Data Analysis and Decision Making - 3
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Lecture 06

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe.

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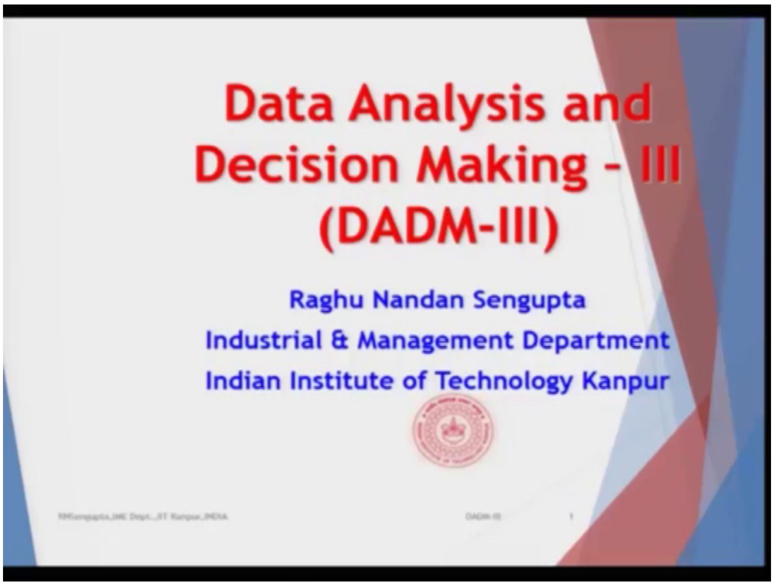

And this is the DADM which is Data Analysis and Decision Making-3 course under the NPTEL MOOC and as you know this total course duration is for 12 weeks which is 60 lectures, totally if you count out the hours contact hour is basically 30 hours and each week we have 5 lectures, after each week we have assignments.

So we have already finished the first week and I am sure you will be by the time when you are listening to these lectures you would definitely you would have understood the concepts of DADM-3 the first week and attempted the questions accordingly, my name is Raghu Nandan Sengupta from the IME department at IIT Kanpur.

So if you remember we had just started the concept of utility theory and I did mention the concept of utility theory can be utilized in a very, very nice and a general sense, where you try to utilize the concept of utility that is a net worth of value which is occurring to you or to the decision maker based on which you are going to optimize, optimize means? Whether you are trying to maximize and minimize that is not the important question immediately, you will see it how it can be done.

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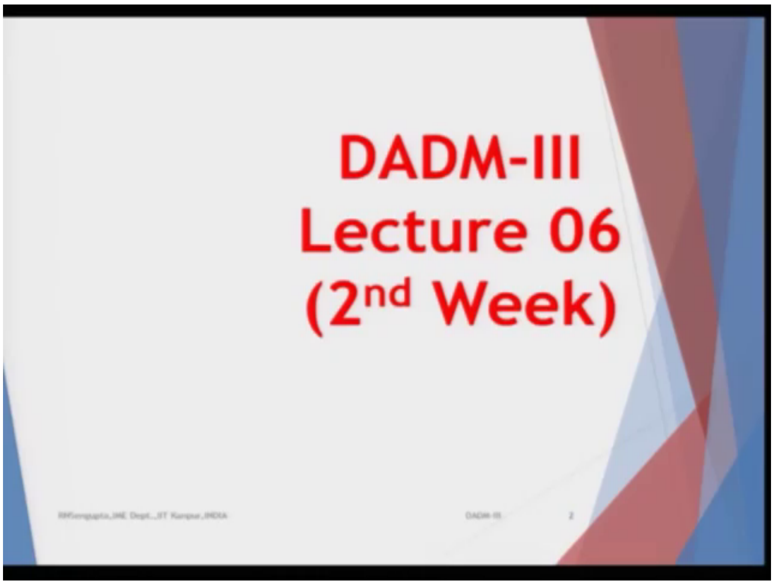

So we are discussing about different utility functions and I continue discussing that, so this is DADM-3 again just I am dating this is lecture number 6 which is the first lecture for the second week. Now when we are discussing the utility theory I did mention two important properties, one was the non-satiation property that means more you give to that person more he or she wants, the demand basically keeps increasing and based on that fact we (one) can easily deduce that the first derivative is greater than 0.

