Artificial Intelligence (AI) for Investments

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Lecture – 27

In this lesson, we will revisit the problem of portfolio construction, computation of expected returns, risk, correlation and covariance. We will also summarize our learnings of portfolio optimization with two-security case and multi-security case. We will construct portfolio possibility curve and examine the feasibility region where all the possible combination of securities in the form of portfolios are available to investors.

We will examine various properties of the feasible region with and without short sale and minimum variance portfolio. We will try to find the most efficient portfolio on these available set of portfolios. Subsequently, we will introduce risk free lending and borrowing and examine the implication of the efficient set of portfolios. Lastly, we will introduce the concept of market risk and beta.

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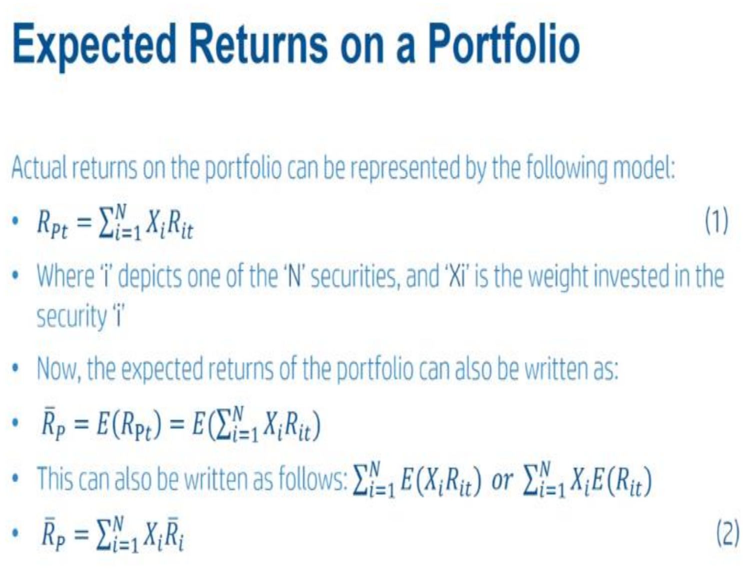

Portfolio construction recap one: In this video, we will discuss the expected return risk and correlation structure for multi-security portfolio. The actual return on a portfolio of assets for any given period is the weighted average of the actual return on the individual securities for that given period. The weights are the fraction of the portfolio invested in securities. Now, we will try and compute the expected returns of the portfolio.

But what is the need to compute the expected returns? Because it is the expected returns that are matched with the risk profile in various portfolio applications, not the actual returns. Now, keeping this rule in mind, we know that expected returns on any security for any period are the simple average if all the return observations are equally likely of the returns sampled from the past periods. What does it mean and what are its implications? This simply means the following.

If you have three observations for example 10 percent, 20 percent and 30 percent and also that the returns on a security are sampled in an unbiased manner or equally spaced periods like daily, weekly and so on, then the expected returns for any given period are the simple averages, for example 10 + 20 + 30 percent divided by 3 which is equal to 20 percent. Mathematically, this can be very easily written as return R i bar which is the expected returns equal to 1 upon T summation T = 1 to t Rit.

Now consider the actual return on the portfolio for period t as shown here. $R_{pt} = \sum_{i=1}^{N} X_i R_{it}$ Where i depicts one of the N securities and Xi is the weight invested in the security i. Now, the expected returns of the portfolio can be easily written as $\bar{R}_P$ = E (R_{pt}) = $\sum_{i=1}^{N} X_i * R_{it}$this can be also written in the following manner.

$$\sum_{i=1}^{N} E(X_i R_{it}) \sum_{i=1}^{N} X_i E(R_{it})$$

Summation of i = 1 to N expectations of X_i R_{it} or summation of i = 1 to N X_i times expectation of R_{it} because X_i constant and this can be effectively written as $\bar{R}_P = \sum_{i=1}^{N} X_i \bar{R}_i$. Even if you are not comfortable with the derivation above, keep the last result in mind. In simple English, this result suggests that the expected returns on the portfolio are simply the weighted average of the expected returns from the individual securities.

Here the weights are the proportion of the portfolio invested in the securities. Please note that while equation 2 looks similar to equation 1, there is a fundamental difference. Equation 1 describes the relationship between the actual return on portfolios and individual securities in that portfolio, while equation 2 describes the expected returns.

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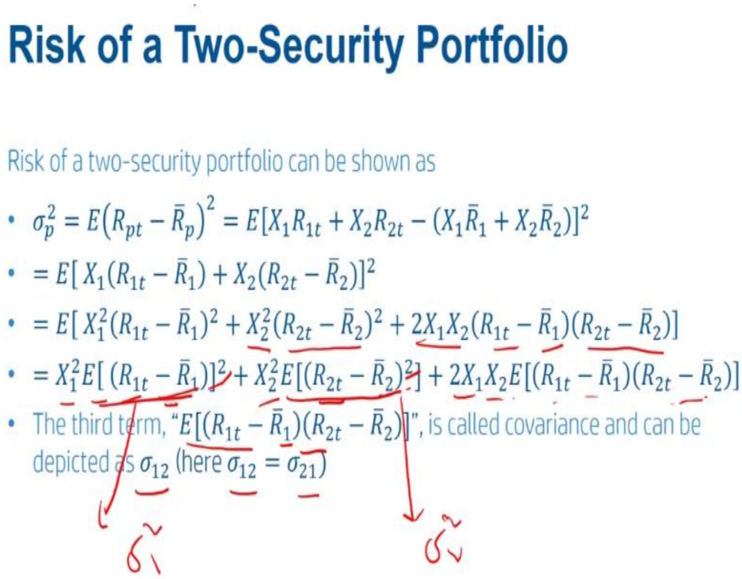

Now, let us talk about computing the risk or standard deviation for a portfolio of assets. Now that we have understood the return part, we will move to the risk part. However, it is not so simple for a multi-security case, so let us start with the two securities case with returns being R 1, R 2. Therefore, σ_p^2 = E $\left(R_{pt} - \bar{R}_P\right)^2 = E[X_1R_{1t} + X_2R_{2t} - (X_1\bar{R}_1 + X_2\bar{R}_2)]^2$

Which is also equal to $E[X_1(R_{1t} - \bar{R}_1) + X_2(R_{2t} - \bar{R}_2)]^2$. This expression can be further expanded to σ p square times $E[X_1^2(R_{1t} - \bar{R}_1)^2 \;\; + X_2^2(R_{2t} - \bar{R}_2)^2 + \; 2X_1X_2\,(R_{1t} - \bar{R}_1)(R_{2t} - \bar{R}_2)]$, which can be further simplified like this $[X_1^2E[(R_{1t} - \bar{R}_1)]^2 \;\; + X_2^2E[(R_{2t} - \bar{R}_2)]^2 + \; 2X_1X_2\,E(R_{1t} - \bar{R}_1)(R_{2t} - \bar{R}_2)]]$

This third term here which is denoted as $E[(R_{1t} - \bar{R}_1)\,(R_{2t} - \bar{R}_2)$ is also called the covariance between securities 1 and 2 and it can be easily depicted as σ_{12} which is the covariance between security 1 and 2 and please note a very fundamental property that $\sigma_{12} = \sigma_{21}$. Also, we must notice that this expectation of $R_{1t} - \bar{R}_1$ raised to the power 2 is nothing but σ_1 square, this is by definition σ1 square. Similarly, expectation of $R_{2t} - \bar{R}_2$ raised to power 2 is nothing but σ_2 raised to the power

2.

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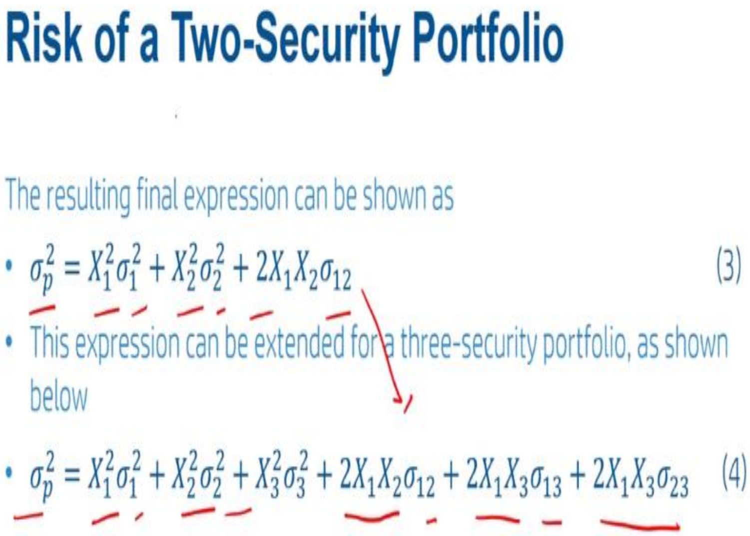

Therefore, a resulting expression can be written here. This is our resulting expression of risk for a two-security portfolio that is $\sigma_p^2 = X_1^2 * \sigma_1^2 + X_2^2 * \sigma_2^2 + 2*$ (X_1* X_2* σ_{12}). Now, this expression can be further extended for a three-security portfolio and here it is $\sigma_p^2 = X_1^2 * \sigma_1^2 + X_2^2 * \sigma_2^2 + X_3^2 * \sigma_3^2 + 2*$ (X_1* X_2* σ_{12}) + 2* (X_1* X_3* σ_{13}) +2* (X_2* X_3* σ_{23}) which is very simply an extension of three-security case.

