Artificial Intelligence (AI) for Investments

Prof. Abhinava Tripathi
Department of Industrial and Management Engineering
Indian Institute of Technology, Kanpur

Lecture - 02

Cash flow discounting. Till now we have understood that corporations want to maximize their value and share price. This requires them to invest in real assets that are worth more than their cost. However, to understand whether an asset is worth more than its cost, we need to develop an understanding of valuing these assets. In some cases, like real estate or shares of a company there is an active market available.

With the help of this market the value of that particular asset can be easily estimated. However, for many real assets for example plant and machinery such markets do not exist. Therefore, such assets need to be valued in a more fundamental manner. Moreover, even those assets where a full functioning secondary markets like stock markets exists it often adds to the analysis if they are valued in a fundamental manner.

In this lesson we will understand the concept of time value of money and cash flow discounting, one of the most important consideration in cash flow discounting is estimating the discount rate or opportunity cost of capital. We will discuss and understand the application of opportunity cost of capital and discounting cash flows. We will learn how to compute present values of annuities and perpetuities with constant cash flows and cash flows with a steady growth rate.

We will also discuss the concept of compounding with a particular focus on continuous compounding. We will understand the practical application of these concepts through various numerical examples.

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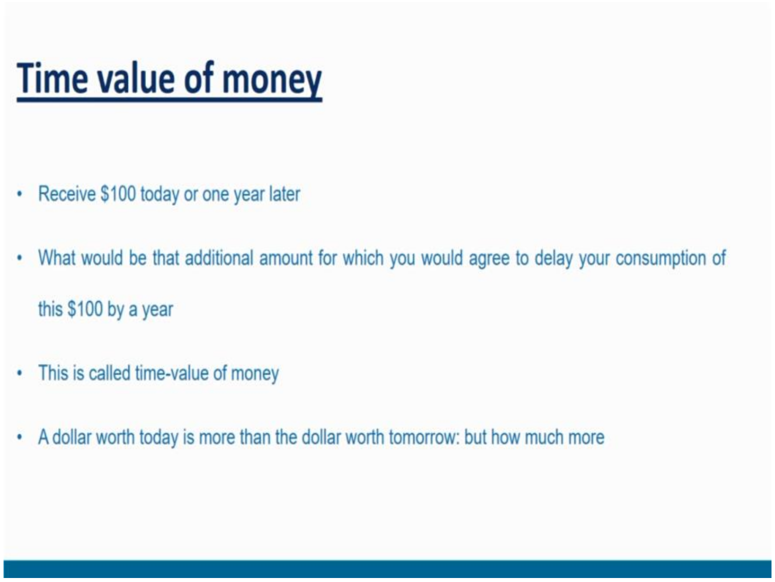

Time value of money. In this video we will discuss the basic theory behind the concept of the time value of money. We will also introduce the concept of cash flow discounting to compute present values from future values. One can invest money to earn an interest so it is not very difficult to see that if you have a choice to receive 100 dollars today or 100 dollars one year later you would prefer to receive 100 dollars today.

In case you are required to wait for one year to receive this money back you would require some additional amount for this delay in your consumption of 100 dollars. Theoretically this additional amount often called the interest should reflect the cost of delaying your consumption by one year this is called the concept of the time value of money. The fundamental concept here is that a dollar today is worth more than dollar tomorrow but how much more? All the discussion in this lesson pertains to quantifying this number.

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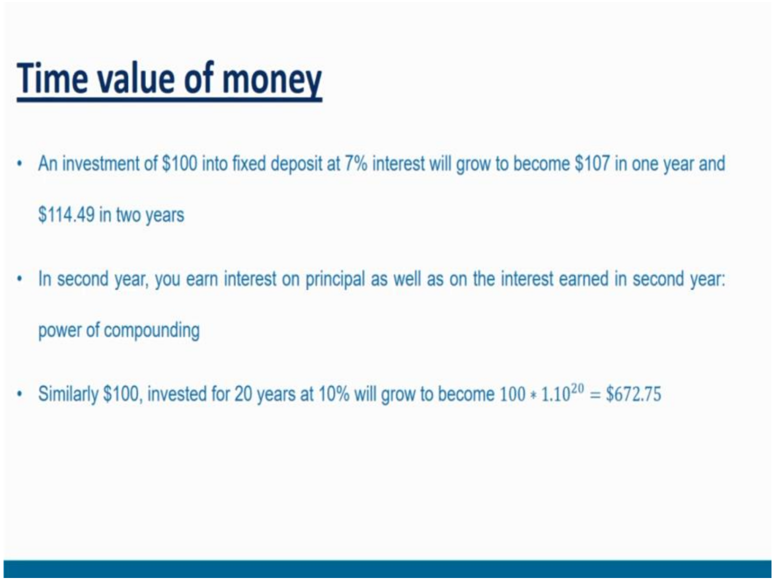

Consider investment into fixed deposit of a bank that pays an interest of 7 percent per annum. So, by the end of the year you will earn an interest of 7 percent multiply by 100 dollars = 7 dollars. So, your overall investment will grow to become 107 dollars, this is a simple but a very powerful and fundamental concept, by the same argument your investment in year 2 will become $100 * 1.07^2 = \$114.49$.

Also please notice that in the second year you are earning interest on the principal amount as well as the 7 dollars interest earned in the first year. Thus, your wealth grows in a manner called compounding, the interest earned through this process is called compounding interest. So, this will help us in defining the future value concept.