But now the question I also answered that is there are 3 properties of risk a person's risky attitude, one would be very cautious, does not want to take the risk, one is indifferent to the level of risk and another person who basically wants to take the risk. Now when you consider these three sets of persons obviously it will have some implication from the second derivative.

So, if U prime is greater than 0 which is the first property, U prime also would give you the information and I am also shown you the graphs that in one case that U double prime which is the second derivative can be increasing at an increasing rate (and the) which means that it was increasing in the increasing rate that means I want to take the risk, if it is constant second derivative which means that I am indifferent and if it is decreasing at an increasing rate, so obviously that means it is decreasing, so the second derivative is decreasing obviously I am risk averse, I want to run away from risk.

Based on that I also mentioned the property of absolute risk aversion and relative risk aversion, in the concept of absolute risk aversion that means on absolute sense, what is your attitude toward risk? Do you want to run away from risk or you want to basically go towards risk or you would be indifferent? In the similar way when we come to the relative risk aversion property the conceptual framework remains the same but it is on a relative sense.

That means as you keep increasing or decreasing or a wealth both in the case of absolute risk aversion property and the relative risk aversion property you will see that as you keep increasing or decreasing the wealth in the absolute terms if the riskiness increases-decreases or remains constant will utilize the concept of a W which is absolute risk aversion property and if on the account of relative riskiness property that means in the relative sense percentage increase or decrease, it increases decreases or remain the same we will use the concept of relative risk aversion property.

We also said a very simply without going to the details that how we can basically derive both the absolute risk aversion property and the relative risk aversion property using simple Taylor series expansion and you have to, in one hand you have a risky asset (another) and you have a gamble where the expected value can be 0 and 1 and you have a fixed decision. So based on that how you balance that and you can basically utilize Taylor series expansion to derive the expression of A (and b) and R.

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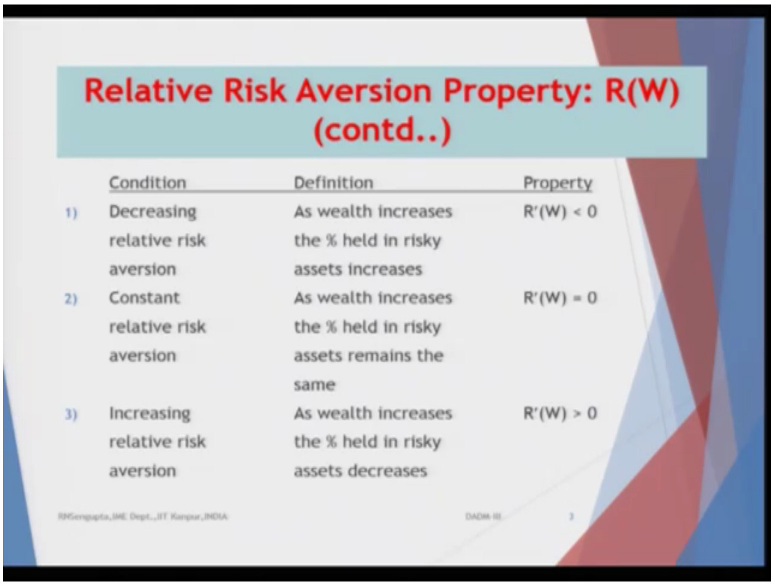

Now coming back to the concept of relative risk aversion property, so I will just explain the concepts in the simplest terms from the theoretical sense and then I will basically go to the why these properties are important. So if we consider the decreasing relative risk aversion property, so in that case as wealth that is the total amount of decision or the amount of wealth and amount of money or amount of total resources which you have as that increases percentage wise.

If it is increases in the risk asset, risk asset means? There is some risk in that in the asset or the decision whatever it is, if it increases then the percentage held in the risk asset also increases, so then in that case you will have a relative risk aversion property which is less than 0. Constant relative risk aversion property would mean that as wealth increases the percentage held in risk asset remains the same. in that case R prime is basically equal to 0 (and in the sense) and in the case when the there is increasing relative risk aversion property it will mean that as wealth increases the percentage held in risk asset decreases.

