Data Analysis and Decision Making - 3
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Lecture 08

Welcome back my dear friends, a very good morning, good afternoon and good evening to all of you wherever you are in this part of the globe. And as you know this is the DADM-3 which is Data Analysis and Decision Making-3 course under the NPTEL MOOC series and as this course I give a general outline on the course not the syllabus or the what is you are going to teach but this course is as you all know is basically for 12 weeks, 60 lectures, 30 contact hours each week we have 5 lectures and after the 5 lectures you have that assignment, one assignment is already to be solved by the time (you) your reading, you are listening to this you have already solved assignment 1 and in totality will have 12 assignments and then the question and answer for the final examination.

So if you remember we were discussing about utility, the 4 different utility functions obviously there are others also and did not go into the details of about that, we discussed about absolute risk aversion, relative risk aversion how they are derived conceptually then I also mentioned that how are these two you this absolute and relative risk aversion properties can we utilize to find out what type of properties the utility functions have? That was from the theoretical point of view then practically considering of a hypothetical example we give and using very simple excel sheet the values of A, A prime, R, R prime and before that you get to find out U prime and U double prime and based on that we saw that the results which we were getting theoretically can we also we obtained practically when you are using the excel sheet.

And then we considered about the certainty value, what is the concept of certainty value, how certainty value can be utilized to rank different decisions. And the certainty value will definitely print on the utility function or other parameters so they will change for the same human being of the utility function changes, the certainty value also changes depending on the situation which you have.

So each is gamble when you have a certainty value, even if the utility function is same. So the gamble changes whether corresponding the probability of the net of value output. So that will have a consequence on the UW value and that would basically have an effect on the expected value of the gamble which will basically have a bearing on trying to find out the certainty value for which the person would be indifferent.

Technically if a rational decision is to be taken but obviously human beings, decision makers can either love the gamble or be indifferent within the gamble and the certainty value or basically take the certainty value and avoid the gamble. So we will classify human beings as risk lovers, risk indifferent person and risk haters. So with this well we will start the 8^{th} lecture which is in the second week which is the third lecture in the second week and we will try to basically give some a very simplistic outline and that how you can analyse using very simple pictorial diagram about the values of U, U prime and also get some information on the value of A, A prime without being aware of the utility function.

So this is a very simple thought experiment. It may be simple and my way of explanation (would) I will try to try to stick it as, make it as simple as possible and then why it is being utilized all these things will come up as we discuss the optimization concepts. So consider you draw a 2 dimension graph. So on a Cartesian coordinate on a graph paper along the y axis you have the values of C, so C is basically the certainty value. So how you market I am going to come to that later. And along the x axis you mark the different values of the wealth.

So here I have taken and we can see these values are AB, so they will keep changing and what you do is that you basically draw 45 degrees line which is the blue one. So, I would not mark it by just show it here.

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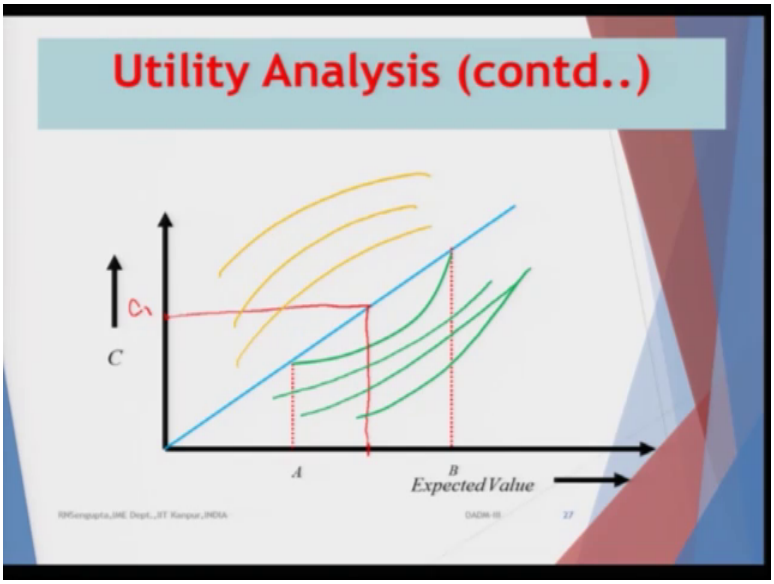

So, this is the 45 degrees line which goes and you mark any two values of A and B on the x axis. Now considered A is the total wealth and other total so called the output, output based wealth. What is that I am going to come to that later. Consider that is the output only, do not consider the wealth for the simple case and even if it is a wealth I will give you a simple idea that how it can be converted into a, the overall utility based on the wealth.