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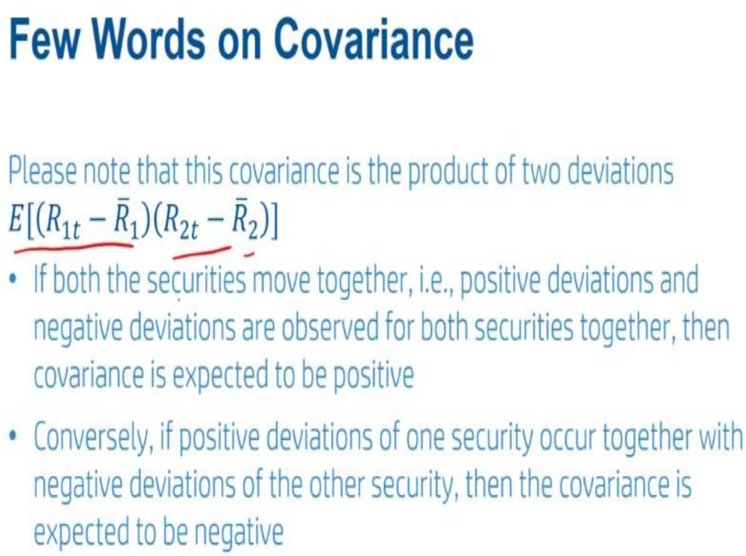

Let us have a few words on the covariance. Please note that this covariance is a product of two deviations. For example, as you can see here it is the $E[(R_{1t} - \bar{R}_1)\,(R_{2t} - \bar{R}_2)]$. Since the deviations are from two different securities, unlike the variance we may expect it to be negative as well, though that is a less practical scenario.

If both the securities move together that is large positive outcomes and large negative outcomes are observed for both of these securities together, then the covariance is expected to be high and positive. Conversely, if large positive outcomes of one security occur together with a large negative outcome of the other security, then the covariance is expected to be high and negative.

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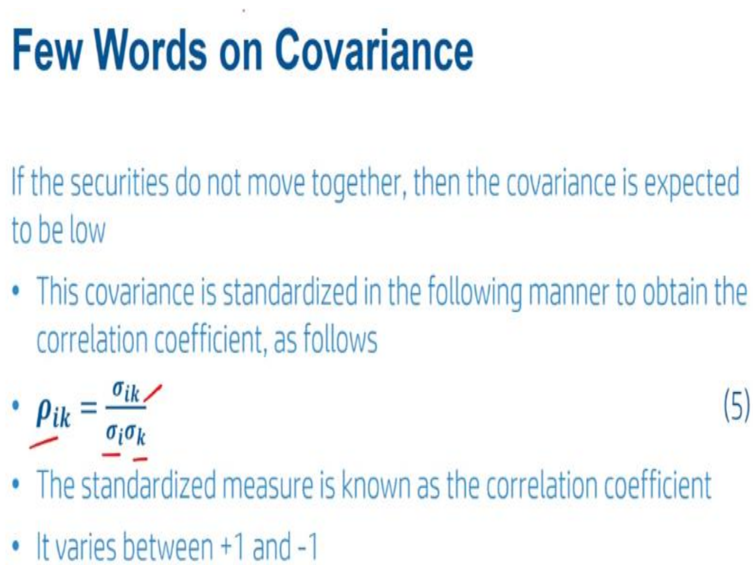

If the two securities do not move together, then the covariance is expected to be low. This covariance between security i and k is standardized in the following manner to obtain the correlation coefficient as shown here, we simply divide the covariance between security i and k that is σ_{ik} by their respective standard deviation σ_i σ_k to obtain the correlation between these two securities ρ_{ik}.

Here we have simply divided the covariance by individual standard deviations to obtain the standardized measure which is also known as correlation coefficient. This correlation coefficient is very easy to interpret and it is free of any skill bias, it varies between +1 and –1. To summarize, in this video we discussed the expected return, risk and covariance of securities in a portfolio. We also discussed that covariance can be standardized to obtain a more clean measure of correlation between securities in a portfolio and this measure ranges from plus +1 to –1.

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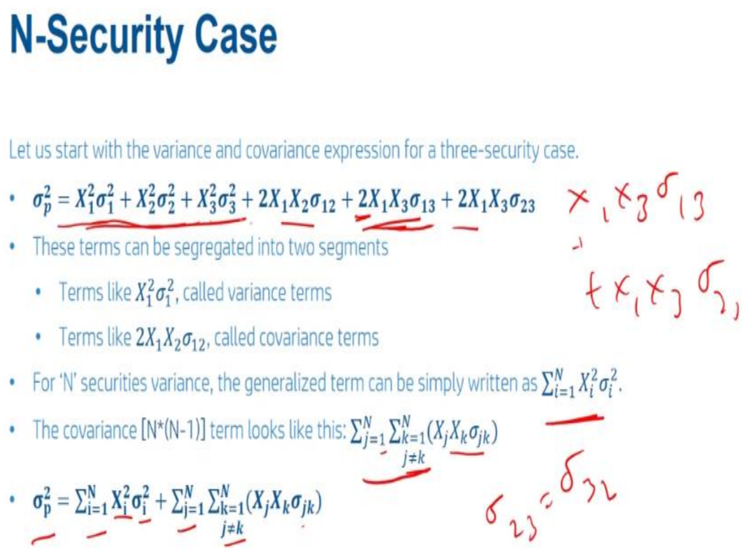

Portfolio construction recap two. In this video, we will talk about the risk of a more generic N-security portfolio case. Now, moving to a more generic case of N securities while solving these cases request computer programming, however, a small discussion will hurt nobody. If we carefully examine the terms in the equation for shown earlier, we will find that these terms can be segregated into two categories.

Terms that are similar to X1 square σ 1 square, for example, you can see here in the three-security case, you have σ p square as X1 square σ1 square + X2 square σ2 square + X3 square σ square these are the terms and another set of terms like two times X1 X2 σ12, times X1 X3 σ13 and so on.$\sigma_p^2 = X_1^2 * \sigma_1^2 + X_2^2 * \sigma_2^2 + X_3^2 * \sigma_3^2 + 2*(X_1 * X_2 * \sigma_{12}) + 2*(X_1 * X_3 * \sigma_{13}) + 2*(X_2 * X_3 * \sigma_{23})$. Terms like X_1 square σ 1 square these are called variance terms and terms like X_1 X_2 these are called covariance terms.

Now, for N-security variance, the generalized term can be simply written as summation of Xi square σ square, i = 1 to N. Let us confirm the correctness of this term by applying it for two or three security case or even more. A three-security case is already provided here as you can see. Understanding the second term, which is the covariance term, is slightly more involved. The term looks like this, it is $\sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \, \sigma_{jk}{}_{j\neq k}$

Please note this j is not equal to k, for all the cases where j = k it becomes the variance terms. If you are looking for the coefficient of security two which is multiplied with all the covariance terms, please remember that j = 2, k = 3 and j = 3 and k = 2 will give the same two results that is $\sigma_{23} = \sigma_{32}$ and that is where we have a multiplication of two here. So, this would have been X1 X3 times σ_{13} + X_1 X_3 time's σ_{31}.

And that is why summation of these two will become two times X 1 X 3 σ 13. Also, the symbol j is not equal to k ensures that the k is not the same as j which otherwise would result in the variance term. So, now, we can write the full equation for the generalized N-security case very simply as follows.

$$\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \, \sigma_{jk}{}_{j\neq k}$$

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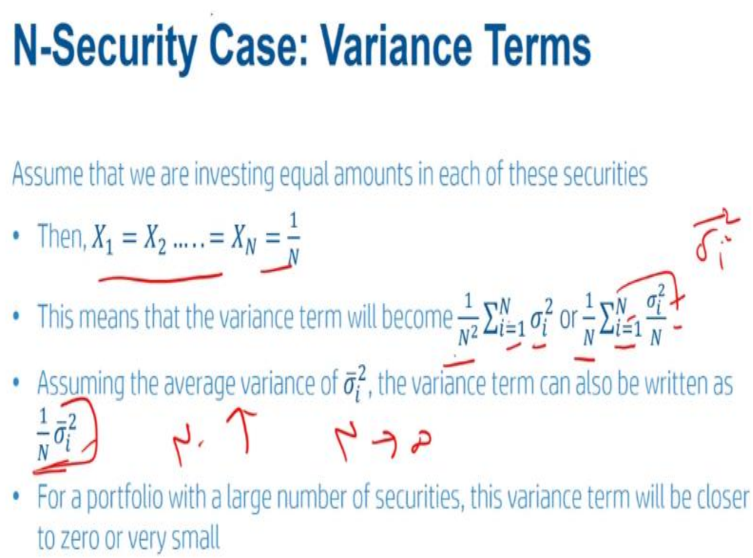

Let us examine this formula further. Assume that we are investing equal amounts in each of these securities and in that case $X_1 = X_2 = X_N = 1/N$. This means that the variance term will $(\frac{1}{N^2})\sum_{i=1}^{N}\sigma_i^2$

or we can take$(\frac{1}{N})\sum_{i=1}^{N}\frac{\sigma_i^2}{N}$ represents the average of variances or average variance that is σ i bar square.