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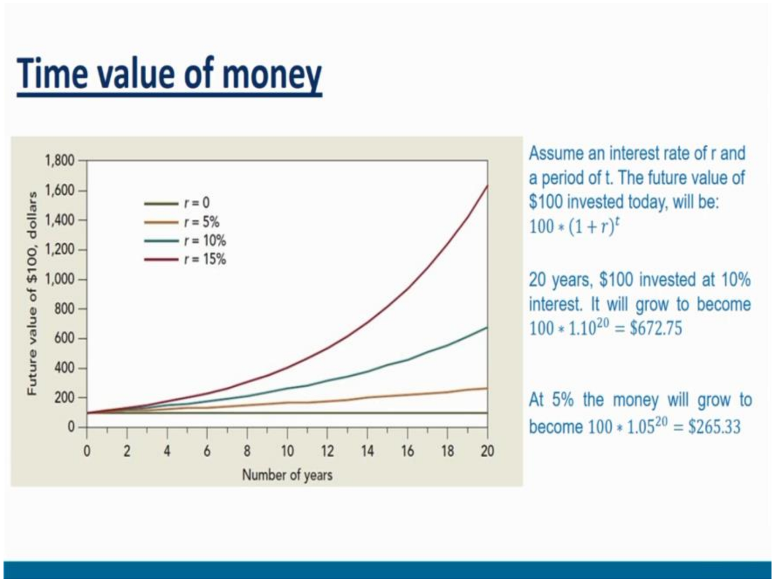

Assume an interest rate of r in a period of t, the future value of hundred dollars invested today will be $100 * (1 + r)^t$, you can also notice here as the value of r or t increases the future value of your wealth goes up exponentially. Consider the figure shown here for changes in interest rates and duration of investment and how the future value of your initial 100 dollars increases.

Consider the following two cases 100 dollars invested for 20 years at 10 percent interest. It will grow to become $100 * 1.10^{20} = \$672.7$. At 5 percent interest rate, the money will grow to become $100 * 1.05^{20} = \$265.33$ .Often this r is the interest rate that is available to you in instruments such as bank FD's and financial markets, remember the concept of opportunity cost.

To summarize in this video, we discussed that the dollar today is worth more than a dollar tomorrow. Because of this basic principle, cash flows received in future need to be discounted to compute their equivalent present values today. This discount factor $df = (1 + r)^t$
here r is the appropriate rate of discounting often referred to as the opportunity cost of capital or hurdle rate and t is the time horizon.

An increase in this interest rate results in a lower PV and an increase investment horizon also results in our lower PV, a larger interest rate reflects a higher preference for consuming now than later. Also, a higher t means delaying the consumption further away and therefore a higher cost of delay in consumption, this also means a lower present value PV.

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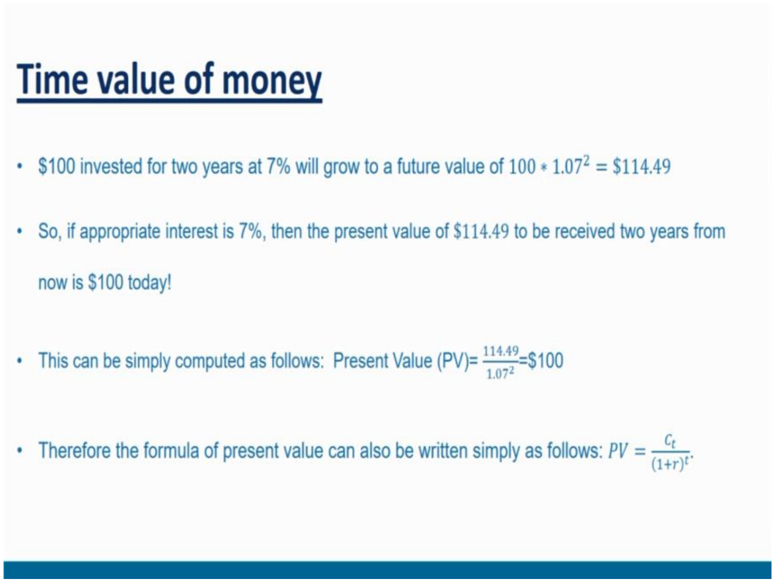

In this video we will extend our understanding of the time value of money and cash flow discounting to compute the net present values. With the help of examples, we will also see how the appropriate discount rate that is opportunity cost of capital is estimated. The previous discussion should also help us in developing the logic for computing the present values for given future values.

Consider this another example 100 dollars invested for two years at 7% will grow to a future value of $100 * 1.07^2 = \$114.49$. So, if this future value of 114.49 dollars is known to us along with the interest rate and the time duration then, we can also compute the present value corresponding to this future value. This can be simply computed in the following manner present value $PV = \frac{114.49}{1.07^2} = \$100$.

Therefore, the formula of present value can also be simply written as provided here $Present\ value = C_t/(1+r)^t$

where C_t's are the cash flows at the end of year t, r is referred to as the discounting rate of interest. The term $\frac{1}{1+r}$ is often referred to as the discounting factor. This expression shows the present value of 1 dollar received in year t, in this case the value of this discounting factor $df = \frac{1}{1.07^2} = 0.87$

This is the value of one dollar received two years from today and at an interest rate of 7%. So, one can multiply this discount factor with the future value of money to arrive at the present value. For example, if the future value is 114.49 dollars then the present value is (d f *114.49) = 100 dollars.