That means I am the riskiness property that means I am willing to take more risk accordingly, so in that case the concept of R prime would be greater than 0. So in and again A prime and R prime would be greater than 0 individually, equal to 0 individually and less than 0 individually depending on what the concept of risk aversion that person has. But obviously it will be termed on an absolute sense and a relative sense depending on what riskiness property you are trying to implement.

Now, these may be more theoretical in nature but you will understand that how we will try to utilize that when we use the concept of different utility functions and utility functions would have some implication of loss functions and loss functions can be utilized in a very big way and a nice way when you are trying to formulate the optimization problem accordingly. So we will consider the first very simply the utility functions would be divided into four groups, one is quadratic, one is power, one is logarithmic, one is exponential.

So we will go one by one and try to understand that how the property of A, A prime, R, R prime can be utilized here to an in order to understand that what type of utility functions which you have.

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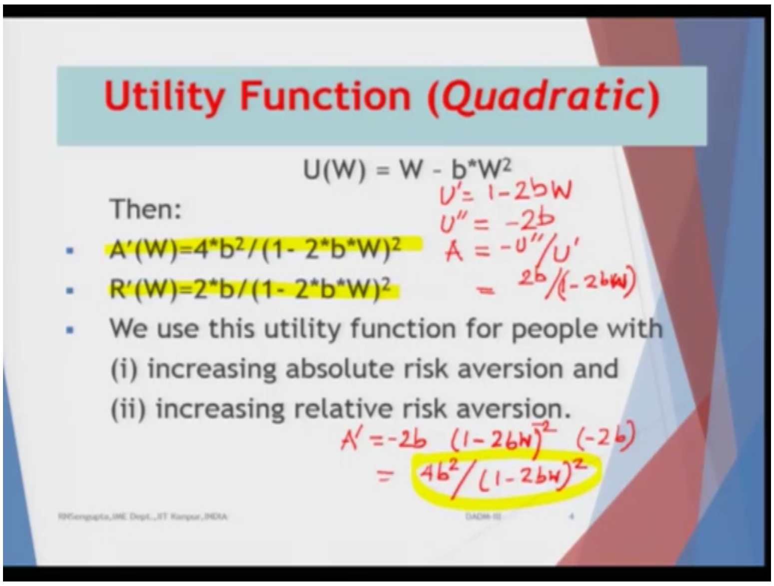

Now if U W which is the utility function is quadratic in nature, so there is one term where you have W square and in other term you have basically W but the parameters like if you have a quadratic equation ax square plus bx plus c, so the values of a, b, c would dictate that the parameters based on which you are trying to find out whether you have absolute risk aversion property increasing, decreasing or constant.

Similarly the relative risk aversion property equal to a greater than 0, less than 0 and constant would basically be dictated as I said by the parameters and it will also give you the shape of the utility functions. Now let us look at the utility unction, so if you remember the property of A was basically or the equations of A was minus U double prime by U Prime and I have mentioned time and again that will always consider and rightly so considering the assumptions that U prime is basically greater than 0.

So, the properties of A prime which you have would be dictated by obviously the minus sign is there but it technically would be dictated by the property of U double prime. So if I basically utilize the formula of A, A prime and R, R prime so your U prime so would be 1 minus 2bW and if I consider the property of U double prime comes minus 2b. Now what is the formula? You know, A I am only writing A I am not writing A W, similarly I have only written U prime and U double prime.

So A is basically minus U double prime by U prime so minus of this would basically become minus-minus 2b and U prime is 1 basically 1 minus 2bW. Now if I want to find out the concept of A prime so 2b remains and this when you multiply it becomes 1 minus 2bW with the minus sign, so this minus sign will be taken care later on, so this becomes square of minus 2 and inside you will basically have minus 2b, so this becomes minus-minus basically becomes plus 4b square divided by 1 minus 2bW whole square. So if I look here, the value which I have here basically matches with this, so this is 4b square by 1 minus 2b into w whole square.

