the bond under discussion has a present value of $120.44, then what is the current interest rate or yield of this bond to the buyer? Bond being an instrument that is traded in financial markets, its price fluctuates with the interest rates. As the interest rates rise, bond prices fall, and when the interest rates fall, bond prices rise.

They should be very easily understood from the discounted cash flow valuation of the bond that we saw earlier. Let us compute the YTM or yield-to-maturity of this bond that is presently selling at $120.44. Using our knowledge of discounted cash flow valuation technique, we can write the bond price expression in YTM yield to maturity terms as shown here,

$$120.44 = \frac{8.5}{1 + ytm} + \frac{8.5}{(1 + ytm)^2} + \frac{8.5}{(1 + ytm)^3} + \frac{108.5}{(1 + ytm)^4}$$

Since we just saw that at a 3% discount rate, the PV of this bond worked out to 120.44. Therefore, its YTM at this price should be only 3 percent. This also means that if you buy this bond and hold it to maturity, you will earn a return of 3 percent per annum. The interesting question is why this bond has a YTM that is less than its coupon of 8.5 percent. A simple answer to this question is that you are paying 120.44 dollars for a bond with a face value of 100 dollars.

That is, the bond is selling at a premium. Many software packages and financial calculators are available that help you to calculate yield to maturity easily.

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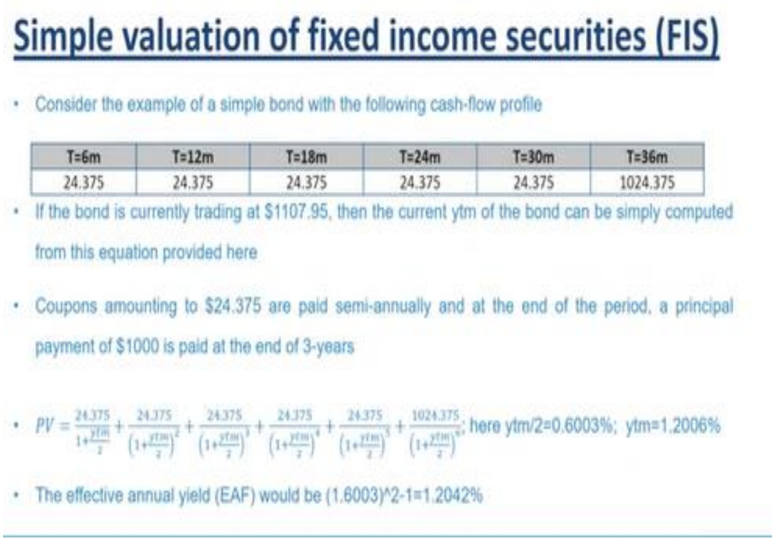

Consider the example of a simple bond with the cash-flow profile as shown here. Coupons amounting to 24.375 dollars and are paid semi-annually, and at the end of the period, a principal payment of 1000 dollars is paid at the end of 3 years. If the bond is currently trading at 1107.95 dollars, then the current yield to maturity of the bond can be computed from this equation provided here.

$$PV = \frac{24.375}{1+\frac{ytm}{2}} + \frac{24.375}{\left(1+\frac{ytm}{2}\right)^2} + \frac{24.375}{\left(1+\frac{ytm}{2}\right)^3}\frac{24.375}{\left(1+\frac{ytm}{2}\right)^4}\frac{24.375}{\left(1+\frac{ytm}{2}\right)^5}\frac{1024.375}{\left(1+\frac{ytm}{2}\right)^6}$$

Here $YTM\ /\ 2\ =\ 0.6003\%$ or yield to maturity, $YTM\ =\ 1.2006\%$.

The division by a factor of 2 indicates the semi-annual compounding aspect. So, the yield is quoted in annual terms at 1.2006%. However, given the semi-annual compounding, the 6-month yield is 0.66003%. The effective annual yield EAF would be $(1.6003)^2\ -\ 1\ =\ 1.2042\%$. This yield, of course, is higher than the quoted annual rate of 1.2006%. To summarize, in this video, we examined the cash flow profile of fixed-income security, such as bank loans and bonds. The cash flow on these fixed-income securities included regular interest payments and principal payments or face value at maturity. We also discussed the concept of YTM, which is yield to maturity. If the cash flows from a fixed-income security FIS are discounted at the yield to maturity, one obtains the current price of the fixed-income security.