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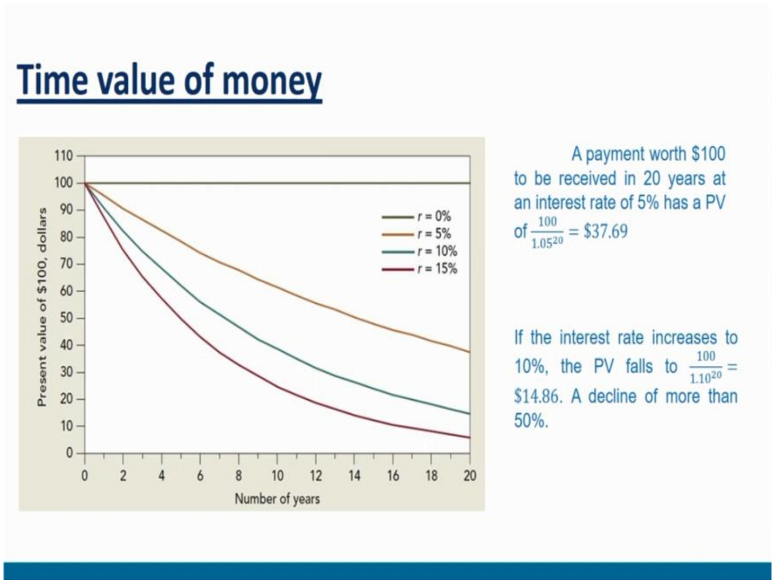

From this PV computation it is quite obvious that the longer you have to wait for your money the lower its present value PV, even small variations in interest rate can have a considerable effect on the present value particularly for long durations. Consider the example in the figure shown here, a payment worth 100 dollars to be received in 20 years at an interest rate of 5 percent has a PV of $\frac{100}{1.05^{20}} = \$37.69$.

If the interest rate increases to 10 percent the PV falls $\frac{100}{(1.10)^{20}} = \$14.86$, a decline of more than 50 percent.

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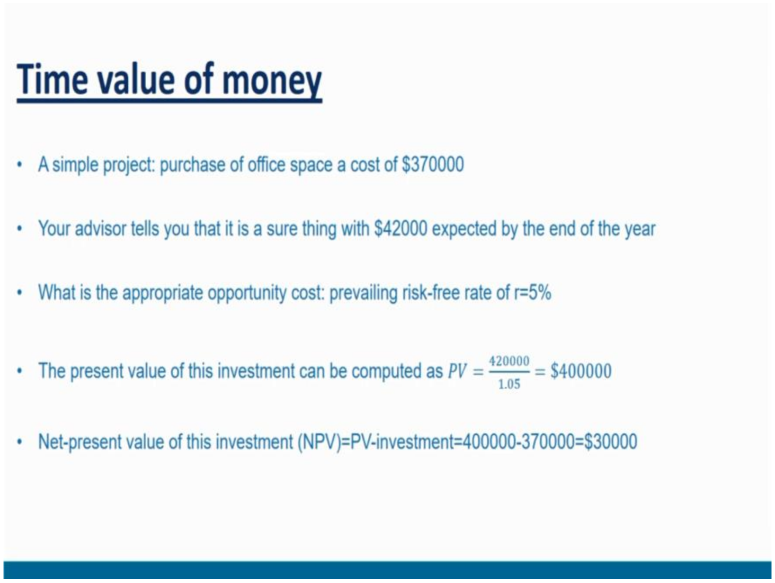

Let us discuss a simple example on how to decide whether investment opportunity is worth undertaking. Suppose you want to purchase an office space and fit outs worth 3,70,000 dollars. Your advisor tells you that if you buy this building today you will be able to sell this within a year at 4,20,000 dollars. He tells you that this is a short sale with almost no uncertainty. If you really believe in his argument what would be an appropriate rate to discount the cash flows.

Remember our opportunity cost of capital concept, instead of investing in this office space you can invest the money in a financial market instrument that has the same risk as this building. Since you believe in your advisors argument and consider this building risk free you would consider the risk free rate available on government securities as the appropriate interest rate. You can invest in risk-free government securities through money market mutual funds.

Also, you can consider bank FD’s to be an instrument closer to a risk free rate. Assume that this risk-free rate available to you is 5 percent, then the present value of this investment to you can be computed as shown here present value equal to $4{,}20{,}000/1.05 = 4{,}00{,}000\ dollars$. This is interest rate r which is often called as discount rate hurdle rate or opportunity cost of capital. In this case you can compute something called net present value of your investment.

The net present value NPV of your investment is NPV = present value - investment = 4,00,000 - 3,70,000 dollars = 30,000 dollars. These 30,000 dollars is the value that you would generate by investing in this office space. If everything works out as planned then you are richer by 30,000 dollars in present value terms.

