

Artificial Intelligence (AI) for Investments
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Lecture - 03

Making investment decisions. Shareholders of a firm would like managers to invest in positive NPV projects to increase the firm value. Thus, managers can support the cause of owners by selecting positive NPV projects and rejecting negative NPV projects. We start the discussion by reviewing our good old NPV rule then, we turn to some other employing decision rules for making investments.

These first two rules are the payback period and book rate of return. These methods though simple but are empirically less rigorous and act as the rule of thumb that is easy to compute and communicate. However, multi-million dollar businesses need more than just a thumb rule for making large capital investment decisions. Another, very important measure that we discuss here is the internal rate of return measure.

This measure is often compared with the expected rate of return by shareholders for investing in similar-risk projects. In this discussion, we will also observe that various times these measures contradict each other. Many times, capital is a scarce resource and even good positive NPV projects need to be rejected in favor of other better projects. In this lesson, we will also discuss ways to resolve such contradictions.

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Review of NPV basics

- Consider yourself in a position of a CFO where you are analyzing \$1 million investment in a new venture called project P
 - That the current market value of your firm is \$10 million, which includes \$1 million cash that you plan to invest in project P
 - You find the NPV of this project by discounting the cash flows, adding them up to compute their PV, and subtracting the initial investment of \$1 million
 - It is easy to understand if $PV > 9$ this project has a positive NPV
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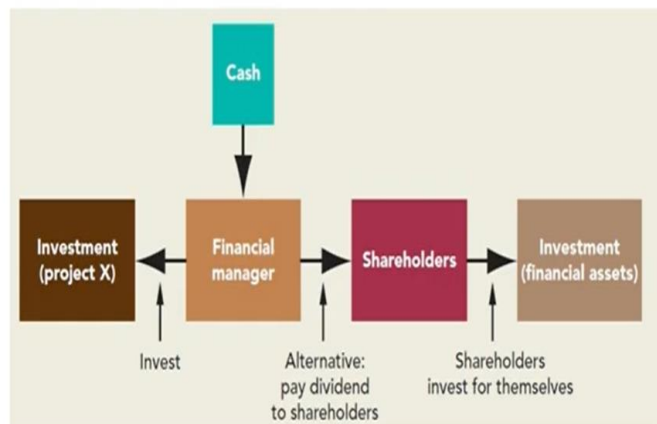
Review of NPV basics, in this video we will review our learning of NPV criteria as a decision rule for the selection and rejection of projects. Consider yourself in a position of a CFO where you are analyzing one million dollars investment in a new venture called project P. You project the cash flow generated by the project over its useful life. You also estimate its appropriate opportunity cost of capital R .

You find the NPV of this project by discounting the cash flows adding them up to compute their present value PV and subtracting the initial investment of 1 million dollars. It is given to you that the current market value of your firm is 10 million dollars which includes one million dollars in cash that you plan to invest in project P. The other assets and opportunities with the firm must be valued by the market at nine million dollars.

Therefore, should you decide to accept this project the value of your firm will be $10 - 1 + PV$ that is $= 9 + PV$. It is easy to understand if this present value PV is greater than 9 this project has a positive NPV.

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Review of NPV basics



Also, instead of investing in this project, you can distribute the cash to shareholders who can themselves invest in capital markets on their own. So, essentially the opportunity cost is investing in the project rather investing in the capital markets. Thus, the project opportunity cost is precisely the rate of return the shareholders can obtain by investing in financial market instruments.

This is summarizing the diagram shown here, but what kind of financial market instruments to choose for the selection of opportunity costs? Ideally, a set of equivalent risks as your project should be chosen to estimate the opportunity cost or expected returns on your project.

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Review of NPV basics

- NPV rule recognizes that a dollar today is worth more than a dollar tomorrow
- Any decision rule that is affected by managers' tastes, choice of accounting method, profitability of existing business, or that of other projects will lead to an inefficient decision
- $NPV(A+B) = NPV(A) + NPV(B)$
- Book incomes are not necessarily the same as cash flows
- Profitability measures such as book rate of returns, heavily depend on the classification of various items as capital investment and their rate of depreciation

The following are key aspects of the NPV rule, first NPV rule recognizes that a dollar today is worth more than a dollar tomorrow. This is so because the dollar today can be put to use by investing it and start earning interest on it. NPV heavily relies upon the forecasted cash flows of the project and its opportunity cost of capital. Present values are measured in today's dollar term so there can be added.