Now consider you are tossing a coin that is a gamble and is an unbiased coin probability is half and half. Now moreover consider that your utility is a simple linear function that means UW is equal or W, so A is the wealth, so the utility function will be U of A which is again A. In the other case if B is the wealth, utility is again UW so that the utility based on the value of B is B. Now consider when you play the gamble, the probability of getting a A is half probability of getting a B is half.

So if I want to find out the expected value it will be A into half plus B into half which would be now I will mark, would be a midpoint between A and B. So A plus B by half, now take a vertical up from here. It touches the line 45 degrees. Now go horizontally onto the left. So, consider this value as C1, now what you do is that in actually, in practicality you place on one table of value of C1 and no other values which is a certainty case. And on the other table you place the values of A and B. Now (that) means you say that if you toss a coin the probability comes half you get head comes you get A and if and the tail comes you get B.

Now you ask the person which one would he or she take? Not consider the person, technically it would mean that if the person is indifferent between this C1 (and) or the other gamble which is A and B which means the certainty value for this person is sized that it exactly matches to C1 which basically would give you a hint that the utility function which the person utilizing to arrive that answer is a linear one as we have discussed.

Now consider the other two cases, in case 1 the person says that the value of C1 which is given to him or her to take with respect to the gamble A and B, he or she wants to take a higher value such that he or she is different between that higher values which is C2 consider with respect to the expected value of A and B. So if that is the case then obviously you would mean that the utility function is such that for as you keep changing A and B without changing the probability and half and half you will get different values of so called C2s which you technically would be marked on the upper portion on the graph which is above the blue line in this areas I am not going to mark it, it will become too cluttered, so it will be marked here.

Now on the other case, consider the person says that no he or she would basically go for a lower value in the case since that the expected value of the gamble which is A plus B by 2 that is actual certainty value is less than C1. So that consider that those sets are as you change the values A and B they are C3. So obviously in that case this part of the portion of the graph would be lower that which I have shown in the green line.

Technically (would) it would mean that as you are able to plot it you will have one set of graphs which is above the 45 degree line which is the blue one and another set of the graphs would be below the 45 degrees line which is the blue one again. Now if you can think about the utility function which you have discussed based on which we found out that we came with the conclusion with non-session property that U prime would be greater than 0, but the value of U double prime would be greater than 0, equal to 0 or less than 0 depending on what type of property the human being has whether he wants to take a risk, we be indifferent with respect to risk or basically try to avoid risk.

So the sets of graph its properties are convex and concavity, you will have one sets of graph let me use a different colour (sorry-sorry). So sets of graphs which are here and the sets of graphs which are here they would basically portray two different concepts, in one case where the person is risk averse and in another case the person is basically risk lover. So in one case you want to

avoid and another case you want to take that risk and the 45 degrees line would basically mean that using the utility function to be very simple linear that means UW is equal to W you will basically be able to plot all the values of C1 such that C1 is equal to A plus B by 2.

So in this way you are able to characterize what type of utility function the human being has without going into details of the utility function as such.

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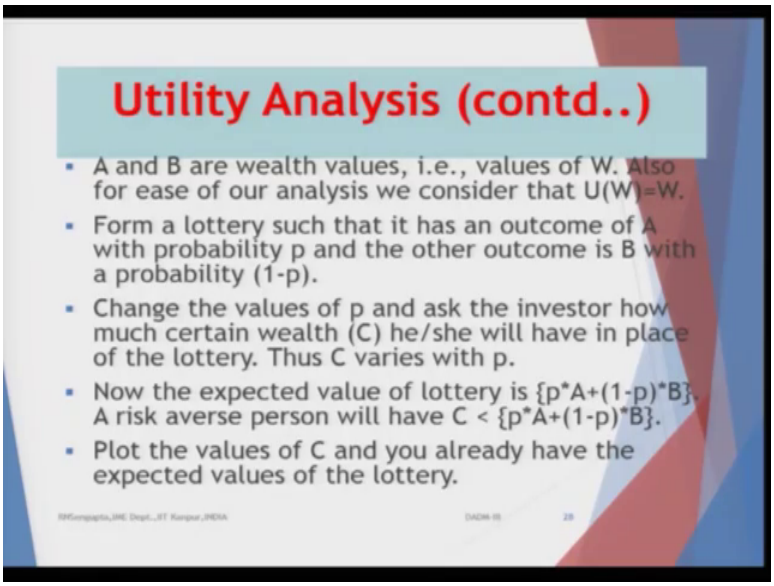

So let me explain, so what I have said I am going to basically now explain that with the PPT slide. So A and B are the wealth values as the values of W which is the wealth also for ease of our analysis we consider the utility function to be W. So now this 45 degree line was done as an experiment it can with different graphs or curves depending of what type of utility function you are using says that the value expected value of A and B exactly matches the expected value the certainty value depending on the utility function.