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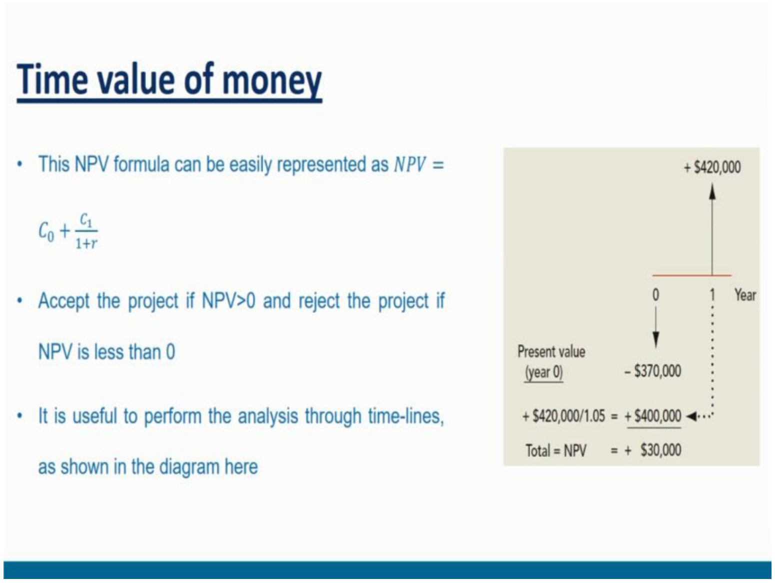

In this particular case the NPV formula can be easily represented as net present value $NPV = C_o + \frac{C_1}{1+r},\ C_o = initial$ investment at time t = 0, C_o will be negative because it is investment, $C_1 =$ the cash inflow at the year end. The decision rule here is that you will accept the project NPV is greater than 0 and reject the project if NPV is less than 0. Many times, things are not simple as presented here.

For example, there may be cash inflows and outflows occurring at different times. In such cases it is useful to perform the analysis through timelines as shown in the diagram here. To summarize in this video, we discussed how to compute the NPVs of project investments that is real assets. We also discussed how to estimate the appropriate opportunity cost of capital by examining the instruments of similar risk available in financial markets.

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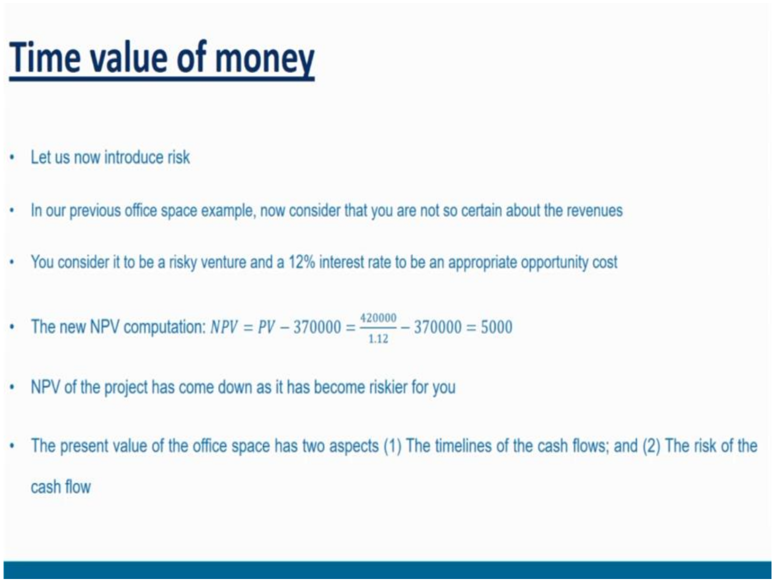

In this video we will examine the role of risk in cash flow discounting, we will also discuss the competition of NPVs for risky projects and the appropriate decision rule for selection and rejection of projects. Let us introduce risk. In the previous example we assumed that the sale value of the office space was certain, let us consider another case where you are not so certain about these cash flows.

In that case the interest rate or opportunity cost for you is not comparable to that of risk free government securities. Should it be more or less? To answer this important question, you have to consider a basic financial principle that a safe dollar is worth more than risky dollar. Investors demand a high return for the instrument that carries more risk this comes from the basic principle of risk averseness.

A normal investor is considered as a rational risk averse person that means he wants incremental returns for bearing more risk. Therefore, your opportunity cost of capital and the appropriate discount rate for this instrument would be higher. Let us suppose that you consider a stock available in financial markets with similar risk as that of your investment in office space building. This stock offers an expected return of 12 percent.

Thus 12 percent is the appropriate discount rate or hurdle rate or opportunity cost of capital for you that is if you are investing in the office space building you will not be able to invest in the stock of the same risk and you are foregoing this return. The appropriate net present value NPV for this project with appropriate risk is now computed as follows net present value NPV = present value (PV) - 3,70,000 dollars = (4,20,000/1.12) - 3,70,000 = 5,000.

Notice that your profit has come down significantly from the earlier risk free scenario that also means that the NPV of the project has come down as it has become riskier for you. Its contribution to your overall wealth has come down and has become much less than that was earlier. In real life projecting the cash flows and risk is easier said than done, therefore estimation of the discount rate or opportunity cost of capital is rather more difficult.

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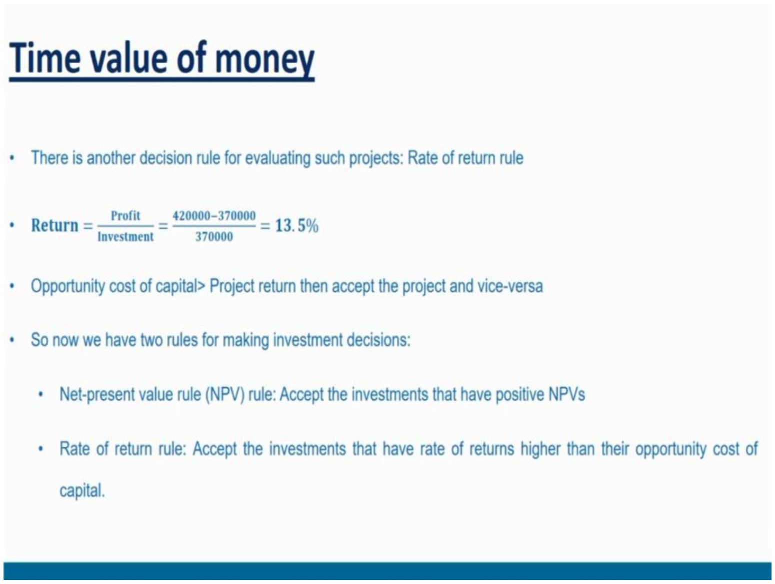