So, we can write this term as $\bar{\sigma}_i^2$ sort of average variance and thus the summation of the variance term can also be written in the form of average variances$\frac{\bar{\sigma}_i^2}{N}$, this is average variance multiplied by 1/N. So, for a portfolio with large number of securities, this variance term will become close to 0 or very small as N increases, this overall term or summation of variances will become very small. In fact, if you assume N tends to infinity, this term will become close to 0.

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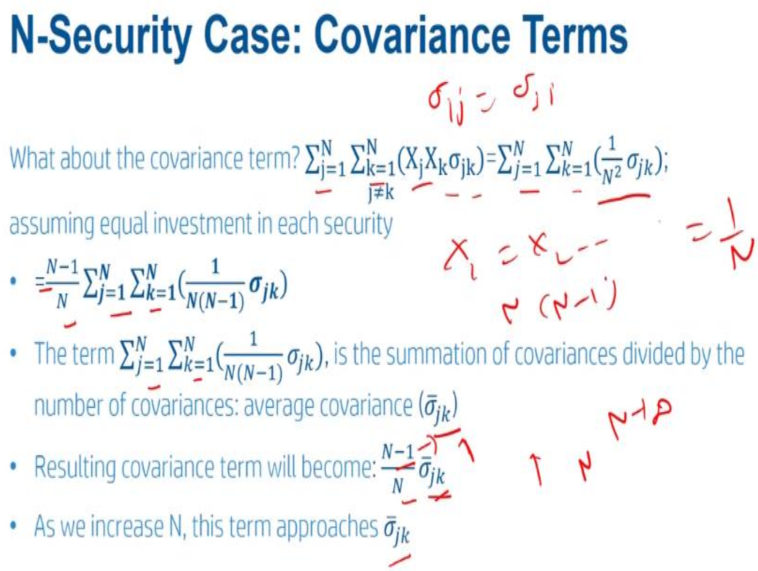

What about covariance terms, summation of covariance terms? Since there are N securities, there will be N into N – 1 covariance terms. For example, with three security there will be 6 such terms. However, since half of them are the same that is $\sigma_{ij} = \sigma_{ji}$ we use a multiple of two and only 3 terms are left, therefore if we present the covariance terms like this that is

$$\sum_{j=1}^{N}\sum_{k=1}^{N} X_i X_j\, \sigma_{jk}{}_{\;j\neq k} = \sum_{j=1}^{N}\sum_{k=1}^{N}\frac{1}{N^2}\sigma_{jk}$$

This also assumes equal investment in all the securities that means $X_1 = X_2$ and so on = 1/N equal investments and this can be further simplified like this.

$$\frac{N-1}{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\frac{1}{N(N-1)}\sigma_{jk}$$

The term $\sum_{j=1}^{N}\sum_{k=1}^{N}\frac{1}{N(N-1)}\sigma_{jk}$ is the summation of covariances divided by the number of covariances, remember there are N into N – 1 covariance terms.

So, this is also called the average covariance or $\overline{\sigma}_{jk}$. Thus, all the covariance terms can be denoted using the average covariance term as (N – 1)/N *$\overline{\sigma}_{jk}$. In this case, please note as long as much as you can increase N, this term will approach to unity. So, if N tends to infinity, this term will approach to 1 and therefore the resulting variance or summation of all the covariance terms will approach $\overline{\sigma}_{jk}$.

Which is precisely the average covariance term and it is not equal to 0 as happened in the case of variances. So, this summation of all the covariance term with a very high value of number of securities, this approaches average covariance $\overline{\sigma}_{jk}$.

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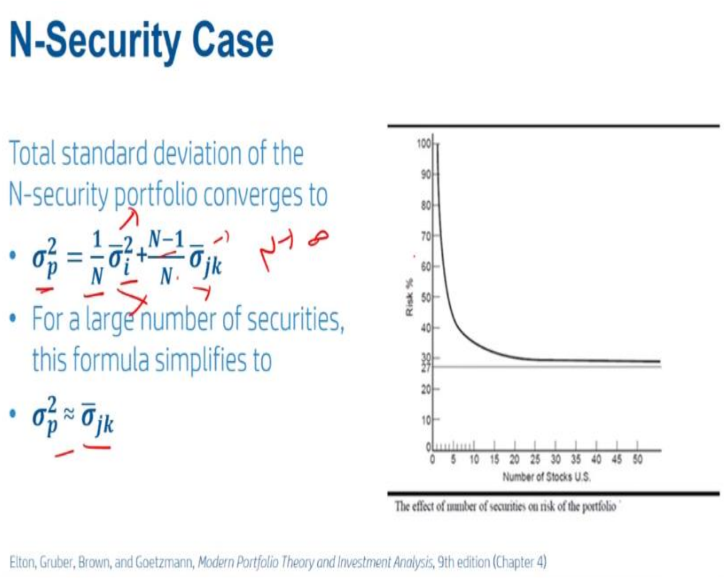

So, what happens to the overall risk of the portfolio that is summation of variance terms plus covariance term when we have large number of security? So, look at this overall total standard deviation of N-security portfolio, it converges to

$$\sigma_p^2 = \frac{1}{N}\overline{\sigma}_i^2 + \frac{N-1}{N}\overline{\sigma}_{jk}$$

which is average covariance.

Now, please note for very large number of securities that is N increasing to a very large number, this term will approach to 0 or a very small value and this N – 1 upon N will approach to unity and therefore we can say that σ_p^2 will simplify our approach to this value $\overline{\sigma}_{jk}$. This is precisely what we observe in this diagram. Let us discuss this diagram in slightly more detail.

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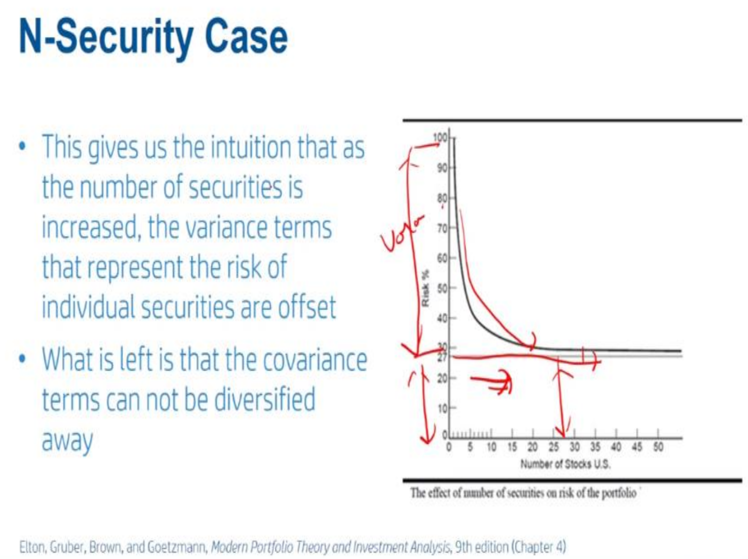

The diagram gives us the intuition that as the number of securities are increased, as you can see here the number of securities are increasing, the variance term that represent the risk of the securities are offset. For example, till here you have the variance terms component and here you have covariance component. As the number of securities are increased, this variance term drastically decreases and for even a small number of security like 30 it becomes close to 0.

So, what is left is the covariance risk that cannot be diversified away. That means this is also called the bedrock of risk as we discussed earlier and as more and more securities are added as figure shown here depicts how the specific risk or the variance term, this variance term, dies away and only the covariance risk, this risk, remains. These are the covariance terms that are left to contribute to the risk of portfolio or the average covariance.

With a fairly large number of securities, this remaining risk is equal to the average covariance of the portfolio often called as market risk or systematic risk or non-diversifiable risk. To summarize, in this video we discussed the risk or standard deviation of an N-security portfolio. We found that as the number of securities increases, the variance term or the idiosyncratic stock specific risk is eliminated.

And the only risk remaining is non-diversifiable and is on account of covariances which is driven by correlations across securities. This is the benefit of diversification, simply by adding more securities to a portfolio we are able to eliminate the stock specific idiosyncratic risk. In this video, we will introduce the Harry Markowitz mean variance framework and its role in portfolio optimization. Since 1952 Harry Markowitz's seminal work on portfolio selection, the practice of portfolio diversification is well known and documented.

In short, by choosing stocks that do not move together or with low correlation, investors can reduce the standard deviation of portfolio returns. If a sufficiently large number of past return observations are taken, their distribution is plotted, then this distribution is fairly closer to a normal distribution that we saw earlier. Though there are certain issues that tail, for example there are observations related to extreme positive and negative values.

That may cause deviations from the normal distribution and other issues such as excess kurtosis and practice, the normal distribution still is considered to define the return distributions very well and uses only two particular parameters, which is mean and standard deviation or variance. In nutshell, all the past returns can be defined by these two numbers that is mean and standard deviation and that is the reason we have discussed the normal distribution in some of the previous discussions.