You form a lottery such that there is an outcome of A with probability p and an outcome B with a probability 1 minus P. So now here I said that the probabilities are half and half that means what we are doing considering is that we are considering that the P and 1 minus P are both half. Change the values of P so it can be done but for our case we did not consider it in this thought out experiment whatever I told, I consider only changing in A and B. But still let me read it, so change the values of p and ask the investor how much certain wealth or C he or she will have in place of the lottery? Hence the value of C will keep changing with the probability.

But we are here we are not changing the value of p and 1 minus p (we are) only changing the values of A and B. Now the expected value of the lottery would be what? We know this P into A plus 1 minus P into B which is basically if P and 1 minus P are half and half it will be A plus B by 2. Risk averse person will basically have a certainty value which is less than this expected value of the gamble and will plot the values of C and you have already have the expected value of the lottery based on that you can basically compare that whether C is greater whether it is in the in the subset C2 or in the subset C3 such that you can basically classify what type of the utility function that person has.

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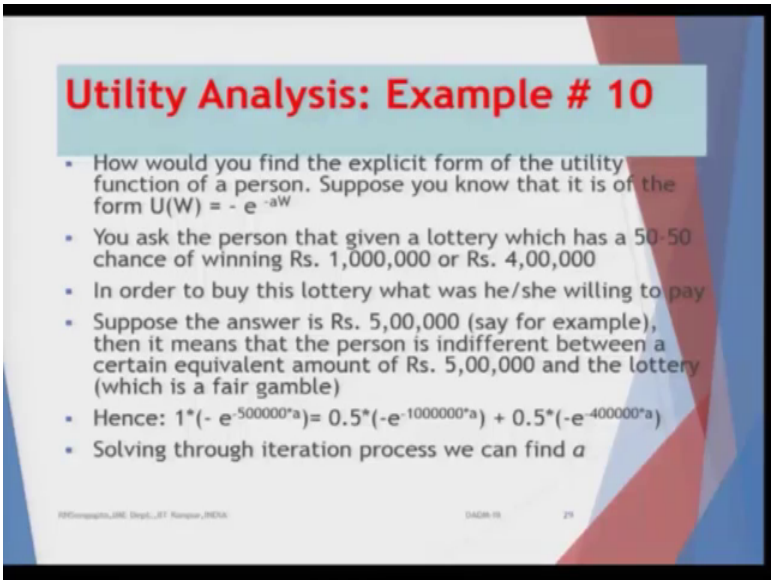

Now the next question will try to basically tackle is that how would you find the explicit form of the utility function of a person? Suppose we know that it is of the form UW is equal to the exponential function which is minus A, E to the power minus AW. So you ask, you will keep asking the person so you know his or her utility function but you do not know the parameters A which you have to find out. So you ask other person that you are given a lottery which is a 50-50 chance which is P is equal to half, Q is equal to half of winning in one case it is a basically 10 lakhs and another case it is basically 4 lakhs.

So I will win 10 lakhs with the probability half, I will win basically 4 lakhs with probability half. So in order to buy this lottery what is he or she willing to basically pay? Now in that case if the lottery basically has a utility function which is linear then obviously the expected value which is

(what) 10 lakhs plus 4 lakhs divided by 2 which would basically be a linear line would (basically) will be utilized to find out the value of C for the certainty value for that person. So based on that we will proceed., so let us continue reading it.

So in order to buy this lottery what he or she will be willing to buy? Pay is basically the main question. Suppose the answer is say for example 5 lakhs then it means the person is indifferent between a certain p value amount 5 lakh and the lottery which is basically a fair gamble. So in that case the person is indifferent and based on that you can take a decision that what the actual person is thinking about the certainty value. So once you equate that you will basically have that 5 lakhs which came out to be the certainty value based on which you are indifferent.