There is another rule for evaluating such projects this is called rate of return rule that we will discuss now. We all know that simple written computation process for example if the previous office space project that we discussed earlier the initial investment was 3,70,000 dollars and the payoff was 4,20,000 dollars. The simple return on this project can be easily computed as follows, return = profit / investment = (4,20,000 - 3,70,000) / 3,70,000 = 13.5%

It appears that this project offers a return of 13.5%. Therefore, if its opportunity cost of capital that is the written foregone by not investing in financial markets is less than 13.5% then this project seems to be profitable. For example, we said that the appropriate opportunity cost for this project was 12 percent to which is less than the project returned. So, as per this rate of return rule we should go ahead with the project.

To summarize in this video, we found that a risky dollar is worth less than a risk-free dollar, we also discussed how this increase risk is reflected in a higher opportunity cost of capital and thus a lower discount factor d f. So, now we have two very important rules for making investment decisions. One, net present value rule and PV rule accept the investments that have positive NPVs and vice versa.

Second, rate of return rule accept the investment that have the rate of returns higher than their opportunity cost of capital and vice versa.

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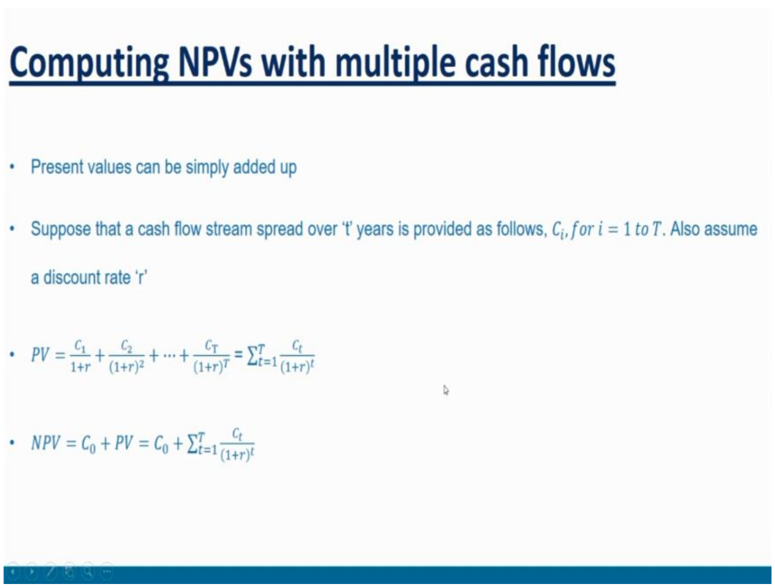

Computing NPVs with multiple cash flows. In this video we will examine the computation of present values for complex cash flows structured over different years. We will also consolidate our understanding of NPV computation with the help of examples. Present values are often expressed in current dollars terms that means they can be very simply added up, consider a stream of cash flows C_i is spread over t years as shown here for i = 1 to T years.

Also assume a discount rate r. The present value of this set of cash flows can be simply represented as shown here present value

$$PV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_r}{(1+r)^T} = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

$$NPV = C_o + PV = C_o + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

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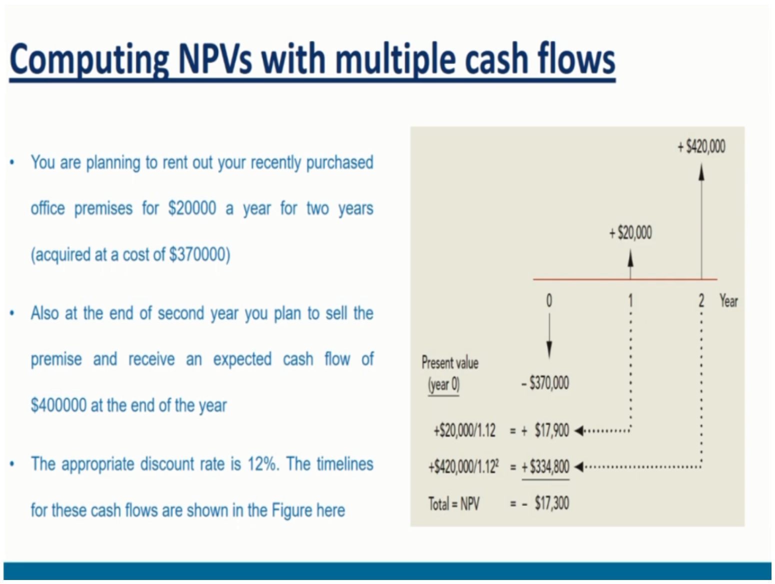

Consider simple example, you are planning to rent out your recently purchased office premises for 20,000 dollars a year for 2 years acquired at a cost of 3,70,000 dollars. The rent will be received at the end of the year. Also, at the end of second year you plan to sell the premises and receive an expected cash flow of 4,00,000 dollars at the end of the year. The appropriate discount rate is 12%. The timelines for these cash flows are shown in the figure here.