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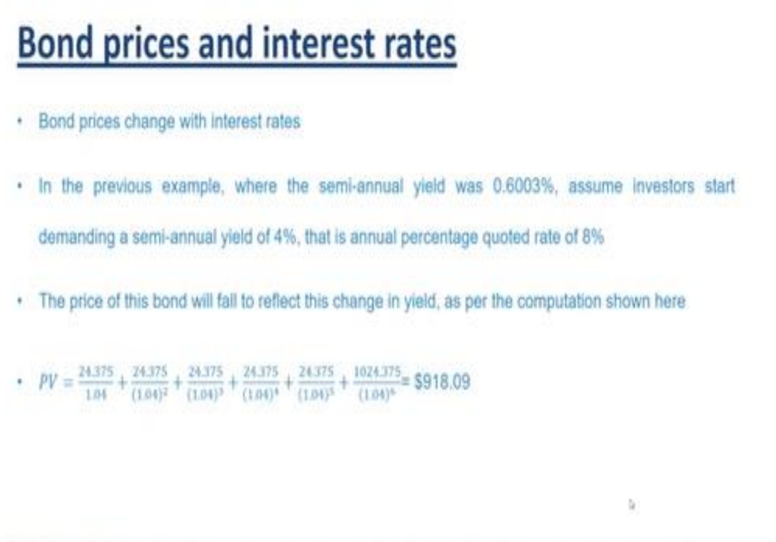

Bond prices and interest rates. We will discuss the discounted cash flow valuation of fixed-income securities. We will also examine the relationship between interest rates and fixed-income security prices. Bond prices change with interest rates. In the previous example, where the semi-annual yield was 0.6 percent, assume investors start demanding the same annual yield of 4%, an annual percentage quoted rate of 8 percent. The price of this bond will fall to reflect this change in yield. Saying that the price has fallen is equivalent to saying that yields have risen. The new price can be easily computed with discounted cash flow method, as shown here,

$$PV = \frac{24.375}{1.04} + \frac{24.375}{(1.04)^2} + \frac{24.375}{(1.04)^3} + \frac{24.375}{(1.04)^4} + \frac{24.375}{(1.04)^5} + \frac{1024.375}{(1.04)^6} = \$918.09$$

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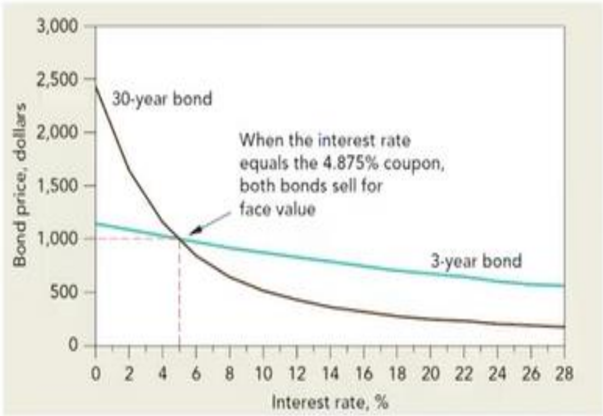
Bond prices and interest rates



The cash flow discounting relation makes it obvious that with rising interest rates, prices would fall and prices would rise with falling interest rates. This explains why bond prices and interest rates move in the opposite direction. Let us examine the price-interest rate relationship for two bonds with the same coupon of 4.875 percent. The relationship is shown in the figure here. It must be noted that the impact of interest rate is only modest on near-term cash flows, while it is more drastic on distant cash flows. This is precisely the reason why the price of a 30-year bond is much more sensitive to interest rate movements than a 3-year bond. In fact, for the 30-year bond, the price fall is particularly sharp at lower interest rates. To summarize, in this video, we examined the relationship between interest rates and fixed-income security prices. We found that the price of a fixed-income security FIS is inversely related to the interest rates. If interest rates rise, prices fall, and if interest rates fall, prices rise.

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Duration of a bond

- We saw that changes in interest rates have greater impact on the prices of long-term bonds that short-term bonds
- Separate Trading of Registered Interest and Principal Securities (STRIPS) are special instruments, created by stripping the cash flows from treasury instruments and government securities
- These are often called as zero-coupon bonds, and have the maturity same as duration

Duration of a bond. We will discuss the concept of duration for fixed-income securities. We will also understand the computation of duration measures and their role in interest risk management. We saw that changes in interest rates have a greater impact on long-term bonds' prices than short-term bonds. However, for fixed-income securities such as bank loans or bonds, short-term or long-term is a vague reference.