So in the left hand side this 1 would basically mean the probability for the certainty value and this function which I am just hovering in the pen is basically the utility function based on the value that the certainty values is being utilized. So, certainty value as I say this 5 lakhs. So that will be equated to the expected value of the gamble where 0.5, 0.5 are the heads and tails and the corresponding utilities for 10 lakhs is given by where I again hovering my pointer and the second case would be as pointed out is basically the utility function based on the fact when 4 lakhs comes out with that value.

So solving through iteration process we can find out the value A and then proceed to give some resemblance of whatever utility function that human being has.

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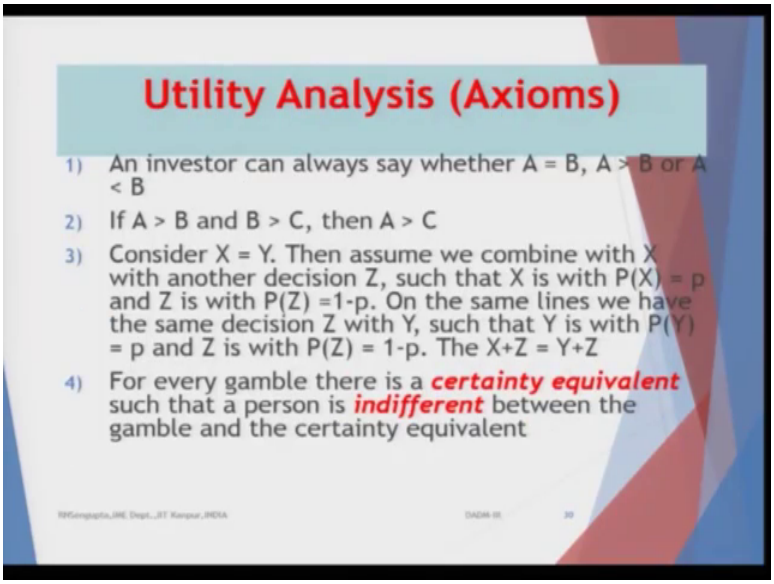

So an a investor can always say whether A and B so for an investor I can categorized that means A is better than B, in the sense that I am getting more benefit. In other sense we can say A is equal to B, such the benefits are equal in another case you can say that A is less than B. So here we can (classi) we can rank them and this ranking would basically depend on the utility function or the or the ranking system we are trying to utilize.

Now consider X and Y are equal then suppose combined X with another decisions Z such that X has a probability of P and the probability for Z is basically 1 minus p so in one case you would basically consider X and in another case you consider Z. On the same lines we have the same decision Z with Y also such that Y is with probability P and Z is again is basically probability 1 minus p . So once I find out try to find out the expected value for the corresponding to the decision X and when I basically try to find out the expected value of corresponding to the value of Y considering Z has been added in both the cases.

So this will give us some idea about the equivalence between X and Y. So for every gamble there is a certainty equivalence such that the person is indifferent between the gamble and the certainty value based on which he or she would make a decisions accordingly.

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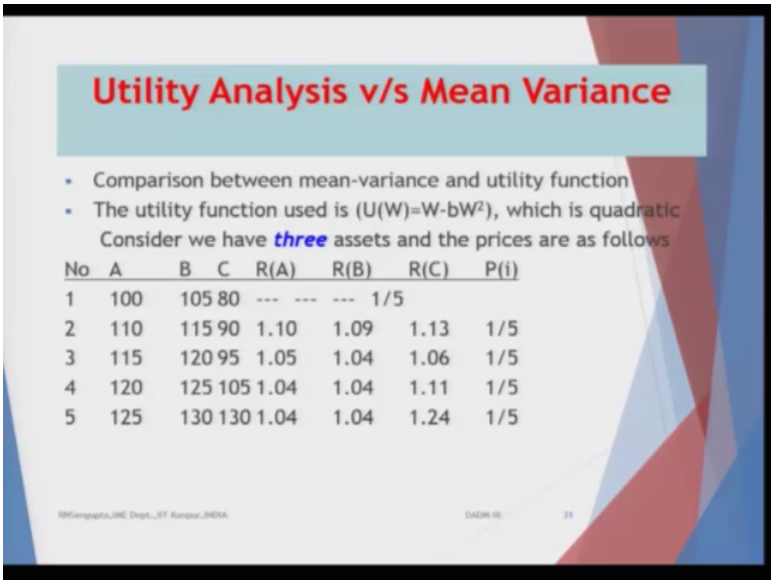

Now will consider the comparison between mean variance and the utility (theory), so the mean variance will try to come back when you solve the problem from the optimization point of view from the finance so that is the mean variance theory is utilized. So the utility function and the mean variance theorem, so utility function based on the fact of is quadratic so we will basically have three assets and the prices are as follows. So this serial number gives you different instances are or the examples.