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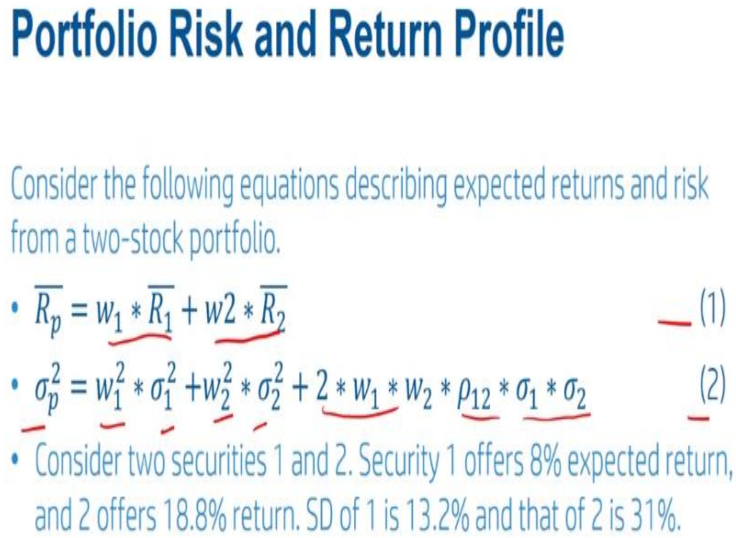

Now, let us combine two securities to construct a portfolio. Consider two securities A and B security A offers 8 percent expected return and B offers 18.8 percent return, standard deviation of A is 13.2 percent and that of B is 31 percent. A portfolio comprising these two securities has its expected return and risk are governed by these two equations, equation 1 and 2 which can be seen here.

$\bar{R}_P = w_1 * \bar{R}_1 + w_2 * \bar{R}_2$

For example, expected return of the portfolio can be easily written as w 1 times R 1 bar which is the expected return on security 1 and w 2 times R 2 bar which is equation 1 for the expected return on the portfolio, this we have seen earlier. Similarly, the variance of the portfolio is $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * (w_1 * \sigma_1)(w_2 * \sigma_2)\rho_{12}$, this is equation 2 which also we have seen earlier.

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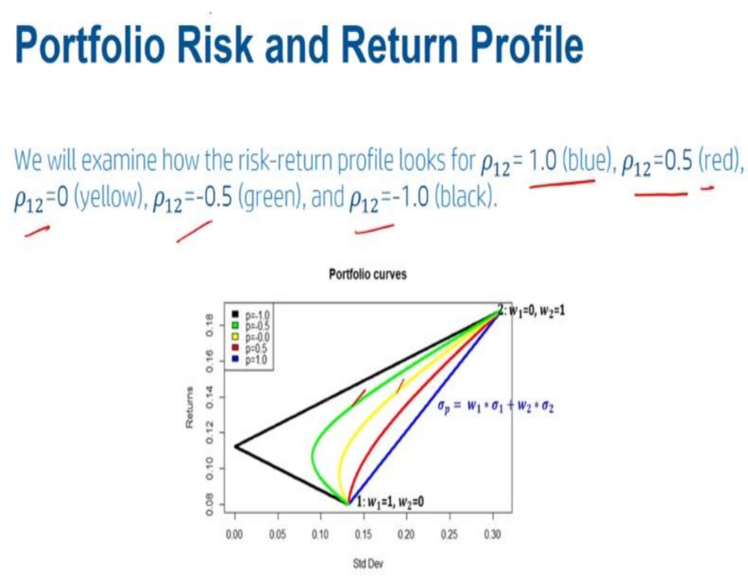

Now, we will examine how the behavior of returns and standard deviation for different levels of correlations and different investment proportions that is w_1 and w_2 for these two securities. We have plotted all the possible portfolios of correlations corresponding to $\rho = 1$ in blue, $\rho_{12} = 0.5$ in red row, $\rho_{12} = 0$ in yellow, $\rho_{12} = -0.5$ in green and $\rho_{12} = -1$ in black.

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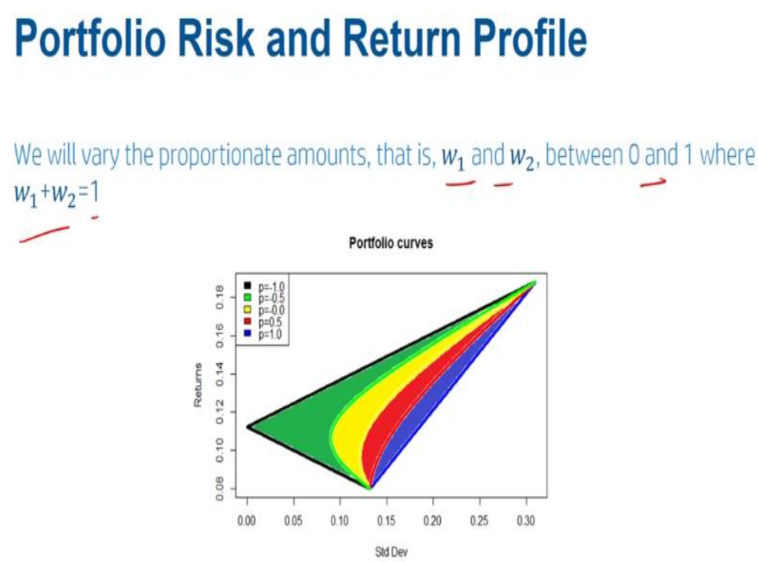

Here, each curve with a given colour indicates all the possible combinations of investment that is each possibility is a point here in either of these securities from w1= 0 to w1 = 1 where w1 + w2 = 1. So, basically we are varying the proportionate amounts that is w1 and w2 between 0 and 1 and ensuring that w1 + w2 = 1 constraint is followed. These graphs are essentially governed by the tool return and risk equations that we have already seen.

Which is Rp = w1*R1 + w2*R2 and $\sigma p^2 = w_1^2 \sigma_1^2 + w_2^2\sigma_2^2 + 2*w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$, the equation 1 and 2 that we saw earlier. P 12 here was the correlation coefficient between the two securities 1 and 2 and then a very important parameter.

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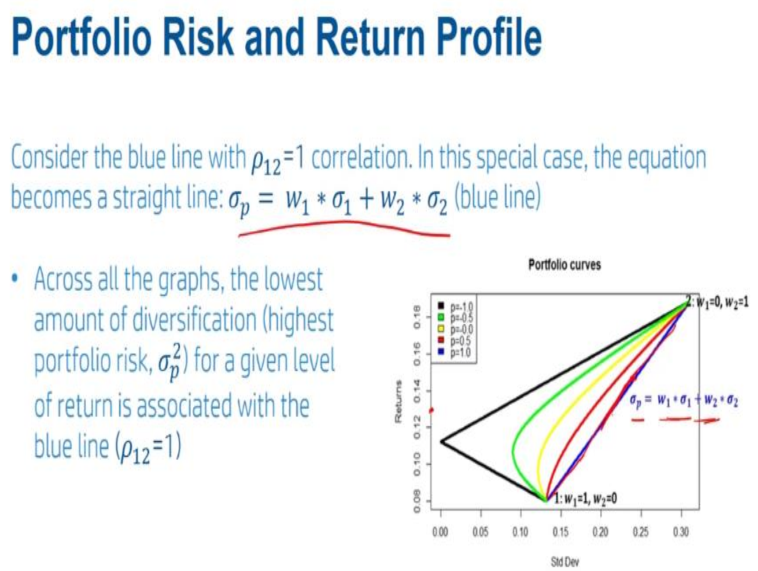

Now, please consider the blue line here with ρ_{12} = 1 correlation. In the special case the equation becomes a straight line with $\sigma p = w_1*\sigma_1 + w_2*\sigma_2$, this is the blue line case, this case, and it should be clear from the graph itself that across all the graphs the lowest amount of diversification that is highest portfolio risk σ_p square is achieved for a given level of return associated with this line.

By so, please notice that for this blue line for all the combination of w_1, w_2 the amount of risk for a given level of return is maximum for all the combinations. For example, if we look at this level of returns, the maximum risk is obtained on this blue line for all the different correlation combinations. We will discuss the minimum and other combinations also, but it is very easy to see here that the maximum risk is observed or obtained for this particular line.

And in this particular line, there is more diversification and the equation of the σ_p is simply the weighted average of risk between two securities that is $w_1*\sigma_1 + w_2*\sigma_2$, something similar to the weighted average of returns as we computed.