Now look at it very carefully whatever the sign is consider for the (same) time being b is positive if negative whatever but obviously it will have a particular range of values. So in that case if you consider A prime the numerator is always positive because 4b square and the denominator is also positive is b square. So you can immediately tell the value of A prime is greater than 0 which would mean that an increasing absolute risk aversion property would be true.

Now when I go to the differentiation of and find out the of R and R prime, so R we know basically would be a formula where this A is being multiplied by W and remember W values are all positive, when we use the formula do the differentiation or R prime you do the differentiation on R to find out R prime. The actual formula comes out to be, I am not go into the detail you can just simply solve it, is very simple straightforward problem, it comes out to be 2b divided by a square term, so the square term in the denominator of always remains greater than 0.

So now depending on the value of P your value of R prime would be dictated. So if b is positive the numerator also remains positive that means (pos) denominator is also positive, so R prime would basically be greater than 0. Now in case see for example b is negative, so in that case the numerator for R prime is negative but the denominator continues to be positive, so obviously you have to take a decision accordingly whether its positive or negative.

So in this case it basically means that as b is positive, so hence the numerator and denominator both are positive for R Prime and you have increasing relative risk aversion property that means both from the absolute risk aversion property sense and the increasing relative risk aversion property sense this quadratic utility function is basically positive. Now remember one thing we will be referring to and I did mention that in one of the DADM-2 classes or few of the DADM two classes repeatedly.

So whenever you are considering the quadratic utility function remember that this is something to do with the decision which you are making that means I invest money, I take a decision and the returns are basically considering that the overall net value which accrues to me, accrues to me is basically quadratic which underlining means that the investments or the distribution of the investments are basically considered to be normally distributed.

That means utility function quadratic and the returns to be normal distribution are a one to one correspondence which means that if utility function is quadratic then b then your returns or the investments returns or the decision returns are normal distribution and vice versa. So you will be utilizing this later on also and technically we will consider for generally for the stochastic programming and so on and so forth the distribution would be normal in nature even though that may not be true but we will try to utilize that accordingly.

So here it will be it has been shown indirectly that if you if the property of absolute risk aversion is increasing it is given it is increasing and the relative risk aversion property is also increasing then we rest assured that the utility function would be quadratic in nature.

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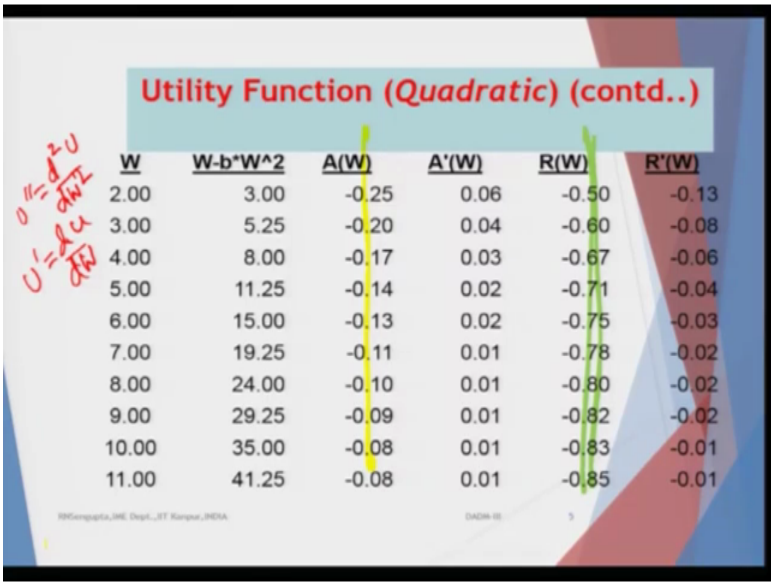

W	W-b*W^2	A(W)	A'(W)	R(W)	R'(W)
2.00	3.00	-0.25	0.06	-0.50	-0.13
3.00	5.25	-0.20	0.04	-0.60	-0.08
4.00	8.00	-0.17	0.03	-0.67	-0.06
5.00	11.25	-0.14	0.02	-0.71	-0.04
6.00	15.00	-0.13	0.02	-0.75	-0.03
7.00	19.25	-0.11	0.01	-0.78	-0.02
8.00	24.00	-0.10	0.01	-0.80	-0.02
9.00	29.25	-0.09	0.01	-0.82	-0.02
10.00	35.00	-0.08	0.01	-0.83	-0.01
11.00	41.25	-0.08	0.01	-0.85	-0.01

Now you may be thinking that you need to draw them and find out. So, obviously quadratic means it would look like a parabola. So I have taken a hypothetical value, so what I do is that I marks in excel sheet, I mark few values on the left first column which is W. So these values are the wealth, so they are 2, 3, 4, 5, 6, n and so on and so forth till 11 and I also consider a value of b.

