**Data Analysis and Decision Making - 3**
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**Indian Institute of Technology Kanpur**
**Lecture 07**

Welcome back my dear friends and students, well it is a very good morning, good afternoon, good evening wherever you are in this part of this globe and as you know this is the DADM which is Data Analysis and Decision Making free codes under the NPTEL MOOC series.

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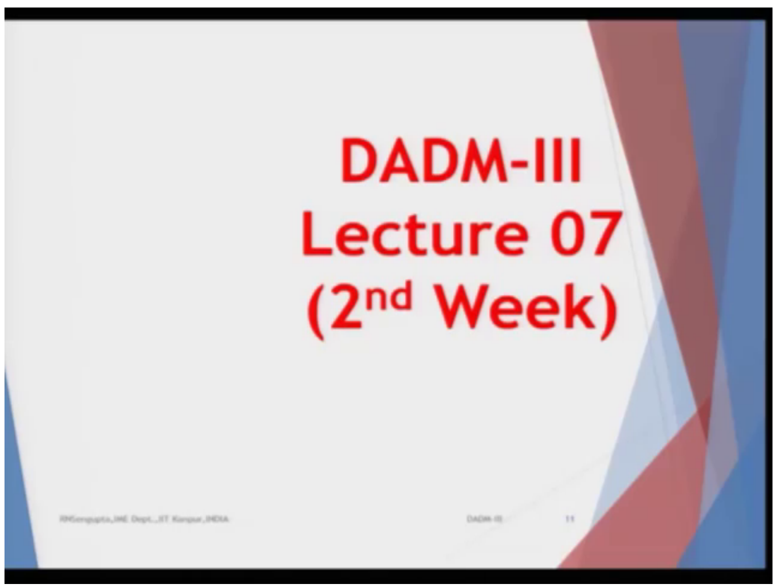


And as you can see from the slide this is the second week and the seventh lecture going on which is the second lecture in this and the second week and seventh lecture in totality. And this course total duration as you know is for 30 hours which gets converted into 60 lectures, 30 contact hours and spared over 12 weeks and after each week which consist of 5 lectures we will have 1 assignment and we have already completed 1 assignment and we are going to do the second one once 10 lectures are over that means 6th, 7th, 8th, 9th, 10th are over. In course you will basically have the question paper, examination for this DADM-3 and my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur.

So we will discussing the 4 utility functions and I did mention that the utility or no I would not use the word utility, the real requirement of A-A prime, R-R prime is that basically want to understand the property of the utility function and based on that property of that utility function you want to basically have a understanding how the utility function is, what is the shape size and the scale parameters of the utility function.

So this word of shape, size and scale parameter I am talking from the point of view of statistics so we use alpha, beta, gamma, so here it would be A, B, C accordingly. Now we were discussing in the exponential utility function and if you remember in the exponential utility function, we found out that the value of A prime was 0 and we will or prove it using the simple (example) hypothetical example.

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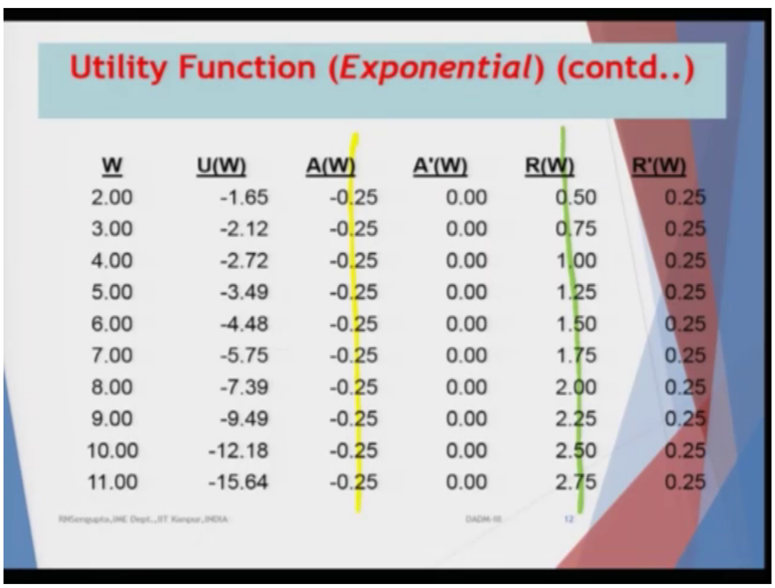


| W | U(W) | A(W) | A'(W) | R(W) | R'(W) |
|---|---|---|---|---|---|
| 2.00 | -1.65 | -0.25 | 0.00 | 0.50 | 0.25 |
| 3.00 | -2.12 | -0.25 | 0.00 | 0.75 | 0.25 |
| 4.00 | -2.72 | -0.25 | 0.00 | 1.00 | 0.25 |
| 5.00 | -3.49 | -0.25 | 0.00 | 1.25 | 0.25 |
| 6.00 | -4.48 | -0.25 | 0.00 | 1.50 | 0.25 |
| 7.00 | -5.75 | -0.25 | 0.00 | 1.75 | 0.25 |
| 8.00 | -7.39 | -0.25 | 0.00 | 2.00 | 0.25 |
| 9.00 | -9.49 | -0.25 | 0.00 | 2.25 | 0.25 |
| 10.00 | -12.18 | -0.25 | 0.00 | 2.50 | 0.25 |
| 11.00 | -15.64 | -0.25 | 0.00 | 2.75 | 0.25 |

Again we have the values of W as given in the first column, the second column is the basically the utility function which is l n of W. And one thing needs to be done (oh) it is the exponential not l n W is minus e to the power Aw, so we have the values of U W.