Lastly, any decision rule that is affected by managers' taste, choice of the accounting period, the profitability of the existing business, or that of other projects will lead to an inefficient decision rule. Therefore, the net present value $(A+B) = NPV(A) + NPV(B)$. In this expression, if project B has a negative NPV then the PV of A+B though positive is lower than the NPV of A. Therefore, NPV will identify that and you would not take project B just because it is packaged with a good project A.

Those decision rules that do not have this property or capacity are considered inferior. Companies report their incomes frequently these book incomes are not necessarily the same as cash flows. Take for example the case of depreciation expenses even though it is a non-cash expense it is subtracted from book income to arrive at profits. Therefore, profitability measures such as book rate of returns heavily depend on the classification of various items as capital investment and their rate of depreciation.

However, it should be very clear by now that a project selection or rejection should not depend on how accountants classify cash flows or expenses. Thus, measures such as book rate of return are not considered robust. Summarize in this video, we discussed that the NPV rule is a simple but very powerful rule for the following reasons. First, it considers the time value of money, second, it reflects the fact that a dollar today is worth more than a dollar tomorrow.

Present values can be simply added up in today's dollar terms however the efficiency of the NPV rule depends on the ability of managers to project cash flows in the future and estimate the appropriate opportunity cost of capital which is the discount rate.

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Alternatives to NPV rule – Payback period method

- A project's payback period is simply found by estimating the years it takes for the project cash flows to meet the initial investment
- A washing machine is costing \$800. You spend \$300 a year on washing your clothes. As a thumb rule, if this machine is purchased, it will recover its expenses in 3 years
- The payback rule states that a project should be accepted if its payback period is less than some cut-off period
- Consider a simple example here

Project	C0	C1	C2	C3	Payback Period (years)	NPV at 10%
A	-2,000	500	500	5,000	3	+2,624
B	-2,000	500	1,800	0	2	-58
C	-2,000	1,800	500	0	2	+50

Alternatives to NPV rule payback, period method. In this video, we will discuss the pros and cons of the payback period method. A project's payback period is simply found by estimating the years it takes for the project cash flows to meet the initial investment. For example, a washing machine is costing 100 dollars, you spend 300 a year on washing your clothes as a thumb rule if this machine is purchased it will recover its expenses in three years.

The payback rule states that a project should be accepted if its payback period is less than some cut-off period. For example, in the previous case if the cut-off period is four years, then the washing machine will be purchased. Consider a simple example shown here, cash flows corresponding to projects A, B, and C is provided along with their payback period and net present values at a 10% discount rate.

The NPV that is net present value rule suggests accepting projects A and C but rejects project B, contrasting results are obtained as per the payback rule. If a two-year cut-off period is selected then as per the payback rule only projects B and C would be selected. Thus, the payback rule offers misleading results for the following reasons. It ignores the cash flows after the cut-off period it does not consider the time value of money.

And therefore, gives equal weight to all the cash flows before the cut-off date. Therefore, if you entirely rely on the payback rule you will miss good projects with long life and may accept poor

projects that are short-lived. However, managers often use this rule to select projects due to quick profits and early investment recovery. Remember our discussion of the separation of ownership and control and its implication for managers' incentives being not aligned with those of shareholders.

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Alternatives to NPV rule – Discounted Payback period method

- An improved version of payback period is to employ discounted cash flows
- This discounted payback rule examines that how many years it takes for the discounted cash flows to recover the initial investment, i.e., become NPV positive
- Let us examine our previous example, with the help of discounted cash flows

Project	C0	C1	C2	C3	Discounted Payback Period (years)	NPV at 10%
A	-2,000	$\frac{500}{1.1} = 455$	$\frac{500}{1.1^2} = 413$	$\frac{5,000}{1.1^3} = 3757$	3	+2,624
B	-2,000	$\frac{500}{1.1} = 455$	$\frac{1,800}{1.1^2} = 1488$	-	-	-58
C	-2,000	$\frac{1,800}{1.1} = 1636$	$\frac{500}{1.1^2} = 413$	-	2	+50

An improved version of the payback period is to employ discounted cash flows. Let us examine our previous example with the help of discounted cash flows. This discounted payback rule examines how many years it takes for the discounted cash flows to recover the initial investment that is net present value becomes positive. Unlike the simple payback rule, this rule would not accept negative NPV projects.