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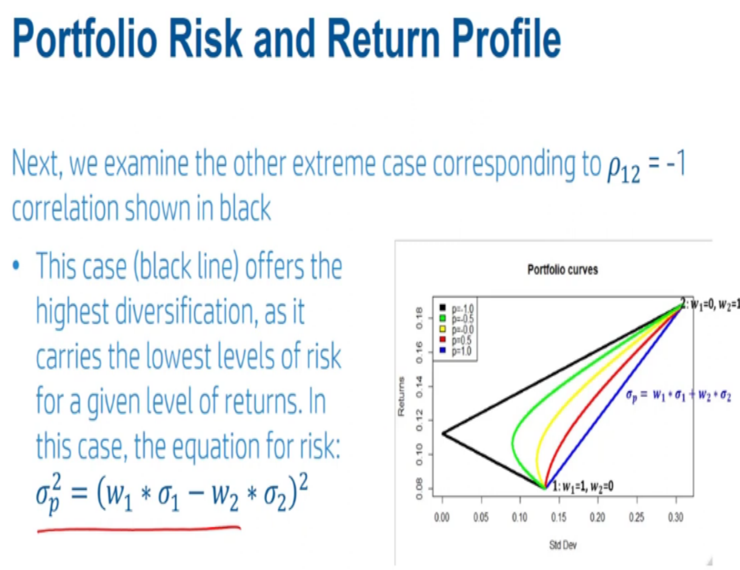

Next, we examine the other extreme case corresponding to $\rho_{12} = -1$ and that correlation is shown here in green. This is the case where highest diversification is observed as it carries the highest levels of return for a given level of risk or minimum level of risk for a given level of return. In this case, the equation for risk effectively becomes described by this equation which is σ_p^2 = (w_1*σ_1 – w_2*σ_2)^2

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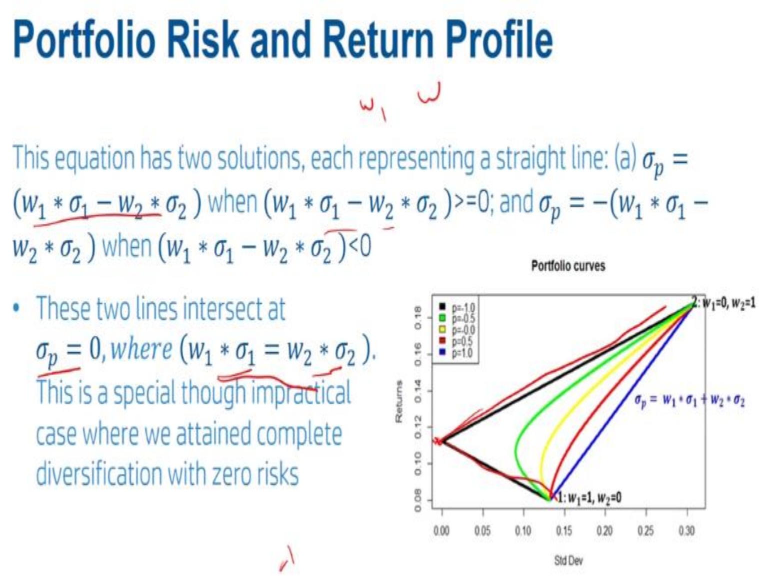

Please remember this particular equation has two solutions, each representing a straight line that is line number 1 is σ_p = (w_1*σ_1 – w_2*σ_2) which is this line and w_1*σ_1 – w_2*σ_2 which is this line. So, in this case, when σ_p = (w_1*σ_1 – w_2*σ_2) when w_1*σ_1 – w_2*σ_2 is greater than or equal to 0 and σ_p = – w_1*σ_1 – w_2*σ_2 when w_1*σ_1 – w_2*σ_2 is less than 0.

So, these two line segments can be represented by this first this black line and the other one is by this black line. So, these are two line segments and these two lines intersect at $\sigma_p = 0$ where precisely $w_1*\sigma_1 = w_2*\sigma_2$. Now, this is a very special but an impractical case where you have attained complete diversification with zero risk. Such cases of negative correlation are rarely observed and almost okay from theoretical point of view, but not observed practically.

However, if we assume them to be true in theoretical sense, then there is a possible combination of weights w_1 and w_2 such that we can obtain this condition $w_1*\sigma_1 = w_2*\sigma_2$and the risk of the portfolio such hypothetical portfolio complete risk becomes 0 or neutralize. This is a special case where minimum risk $\sigma_p = 0$ is observed.

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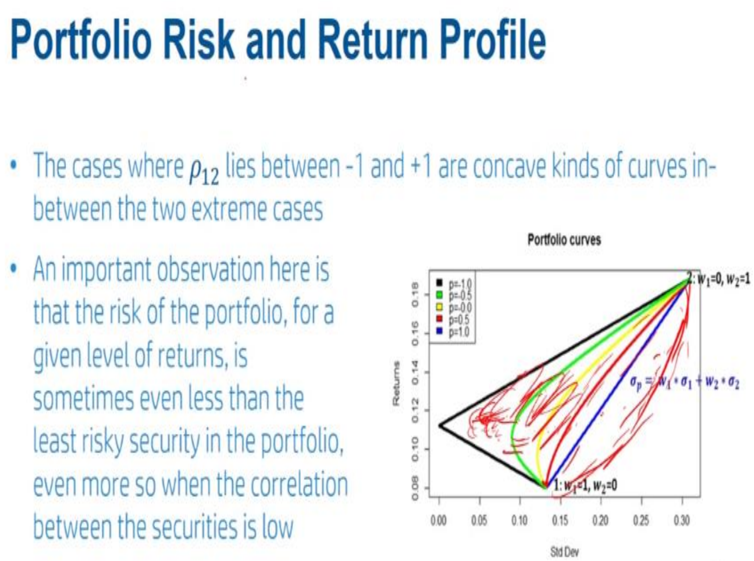

For all the other cases where ρ_{12} lies between –1 to +1 are concave kind of curves in between these two lines. For example, notice all the cases will be between this blue or black line, you will not observe any case such as this, such cases will not be observed. So, all these are concave kinds of cases between blue and black line, the extreme cases that we have discussed. Any point here would suggest even higher level of risk for a given level of return as compared to maximum correlation equal to 1 which is not feasible.

At any point beyond this black curve would suggest a risk which is lower than even this negative

–1 correlation which is also not possible. So, all the possible combinations are expected for different correlation levels are expected to lie only between these two black and blue curves and of concave nature. So, there are no convex cases like this which will be outside as the lines will strictly lie between these two extreme cases of black and blue lines.

Also imagine a concave curve, it would suggest a completely opposite situation to what we have discussed about diversification. It would suggest the existence of curves where expected returns will be lower for a given level of risk as we have considered portfolio of securities. It would suggest more risk here if it is of this nature. If the curve is of this nature, it would suggest more risk for a given level of return or less expected return for a given level of risk which is spurious and not possible.

So, our region of possibilities is strictly line between these black and blue lines. So, this is the region of possibility for all possible combination of correlations and w 1, w 2 weights. An important observation here is that the risk of the portfolio for a given level of returns is sometimes even less than the least risky security in the portfolio, so the risk is sometime even less then. For example this is the least risky security, then there are certain combination where risk is even less on the left of it.

And this is particularly the case when the correlation between the securities is very low, for example on this side here correlation is low, on the side we are expecting low correlation points. In actual terms, there may be a security that exhibits returns outside this region of possibilities, then what happens? Surely expectations cannot stay away from the actual returns for a long time if markets are reasonably efficient.

That means if any securities lying outside of this feasibility region, then market forces of arbitrage and market efficiency will drive down these points or these returns back to the normal reverse by correcting the points. So, there will be a correction witnessed in the point that will drive down the observed level of prices to align them with the expected returns.

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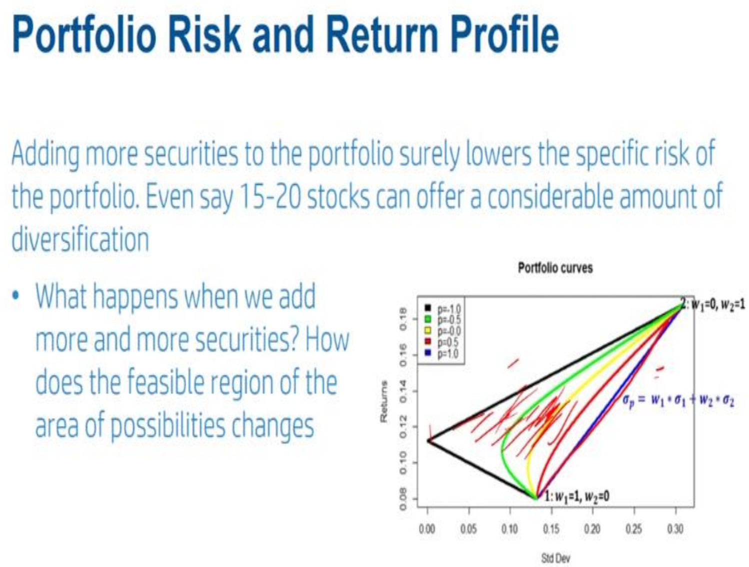

So, we have seen that adding more and more securities to the portfolio here surely lower the specific risk of the portfolio. Even with 15 to 20 stocks, we can achieve a considerable amount of diversification by neutralizing the stock specific or variance risks. What happens when we add more and more securities to this two-stock portfolio? How does the feasible region as we saw here earlier, this region changes, which is the area of all the possibilities.

In the next video we will discuss these possibilities when we add more securities to this two-stock portfolio how this feasibility region changes. To summarize, in this video we saw the mean variance framework of Harry Markowitz. We found that for correlation equal to 1, there is no diversification in risk and the total risk of the portfolio is simply the weighted average of individual risk or standard deviations of the two securities, weights being the proportionate amount invested in these.

We also saw that for a theoretical and hypothetical case where correlation is strictly equal to –1, maximum diversification is achieved which is represented here by two line segments in the black colour. Theoretically, there is a possibility of a point where overall risk becomes 0 for a given correlation equal to –1, however, that is more of a theoretical possibility. For all the possible combinations of correlation and proportionate amounts invested in the portfolio, the feasibility region lies between these two curves.

That is between this blue which is corresponding to correlation equal to 1 and the black curve which is corresponding to correlation equal to –1. So, all the feasible points lie in this region. Any other points that are outside this curve, whether on this side or this side are not possible and if there is a stock for which prices are such that that the stock lies on this point, market forces of arbitrage and market forces of efficiency will drive that point or price of that security within this region in a short frame if the markets are reasonably efficient.