Now remember we follow the same step, so for finding out A and R we need the values of u prime and u double prime, so u prime and u double prime would be found out using the concept of del u by del w which is the change of utility function divided by the change of the wealth function and the value terms. So this is the first difference, ratio of first difference then we will have the ratio of second difference because u double prime is d2 u by dw2, so you will find out the difference accordingly. And that would be basically will give us the values of A and R.

So if I plot it, if I show it, so this is A, this is value R. Now remember here, see very interesting the A values comes out to be constant so obviously it will immediately point to the fact that when I want to find out the first derivative of A which is A prime which is the absolute risk aversion property, it will come out to be 0 which matches with the theoretical point which you have just discussed.

So A prime is 0 and similarly if you remember the value of R prime was coming out to be the value of small a, that means if you remember the formula was minus e to the power A W, so that value is 0.25 so R prime comes out to be 0.25 or a value, small a value. Now from this you can immediately deduce that the absolute risk aversion property is constant and the relative risk aversion property is increasing, so based on that you can immediately comment what type of utility function which is there, which is basically the exponential utility function.

Now, the property if you remember when we discussed the quadratic utility function I did mention that we are going to use the concept that utility function in quadratic and the investment being a normal distribution there is one to one relationship based on which we will try to proceed and give some feel that when we do the optimization from the finance prospective.

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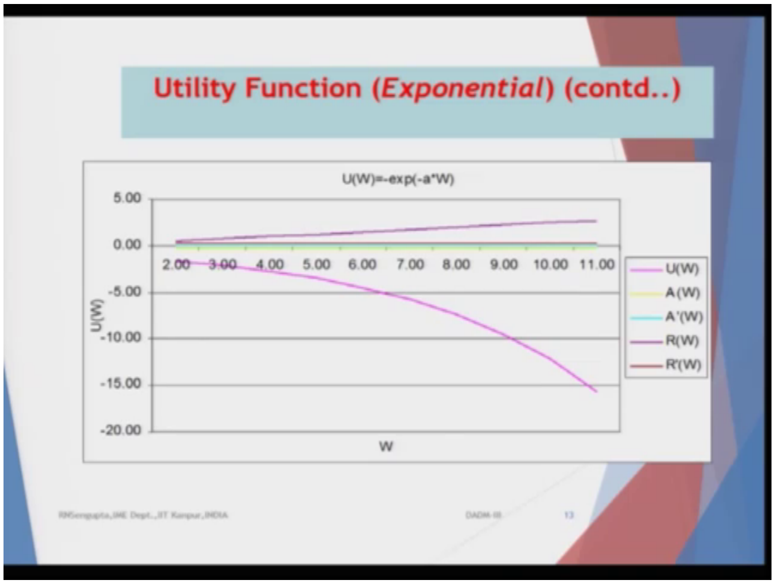


So now if we have plot the utility function which is exponential, the pink one shows again I am not going to draw it but only highlight it. The pink line shows the utility function which is the U W, the yellow and turquoise green or the bluish green one would give you the respective values of A and A prime, so they are almost equal nearest the zero line, so the yellow and the green if you are able to see, you can plot it using very simple excel sheet you just generate only generate the W values and consider the utility functions. We have already considered the quadratic, we have already considered the logarithmic, we already considered the exponential and (you will) later on will see the power of utility function. Just spend 10 minutes draw it in excel sheet and understand.

So once this values of A-A prime and R- R prime are plotted, the information which you proved it theoretically then you basically plotted on your excel sheet you got it, you will basically trying to double verify, triple verify plotting it as a graph. So that will give you what type of property the utility function has.

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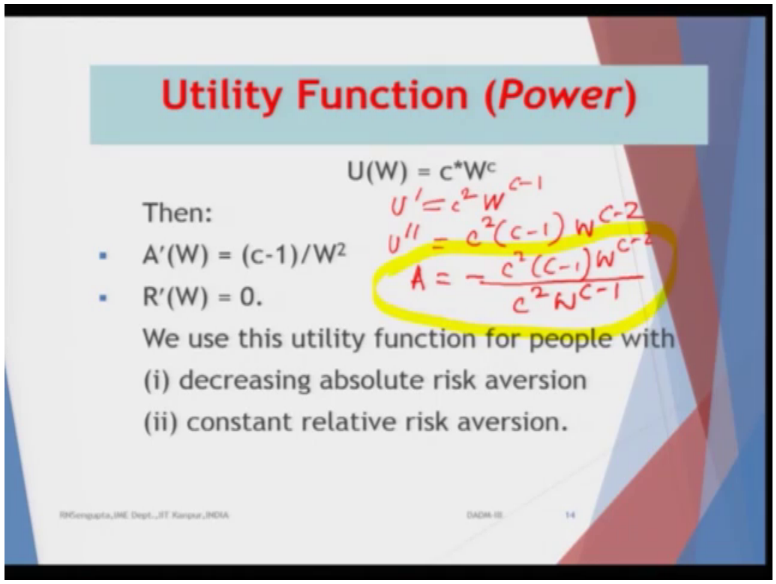


Now we will go into the utility function which is power, this is the fourth one, obviously there other utility function also this CARA, HARA and all these things hyperbolic absolute risk aversion property. So we are not going to go into that until and unless required for the genuine problem solving, I am just mentioning it.