So if I know the value of b positive or negative whatever it is true, so I can immediately find out the utility function because the utility function is given based on the values of b whatever it is. Now what we need to do is that we need to find out step by step actually we need to find out A prime and R prime. So in this case what we will do is that will go step by step.

First we will find out U and U prime U, utilize the U and also find out U double prime, so you utilize the concept on the values of U double Prime and U prime to find out A, utilize the values of U double prime and U prime and W to find out R and then differentiate the values of A W which is absolute risk aversion property in R which is relative risk aversion property to find out their first derivatives and then basically comment accordingly.

Now what I have done is that I may not have covered the all the cells or the columns accordingly but they are easily discernible and can make a note. So on as I said the first columns are the W's, the next column depending on the value of B you have basically the quadratic utility function. Now what is missing here is that (you will) you will basically find out U, the value of U prime and U double prime, so how do you do that? What is U prime?

Remember U prime is basically the difference in the utility defined by the difference in the wealth.

So what we will do is that we find out the difference of the utility which is 5.25 minus 3 and in the denominator you will basically have the delta W or the rate of (change) the change of the W values based on which you will want to find out. So what we will do is that, find out the difference of 5.23 divided by 3 and divide it by 3 minus 2 that means in the numerator we have delta U which is D and in the denominator you have delta W.

So if you basically find out in the long run it is basically dU by dW. Now once dW is dU, dW is formed you can basically find out the next values accordingly. So what you need to do is that, you next find out the differences, first differences between the values of U prime and then divide by the differences of W, so what you are doing? You are trying to basically find out the W, I would not say use the word W I am going to come to that later.

So what you are trying to do is this, you have U double prime is equal to d 2 U by dW 2, similarly you have U prime is equal to dU by dW. So once you have the second difference of U's and also use the second differences of W that means first difference is basically 3 minus 2 then second difference would basically be found accordingly, so if you find out these values those columns which are not given would give you the values of U double prime and U prime.

Now what you do is that immediately you utilize the formula, so the formula for A is minus U double prime by U, you replace that and the values which you get in the third column are the values of A W, so you have got the A values A W. Now skipping it A prime let us go to the second last column which is R, so we also know the formulas for R which is minus W into U double prime by U prime you plug that value and find out. So if you double check that the value of R W which is minus 0.5, so which means that I am multiplying A W which is minus 0.25 into 2 and you would basically get the value of minus 0.50.

Similarly, you find out the next value of R and the values which are given here I will just mark it. So this would be the value of A, this is the value of R, so once, ok so sorry-sorry this would not be the value of A, this is A and this is R, so once A and R are known you can find out the utilize the formula to find out A prime and R prime. But only remember in the formula be careful to find out U prime and U double prime carefully such that the later calculations are easy for you to handle in the sense it will give you an idea that how the calculations are going.

Now once you find out A prime and R prime, so they would give you the values of the absolute risk aversion property and from that you can genuinely find out that what type of utility functions which you have. Now if you look at the values of A prime carefully, so they are all positive which will give you that the absolute risk aversion property is greater than zero based on which you will we can comment that whatever utility functions it is.

Similarly when you find out the values of a relative risk aversion property you can basically mention the, if you remember the in the quadric utility function I did mention that the depending on the values of b you can say that what is the actual value of R prime depending on that in the R prime formula the numerator would be dictated by the value of b while the denominator continues to be the quadratic one, quadratic one or the square one.