For example, consider a 30-year maturity bond with coupons spread evenly across the 30-year period with principal payment occurring on the maturity, that is, at the end of the 30-year period. It would be misleading to describe this bond as a 30-year bond. Let us consider an example of strips that is a separate reading of registered interest and principal securities. Strips are special instruments created by stripping the cash flows from treasury instruments and government securities, where each cash flow trades as a bond.

For such instruments, only one cash flow occurs, that is, at maturity and no coupon payment. These are often called zero-coupon bonds. Consider a strip with a 30-year maturity that is a single payment occurring at the end of 30 years. It would not be misleading to say that this bond has a duration of 30 years.

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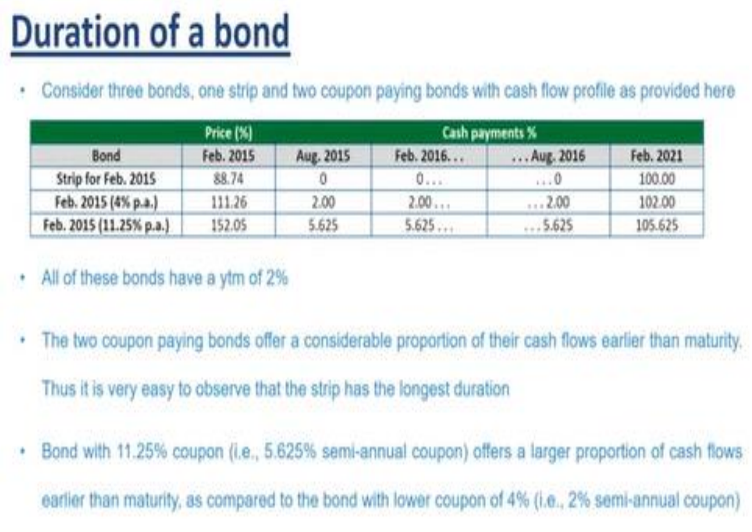

	Price (%)	Cash payments %			
Bond	Feb. 2015	Aug. 2015	Feb. 2016…	…Aug. 2016	Feb. 2021
Strip for Feb. 2015	88.74	0	0…	…0	100.00
Feb. 2015 (4% p.a.)	111.26	2.00	2.00…	…2.00	102.00
Feb. 2015 (11.25% p.a.)	152.05	5.625	5.625…	…5.625	105.625

Consider three bonds. One strip and two coupon paying bonds with cash flows profile as provided here, all of these bonds have a YTM, that is, yield to maturity of 2%. Let us examine their time pattern of cash flows. Which one of these can be considered the longest-term investment? Although all of them have the same maturity in February 2021. The cash flow profiles that is proportionate amounts distributed over the years is different for each of these bonds.

The two coupon-paying bonds offer a considerable proportion of their cash flows earlier than maturity. Thus, it is very easy to observe that the strip has the longest duration, as explained earlier. Also, it is not too difficult to see that the bond with an 11.25 percent coupon, that is 5.625 percent semi-annual coupon, offers a larger proportion of cash flows earlier than maturity as compared to the bond with a lower coupon of 4 percent that is 2 percent of semi-annual coupon.

Therefore, the average maturity in cash flow terms should be higher for the 4% bond, if these two bonds are compared because the larger proportion of cash flows comes earlier for the 11.25 percent coupon bond. It can be easily understood that this bond has the lowest average maturity of cash flows across all three bonds.

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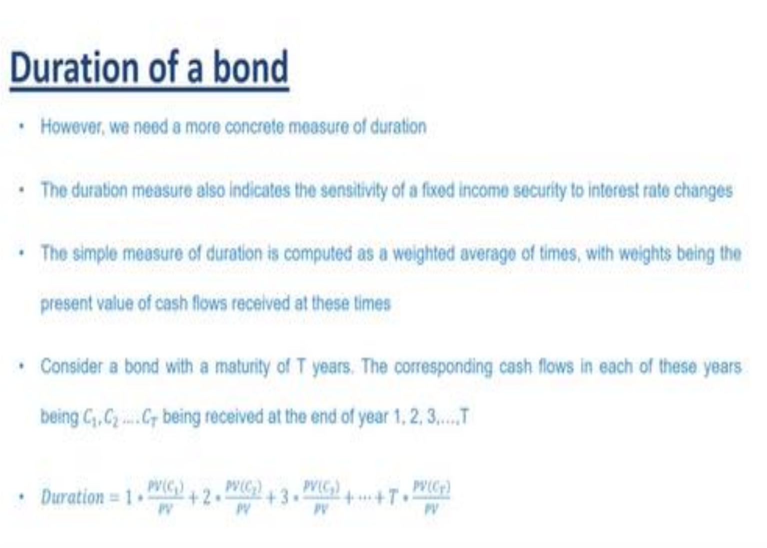