So now if I want to find out the value of U and U prime so again U value is given is c W to the bar c and c value would be greater than 1 and if you have differentiated it will be found out accordingly, so consider c is greater than 1, so it would be c square this is the first value so you can basically find out A minus c square c minus 1 w c minus 2 c square w c minus 1 so utilize this equation and you can find out the value of A prime, similarly you want to find out R, R formula is basically minus of w, w remember I am repeating it time and again but please bear with me w is positive.

So is minus w, U double prime by U prime, find out the value, differentiate that, find out R prime and this comes out to be 0. Now here W square or W is as such positive, W square is positive so the denominator which in formula of A prime is positive, C is positive so C and greater than 1, so it is C minus 1 is greater than 0, so the whole value of A prime is positive. And if it is C is less than 1 obviously it will be negative so we will take it accordingly.

So use the utility function for people with decreasing absolute risk aversion property, here decreasing because C value is less than 1, so it will be decreasing absolute risk aversion property but here the R prime value would remain as 0 as it is. So it will be a constant relative risk aversion property as shown here and it can be easily proved just spent 2 minutes and can deduce. So this are mathematical one then we will go to using the excel sheet.

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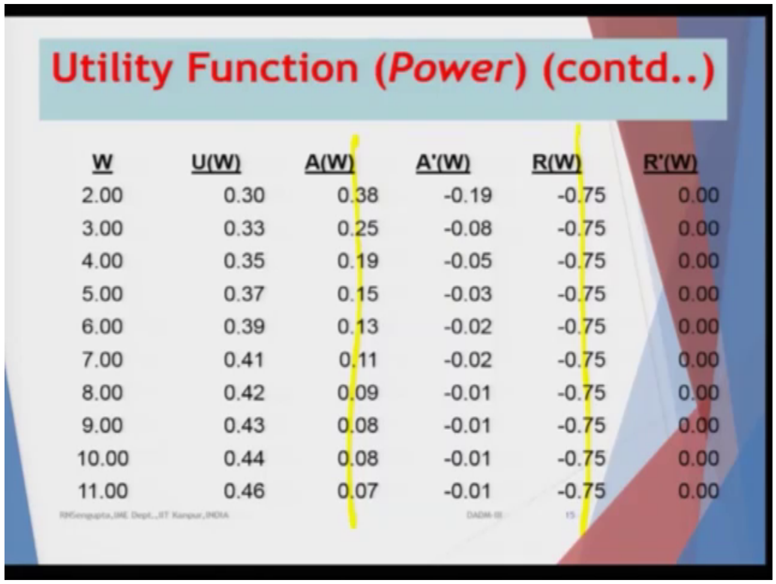


| W | U(W) | A(W) | A'(W) | R(W) | R'(W) |
|---|---|---|---|---|---|
| 2.00 | 0.30 | 0.38 | -0.19 | -0.75 | 0.00 |
| 3.00 | 0.33 | 0.25 | -0.08 | -0.75 | 0.00 |
| 4.00 | 0.35 | 0.19 | -0.05 | -0.75 | 0.00 |
| 5.00 | 0.37 | 0.15 | -0.03 | -0.75 | 0.00 |
| 6.00 | 0.39 | 0.13 | -0.02 | -0.75 | 0.00 |
| 7.00 | 0.41 | 0.11 | -0.02 | -0.75 | 0.00 |
| 8.00 | 0.42 | 0.09 | -0.01 | -0.75 | 0.00 |
| 9.00 | 0.43 | 0.08 | -0.01 | -0.75 | 0.00 |
| 10.00 | 0.44 | 0.08 | -0.01 | -0.75 | 0.00 |
| 11.00 | 0.46 | 0.07 | -0.01 | -0.75 | 0.00 |

Now again I plot the values of W and I have on the second column the U W values which is basically C W to the bar C. These (value) the value are taken, you take any value of C and consider C is less than 1. Now again we follow the same step and please bear with me I am repeating it time and again. So we will first find out U prime, U prime would basically be delta U by delta W, that means the differences of U into 2 into cells, one is 0.33 (divide) minus 0.3 divided by 3 minus 2, so here in the numerator your delta U and in denominator your delta W.

So those values would give you U prime which I have not drawn here then I will basically find out U double prime, again U double prime we know it is d 2 u by d w 2, so find out the second difference for the value of U, find out the difference for the value of W, find out the ratio and that will give you U double prime. Utilize this values with the formula that A is equal to minus U double prime by U prime and find it and once this value are found out so you will basically have A W. Now if I want to go to R W, so R W value formula is again the same or just you multiply by W, so in minus W U double prime by U prime you put it, you find out this value, so very interesting note down, so the R values are all constant, so if it is constant what it would be and i will come within few seconds.

Given A (prime) so again you will find out A prime would be d of A by d of W which is delta of A, delta means the difference of A two consecutive values divided by the difference of W. So that will give you A prime which is given in the fourth column I would not highlight it when just take my curser accordingly. Now if you see the second last column which is R W, so that gives us the value as constant, so if you have differentiate that, that will give me the values of R prime which is 0. So once you have 0 you immediately know that the relative risk aversion property is constant and corresponding to the value of C you know what is the absolute risk aversion property, so we have been able to define ok let me come to the graph then I will basically summarize sorry for that.