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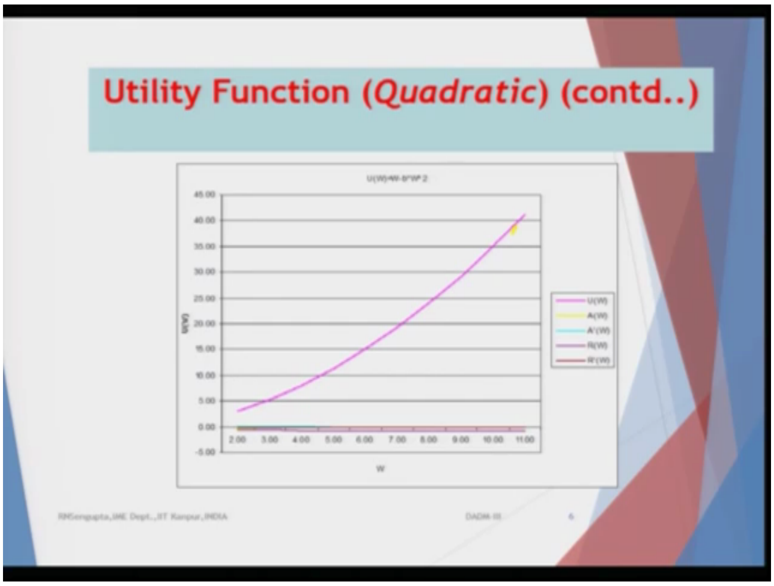

So continuing with the utility function of the quadratic one, we plot the values may now it may not be feasible here but let me try to highlight, so what do I plot R is final falling? I plot the W values on the X-axis and different values of the utilities and what are the values I would have to come to that later on the Y-axis. So, first we will basically plot U W which (I) would not hide I will just show it, so this is U W which is a quadratic one then if you zoom in here all the other values are there.

So what we will also plot is the yellow line will give you (A per) A and like greenish blue one or the seek green sort of coloured or the blue one it would give you basically the A prime and similarly you can find out R and R prime and based on the formulas or do the calculations in simple excel sheet you can find it accordingly.

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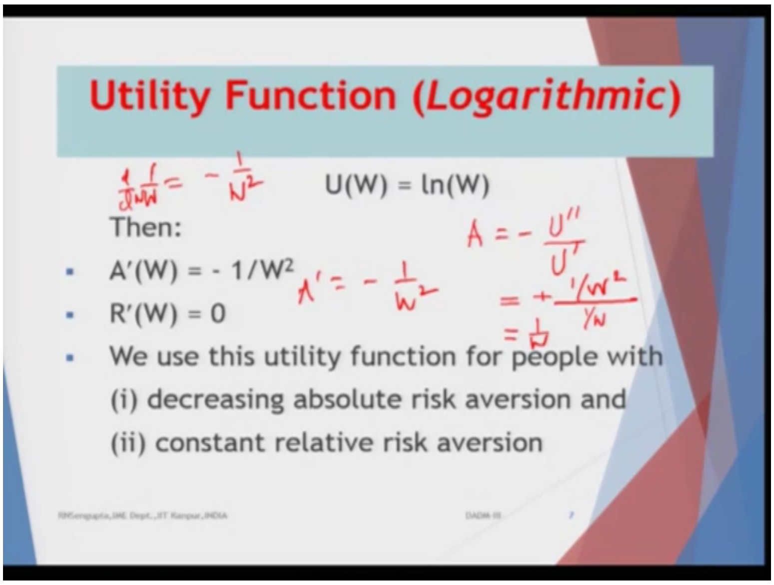

Now consider we have the logarithmic utility function, so the logarithmic utility function is based not on 10, on the base 10 is basically on the base e, so you have basically the utility function given an L and W, now pause here and let us find out the properties. So if I want to find out A and R first I have to basically find out U prime and U double prime based on that we will proceed.

So if I find out U prime it is (minu) 1 by W, if I want to find out U double prime it is minus 1 by W square, put them these values and in order to find out A. So A would basically be given by the formula as minus U double prime by U prime, so this is minus 1 of 1 by W and then again 1 by W square, so this basically this is 1 by W and then if you differentiate it minus 1 by W square minus-minus plus 1 by W square and in the numerator you already have the values of U prime which is given as 1 by W.