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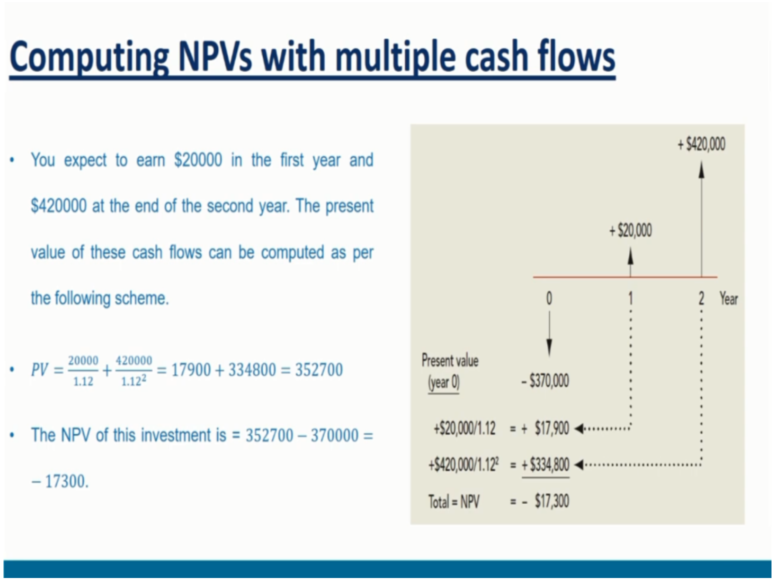

You expect 20,000 dollars in the first year and 4,20,000 dollars at the end of the second year. The present value of these cash flows can be computed as per the following scheme,

$$PV = \frac{20000}{1.12} + \frac{420000}{1.12^2} = 17900 + 334800 = 352700$$

The net present value of this investment is 3,52,700 - 3,70,000 dollars = (-)17,300 dollars.

The negative net present value NPV indicates that this project should not be considered as it is has a negative contribution to the present value of your wealth. By investing in this office building, you gave up the opportunity to earn 12 percent in the stock market. Therefore, your opportunity cost of capital is 12 percent, essentially when we are discounting the expected cash flows from the project, we are trying to estimate the value of a security in financial markets that has the same risk and produces the same stream of cash flows. The computation suggests that an investment that produces 20000 dollars in year one and 4,20,000 dollars in year two has a present value of 3,52,700 dollars if the opportunity cost of capital is 12 percent. Now consider a scenario where your banker approaches you and tells you that she can fund your project at 8% interest loan, does it mean your opportunity cost of capital is 8 percent now?, No it is not.

You can still borrow at 8 percent and invest the money in financial markets at 12 percent in the instrument which is of similar risk as your project. If another project offering 14 percent but with similar risk is available to you, you would rather invest in that project and therefore 14 percent would be your opportunity cost. To summarize in this video, we have applied our understanding of cash flow discounting technique to compute net present values of projects with cash flow spread over different years.

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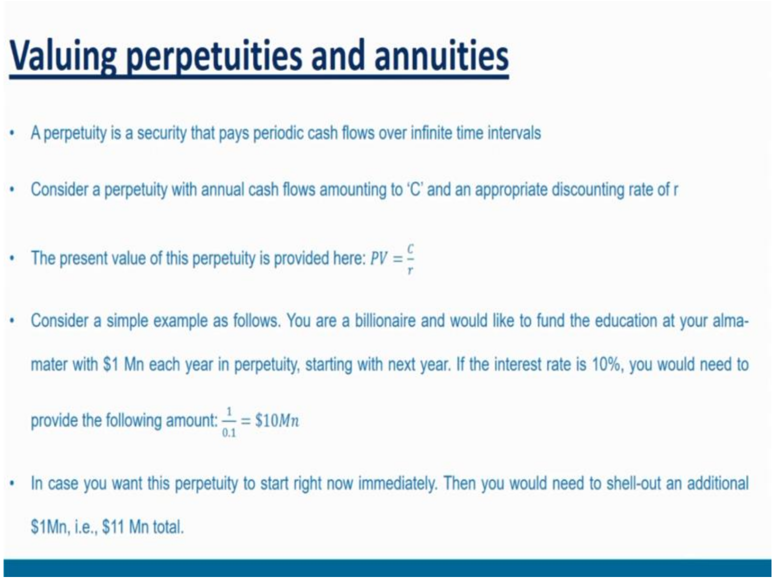

Valuing perpetuities and annuities. In this video we will discuss the computation of perpetuities and annuities. A perpetuity is a security that pays periodic cash flows over infinite time intervals. We will consider annual perpetuities. Consider a perpetuity with annual cash flows amounting to C and an appropriate discount rate of r. The present value of this perpetuity is computed with the help of the infinite geometric progression formula as provided here.

Present value $PV = \frac{C}{r}$. Consider a simple example as follows you are a billionaire and you would like to fund the education at your alma-mater with one million dollars each year in perpetuity starting with the next year. If the interest rate is 10 percent you would need to provide the following amount 1/0.1 = 10 million dollars. In case you want this perpetuity to start right now immediately then you would need to shell out an additional one million dollars that is 11 million dollars total.