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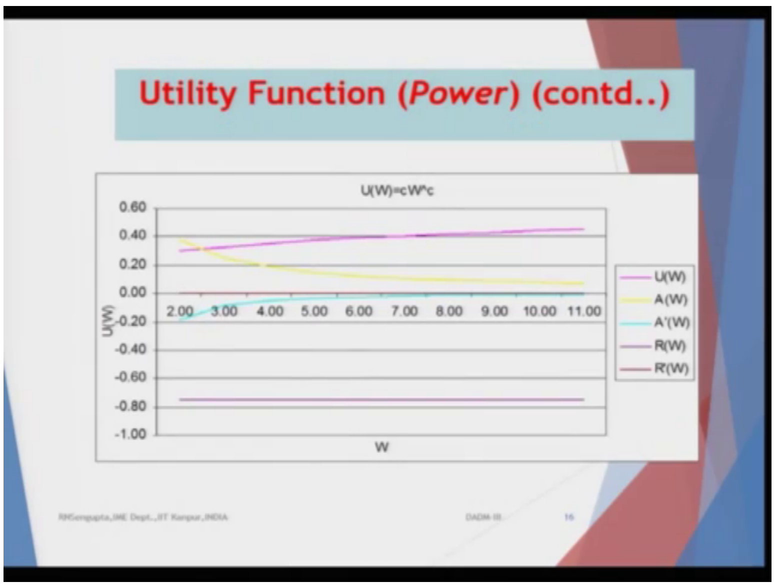


So again I basically plot the values of power utility function that I follow the same in the index the same coloring scheme, the pink one I am not going to highlight it, pink one is basically for the utility function, the yellow and greenish, bluish green are basically for A and A prime and the darkish one, the dark, very dark violet and the brown one is basically for R and R prime, so you can plot it.

Now what we have done is that, we have basically given the properties of A, A prime and R, R prime derived it theoretically for four utility functions and then double verified that using a hypothetical set of values for W and U w and then shown it that the calculation which you are doing theoretically using calculus matches with the fact that what you can prove it pictorially and that basically classifies the four utility functions into the categories, categories means that I am able to understand what type of utility function I used provided I know A, A prime its properties R and R prime and its properties and I also know something about the parameter values which I am utilizing.

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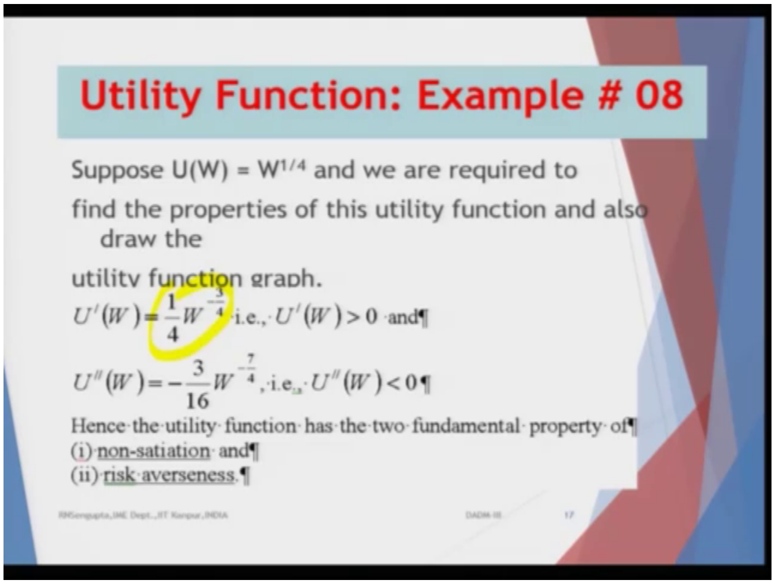


So consider an example, suppose U which is the utility function is W to the power one fourth some power and we have required to find properties of this utility function, also draw the graph. So or the utility function so u prime, u prime would be, you differentiate that it becomes one fourth or w to the power minus three fourth.

So obviously as W is positive ,so this whole equation because one fourth is obviously positive so this whole value is greater than 0, so obviously the first derivative is greater than zero which basically satisfies the first property of non-satiation. Now when I go into the second property again I differentiate the second time, the value comes out to be minus 3, 16 by W to the power minus 7, 4, W value is always positive so that value W to the power minus 7, 4 is positive, 3 by16 is positive, but there is minus sign which immediately point to the fact that U double prime is less than 0 that means you have not, have a risk aversion property in that human being or decision maker.

Hence the utility function has shown the two fundamental property of non-satiation and risk aversion and based on that we can say, that the person has the non-satiation property holds true and risk aversion property also holds true, (where) the risk aversion property is 1of the 3 which is I will obvious, I am indifferent to risk and I am basically averse to risk.