However, we need a more concrete measure of duration. The discussion here offers the intuition for that measure of duration. Also, there is another important application of this duration measure, which is why investors and financial managers keep track of the duration measure. The duration measure also indicates the sensitivity of a fixed-income security to interest rate changes.

The simple measure of duration is computed as a weighted average of times, with weights being the present value of cash flows received at these times. Consider a bond with the majority of T years. The corresponding cash flows in each of these years being $C_1, C_2, \ldots, C_T$ being received at the end of years 1, 2, 3 up to T, and the present value of these cash flows as PV.

The duration measure for this bond can be simply computed with the formula shown here, that is

$$Duration = 1 * \frac{PV(C_1)}{PV} + 2 * \frac{PV(C_2)}{PV} + 3 * \frac{PV(C_3)}{PV} + \cdots + T * \frac{PV(C_T)}{PV}$$

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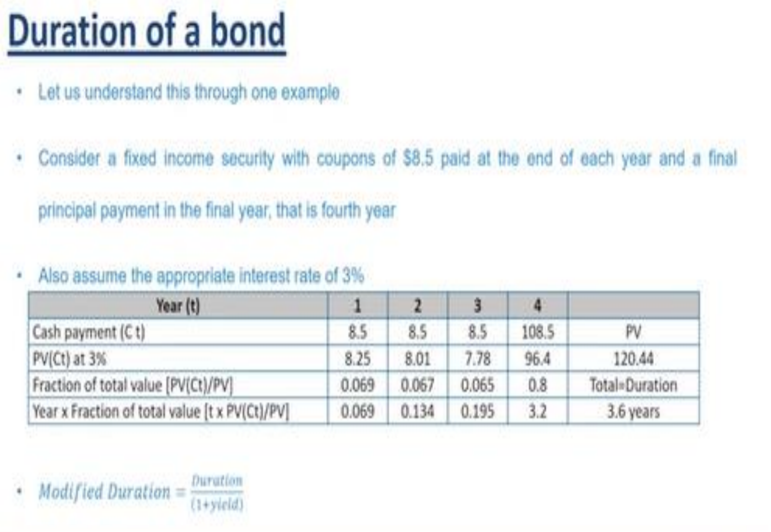

Year (t)	1	2	3	4	
Cash payment (C t)	8.5	8.5	8.5	108.5	PV
PV(Ct) at 3%	8.25	8.01	7.78	96.4	120.44
Fraction of total value [PV(Ct)/PV]	0.069	0.067	0.065	0.8	Total=Duration
Year x Fraction of total value [t x PV(Ct)/PV]	0.069	0.134	0.195	3.2	3.6 years

Let us understand this through one example. Consider a fixed income security with coupons of 8.5 dollars paid at the end of each year and the final principal payment in the final year, that is, the fourth year. Also, assume the appropriate interest rate of 3%. Let us examine the computation of duration in the table provided here. Using this duration measure, a measure of the sensitivity of fixed-income security prices to interest rate changes is computed as shown here.