Still, it does not take into account of good long-term projects such as A thus, this method can be considered as a warning signal to put a red flag where projects are showing very optimistic forecasts in the distant future. To summarize in this video, we learned that discounted payback period method is a simple thumb rule to examine how many years it takes to recover the initial investment. However, this rule does not consider the time value of money.

Also, it ignores cash flow, that is after some pre-specified cut-off date. An improved version of the payback period method is discounted payback method wherein discounted cash flows are considered for project evaluation. However, these discounted cash flows also give zero weight to

cash flows after the cut-off date. Thus, while these payback rules may act as a simple thumb rule they are still much inferior to the NPV criteria.

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Alternatives to NPV rule – Internal rate of return (IRR) method

- IRR rule comes from the simple return measure

$$\bullet \text{ Project return} = \frac{\text{Profit}}{\text{Investment}} = \frac{\text{Payoff}}{\text{Investment}} - 1; \text{ or } -\text{Investment} + \frac{\text{Payoff}}{1 + \text{Project Return}} = 0$$

- IRR is the return or discount rate at which NPV=0

$$\bullet \text{ NPV} = C_0 + \frac{C_1}{(1+IRR)} + \frac{C_2}{(1+IRR)^2} + \dots + \frac{C_T}{(1+IRR)^T} = 0$$

C_0	C_1	C_2
-4000	+2000	+4000

$$\bullet \text{ NPV} = -4000 + \frac{2000}{1+IRR} + \frac{4000}{(1+IRR)^2} = 0; \text{ solving for this, we get IRR} = 28.08\%$$

Alternatives to the NPV rule, the internal rate of return is the IRR method. In this video we will discuss the IRR rule we will understand the computation of IRR and its application in project evaluation. Internal rate of return IRR is one of the often-used measures as a project evaluation criterion. The ancestry of the IRR rule comes from the simply written measure shown here. For a one-period project, the return can be simply computed as shown here.

$$\text{Project Return} = \frac{\text{Profit}}{\text{Investment}} = \frac{\text{Payoff}}{\text{Investment}} - 1; \text{ or } -\text{Investment} + \frac{\text{Payoff}}{1 + \text{Project Return}} = 0$$

It should not be very difficult to see here that the NPV of the project is 0 at this return. This can be generalized to a multi-period project to define a more generic IRR formula.

That is IRR is the return, a discount rate at which NPV = 0. It is a very useful measure though it sometimes contradicts inferences from the NPV rule, with this definition the IRR rule can be expressed as shown here.

$$\text{NPV} = C_0 + \frac{C_1}{1 + IRR} + \frac{C_2}{(1 + IRR)^2} + \dots + \frac{C_T}{(1 + IRR)^T} = 0$$

Several software including Microsoft Excel is available to solve this expression.

Consider the cash flows from a project as provided here, the IRR computation for these cash flows on as a net present value

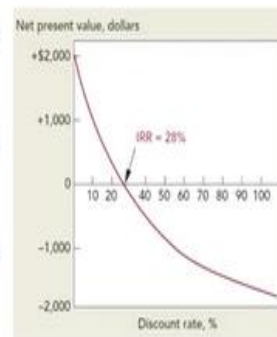
$$NPV = -4000 + \frac{2000}{1 + IRR} + \frac{4000}{(1 + IRR)^2} = 0$$

Solving for this, we get an IRR = 28.08%.

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Alternatives to NPV rule – Internal rate of return (IRR) method

- If the opportunity cost of capital is less than the 28.08% IRR, then the project has a positive NPV
- If opportunity cost of capital is greater than the IRR, the project has a negative NPV
- Please note that IRR is a profitability measure and depends solely on the timing of the project cash flows
- The opportunity cost of capital is the standard of profitability to judge the worth (or NPV) of the project



Let us examine this in more detail, what happens to NPV as the discount rate changes? Although we already know that NPV becomes 0 at IRR = 28.08 percent this is shown here in the diagram as well. In the diagram, we can easily see that if the opportunity cost of capital is less than the 28.08 percent IRR. Then the project is approximate NPV, of course, if the opportunity cost is the same as IRR, then NPV = 0. If the opportunity cost of capital is greater than the IRR the project has a negative NPV.