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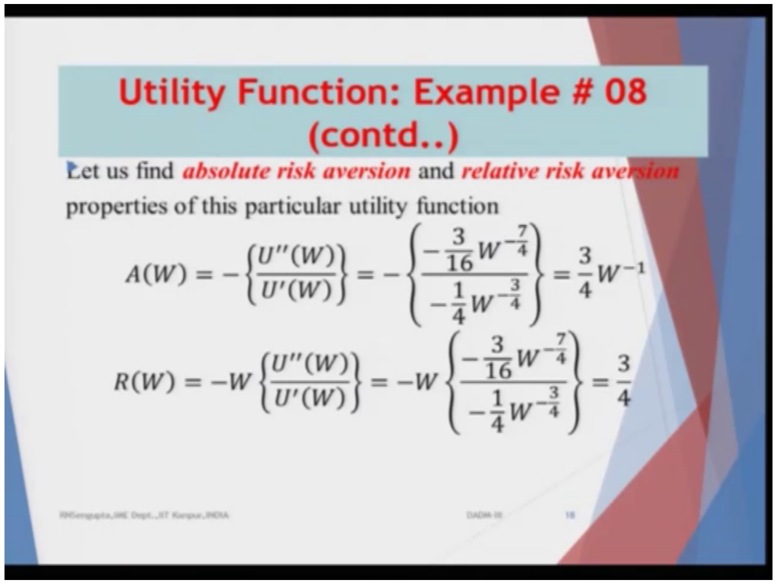


So now let us find out the absolute risk aversion and relative risk aversion property, just put them in the formula, find out A which is minus U double prime by U prime and that comes out to be three fourth by W to the power minus 1, so is just simple calculation what you do if you check (you just) I am just putting the values of U prime and U double prime.

Then again I put the values of R, now R from formula is only I multiplied by W with respective A which I have and I put it, the value come out to be three fourth. Now remember one thing, the moment R comes out to be constant, you immediately know that the R prime is 0 that means you have constant relative risk aversion property that is finalized. When I go to A, its given by three fourth by W to the power minus half, three fourth is always positive, W is always positive, you differentiate that but with a minus sign which immediately give us the information that A prime which is absolute risk aversion property is decreasing.

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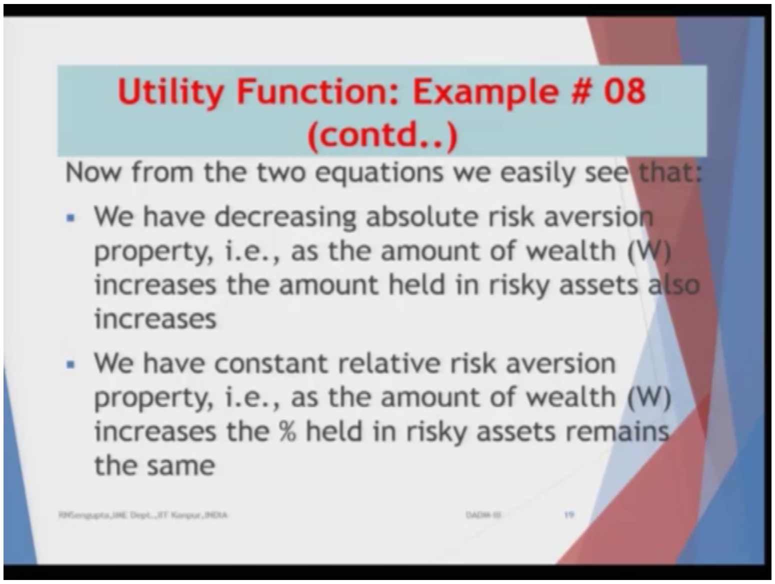


Now from the two equations, which I have already summarize I am just going to read it, we have decreasing absolute risk aversion property that is the amount of wealth increases, the amount held in risky asset also increases. That means you are decreasing absolute risk aversion property that means I want to take risk. On other hand you have constant relative risk aversion property that is the amount of wealth increases the percent held, the percentage held in risky asset remains the constant, so percentage wise it remains fixed or in the absolute sense I am a going away from being a risk, a decreasing risk aversion that means my property of trying to run away from risk is basically decreasing that means I want to take risk.

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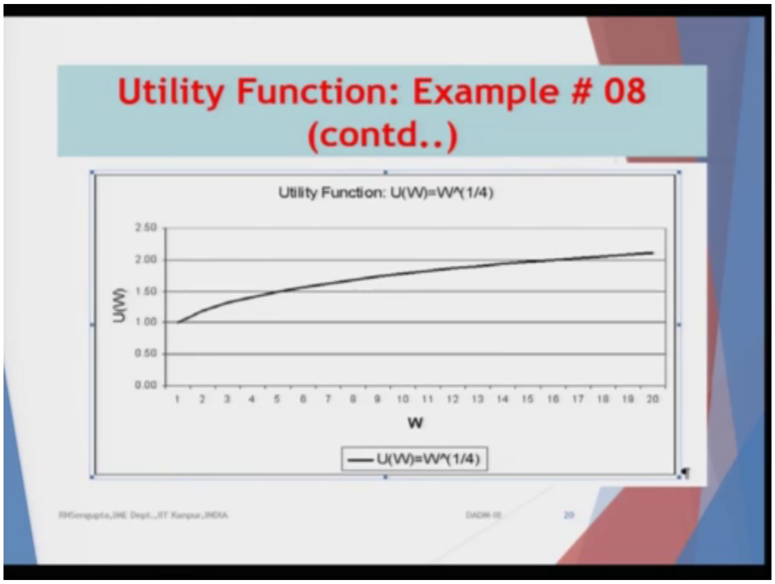


So I plot this graph in simple excel sheet, I am not done the values of U, U prime, A, A prime, R, R prime I am just going to draw the, because it becomes too cluttered you can use utilize different excel sheet to find it out. I plot the values of, value of U on the y axis and W along the x axis if I plot it, the value comes out to be as it is. So it is increasing, but increasing at a decreasing (rate).