This measure is often referred to as a modified duration measure.

$$Modify\ duration = \frac{Duration}{1 + yield}$$

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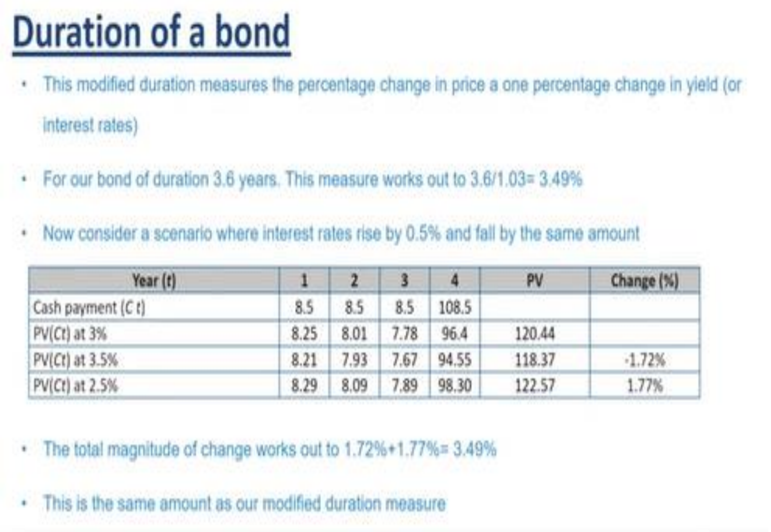

This modified duration measures the percentage change in price for a 1% change in yield or interest rates. For our bond of a duration of 3.6 years, this measure works out to $\frac{3.6}{1.03} = 3.49\%$. Now consider a scenario where interest rates rise by 0.5% and fall by the same amount. Let us compute the corresponding price changes as shown in the table here. The total magnitude of change works out to $1.72 + 1.77 = 3.49\%$.

This is the same amount as our modified duration measure. This also indicates that duration is an important measure of the sensitivity of an instrument to changes in interest rates. Moreover, as the duration of an instrument increases, its sensitivity to interest rate changes also increases. Therefore, the interest rate risk of fixed-income securities is computed with the help of this duration measure.

Another important application of this duration measure is in managing the interest rate risk of portfolios. Often while designing portfolios, interest rate risks are suitably held or actively monitored to take a view on the interest rate changes. To execute these strategies, portfolio managers often rely on this duration measure for the fixed income securities. To summarize, in this video, we computed the duration measure of fixed-income securities such as bonds. We found that a bond with a larger duration is more sensitive to interest rate movements, while a bond with a relatively smaller duration is less sensitive to interest rate movements. Thus, the duration of a bond is an important measure of its sensitivity to interest rate risk. Also, with the help of an example, we understood the computation of the modified duration measure.

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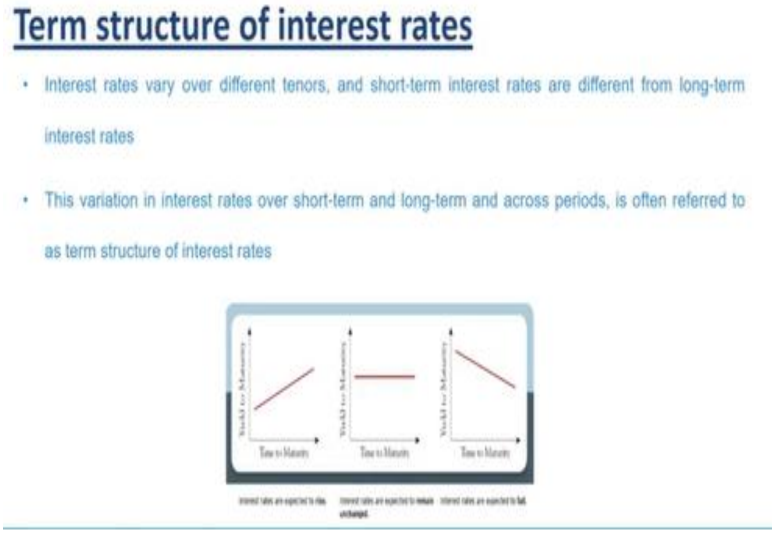

Term structure of interest rates. We will discuss the term structure of interest rates. We will also understand the important theories that explain different term structures. Till now we have been using the same discount rate to calculate the present values for all the cash flows in each of the periods. A single yield to maturity was employed to compute the present value of fixed-income securities like a bond. This kind of approximation is very useful in many situations.

However, we need to remember that interest rates vary over different tenors, and short-term interest rates are different from long-term interest rates. This variation in interest rates over the short term and long term and across periods is often referred to as the term structure of interest rates. The term structure is most often rising but sometimes can be falling as well. The figure provided here indicates all the possible term structures that are rising, falling, and constant.