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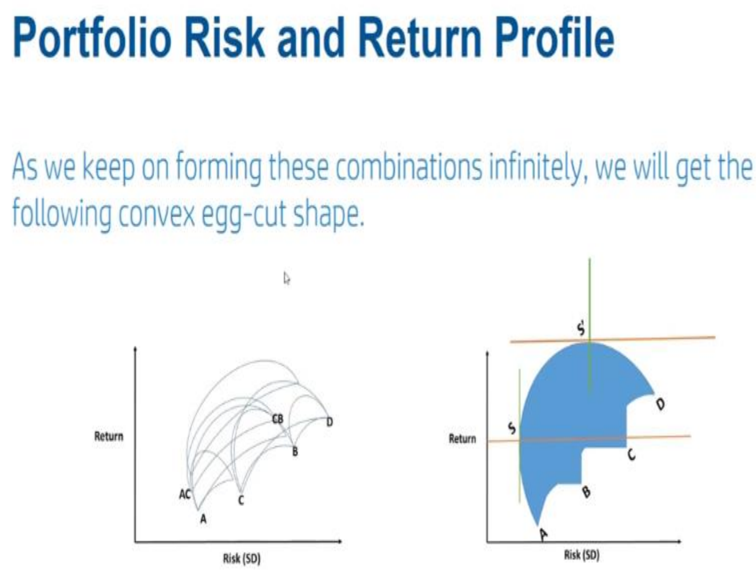

In this video, we will talk about the feasible region and various portfolio possibilities with multi-security case. We start with four hypothetical portfolios, different combinations of A, B, C, D that means A, B, C, D, AC and so on with different investment mixtures can lead to large number of portfolio possibilities as shown here. These possibilities are shown in the form of curves in the figure.

As we add more and more securities, we get the first diagram like a net cut shape. As we keep on forming these combinations infinitely, you can add more and more securities in addition to these showed securities, we will get the convex egg cut shape as provided here.

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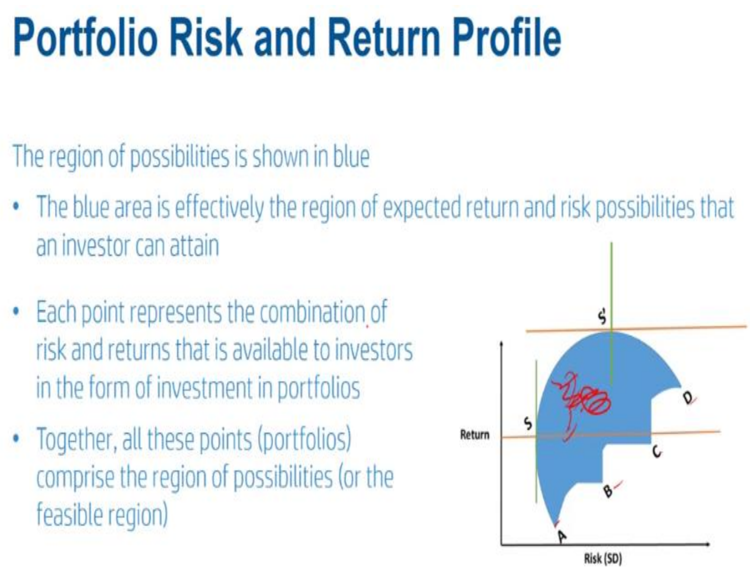

This region of possibilities is shown here in blue. The blue area is effectively the region of expected return and risk possibilities that an investor can attain. Each point here represents a combination of risk and returns that is available to investors in the form of investment into portfolios. Together, all these points or portfolios comprise a region of possibilities or the feasible region as you can see here.

This is your feasible region of all the possible different combinations. We started from four securities A, B and C and D, then we kept adding more and more securities to obtain this region of possibility.

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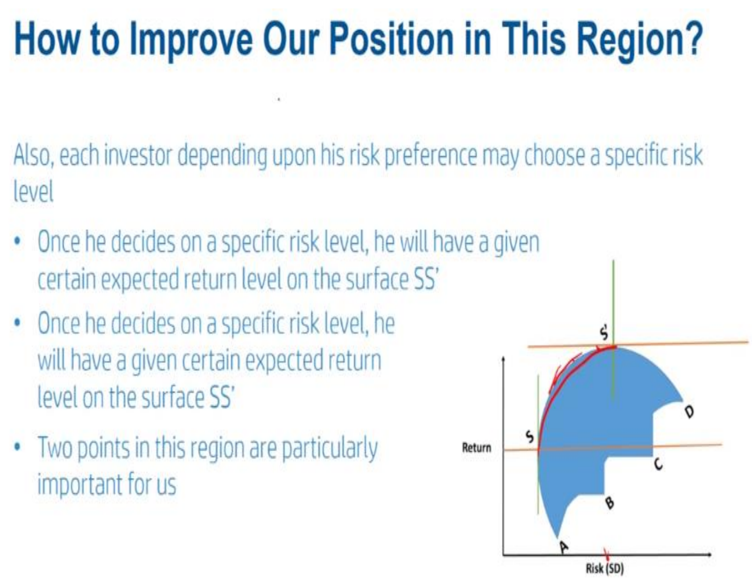

Now, on this region of possibility, our goal is clear, we want to move up or increase the returns that means we want to move in this direction where for a given risk we are increasing the expected returns and we want to move to left. That means for a given level of return, we want to reduce the risk. As we do this, we reach the top surface of the region of possibilities which is this surface or we can say S S dash on this region of feasibility.

There are no more points where we can move further left or up on this curve. So, this is the extreme available to us the best possible combination and that is why this region is called efficient frontier. All the points on this region offer the highest return for a given level of risk or lowest risk for a given level of return. For example, if you pick this level of return, the lowest amount of risk available for all the points available in feasible region would be on this that is on the surface S S dash.

Similarly, if you pick a level of risk, let us say this level of risk, the maximum level of return available could be on this S S dash line here. Now, on this S S dash or efficient frontier, each investor depending upon their risk preference may choose a specific risk level. For example, investor may choose this point if he prefers this level of risk or if he prefers a lower level of risk, they can move left or left on this particular efficient frontier.

Once the investor decides a specific risk level, they will have a given level of fixed return on the surface S S dash. So, we hope and we can clearly see here that once the investor reaches the surface of risk return, they cannot do any better by moving left or up.

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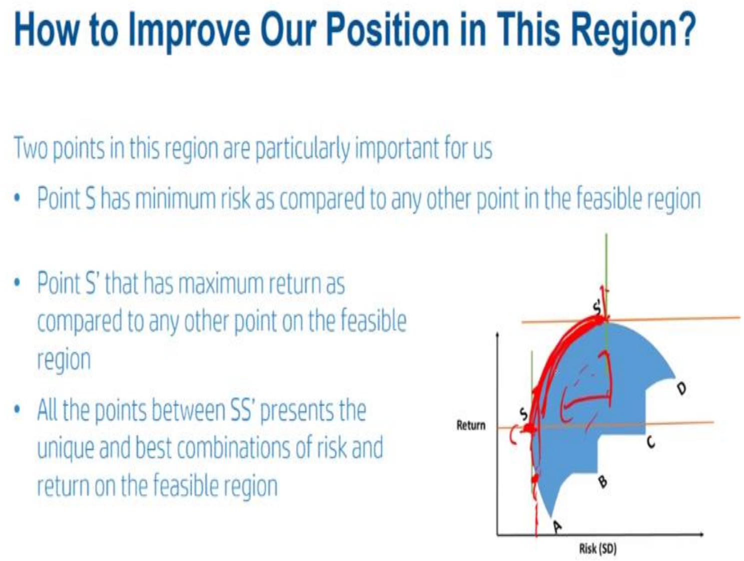

Now, if you focus on this efficient frontier which is S S dash, two points are particularly important for us here. First point S, this point is the minimum risk point. So, if you look at this efficient frontier on the feasible region, this point has the minimum risk. There is no point on the left of this in the feasible region, so this is the leftmost point and therefore represents the point of minimum risk. Similarly, point S dash which is at the top of it is the point of maximum return.

So, there is no point on this feasible region which is above this point S dash and therefore this point offers you the maximum return. All the combinations of this surface S S dash are called the efficient frontier on this entire feasible region because these points offer you the best combinations of risk and return possible in the entire feasible region. So, this becomes our efficient frontier.

One interesting point to note here or observe that if you go below this point as there are some points that offer you lower return, but these points are not efficient because for this level of risk you can also obtain this level of return. So, as compared to this point you would like to be on this point. So, in this manner the S S dash means the efficient frontier, the best combinations of risk-return that are available to you.

To summarize, in this video we saw that as we keep on adding multiple securities to a portfolio, we obtain a region of possibilities which is called feasible region as shown here in the blue. This appears to be an egg-cut shape. In order to improve our position or profile on this feasible region, we would like to move on the left to improve or lower our risk for a given level of return or we want to move up to increase our expected return for a given level of risk.

As we keep on doing that, we obtain a region called efficient frontier or a surface which is the efficient frontier which offers us the best possible combinations of risk-return scenarios. And this surface or this region which is called efficient frontier start from the point with the minimum risk, global minimum risk portfolio which is S here and the portfolio which offers the maximum expected return S dash here and this S S dash presents us the efficient frontier.