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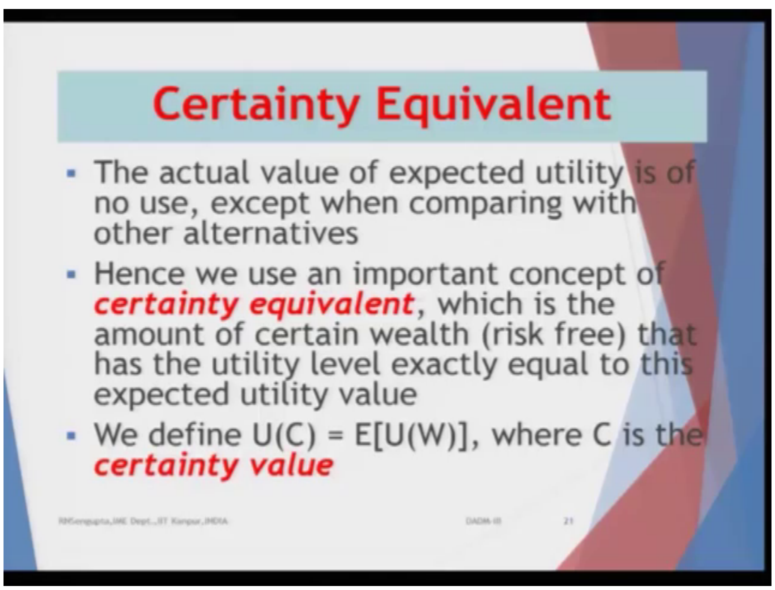


Now we have been doing trying to find out what is utility? The excepted utility and why it is important? Then the concept of absolute risk aversion, relative risk aversion, U prime, U double prime, non-satiation, all this things. What actually they are utilize, I will come to that later and if you remember i did mention expected value and variance of utility function which was basically sort a positive and negative benefit we want to optimize and would become a (())(17:27). The actual value of expected utility is of no use, expect except when comparing with other alternative, so you want to rang the alternatives obviously you need the expected value.

Hence we use an important concept of certainty equivalent which is amount of certain wealth or risk free wealth that has the utility level exactly equal to this utility value. So based on that once you are able to find out the expected value and the certainty value are matching, certainty value means it is a non-probabilistic decision, it is a deterministic decision. While in the utility case you have a probability hence you based on that you want to find out and find out the expected value. Thus the utility level is equals exactly equals to the expected utility value, so we define the certainty value to be U c, c is the utility function as E expected value of the utility function.

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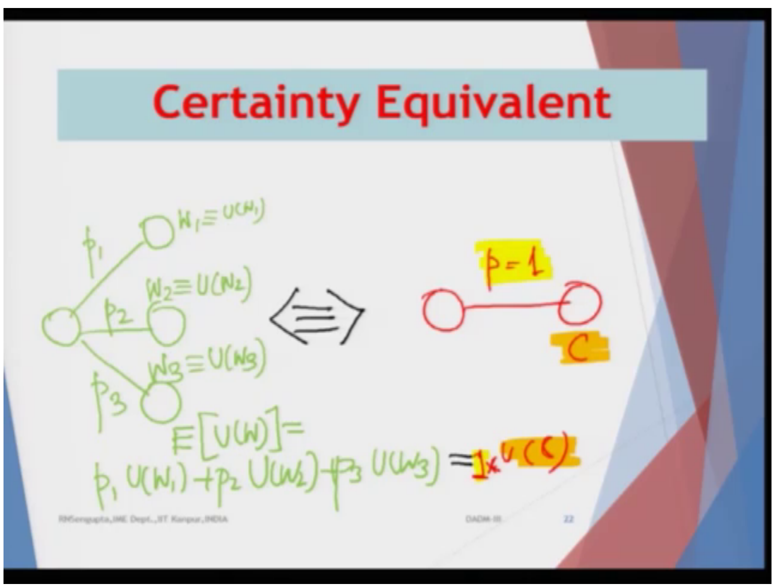


So what we are doing is like this, so this is the, let me use the different color sorry, so consider there are 3 arms, so probabilities are given P1, P2, P3 and I am saying it is equivalent. So the probability P is equal to 1 and C .So what ok, and also remember (I have not marked) these are W1, so obviously it will have an equivalent of U W1, W2 which will have an equivalent of U W2, W3 which will have an equivalent of U W3. If I want to find out the expected value for this, expected value of U W would become I am writing in the bottom part so it will be easier for me to compare so it will be P1 into U W1 plus P2 into U W2 plus P3 into U W3.

And on the right hand side I have, so here the probability is 1 but I will use the same utility function, do not change the utility function. It is U c into 1, so this 1 value is coming for this one, 1, this value is coming from here and you know the utilities have been formed out accordingly. So if you balance that, the c value based on which you want to find out will come out immediately. So based on that immediately take the decisions.