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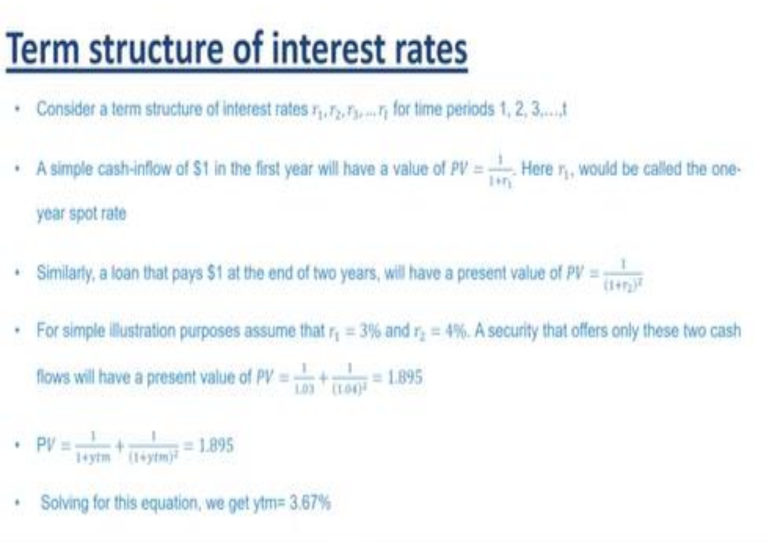

Let us examine this behavior of the term structure of interest rates. Consider the term structure of interest rates $r_1, r_2, r_3, \ldots, r_t$ for time periods $1, 2, 3, \ldots, t$. A simple cash flow of $1 in the first year will have a value of present value $PV = \frac{1}{1+r_1}$. Here r_1 would be called the 1-year spot rate. Similarly, a loan that pays $1 at the end of 2 years will have a present value of

$PV = \frac{1}{(1+r_2)^2}$. .

For simple illustration purposes, assume that r_1 = 3 percent and r_2 = 4 percent. A security that offers only these two cash flows will have a present value of

$$PV = \frac{1}{1.03} + \frac{1}{(1.04)^2} = 1.895$$

Now that we have the present values of the security, we can simply compute the yield to maturity, that is YTM of the security, with the simple formula provided here

$$PV = \frac{1}{1+ytm} + \frac{1}{(1+ytm)^2} = 1.895$$

Solving for this equation, we get YTM = 3.67 percent. Here we saw that spot rates for different maturities help us in determining the prices. Once the price is determined with the help of spot rates yield to maturity, that is YTM can be determined subsequently.

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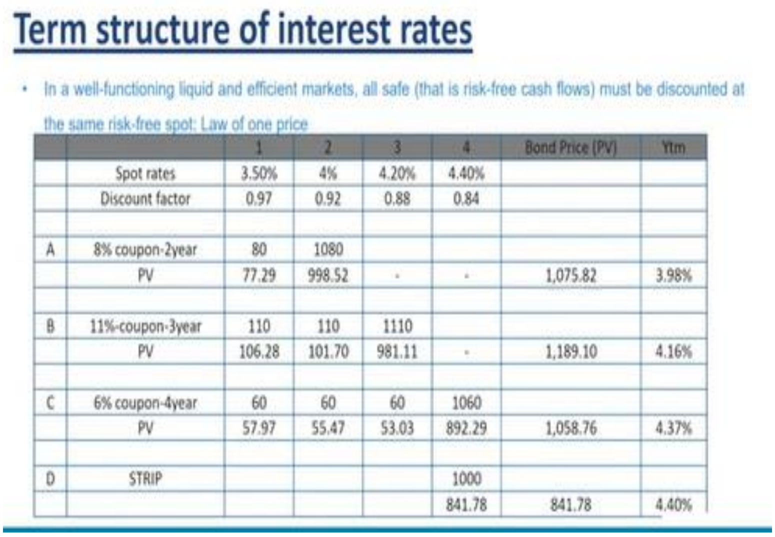

Term structure of interest rates

- In a well-functioning liquid and efficient markets, all safe (that is risk-free cash flows) must be discounted at the same risk-free spot: Law of one price

		1	2	3	4	Bond Price (PV)	Ytm
	Spot rates	3.50%	4%	4.20%	4.40%		
	Discount factor	0.97	0.92	0.88	0.84		
A	8% coupon-2year	80	1080				
	PV	77.29	998.52	-	-	1,075.82	3.98%
B	11%-coupon-3year	110	110	1110			
	PV	106.28	101.70	981.11	-	1,189.10	4.16%
C	6% coupon-4year	60	60	60	1060		
	PV	57.97	55.47	53.03	892.29	1,058.76	4.37%
D	STRIP				1000		
					841.78	841.78	4.40%

In this backdrop, a very important concept is that of the law of one price. In well-functioning liquid and efficient markets, all safe dollars that are risk-free cash flows must be discounted at the same risk-free spot rate. This also holds true for cash flows of similar risk. For example, consider four risk-free government bonds with annual coupon payments, as shown here.