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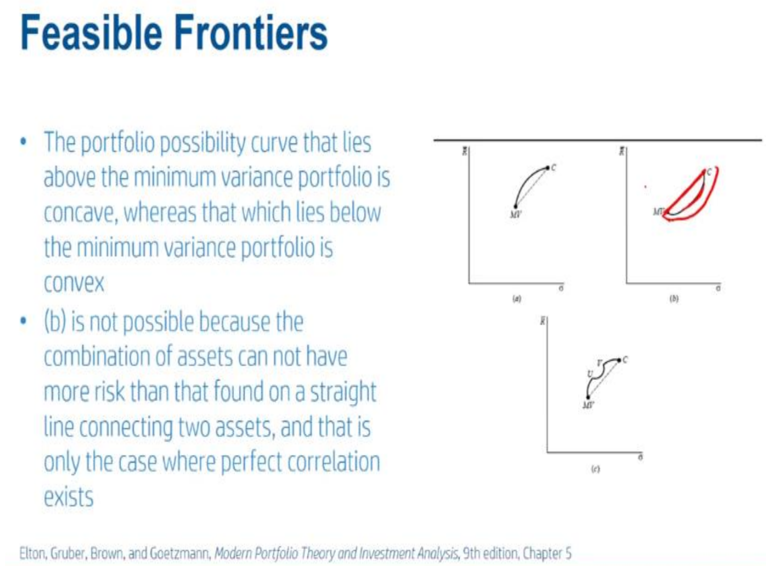

Feasible frontiers: In this video, we will examine the shape of efficient frontiers that are feasible. We will also discuss some of the shapes of efficient frontiers that are not feasible. Look at the possibility curve shown here. From what we already know, the figure b is not a possibility because it is convex in nature. Notice the convex nature of figure b here. We already know that there cannot be points to the right of this line joining MV and C.

We know that there cannot be any points to the right of line joining global minimum variance portfolio which is MV and C here which is above MV by so because for any point that lies on this curve we will have a risk which is higher than those points that have perfect correlation which is equal to 1 that fall on this line. And therefore, in that case the points on the convex curve will have more risk or higher standard deviation than the straight line.

But we already know that the straight line joining MV and C corresponds to the highest risk which is correlation equal to 1 and hence it is not possible.

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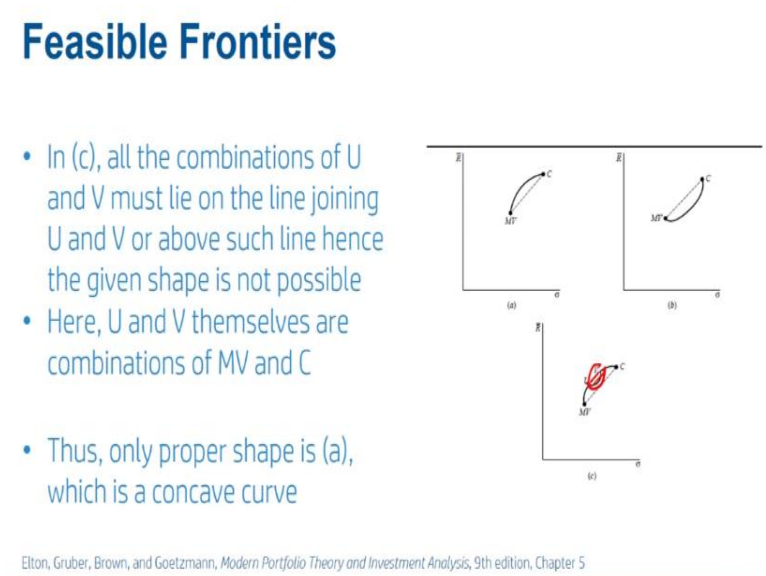

What about figure c, this figure? Please note again our reasoning remains the same, U and V are also portfolios. So, any portfolio that can be found by combining U and V would look something like this and therefore any shape which is like this convex shape like this should not be possible. So, the straight line joining this U V point will be one extreme set of portfolios with the highest risk and therefore the shape of the curve joining U V can only be at max or at best a straight line or a concave or not this kind of convex curve as we have discussed and therefore this figure c is also not a possibility.

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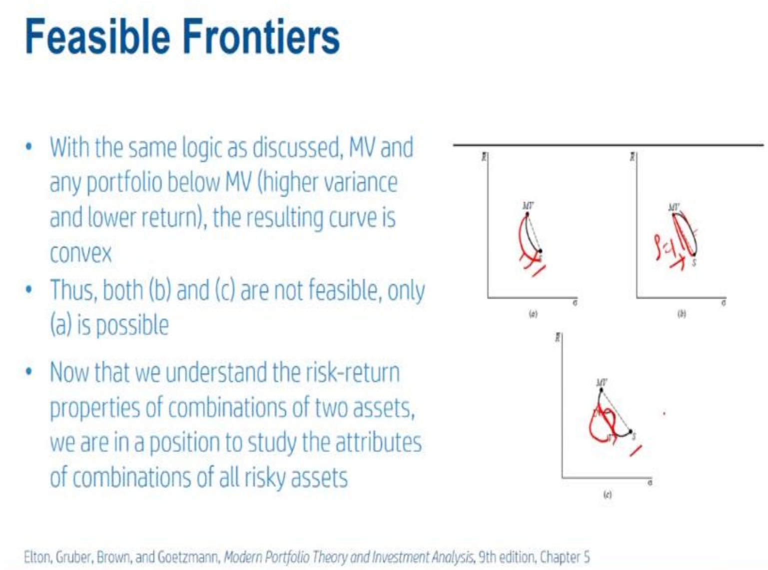

Let us also now discuss the points that are below the minimum variance frontier like here are shown in a, b and c. With the same reasons as we discussed earlier, only a is the feasible option. This one is the feasible one, the curve below the point M V should be convex like this and therefore figure b is also not possible because it would again represent those points that are having higher risk than the straight line with correlation equal to 1.

Now, if U and V are combinations of MV and S, for example, these U and V they are combination of MV and S. If the portfolios U and V are mixed the combination can be a convex curve or at max, a straight line joining U V like this. So, it can be like this or the straight line joining there. It cannot be a concave curve as shown here. So, it cannot be a concave curve and therefore c is also not a possibility. So, now we have understood the risk return profile of a combination of two assets and how feasible frontier would look like.

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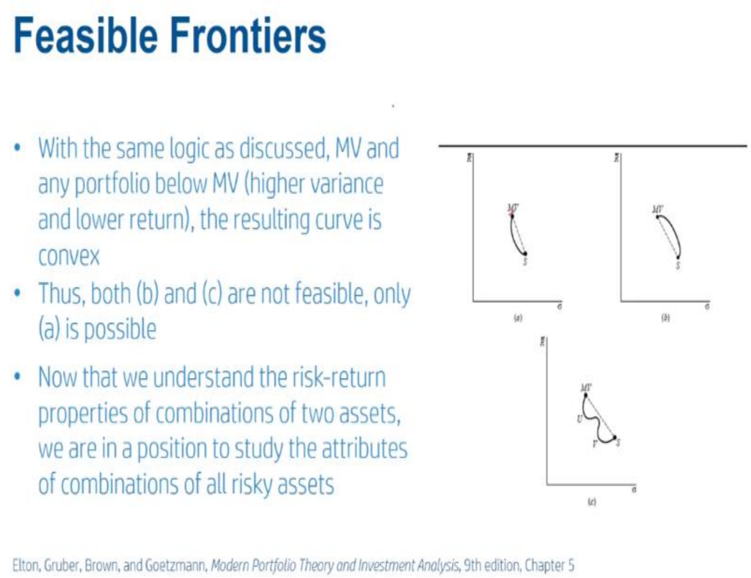

To summarize, in this video we discussed the shape of feasible frontiers. For those points that are lying above the minimum variance portfolio MV, this MV, the shape of efficient frontier should be a concave curve that is towards the left of the straight line joining point MV and S which suggests that there cannot be any portfolio with higher risk for a given level of return.

Similarly, for those points that are below MV, the curve has to be convex and not concave, the shape of feasible frontier should be convex not concave because all the concave curves would indicate those points that have higher risk for a given return than the correlation equal to 1.

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In this video, we will talk about different efficient frontiers for multi-security case, first without short sales and then with short sales.

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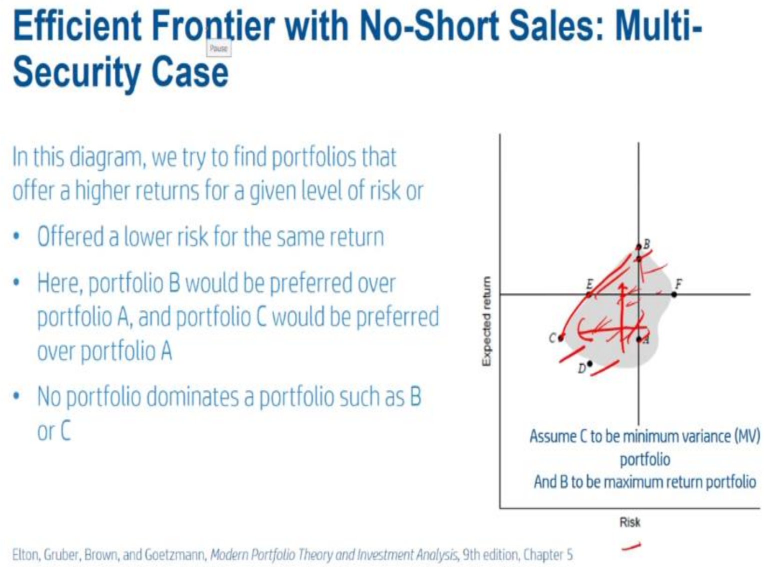

Let us start by examining a case with multiple securities that are distributed in a rather heterogeneous manner as shown in the figure here. So, here we have six securities. These are A, B, C, D, E these six securities, they are with different combinations are plotted here on risk and expected return space. Before we start our discussion, let us remember two important goals, one higher return for a given risk.