So the values of this (func) this value of asset A is 100, 110, 115, 120, 125 which is basically the second column I am just might marking it out showing with the pointer while the corresponding values for the other two assets B and C are given in the third and the fourth column. Now what we need to find out are the returns so we will consider the absolute returns which will be for A it would be 110 minus 100 by 100. Similarly for the next case it would be 115 for A only it will be 115 minus 110 by 100 and based on that fact you find ou the returns for the investment A, similarly you will find out the returns for investment B and C accordingly.]

So consider the probabilities are given for all these occurrences, so serial number which you have got is basically corresponding to the occurrences. Nothing to do with the say for example the utilities so occurrences would be basically lead to the fact that how you are able to find out the probability.

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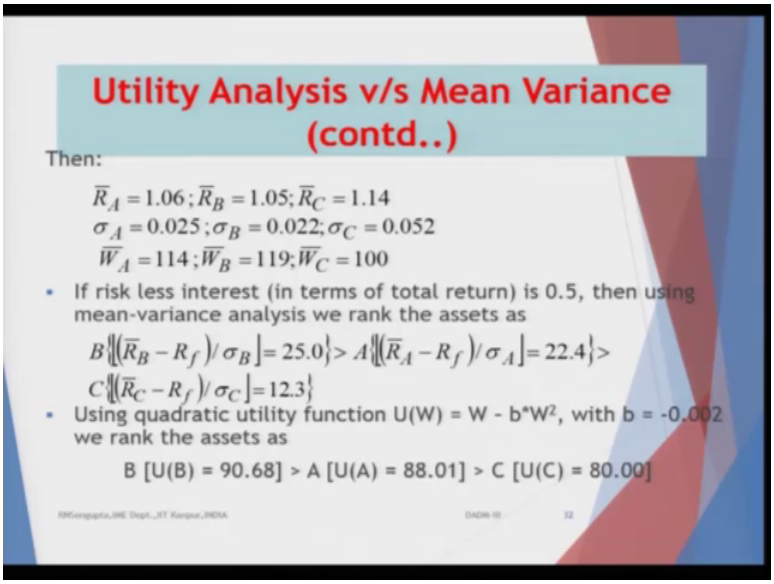

So in this case I will need to find on the average returns, so R bar A is 1.06, R bar B is 1.5, R bar C is 1.14, so in this case the values of standard deviation for A, B, C would be given s 25 return to the power minus 3 for A, 22 into 10 to the power minus 3 for B and 52 into 10 to the power minus 3 for C and will also consider the weights or the so-called amount of investment which is happening on an average for A B C would be given by 114, 119, 110. Now if less risk interest so if a less risk interest (in turn) terms of the total returns is 0.5 then using the mean variance theorem will rank them. So the ranking would be based on the fact that you are using the standard (())(18:57) deviates, so it will be RA minus RF divide by sigma A for the first case.

Similarly for the second case it will be RB minus RF divided by sigma B and the third case would be R C minus RF divided sigma C now remember one thing so once you do that you are basically converting them into the standard normal deviate. So that is the reason, so using quadratic utility function which is quadratic in nature which is W minus BW squared with B is equal to minus 0.002 we can rank this system. So what do we do is that, we find out the values of the standard normal deviate and in this case these values are coming out to be as I mentioned it would be coming out to be 25 for B. 22.4 for A and 12.3 for C so if you rank them B basically would come out the best similarly you will basically have A and C coming out later on.

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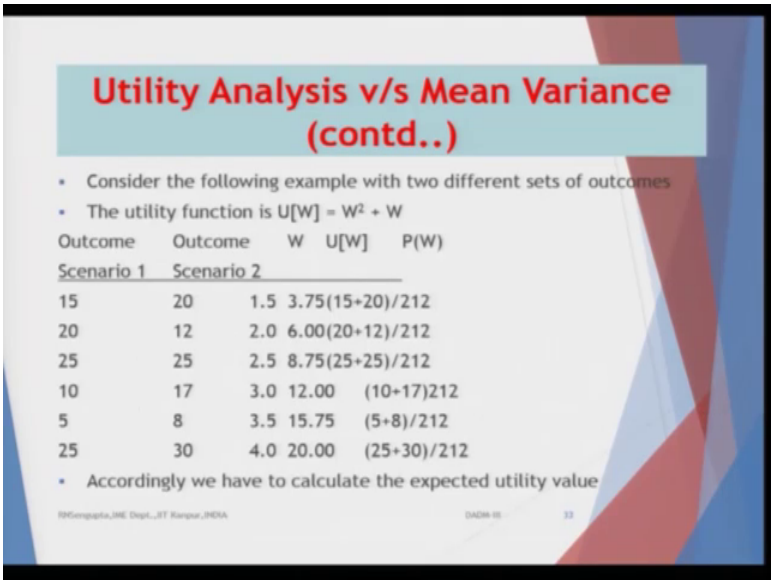