Bond A is the shortest duration bond among all, while strip D is the longest duration. The corresponding spot rates for each duration are provided here. The law of one price says that each of these risk-free dollars should carry the same value if received at the same date. For example, 1 dollar received in year four has the same value, whether from bonds A, B, C, or D.

This also means that one can use the same discount factor for each of the years 1, 2, 3, and 4 for all the bonds. Thus, we compute the PV, which is the present value of all the bonds, using spot rates for each of the years 1, 2, 3, and 4. Once the PV of these bonds is computed, yield to maturity YTM can also be easily computed.

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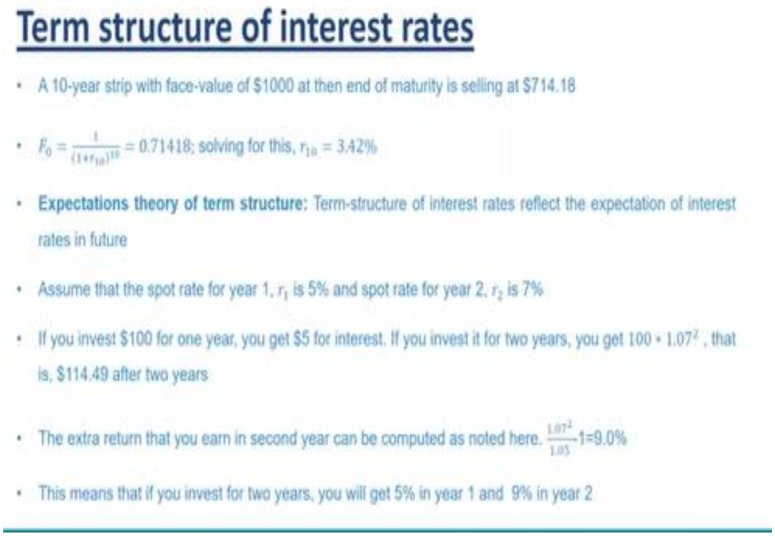

If you have understood the above concept, the procedure to estimate the spot rates for different maturities and, therefore, the term structure of interest rate is not too difficult. All you need is a bond with a single cash flow that is strips for different maturities. Using the price of these strips, one can estimate the spot rates prevailing in the market for that maturity. For example, a 10-year strip with a face value of 1000 dollars at the end of maturity selling at 714.18.

This also means that, for each dollar to be received after ten years, investors are willing to pay 0.714 dollars today. The 10-year spot rate can be easily computed from this information as provided here,

$$F_0 = \frac{1}{(1+r_{10})^{10}} = 0.71418$$

solving for this $r_{10} = 3.42\%$. In this manner, one can estimate the entire term structure of interest rates. Lastly, there are a number of theories of the term structure of interest rates, as explained here.

First expectations theory of term structure. The term structure of interest rates reflects the expectation of interest rates in the future. For example, a rising term structure indicates that market participants expect interest rates to rise in the future. Let us understand this through the example provided here. Assume that the spot rate for years 1 r_1 is 5%, and the spot rate for year 2 r_2 is 7%. If you invest 100 dollars for one year, you get 5 dollars for interest.

If you invest it for two years, you get $100 * 1.07^2$, that is, $114.49 after two years. The extra return that you earn in the second year can be computed as shown here,

$$\frac{1.07^2}{1.05} - 1 = 9\%$$

This means that if you invest for two years, you will get 5% in year 1 and 9% in year 2.