Hence, we compare the opportunity cost of capital with the IRR on our project. This rule holds whenever the NPV of a project is a continuously declining function of the discount rate. This may not always be true. Often, we tend to confuse opportunity costs with IRR. Please note that IRR is a profitability measure and depends solely on the timing of the project cash flows. In contrast, the opportunity cost of capital is the standard of profitability to judge the Worth or NPV of the project.

So, at the opportunity cost of capital projects, NPV can be positive, negative, or 0. However, if project cash flows are discounted at IRR and NPV is necessary = 0. Also, the opportunity cost is

observed and estimated from capital markets by examining the securities with a similar risk as that of the project. To summarize in this video, we discussed the IRR measure of project profitability. IRR measure is the discount rate at which $NPV = 0$.

IRR is intrinsic to the project and unlike the opportunity cost of capital that was determined from well-functioning capital markets IRR is dependent on project cash flows only. The simple decision rule here is to accept an investment project if the opportunity cost of capital is less than IRR or reject the project if the opportunity cost of capital is more than IRR.

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Pitfalls of IRR

- Pitfall 1: Problem of Lending vs borrowing
 - Consider the project cash flows from projects A and B as shown here
- | Projects | C_0 | C_1 | IRR | NPV at 10% |
|----------|-------|-------|-----|------------|
| A | -1000 | +1500 | 50% | +364 |
| B | 1000 | -1500 | 50% | -364 |
- Both of these projects will give you the same IRR
 - In project A, we are paying out \$1000 initially, and getting \$1500 later - Case of lending
 - While in case of B, we are initially getting \$1000 and paying back \$1500 later - Case of borrowing
 - When you lend money, you want a higher return and when you borrow money you want a lower return

Pitfalls of IRR, in this video we will discuss various pitfalls associated with IRR as compared to the NPV rule. We will also see why despite these pitfalls IRR is still an important profitability measure. Lastly, we will also discuss how to improve, IRR in some of these pitfall situations. IRR rule is also not free from pitfalls remember the caveats highlighted earlier during the IRR discussion.

Let us examine these pitfalls one by one. Pitfall 1 problem of lending versus borrowing considers project cash flows from projects A and B as shown here. Both of these projects will give you the same IRR but have a look at their NPV. The NPV suggests that these projects obviously cannot be equally attractive. In project A we are paying out 1000 dollars initially and getting 1500 later- A case of lending or investing.

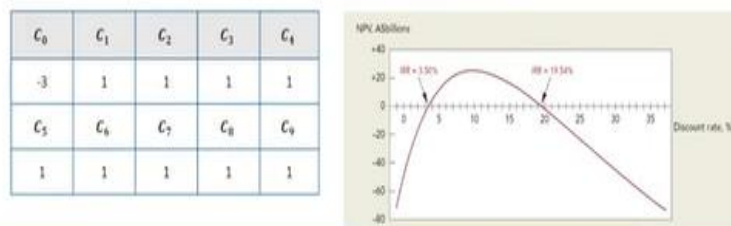
While, in the case of B, we are initially getting 1000 dollars and paying back 1500 dollars later- in the case of borrowing, Quite obvious to note that when you lend money you want a higher return, and when you borrow the money you want a lower return. Therefore, for project B you will find that NPV increases, and a discount rate increases because of the borrowing nature of the transaction.

That means the traditional way of looking at IRR will not work in this case and we need to consider the rule oppositely. Let us select those opportunities for borrowing where IRR is less than the opportunity cost of capital. In this case, the opportunity cost of capital means the opportunity to borrow from other sources available, for example, financial markets, bank borrowings, etc.

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Pitfalls of IRR

- Pitfall 2: Multiple rates of return
- Consider another project that involves an initial investment of \$3 Billion and then produce a cash flow \$1 Billion per year, for next nine years
- At the end of the project, the company will incur \$6.5 billion of cleanup costs



Pitfall 2 multiple rates of return. Consider another project that involves an initial investment of three billion dollars and then produces a cash flow of one billion dollars per year for the next nine years. At the end of the project, the company will incur 6.5 billion in clean-up costs. Such clean-up costs at the end of the projects are customary in projects like mining oil and gas exploration among others.

The projected cash flows are shown here. Let us see the relationship between the project NPV and the discount rate for this project as shown here. As we can see two discount rates make $NPV = 0$ these are 3.5 percent and 19.5 percent. As the discount rate rises, NPV initially rises and then

declines. So, two times NPV crosses the 0 and NPV line first on the rising side and then second on the falling side.