Now consider the following example with two different sets of outcomes. Again its a utility function is quadratic but in this case I basically consider the this outcomes are given and for scenario 1 and scenario 2, so scenario 1 outcomes are I am just reading the mean first column it is 15, 20, 25, 10, 5 and for outcomes scenarios for scenario 2 would be 20, 12, 25, 17, 8 and 30. So the utilities again I consider reliability to be 1.5, 2, 2.5, 3, 3.5, 4 and the corresponding utilities found out using the quadratic utility function. And also we find out the probability based on the fact that both the scenarios need to be counted together such that the total of all the scenarios is (212), 212.

So accordingly we have to calculate the expected utility function and basically try to find out such that this mutual function as it keeps changing you will basically have a different expected value. And once the expected value found out you can basically throw some light on the certainty value also. And obviously it will give you some idea of what type of person he or she is.

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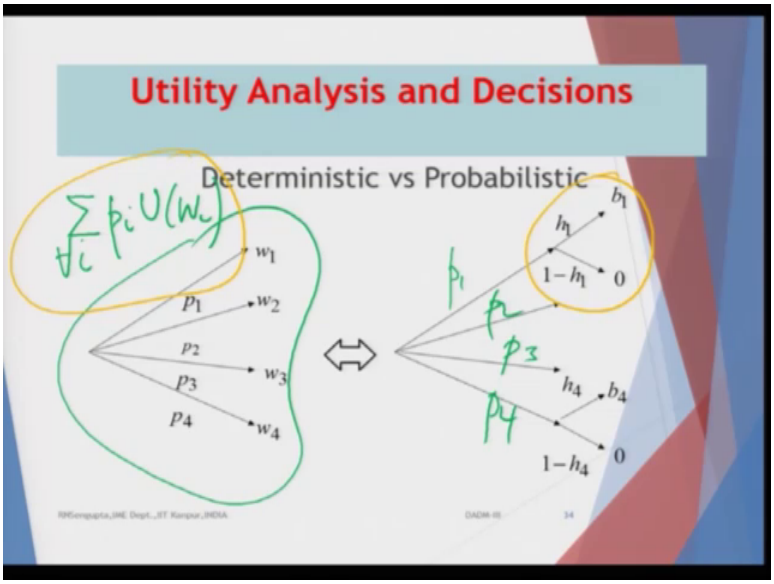

So this is a very simple diagram we have the deterministic versus probabilistic on the left hand side. So both are probabilistic but what you are trying to do is that you are trying to basically combined and find out the expected value on the left hand side. So for this the expect value would be P1 into W1 plus P2 into W2 plus P3 into W3 plus P4 into W4 now mark it here, W1, W2, W3, W4 are technically the utility. So if I want to go into detail and I consider the utility functions to be UW, so obviously for the left hand side I will use the summation for all my i, Pi, U of Wi. So here W1, W2, W3 are basically the corresponding wealth.

Similarly when I find out on the right hand side and consider the probability obviously there would be same this is P1, P2, P3, P4. So you want to find out the expected value, so I have to basically break them up. So what I mean is that first find out utility of this arm. So that will be h1 into U of B1 plus 1 minus h1 into 0. Similarly for the second one will be h2 into B2 U of B2 plus 1 minus h2 into U of 0.

So based on that we find out the expected value for each the binomial so called output which you are getting, so now if the whole thing has to be balanced then the expected value which is P1 into the expected value of the yellow circle line plus P2 into the expected value on the second arm similarly P3 into expected value of the third arm so on and so forth. When you add them up should be exactly equal to the expected value which you have on the left hand side. So based on that you can basically (make) expand it or contract it such that you are expanding means make

more arms such that you are able to understand how the utility function can be managed to find out the certainty value and the expected value.