So it becomes 1 by W A and if I do want to differentiate it because 1 by W square so as found out with a negative sign, so I had found out the values of A prime comes out to minus 1 by W square. Now remember W is positive, so obviously and obviously is rightly so W square is also positive, so the negative sign it will mean that we have a decreasing absolute risk aversion property.

Now when we come to finding out the values of R and R prime, so once you differentiate it the actual value of R comes out to be constant when you differentiate it to find out R prime it turns out to be say for example 0. Now if it is 0 obviously it will give the information that there is constant relative risk aversion property, so on the concept of percentage change of the risk asset held depending on decisions and or the investments or the optimization concept which you have you will basically be able to comment that it has constant relative risk aversion property.

Now again ok, now sorry-sorry I am going to skip to the earlier slide, so just keep make a note of R prime, so that value is 0, so we will come back to the R prime within 2 minutes.

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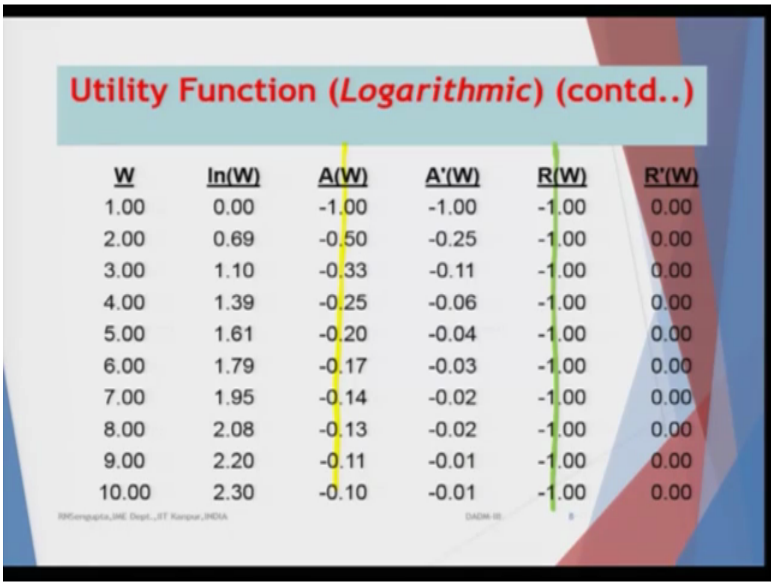

W	ln(W)	A(W)	A'(W)	R(W)	R'(W)
1.00	0.00	-1.00	-1.00	-1.00	0.00
2.00	0.69	-0.50	-0.25	-1.00	0.00
3.00	1.10	-0.33	-0.11	-1.00	0.00
4.00	1.39	-0.25	-0.06	-1.00	0.00
5.00	1.61	-0.20	-0.04	-1.00	0.00
6.00	1.79	-0.17	-0.03	-1.00	0.00
7.00	1.95	-0.14	-0.02	-1.00	0.00
8.00	2.08	-0.13	-0.02	-1.00	0.00
9.00	2.20	-0.11	-0.01	-1.00	0.00
10.00	2.30	-0.10	-0.01	-1.00	0.00

Now again do I do the same here hypothetical thought out experiment, I place the values are W in the first column then I basically place the values of utility which is ln W on the second column, so they basically the values match. Now, I need to find out A and A prime, R and R Prime. So in order to find out A first and also R we need two values which is U prime and U double prime, U prime and U double prime would basically be utilize to find out the values of A and R which is given in the column is A and the values of R is given by R.

So now I want to differentiate it, once I differentiate it I may able to find the values of A prime which is given and this I am not going to highlight it these values and when I basically take the values of R prime or R so all the values are a constant, so the moment I need to find out what is the value of R prime the value comes out to be 0 and this 0 is matching with the theoretical zeros which you have, so theoretical values which you have.

So which means that it has got 0 constant relative risk aversion property and the value of A prime has already been calculated depending on the values of W, so W was positive so it would not have an effect but that minus sign would be coming there.