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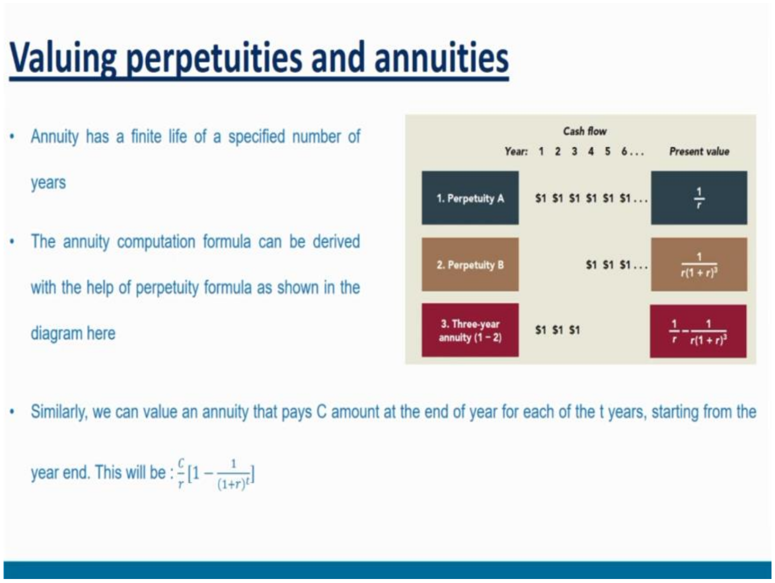

A similar concept here is that of annuity. However, annuity has a finite life of a specified number of years. The annuity computation formula can be derived with the help of perpetuity formula in a simple manner. This is shown in the diagram here, the diagram shows the value of a perpetuity that is of one dollar per year that starts today that is $\frac{1}{r}$. If the perpetuity started from the fourth year, then the value of this perpetuity would have been $\frac{1}{r(1+r)^3}$

The difference between these cash flows that is $\frac{1}{r} - \frac{1}{r(1+r)^3}$ is essentially the value of a three year annuity that pays one dollar at the end of years one, two and three. Similarly, we can value an annuity that pays C amount at the end of the year for each of the t years starting from the year end this will be $\frac{c}{r}[1 - \frac{1}{(1+r)^t}]$

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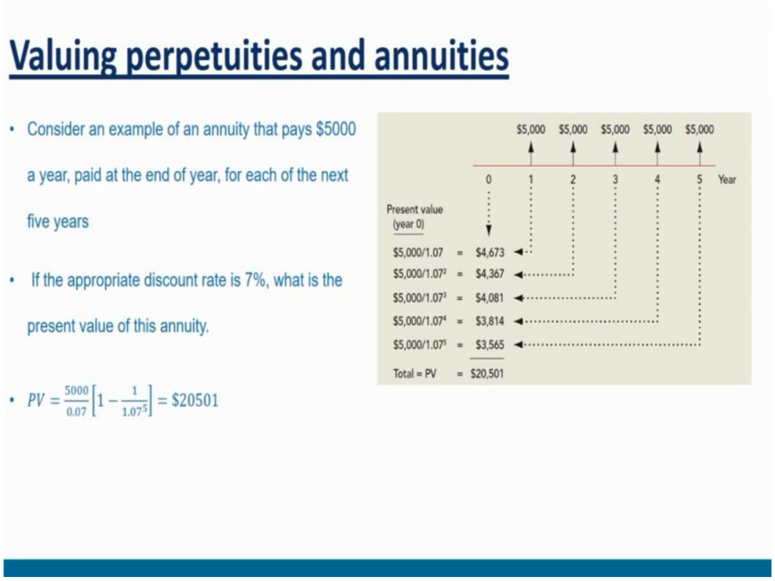

Consider an example of an annuity that pays 5,000 dollars a year paid at the end of the year for each of the next five years. If the appropriate discount rate is 7% what is the present value of this annuity. $PV = \frac{5000}{0.07}\left[1 - \frac{1}{1.07^5}\right] = \$20501$

A simple illustration of these computations in year wise manner is shown here.

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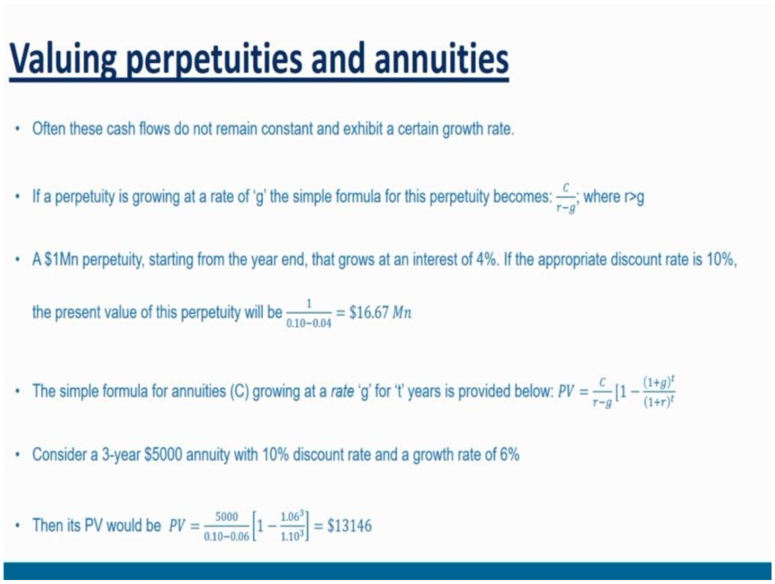

Often these cash flows do not remain constant and exhibit a certain growth rate. If a perpetuity is growing at a rate of 'g' the simple formula for this perpetuity becomes

$\frac{c}{r-g}$; where r > g. Consider the following example, a 1 million dollar perpetuity starting from the year end that grows at an interest rate of 4 percent if the appropriate discount rate is 10 percent the present value of this perpetuity will be $\frac{1}{0.10-0.04} = \$16.67\ Mn$

The simple formula for annuities growing at a rate ‘g’ for ‘t’ years is provided here,

$$PV = \frac{c}{r-g}[1 - \frac{(1+g)^t}{(1+r)^t}$$

Consider a 3 year 5,000 dollar annuity with 10 percent discount rate and a growth rate of 6 percent then its present value PV would be present value

$$PV = \frac{5000}{0.10-0.06}\left[1 - \frac{1.06^3}{1.10^3}\right] = \$13146$$

To summarize in this video, we learned how to compute the present values for cash flows that are perpetual in nature and those that are fixed time period annuities. We also examined the valuation of cash flows that are growing in nature in perpetuity as well as growing annuities having fixed time periods.