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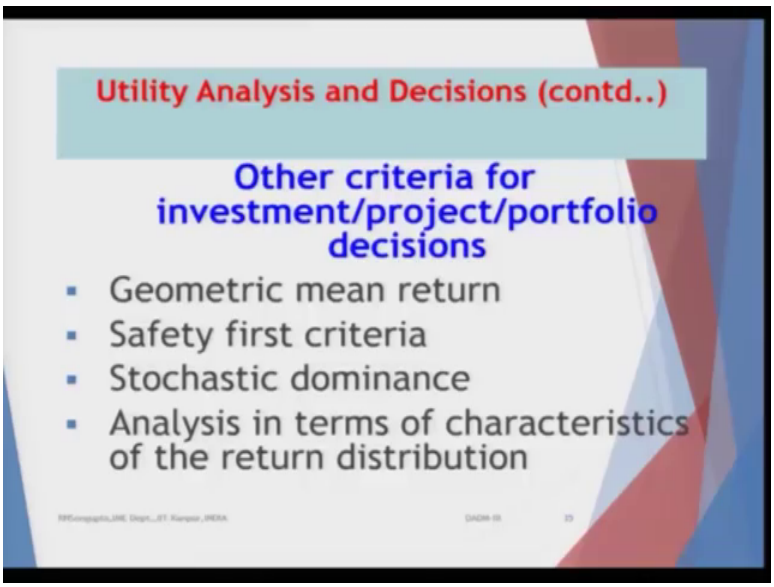

Now will consider some other criterions for investment projects and portfolio decisions and this I will come up later on in the optimization framework. So you have the geometric mean concept, you have the safety first criteria, safety first criteria is basically important to understand, you have the stochastic dominance and the analysis in terms of characteristics of the return distribution.

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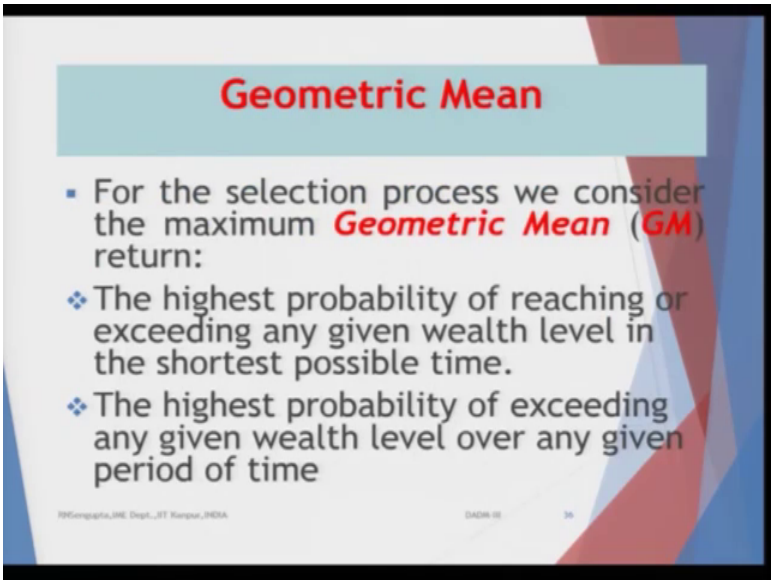

So let us consider the geometry ones, so if you have if you have read all this things I am sure you have done read so the central tendency is given by different formula so one is the mean, arithmetic mean, one is the geometric mean, one is the harmonic mean, so you are talking about the geometric mean. So it will give you basically the highest probability of reaching or exceeding any given wealth level in the shortest possible time and is the highest probability of exceeding any given wealth level over any given period of time.

So basically it will be geometric mean would give you this information such that you take the highest probability and utilize it to find out the when it will be when would be the given wealth level be achieved in the shortest possible time and the highest probability of exceeding and any given wealth level over any given period of time would basically be given by the geometric mean.