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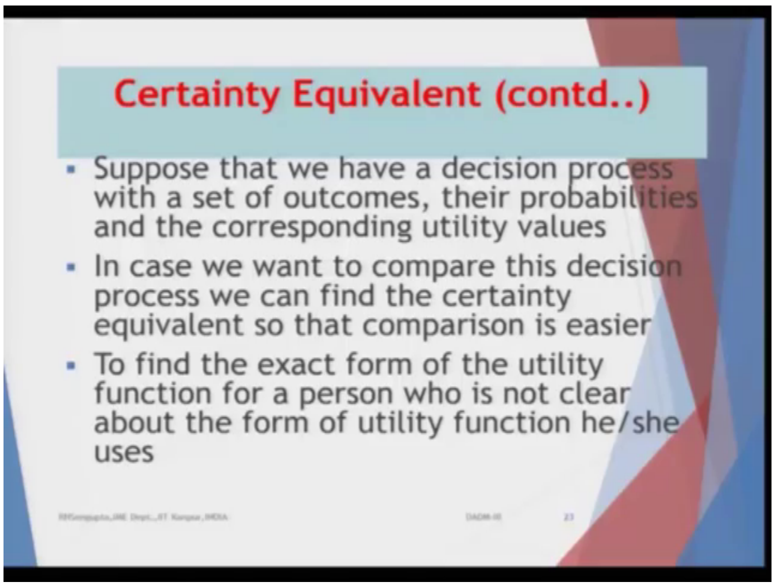


Suppose that if we have a so why it is important, suppose that we have a decision process with a set of outcomes, their probabilities on the corresponding utility values are given. So as I considered in the last (exam) example, in case if you want to compare the decision process we can find the certainty equivalent so that comparison is easier. To find the exact from the utility function for a person who is not clear about the utility function he or she uses this in order to find out how I am going to come back.

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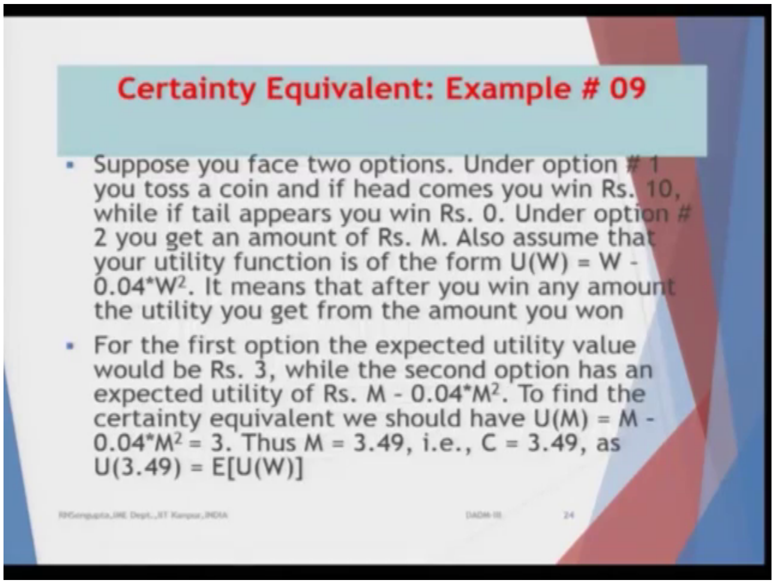


So will first let us understand the example of what is certainty value consider the utility function is given, so let me read out the problem. Suppose there are two options, option 1 and option 2. So under option 1 you toss a coin, a simple unbiased coin and if a head comes with probability half you win 10. But now mark my words, you are winning 10 but it does not it is not giving you, it is not mentioned that what is the net worth which across to you, if you win 10. While if a tail appears you win 0, that means you do not. Again 0 value is the so called wealth, it does not give you any net worth what is the actual value. Under option 2, which is on the other side you get a amount of M, we do not know the value of M.

Now consider for both the cases, the utility functions are same and there is a quadratic one given by W minus 0.04 into W square, its a utility function. Now I need to find out the M which will balance the expected value on both for both option 1 and option 2, hence we can say that value of M which we want to find out is the certainty value. So what do I do? In the first case when there is a head and a tail and the actual value of W is 10 and 0, we will put the value of 10 in the quadratic utility function which is W minus 0.04 and a W square multiplied by half so that is one part and then put again the value of 0 into utility function and multiply it by half.

If we add both of them that will give me the expected value of that option where there are 2 outcomes, the values are 10 which I see and what is the actual accurate value, accurate value to me would be given the utility function once 10 is placed there. And (an) another case when the outcome is value 0, what is the total value is accrues to me would be given by the fact when 0 is put into the utility function. Now what happens to the other option?

In the other option which is deterministic you put the value of W, M in the utility function and multiply it by 1 because it is the certainty value. Equate both the sides of the equation by the expected value which you consider to be same and find out the value of M. So that M value would give you the certainty you cover in such that if that amount of money is kept or the value is kept on one side of the table and other side of the table you will basically have that option, whether a 2 outcomes with a head and a tail. You as a person who has an utility function which is given by the coordinates 1 you will be indifferent.

Now mark my words what I said in the last few seconds that means if your utility function is same, but if your utility function is different obviously the M value will changed. Hence the value which is to be kept on the table will change accordingly and then the certainty value will change depending on both the utility function, both on the risk aversion property or risk loving property, or the risk indifferent property of the human being of the decision maker is going to take the decision.

For the first option the expected utility function would be 3, you can basically find out putting them in the equation that means put (2) 10 into the utility function and multiply half, you put 0 into the utility function, multiply half add both of them and the value is 3.While the second option has a utility which is given by M minus 0.04 M square. To find out the certainty equivalent we should have basically U (M) as given here. So the left hand side and the right hand side should be equal, so we equate you find out the value of W as 3.49 which means that 3 rupees 49 paisa or whatever the unit is, if that is placed on the table and you have other option which is the head or a tail if this depending on the value of 10 or 0 you are getting if you want to be indifferent then the value which will satisfy U to be indifferent is 3.49.