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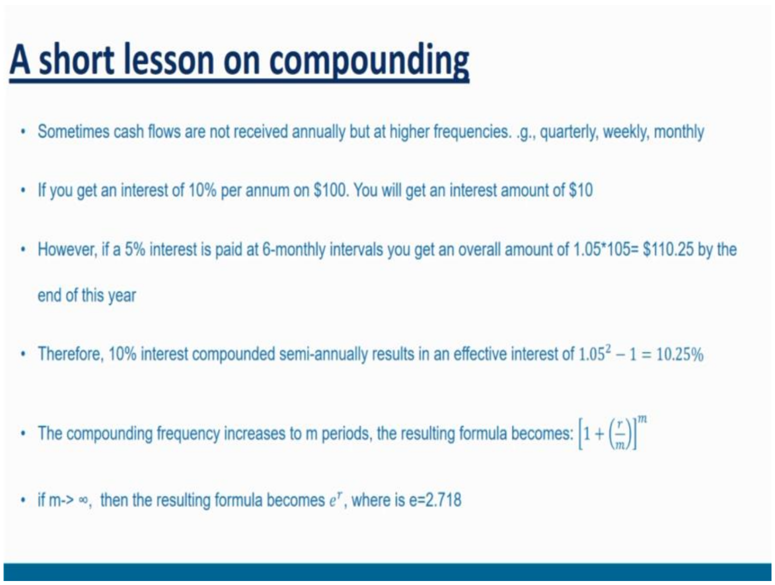

A short lesson on compounding. In this video we will discuss the concept of compounding with the help of examples. We will examine how changes in compounding frequency impact the effective interest rate. Till now we have assumed annual cash flows at the end of the year very often cash flows are received at higher frequencies, for example quarterly weekly monthly. That means you have more opportunity to invest these early and more frequent interest payments.

This results in a higher frequency of compounding. For example, if you get an interest of 10% per annum on 100 dollars you will get an interest amount of 10 dollars however if a 5% interest is paid at 6 monthly intervals, then you get 5 dollars after for 6 months. In the next 6 months you also get an additional interest on this 5-dollar earned in the first 6 months. Thus, you get an overall amount of 1.05*105 = $110.25 by the end of this year.

So, the effective interest rate is 10.25% therefore 10% interest compounded semi-annually results in an effective interest rate of $1.05^2 - 1 = 10.25\%$. If the compounding frequency increases to m periods, the resulting formula becomes $\left\{1 + \left(\frac{r}{m}\right)\right\}^m$;where r is the annual quoted rate and m is the period. For example, if the quoted rate is 12 percent and frequency m = 12 that is monthly frequency.

Then the effective rate is $1.01^{12} - 1 = 12.68\%$. If m tends to infinity, then the resulting formula becomes e^r ;where e = 2.718. This is often referred to as continuous compounding the effective interest rate for annual quoted rate of 12% is $e^{0.12} - 1$ = 12.75 percent. This means that investing 1 dollar at a continuously compounded rate of 12 per annum is same as investing 12.75 percent a year at annually compounded interest rate.

To summarize in this video, we computed effective interest rates at annual quarterly and monthly frequencies and continuous compounding as well. We found that as the compounding frequency increases the effective interest rate also increases, this is ascribed to the fact that more interest is earned on interest itself as the compounding frequency increases.
Cash flow discounting: Cash flows are discounted for two simple reasons, one dollar is worth more than a dollar worth tomorrow.

A saved dollar is worth more than a risky dollar while computing the present values of these cash flows the following factors are important. First, frequency of the payments which represents the compounding aspect that is annual, quarterly, monthly, continuous etc. Opportunity cost of capital that is period of horizon of these payments for example perpetuity 5 years 10 years and so on.

Managers can maximize the firm value by accepting projects with positive net present values or NPVs. To find this NPV we discount all the future cash flows with an appropriate discount rate are usually called the hurdle rate or opportunity cost of capital as per the formula discussed in this lesson that is $NPV = C_o + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} \ldots\ldots and\ so\ on; C_o =$ the initial investment which is expected to be negative, i.e., outflow of cash.

Discount rate 'r' is whereby estimating the prevailing interest rates in financial markets on the instruments of same risk. For example, if the future expected cash flows are almost certain then the appropriate discount rate is the risk-free rate available on government securities. Although retail small investors cannot directly participate in such securities, they can still invest in these securities by investing in money market mutual funds.

If the cash flows are uncertain and risky then the discount rate should be estimated by examining the expected returns offered by the same risk securities. Financial markets are the places where both safe and risk instruments are traded and valued that is the values of risk and returns are easily observed. When we calculate the present values by discounting the cash flows with appropriate discount rates, we essentially estimate the fair and efficient price that alternative securities in financial markets will be available that have the same risk.