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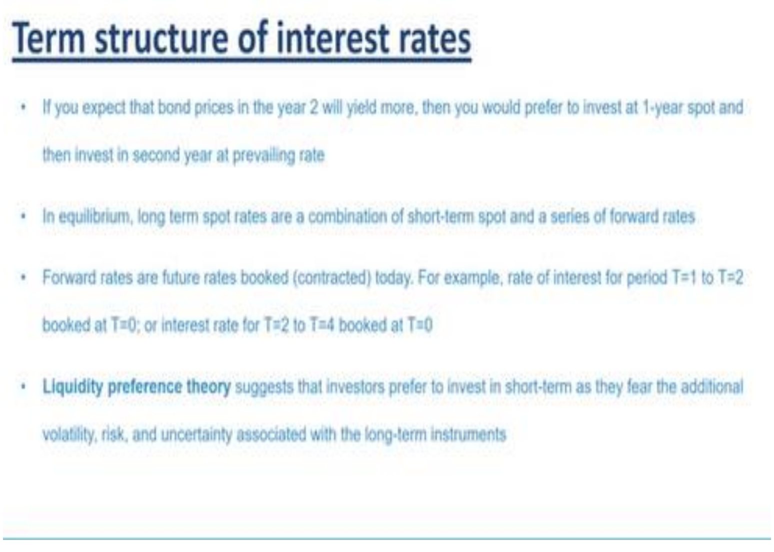

If you expect that bond prices in year 2 will yield more, then you would prefer to invest in one year spot and then invest in the second year at the prevailing rates. If many people would think like that, they will sell the second-year bond and buy the first-year bond. Ultimately, prices of these bonds have to adjust so that people are equally satisfied, whether they are investing in one-year spot or two-year spot.

Then the prices will be at equilibrium. This is often related to the expectations theory of the term structure of interest rates. This theory suggests that in equilibrium, long term spot rates are a combination of short-term spot and a series of forward rates. Forward rates are future rates booked today. For example, the rate of interest for period T = 1 to T = 2 booked at T = 0 or the interest rate for T = 2 to T = 4 booked at T = 0.

Essentially the forward rates reflect the future expectations of market participants. If the expectations theory is to be believed, then these forward rates prevailing today will materialize in the future. To summarize, the expectations theory the only reason for an upward-sloping term structure is that market participants or investors expect the short-term rates to rise.

Similarly, the only reason for a downward-sloping term structure is that market participants expect the short-term rates to fall.
Next, we have liquidity preference theory. The expectations theory here is not a complete explanation. Another important theory in this regard is the liquidity preference theory. Liquidity preference theory suggests that investors prefer to invest in the short term as they fear the additional volatility, risk, and uncertainty associated with long-term instruments.

Therefore, investors require additional premia to hold long-term instruments as against short-term instruments. Both of these theories are often employed to explain the dynamics of the term structure of interest rates. To summarize, in this video, we discussed that interest rates might differ for different maturities. This variation in interest rates across different maturities leads to a very important concept called the term structure of interest rates.

The term structure of interest rates can be rising as well as falling. Two key theories explain these term structures, including expectations theory and liquidity preference theory. To summarize this lesson, fixed-income securities like bonds are simply long-term loans. These instruments include regular interest or coupon payments, and at maturity, you get back the face value or principal amount.

The frequency of these payments can be annual, semi-annual, quarterly, etc. These instruments can be easily valued through discounted cash flow valuation method. Also, it is appropriate to discount each of these cash flows with its own spot rate, corresponding to the duration of the cash flows. The spot rate is observed on the term structure of interest rates. The term structure of interest rates is computed using the strips.

These strips are bonds with single cash flows and prices that are easily observed. These strips are created using government bonds, and therefore spot rates thus calculated reflect the risk-free rates. Finally, once the present value of a bond is computed using bond cash flows, one can also calculate the yield to maturity of the bond. YTM is a single interest rate that equates the bond cash flow with its present value. Another important related concept is that of duration.

Duration reflects the average time associated with cash flows of a fixed-income security. Duration also reflects the sensitivity of the price of a bond with the prevailing interest rates. It has been widely acknowledged that bonds with longer duration are more sensitive to changes in interest rates. The term structure of interest rates is most often upward-sloping. The expectations theory of interest rates suggests that rising interest rates reflect the future expectation of investors and market participants.

That is, they are expecting future interest rates to rise. The liquidity preference theory suggests that investors prefer to invest short term rather than the long term. Therefore, an additional compensation premium needs to be offered to investors to invest in long-term instruments. In addition, the theory of liquidity preference suggests that investors prefer to hold short-term instruments as compared to long-term instruments. An additional premium needs to be offered to make them hold long-term instruments.