That means for a given amount of risk you would like to have higher expected returns that is you would like to move up and lower the risk for a given level of return that is you would like to move on the left. These are desirable portfolio characteristics. Now, if we compare portfolios A and B here, portfolio A and B in this possibility space which is this portfolio possibility space B of course would be preferable to A since it offers a higher return for the same level of risk as we can see here.

Similarly, C would be preferable to A because it offers same level of return with a lower level of risk. So, it may appear to us by this logic that no portfolio dominates portfolio C, B or for that matter portfolio E. So, these are the dominating portfolios. This kind of elimination process where we compare the portfolios for their risk-return profile, we can easily see that portfolio C, B, E they cannot be eliminated and therefore they are part of the efficient frontier.

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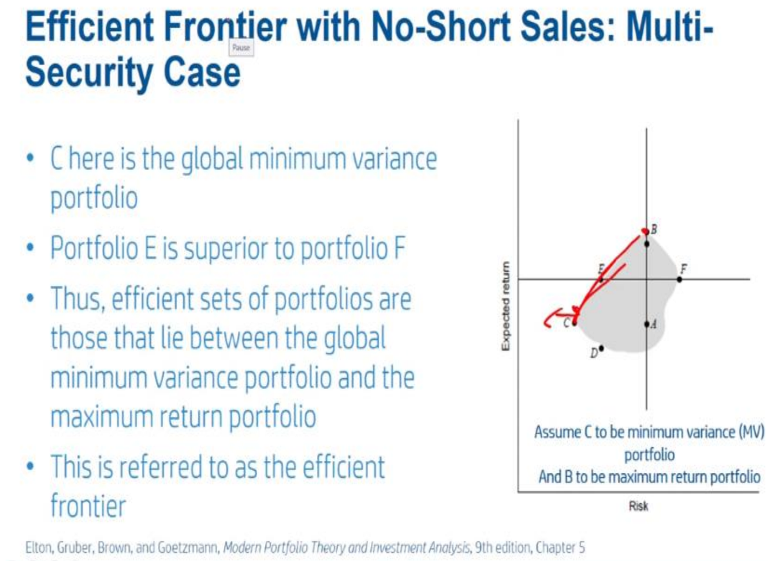

Also, we can clearly see here that C is the point of global minimum variance that means there is no portfolio that has lower risk that is on the left of this point. So, no point on this feasibility region is on the left of point C, so no other portfolio has a lower risk than this portfolio C. And therefore, we can say this surface CEB would comprise the set of portfolios that offer the highest return for a given level of risk or alternatively the lowest risk for a given level of return.

Therefore, this curvature, the set of portfolios that lie on this curvature CEB are called efficient set of portfolios or this is our efficient frontier. Here we also find that the C portfolio is the portfolio with a minimum variance and Portfolio B is the portfolio with maximum return, also we can call C as global minimum variance portfolio.

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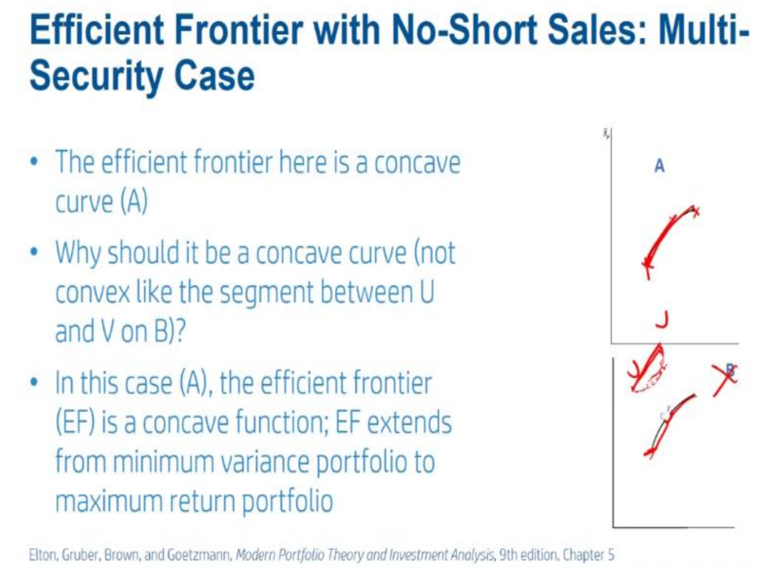

Now, if we take out of the portion of this efficient frontier, it would look something like the graph shown on A, this A curvature. We know from our previous discussions that the shape can only be concave, not convex. The lower end of this frontier represents the portfolio with the minimum variance, this one the minimum variance, the higher end of this portfolio is the portfolio with maximum return across all the portfolio that is security with the highest returns.

Please remember, in this case we cannot have a portfolio looking like B as we have discussed already. Any convex nature like between U and V is not possible why? Because U and V are also portfolios and therefore we have already discussed if all the points are above minimum variance frontier that is above this minimum variance point, any two portfolios U and V their combination can be at best a straight line.

And in that case the straight line would represent portfolios with correlation of 1 that is maximum risk. So, any portfolio on the right side of it would represent a set of portfolios that carry higher risk than those with correlation of 1 which is not possible and therefore any set of portfolios on the right side would not be possible and therefore this convex nature of curve is not possible. Therefore, B is not a feasible scenario. The only feasible scenario for efficient frontier in this case is this one.

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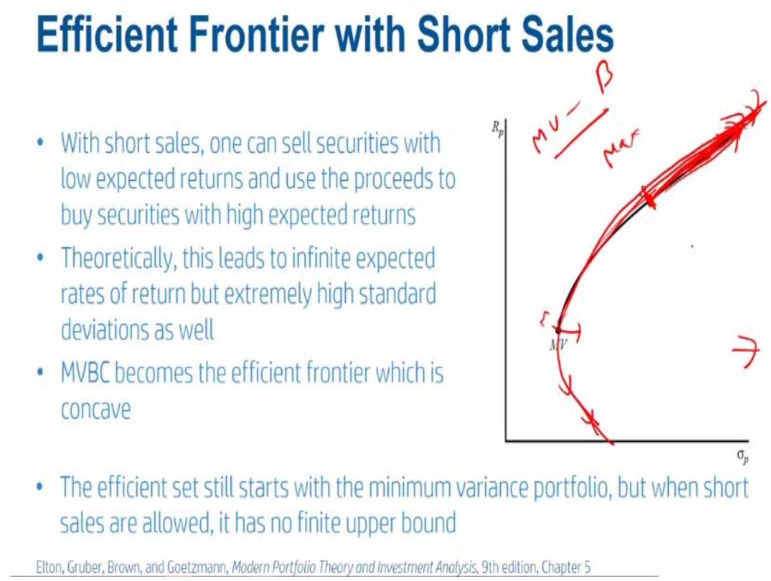

Now, let us talk about the case where short sales is allowed. Short Sale is essentially selling a security one does not own, which is equivalent to taking a negative position in the security. This negative position allows one some additional liquidity or funds that one can invest further in taking a long position or investing in any other purposes. Now, if you are making the short selling, you would like to sell securities that have lower expected returns.

And you would like to buy securities from those proceeds where you have higher expected returns and therefore those not so efficient securities which are lower in the efficient frontier here, you have this opportunity now to short sell these securities because these securities are offering us lower expected returns. This suggest that one can keep on short selling securities that are here while investing in long securities that are here and extending their position beyond B to C and further high.

Theoretically, this suggests that one can achieve infinite expected returns or very large amounts of expected returns, but at the same time when you do that, you also extend the risk of your position that is you also extend the risk of standard deviation to very large levels. And therefore, one can see that now our actual efficient frontier is not restricted at point B and one can take positions that can go beyond B and even further and further.

So now, on aggregate our position becomes MV B and beyond to C. So one can extend their portfolio possibility curve to MV B C which becomes the new efficient frontier and notice that this is a concave curve, which is similar to earlier without short sale scenario, but now with short sales. Our efficient frontier extends from MV not only moves up to the security with maximum return, but it also extend beyond that to large expected returns which have no finite upper bound.

However, please note the caveat, in order to achieve that, we are able to or we are assuming a large amount of short sales, every market have certain restrictions on short sales, but still to a reasonable extent we can assume that one can extend their efficient frontier beyond this point B which was point of maximum return to further with no upper bound. To summarize, in this video we discussed the shape of efficient frontier with and without short sales.

We found that in the absence of short sales, the efficient frontier extends from portfolio of minimum variance to portfolio with global minimum variance to portfolio of maximum return, in this case and MV to B. However, when the restrictions on short sales are removed, one can short sell securities with poor expected return and invest them in higher expected returns that are on the efficient frontier and extend their possibilities beyond this upper point B.

And in that case, the expected returns would be very higher with no upper bound on this efficient frontier. However, at the same time if one were to obtain such positions, the risk or standard deviation of such positions would also be very high.