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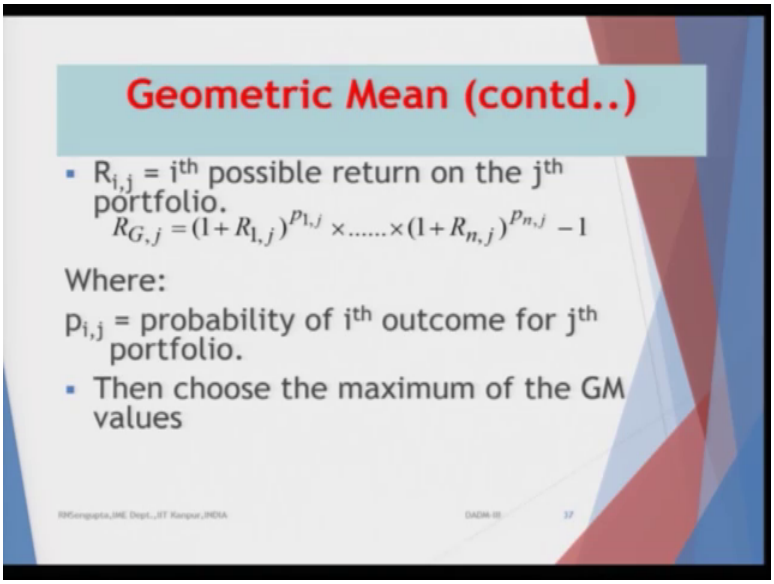

So how does the geometry mean work? So you will basically find out Ri,j which is the Ith possibility return for the Jth portfolio. So I have basically different returns in a basket for the Jth portfolio so J1, J2, J3, J4, till say for example example J10 and each of them would basically have say for example I is equal to 1 2, n number of such decisions. So, if I want to find out the overall expected value I need to find out basically the overall geometric mean and such that the geometry will basically give me the ranking of the utilities.

So the average value of the geometric mean for the Jth portfolio, portfolio is the baskets. So it would be 1 plus R1 r suffix 1 comma J and that would basically be put to the power of Pi,j, so Pi,j is basically the probability while Ri,j is basically the return which you or the so-called output which you are getting from the first decision and you will basically being multiplying each and every term that the suffix would be the probabilities and to the number or the values to which you are changing that, that power would basically be the corresponding returns of the decisions.

Now remember one thing geometry mean can also be formulated based on the fact you are utilizing the concept or returns where small r is equal to P2 minus P1 divide by P1. So what is P1 and P2 remember that? So is basically the P2 is basically the so-called returns coming out from the second day and while P1 is basically investment which is happening. So I want to find out the returns on the issues on the percentage scale.

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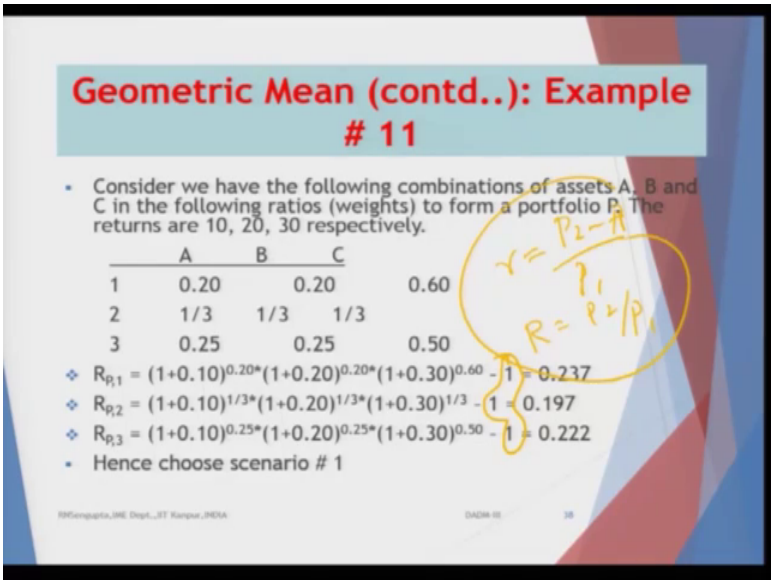

So, consider with the following combination assets A, B and C in the following ratios. So, these on the weights, the corresponding returns are given. And these are for A, it is 0.2 one third 0.22 in the first, second, third decision. For B it is 0.2 one third 0.25 in the first, second, third decision. Similarly for C it is 0.6 one third 0.5 for the third decision. So if I want to find out the return using the geometric mean for the portfolio based on one, so it will be basically so these 10, 20, 30 are given as the returns respectively so what I would note is that one plus the return to the (part) power. So basically I multiply 1 plus 0.1to the power 0, to the power 0.2 so I keep it doing it and obviously minus 1 because it is the returns formulas are given.

One is R is equal to P2 minus P1 by P1. So these P will be on P2 another prices, similarly capital R would be equal to P2 by P1. So that is why this minus 1 is coming here. So based on that we find out is 23.7, 19.7, 22.2and hence will rank that decision 1. So with this I will end this 8th lecture and continue discussing more about utility and how they can be utilized for the optimization problem. Have a nice day and thank you very much.