The reason for this change in direction and therefore the change in NPV sign is that there is a change in cash flow signs. Unlike our previous examples, there is also a change in cash outflow at the end. So, we can see as many IRR as there are changes in signs of cash flows. Particularly so when such outflows for example investment decommissioning cost etc, are substantial expenses.

And sometimes there are cases where there will not be any IRR even, often the most simple solution available in such cases is to consider NPV which is the net present value.

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Pitfalls of IRR

- Pitfall 3: Mutually exclusive projects
- Firms often have to choose from mutually exclusive projects, since it may not be feasible to take all of them
- In the project cash flows shown here, it seems IRR and NPV are contradicting each other

Projects	C_0	C_1	IRR (%)	NPV at 10%
D	-10000	+20000	100	8182
E	20000	+35000	75	11818

- In such cases, IRR can still be salvaged by examining incremental cash flows as shown here

Projects	C_0	C_1	IRR (%)	NPV at 10%
E-D	-10000	+15000	50	3636

At pitfall 3 mutually exclusive projects. Firms often have to choose from several projects available. Many of these projects are good projects with positive NPVs but mutually exclusive projects. That is it is not feasible to take all of them, this may happen for many reasons. The most important of them is capital, capital is a scarce resource that is available only in limited quantities. Many times, here IRR can be misleading, for example, consider the project's B and D here.

Also, it seems that IRR and NPV rules contradict each other. IRR rules suggest that a project in D is more profitable. In contrast, NPV rules suggest that project E is more profitable. In such cases, the IRR rule can be salvaged by examining the IRR on incremental cash flows. Let us look at how

to do it, and compute the difference between E and D cash flows. Then examine the IRR on the additional 1000 dollars being spent.

That is the incremental cash flows from undertaking project D, do they justify the investment? As we can see the IRR on incremental cash flows is 50% which is more than the opportunity cost of capital. This is also confirmed by the positive NPV of the project.

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IRR in Conclusion

- Many things can go wrong with IRR, but it is still a very useful benchmark
- To see its utility, have a look at the project cash flows, NPV, and IRR estimates for two projects X and Y as shown here (\$, thousands)

Projects	C ₀	C ₁	C ₂	C ₃	NPV at 8%	IRR (%)
X	-9.0	2.9	4.0	5.4	1.4	15.58
Y	-9000	2560	3540	4530	1.4	8.01

- Both of these projects offer the same positive NPV of \$1400
- As rational individuals you would select X over Y (Why?)
- The higher IRR associated with X (15.58%) reflects the low risk and efforts involved as compared with Y

As we can see many things can go wrong with IRR. However, IRR is still a very useful benchmark or a decision rule often employed by managers. You would often probably not hear NPVs or dollars in boardroom discussions but return figures when project profitability is discussed. Thus, the IRR figure can convey the profitability of a project more easily. To clarify this argument, have a look at the project cash flows NPV and IRR estimates for two projects X and Y as shown here in 1000 dollars.

It is an interesting example as both of these projects offer the same NPV of 1400 dollars. As a rational individual I am quite sure that you would select project X against Y. Investing in project X is clearly attractive as it offers you 1400 by investing only 9000 dollars as against 9 million dollars in project Y. This also reflects the higher IRR level of X that is 15.6 percent as compared to that of Y that is 8 percent.

A qualitative argument in favor of project X goes like this, setting project X is rather easy, less costly, less manpower maybe less time involved. Project Y involves a lot of costs and expenses and your managerial resources such as time. If unsuccessful there would be much heartburn in the case of project Y. So, it is too obvious to notice that project Y is not probably worth the worry and time compared to project X.

In such cases, IRR offers a good advantage in selecting and rejecting projects. To summarize in this video, we discuss various pitfalls of IRR we discussed the problem of lending versus borrowing and how to interpret IRR in these situations. We also examined the scenario where multiple IRRs, due to changing the sign of cash flows. In such cases, NPV provides more consistent results.

We also discussed the case of mutually exclusive projects where IRR can still be employed by using incremental cash flows. Despite all the shortcomings, IRR is still a useful tool when comparing project investments.