If any value is placed more than that and less than that, your decision will change So more than that obviously it will be mean that will go for the certainty value. (No) not I would not use the word certainty value, the amount which is fixed and kept on the table. If a value is kept less than the certainty value obviously in that case I would rather technically go for the gamble because on the long run the expected value of the gamble is more.

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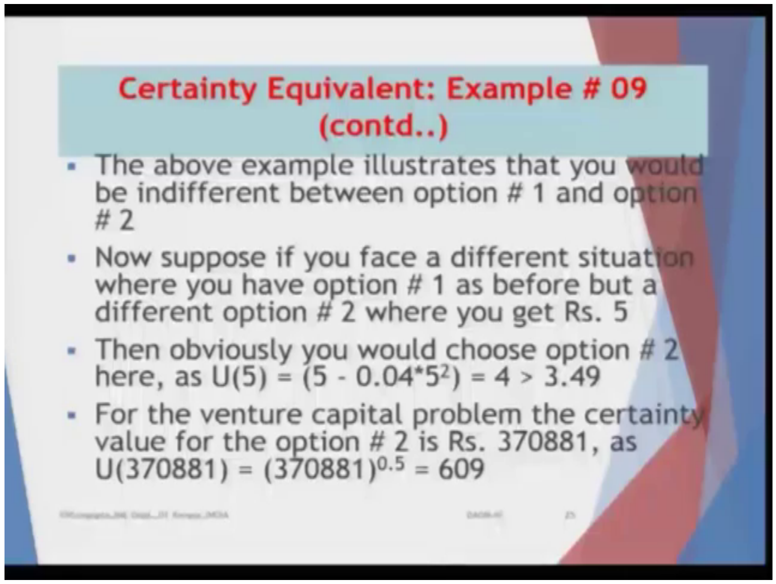


So the above example illustrates that you would be the indifferent between option 1 and option 2 as i said. 3.49 on the other side and the gamble on the other side. Now suppose if you face a different situation where we have an option 1 as before as the gamble but a different option 2 where we have rupees 5, so that means 5 is been given and n is a certainty value, its not the certainty value in the case when it is the deterministic case and then if you want to find out the utility so the deterministic case it will be when you put 5 in the utility function which is 5 minus 0.04 into 5 square, it comes out to be 4. Now that 4 value is greater than 3.49, so obviously in that case you would choose option 2 depending on the expected value which is high.

So if you remember I did mention 3.49 and value more than that you will go for that certainty case. Not certainty that deterministic case and any value less than 3.49 we will go for the probabilistic case. Now if I go to the venture capital problem, where the utility function was half W 2 to the power half I should not use the word half, is basically W 2 to the power half, then in that case you can find on the certainty value, because that would basically make a balance, give you an idea that where you want to basically balance depending on the decision.

So for the venture capital problem the certainty value of the option 2 would be given by the 370881 as the utility considering that you have a utility function which is W 2 to the power half comes out to be 609. So only 609 is being balanced by the government case, with the government actual expected value was 609, so 609 on the left hand side, 609 on the right hand side would give you a value of C which should be kept on the table. 609 you are comprehending which is the actual value which will be coming out, based on that you find out is 370881.

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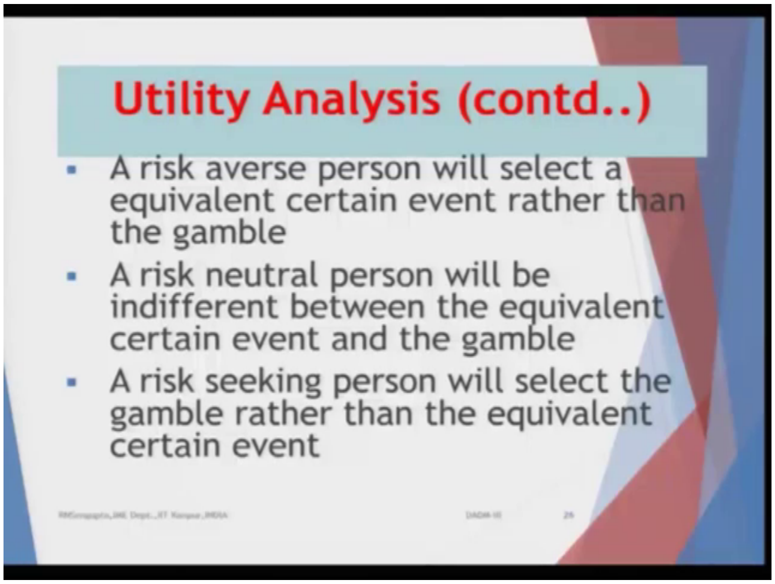


Now if I want to find out and give us some concept of that why that utility function certainty value is important or risk averse property will select a equivalent certainty even rather than the gamble because he wants to make it sure that it does not lose anything. A risk neutral person would be indifferent between the certainty value and the gamble.

When a risk seeking person would select the gamble rather than the certainty value because in one of the arm would give you an higher return so hence the risk loving person thinks in the long run there is a chance that which is true that it may come out he or she will win the and basically take a decision which basically satisfies his or her risk criteria. With this and I will end the 7th lecture and continue more discussion about the utility function and basically then going to the problem of the optimization. Have a nice day and thank you very much.