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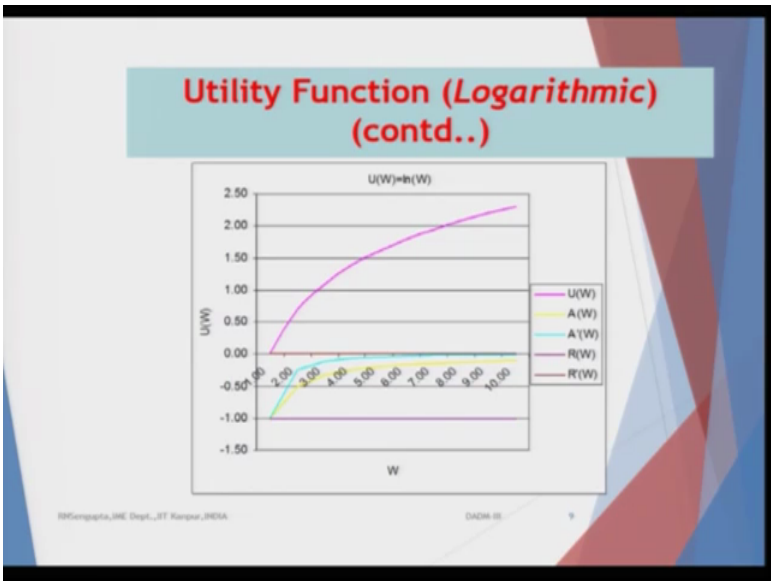

Now if I plot the utility function which is the logarithmic one, so again in the sense that the pink one I am not going to highlight it, the pink one is basically U, the yellow and the turquoise green or the c green would basically be their respective one for A and A prime and brown and the so called (choc) dark violet one is basically related to R and R prime and we will basically utilize the values which you have done and this is the graph which you have, so based on that you can comment enter it is what is required.

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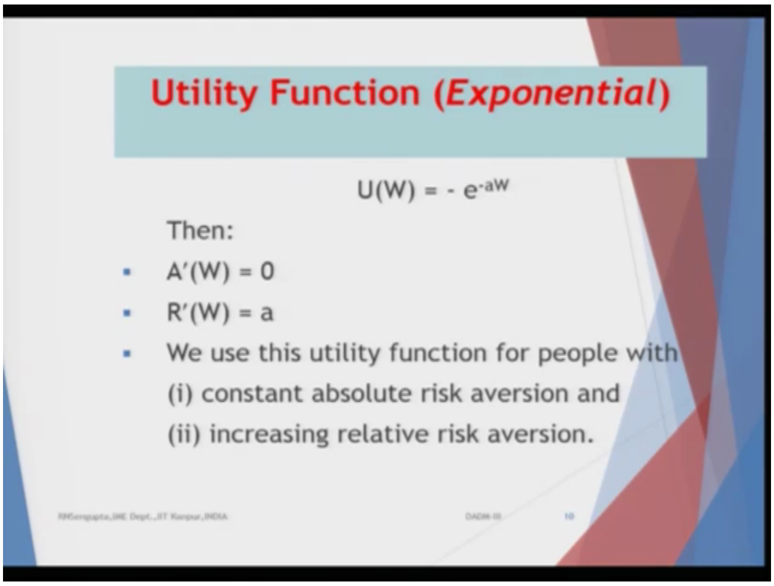

Now I come to the utility function which is exponential, so the exponential utility function is given by minus e to the power minus A W and this A value would be positive it has to take a value positive and negative I will come to that logical sense later on. So again I need to find out here U prime and U double prime utilize these values of U prime and U double prime to find out the values of A and values of R and once I find out the values of A and R I will basically be able to utilize that in order to and I get what is the actual concept of relative risk aversion property and absolute risk aversion property.

So once I find that it comes out to be A double A dash is 0 and R dash is A which means technically the absolute risk aversion property is constant and relative risk aversion property would depend on the values of A and A and B, A can definitely technically you can say that is positive or negative but I will come to that later on. So obviously it will be considering A is positive we will have an increasing relative risk aversion property while A prime is 0, so it will basically have a constant absolute risk aversion property.

So with this I will end the 6 lecture which is the first lecture in the second week and continue discussing more about utility function which will be utilized later on in the optimization techniques, have a nice day and thank you very much.