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Capital investments with limited resources

- Capital is a scarce resource, thus it is not possible to select all the positive NPV projects
- Thus, firms would like to select those projects that offer highest NPV per dollar of investment
- Profitability index (PI) = $\frac{\text{NPV}}{\text{Initial Investment}}$

Project	Cash Flows (\$ Mn)			NPV at 10%	PI
	C0	C1	C2		
A	-10	+30	+5	21	2.1
B	-5	+5	+20	16	3.2
C	-5	+5	+15	12	2.4

Capital investments with limited resources. In this video, we will discuss the project selection criteria when capital is limited. This is often referred to as the problem of capital rationing. Till now our discussions have rested on the proposition that a firm increases its wealth if it accepts the

project with a positive NPV. However, there are various constraints and limitations from undertaking all such projects.

One of the most important issues in this regard is that capital is a scarce resource. Therefore, firms need to solve the problem of what economists call capital rationing. When the capital is limited firms need to select those projects that offer the highest NPV which is a maximum return for each dollar invested. Let us start with a simple problem of capital rationing. Consider the following three projects with cash flows as provided here.

All the projects are attractive but the firm has only 10 million dollars to invest. In that case, it can invest in project A or in projects B and C but not in all three. Although this is a rather easy case as we can see the NPV of project A is less than B and C combined. However, things are not always so easy when the projects are limited, we cannot choose between projects solely on the basis of the net **present** values.

Then we focus on the projects where we get the maximum return for our investment. That is the highest NPV for the given investment. This measure is often called the profitability index and is computed as shown here. Profitability index $PI = \frac{NPV}{\text{initial investment}}$. For these projects ABC the PI is computed as shown in the table here, as per the PI criteria you would first select project B and then C.

Therefore, if your budget limit is 10 million dollars, we should accept only the two projects. However, real-life examples are not so simple.

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Capital investments with limited resources

- Let us add another project D, which needs \$40 Mn investment in second year

Project	C0	C1	C2	NPV at 10%	PI
A	-10	+30	+5	21	2.1
B	-5	+5	+20	16	3.2
C	-5	+5	+15	12	2.4
D	0	-40	+60	13	0.4

- The firm can only raise \$10 Mn in the second year: additional constraint of capital rationing
- The simple way of ranking projects as per PI may not work here
- This particular problem is rather simple, as A and D combined offer a higher NPV than B and C combined
- However, more complex problems are solved with linear programming (LP) techniques

Let us see a more involved example here. In the previous example, another project C is added which comprises investment in the next year. Consider the following projects now and this revised scenario. Suppose the firm can raise only 10 million dollars in each of the years, t equal to 0 and t equal to 1. Now if you consider projects B and C you will miss out on D. The other alternative is to consider A at $t = 0$.

This provides an additional cash flow of 30 million dollars $t = 1$ along with 10 million additional resources the firm can also undertake project D next year. Projects A and D combined have lower PI but have a higher NPV. So, the ranking logic of PI fails in this particular example as the resources are constrained in each of these two periods, or in other words selection of project D is contingent on the selection of project A.

In such cases where additional constraints are present then the simple ranking method with the help of the profitability index fails. Such complex problems are solved with linear programming techniques and LP techniques. To summarize in this video, we highlighted the problem of capital as a scarce resource. Thus, even all the positive NPV projects may not be undertaken. In case the problem of capital rationing is limited to a single period.

Then we can employ the PI or profitability index of projects and rank them. Then one can keep selecting the projects as per the PI-based rank till the time the capital is exhausted. However, when

there are multiple constraints then one needs to consider linear programming to solve this kind of capital rationing problem. To summarize in this lesson, we will learn several decision rules for making investments.

In addition to NPV other rules are also employed to examine alternate investments. These include book rate of return, payback period, and IRR method. The book rate of return is simply computed as book income divided by the Book value of the investment. The payback method examines the project cash flows against a certain specific cut-off period. Only those projects with a payback period less than the cut-off period are considered.

Lastly, IRR is the discount rate at which the firm NPV is 0. As per the IRR rule, firms should accept those projects that have an IRR greater than the opportunity cost of capital. Like the NPV rule, IRR is also based on the DCF technique. There are situations where IRR can give misleading results or contradict the NPV measure. These include lending versus borrowing cases, multiple IRR case, and mutually exclusive projects among others.

Often capital is a scarce source therefore all the projects with positive NPVs may not be accepted, and this becomes a problem of capital rationing. If capital is rationed only in one period, then the firm can use the profitability index method to rank the projects for selection and then keep selecting the projects till the capital is exhausted. PI computes the NPV of the project per dollar of investment.

However, if the capital is rationed in more than one period or there are more constraints then this method may Fail. Then linear programming LP is a more general solution.